Preface

About This Book

Introduction to Physics is an adaptation of *Concepts of Physics*, itself an adaptation based on modules from OpenStax *College Physics*. Some of the modules removed from *College Physics* have been put back in, most notably modules on momentum, rotation and special relativity. Sections on linear momentum has been added from James Rittenbach's derived copy of *Concepts of Physics*. Additional minor edits have been made as necessary.

This textbook is intended for a one-semester introduction to physics course, requiring as little mathematics as possible. Students are expected to have taken some algebra and geometry at high-school level, but no higher level of mathematics is required (and use of algebra and geometry is minimized as much as possible, in order to emphasize the concepts of physics). The courses this textbook is intended for often go by the name of "conceptual physics," "descriptive physics," or "introduction to physics."

This edition is put together for use by College of Alameda's Physics 10 course, starting in Summer 2018 (updated and revised as needed thereafter). Much gratitude is due to Prof. Bobby Bailey for putting together *Concepts of Physics* and to OpenStax for making this Open Educational Resource available.

Andrew Park, Physics Instructor at College of Alameda, June 2018

P.S. Original "Preface" to *Concepts of Physics* follows below.

Preface to Concepts of Physics

Concepts of Physics is an adaptation based on modules from OpenStax College's textbook *College Physics*.

This text is intended for use in a one-semester physical science course that requires algebra but no trigonometry.

Not all physics topics are covered in this work. Of the nearly 300 original *College Physics* modules, only 100 were retained. For these selected modules, content was modified (as needed) to remove/replace any trigonometry-related topics, examples, homework, or figures. Occasionally, original content was edited (or replaced) to reflect the level, and coverage, of the course. Notation was modified in a few cases for consistency between modules. Explicit links between modules (and discussions related to non-retained modules) were removed. Finally, all direct links to PHeT plugin-based (Java or Flash) physics simulations were removed.

The authors of *College Physics* are commended for providing such a comprehensive body of high-quality material. Special thanks to them and to OpenStax. Additional thanks to University of Maryland University College (UMUC) and Debra F. McLaughlin.

Bobby Bailey, January 2015

About OpenStax College

OpenStax College is a non-profit organization committed to improving student access to quality learning materials. Our free textbooks are developed and peer-reviewed by educators to ensure they are readable, accurate, and meet the scope and sequence requirements of modern college courses. Unlike traditional textbooks, OpenStax College resources live online and are owned by the community of educators using them. Through our partnerships with companies and foundations committed to reducing costs for students, OpenStax College is working to improve access to higher education for all. OpenStax College is an initiative of Rice University and is made possible through the generous support of several philanthropic foundations.

Science and the Realm of Physics, Physical Quantities, and Units class="introduction"

Galaxies are as immense as atoms are small. Yet the same laws of physics describe both, and all the rest of nature—an indication of the underlying unity in the universe. The laws of physics are surprisingly few in number, implying an underlying simplicity to nature’s apparent complexity. (credit: NASA, JPL-Caltech, P. Barmby, Harvard-Smithsonian Center for

Astrophysics
)

What is your first reaction when you hear the word “physics”? Did you imagine working through difficult equations or memorizing formulas that seem to have no real use in life outside the physics classroom? Many people come to the subject of physics with a bit of fear. But as you begin your exploration of this broad-ranging subject, you may soon come to realize that physics plays a much larger role in your life than you first thought, no matter your life goals or career choice.

For example, take a look at the image above. This image is of the Andromeda Galaxy, which contains billions of individual stars, huge clouds of gas, and dust. Two smaller galaxies are also visible as bright blue spots in the background. At a staggering 2.5 million light years from the Earth, this galaxy is the nearest one to our own galaxy (which is called the Milky Way). The stars and planets that make up Andromeda might seem to be the furthest thing from most people’s regular, everyday lives. But Andromeda is a great starting point to think about the forces that hold together the universe. The forces that cause Andromeda to act as it does are the same forces we contend with here on Earth, whether we are planning to send a rocket into space or simply raise the walls for a new home. The same gravity that causes the stars of Andromeda to rotate and revolve also causes water to flow over hydroelectric dams here on Earth. Tonight, take a moment to look up at the stars. The forces out there are the same as the ones here on Earth. Through a study of physics, you may gain a greater

understanding of the interconnectedness of everything we can see and know in this universe.

Think now about all of the technological devices that you use on a regular basis. Computers, smart phones, GPS systems, MP3 players, and satellite radio might come to mind. Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above tracks, “invisibility cloaks” that bend light around them, and microscopic robots that fight cancer cells in our bodies. All of these groundbreaking advancements, commonplace or unbelievable, rely on the principles of physics. Aside from playing a significant role in technology, professionals such as engineers, pilots, physicians, physical therapists, electricians, and computer programmers apply physics concepts in their daily work. For example, a pilot must understand how wind forces affect a flight path and a physical therapist must understand how the muscles in the body experience forces as they move and bend. As you will learn in this text, physics principles are propelling new, exciting technologies, and these principles are applied in a wide range of careers.

In this text, you will begin to explore the history of the formal study of physics, beginning with natural philosophy and the ancient Greeks, and leading up through a review of Sir Isaac Newton and the laws of physics that bear his name. You will also be introduced to the standards scientists use when they study physical quantities and the interrelated system of measurements most of the scientific community uses to communicate in a single mathematical language. Finally, you will study the limits of our ability to be accurate and precise, and the reasons scientists go to painstaking lengths to be as clear as possible regarding their own limitations.

Physics: An Introduction

- Explain the difference between a principle and a law.
- Explain the difference between a model and a theory.

The flight formations of migratory birds such as Canada geese are governed by the laws of physics. (credit: David Merrett)

The physical universe is enormously complex in its detail. Every day, each of us observes a great variety of objects and phenomena. Over the centuries, the curiosity of the human race has led us collectively to explore and catalog a tremendous wealth of information. From the flight of birds to the colors of flowers, from lightning to gravity, from quarks to clusters of galaxies, from the flow of time to the mystery of the creation of the universe, we have asked questions and assembled huge arrays of facts. In the face of all these details, we have discovered that a surprisingly small and unified set of physical laws can explain what we observe. As humans, we make generalizations and seek order. We have found that nature is remarkably cooperative—it exhibits the *underlying order and simplicity* we so value.

It is the underlying order of nature that makes science in general, and physics in particular, so enjoyable to study. For example, what do a bag of chips and a car battery have in common? Both contain energy that can be

converted to other forms. The law of conservation of energy (which says that energy can change form but is never lost) ties together such topics as food calories, batteries, heat, light, and watch springs. Understanding this law makes it easier to learn about the various forms energy takes and how they relate to one another. Apparently unrelated topics are connected through broadly applicable physical laws, permitting an understanding beyond just the memorization of lists of facts.

The unifying aspect of physical laws and the basic simplicity of nature form the underlying themes of this text. In learning to apply these laws, you will, of course, study the most important topics in physics. More importantly, you will gain analytical abilities that will enable you to apply these laws far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any professional career you choose to pursue. This module discusses the realm of physics (to define what physics is), some applications of physics (to illustrate its relevance to other disciplines), and more precisely what constitutes a physical law (to illuminate the importance of experimentation to theory).

Science and the Realm of Physics

Science consists of the theories and laws that are the general truths of nature as well as the body of knowledge they encompass. Scientists are continually trying to expand this body of knowledge and to perfect the expression of the laws that describe it. **Physics** is concerned with describing the interactions of energy, matter, space, and time, and it is especially interested in what fundamental mechanisms underlie every phenomenon. The concern for describing the basic phenomena in nature essentially defines the *realm of physics*.

Physics aims to describe the function of everything around us, from the movement of tiny charged particles to the motion of people, cars, and spaceships. In fact, almost everything around you can be described quite accurately by the laws of physics. Consider a smart phone ([link]). Physics describes how electricity interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and

circuit layout when building the smart phone. Next, consider a GPS system. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics equations to determine the travel time from one location to another.

The Apple "iPhone" is a common smart phone with a GPS function. Physics describes the way that electricity flows through the circuits of this device. Engineers use their knowledge of physics to construct an

iPhone with features that consumers will enjoy. One specific feature of an iPhone is the GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (credit: @gletham GIS, Social, Mobile Tech Images)

Applications of Physics

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. It can help you understand how microwave ovens work, why metals should not be put into them, and why they might affect pacemakers. (See [link] and [link].) Physics allows you to understand the hazards of radiation and rationally evaluate these hazards more easily. Physics also explains the reason why a black car radiator helps remove heat in a car engine, and it explains why a white roof helps keep the inside of a house cool. Similarly, the operation of a car's ignition system as well as the transmission of electrical signals through our body's nervous system are

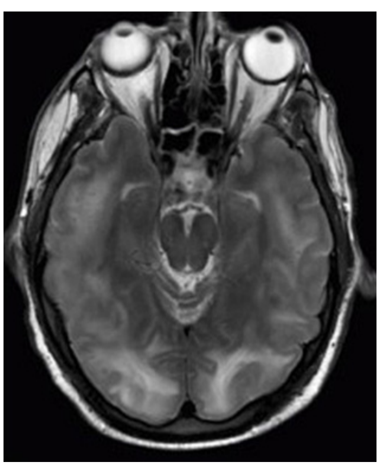
much easier to understand when you think about them in terms of basic physics.

Physics is the foundation of many important disciplines and contributes directly to others. Chemistry, for example—since it deals with the interactions of atoms and molecules—is rooted in atomic and molecular physics. Most branches of engineering are applied physics. In architecture, physics is at the heart of structural stability, and is involved in the acoustics, heating, lighting, and cooling of buildings. Parts of geology rely heavily on physics, such as radioactive dating of rocks, earthquake analysis, and heat transfer in the Earth. Some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

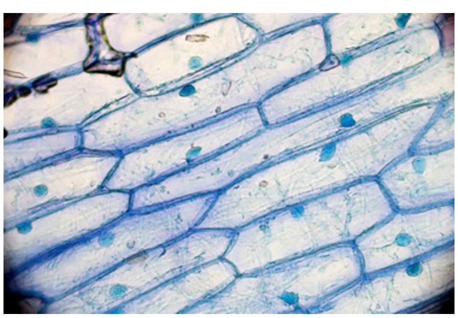
Physics has many applications in the biological sciences. On the microscopic level, it helps describe the properties of cell walls and cell membranes ([link] and [link]). On the macroscopic level, it can explain the heat, work, and power associated with the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements. Medical therapy sometimes directly involves physics; for example, cancer radiotherapy uses ionizing radiation. Physics can also explain sensory phenomena, such as how musical instruments make sound, how the eye detects color, and how lasers can transmit information.

It is not necessary to formally study all applications of physics. What is most useful is knowledge of the basic laws of physics and a skill in the analytical methods for applying them. The study of physics also can improve your problem-solving skills. Furthermore, physics has retained the most basic aspects of science, so it is used by all of the sciences, and the study of physics makes other sciences easier to understand.

The laws of physics help us understand how common appliances work. For example, the laws of physics can help explain how microwave ovens heat up food, and they also help us understand why it is dangerous to place metal objects in a microwave oven. (credit: MoneyBlogNewz)

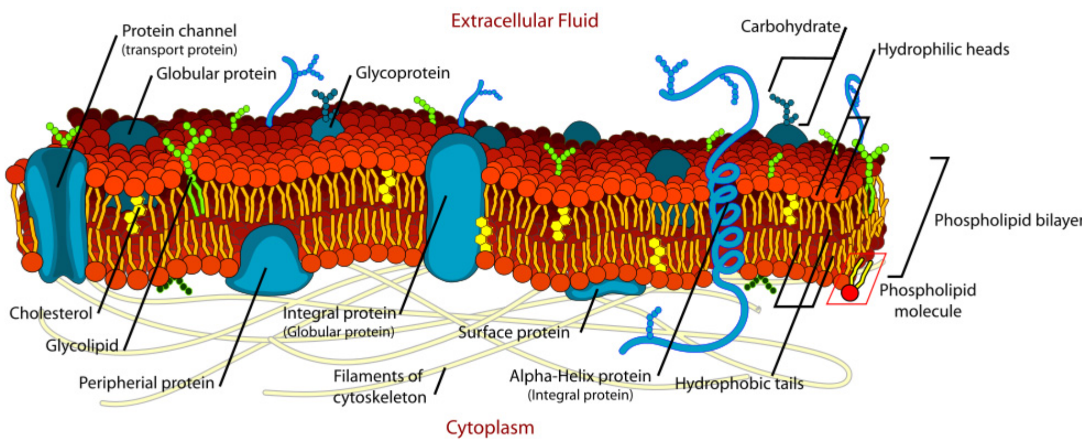


These two applications of physics have more in common than meets the eye. Microwave ovens use electromagnetic waves to heat food. Magnetic resonance imaging (MRI) also uses electromagnetic waves to yield an image of the brain, from which the exact location of tumors can be determined. (credit: Rashmi Chawla, Daniel Smith, and Paul E. Marik)



Physics, chemistry,

and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (credit: Umberto Salvagnin)

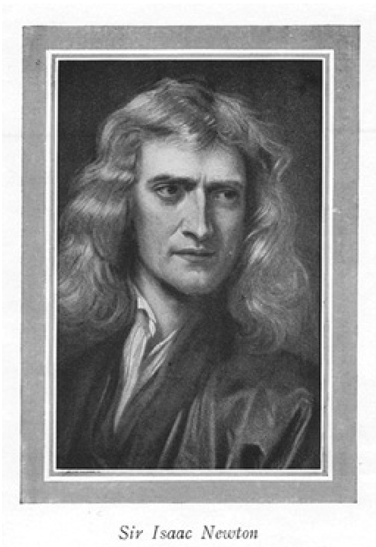


An artist's rendition of the the structure of a cell membrane. Membranes form the boundaries of animal cells and are complex in structure and function. Many of the most fundamental properties of life, such as the firing of nerve cells, are related to membranes. The disciplines of biology, chemistry, and physics all help us understand the membranes of animal cells. (credit: Mariana Ruiz)

Models, Theories, and Laws; The Role of Experimentation

The laws of nature are concise descriptions of the universe around us; they are human statements of the underlying laws or rules that all natural processes follow. Such laws are intrinsic to the universe; humans did not

create them and so cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. (See [link] and [link].) The cornerstone of discovering natural laws is observation; science must describe the universe as it is, not as we may imagine it to be.



Isaac Newton (1642–1727) was very reluctant to publish his revolutionary work and had to be convinced to do so. In his later years, he stepped down from his academic post and became exchequer of the Royal Mint. He took this post

seriously, inventing reeding (or creating ridges) on the edge of coins to prevent unscrupulous people from trimming the silver off of them before using them as currency. (credit: Arthur Shuster and Arthur E. Shipley: *Britain's Heritage of Science*. London, 1917.)

Marie Curie (1867–1934) sacrificed

monetary assets to help finance her early research and damaged her physical well-being with radiation exposure. She is the only person to win Nobel prizes in both physics and chemistry. One of her daughters also won a Nobel Prize. (credit: Wikimedia Commons)

We all are curious to some extent. We look around, make generalizations, and try to understand what we see—for example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how the data may be organized and unified. We then formulate models, theories, and laws based on the data we have collected and analyzed to generalize and communicate the results of these experiments.

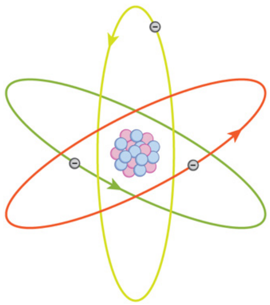
A **model** is a representation of something that is often too difficult (or impossible) to display directly. While a model is justified with experimental proof, it is only accurate under limited situations. An example is the planetary model of the atom in which electrons are pictured as orbiting the

nucleus, analogous to the way planets orbit the Sun. (See [link].) We cannot observe electron orbits directly, but the mental image helps explain the observations we can make, such as the emission of light from hot gases (atomic spectra). Physicists use models for a variety of purposes. For example, models can help physicists analyze a scenario and perform a calculation, or they can be used to represent a situation in the form of a computer simulation. A **theory** is an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers. Some theories include models to help visualize phenomena, whereas others do not. Newton’s theory of gravity, for example, does not require a model or mental image, because we can observe the objects directly with our own senses. The kinetic theory of gases, on the other hand, is a model in which a gas is viewed as being composed of atoms and molecules. Atoms and molecules are too small to be observed directly with our senses—thus, we picture them mentally to understand what our instruments tell us about the behavior of gases.

A **law** uses concise language to describe a generalized pattern in nature that is supported by scientific evidence and repeated experiments. Often, a law can be expressed in the form of a single mathematical equation. Laws and theories are similar in that they are both scientific statements that result from a tested hypothesis and are supported by scientific evidence. However, the designation *law* is reserved for a concise and very general statement that describes phenomena in nature, such as the law that energy is conserved during any process, or Newton’s second law of motion, which relates force, mass, and acceleration by the simple equation $\mathbf{F} = m\mathbf{a}$. A theory, in contrast, is a less concise statement of observed phenomena. For example, the Theory of Evolution and the Theory of Relativity cannot be expressed concisely enough to be considered a law. The biggest difference between a law and a theory is that a theory is much more complex and dynamic. A law describes a single action, whereas a theory explains an entire group of related phenomena. And, whereas a law is a postulate that forms the foundation of the scientific method, a theory is the end result of that process.

Less broadly applicable statements are usually called principles (such as Pascal’s principle, which is applicable only in fluids), but the distinction

between laws and principles often is not carefully made.



What is a model? This planetary model of the atom shows electrons orbiting the nucleus. It is a drawing that we use to form a mental image of the atom that we cannot see directly with our eyes because it is too small.

Note:
Models, Theories, and Laws
Models, theories, and laws are used to help scientists analyze the data they have already collected. However, often after a model, theory, or law has been developed, it points scientists toward new discoveries they would not otherwise have made.

The models, theories, and laws we devise sometimes *imply the existence of objects or phenomena as yet unobserved.* These predictions are remarkable triumphs and tributes to the power of science. It is the underlying order in the universe that enables scientists to make such spectacular predictions. However, if *experiment* does not verify our predictions, then the theory or law is wrong, no matter how elegant or convenient it is. Laws can never be known with absolute certainty because it is impossible to perform every imaginable experiment in order to confirm a law in every possible scenario. Physicists operate under the assumption that all scientific laws and theories are valid until a counterexample is observed. If a good-quality, verifiable experiment contradicts a well-established law, then the law must be modified or overthrown completely.

The study of science in general and physics in particular is an adventure much like the exploration of uncharted ocean. Discoveries are made; models, theories, and laws are formulated; and the beauty of the physical universe is made more sublime for the insights gained.

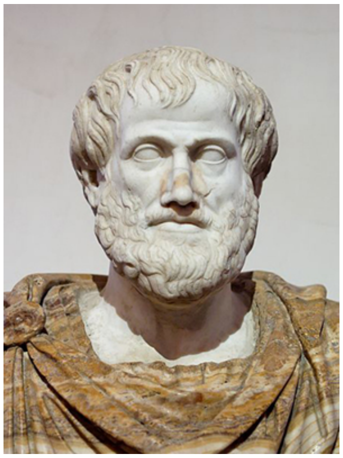
Note:
The Scientific Method
As scientists inquire and gather information about the world, they follow a process called the **scientific method**. This process typically begins with an observation and question that the scientist will research. Next, the scientist

typically performs some research about the topic and then devises a hypothesis. Then, the scientist will test the hypothesis by performing an experiment. Finally, the scientist analyzes the results of the experiment and draws a conclusion. Note that the scientific method can be applied to many situations that are not limited to science, and this method can be modified to suit the situation.

Consider an example. Let us say that you try to turn on your car, but it will not start. You undoubtedly wonder: Why will the car not start? You can follow a scientific method to answer this question. First off, you may perform some research to determine a variety of reasons why the car will not start. Next, you will state a hypothesis. For example, you may believe that the car is not starting because it has no engine oil. To test this, you open the hood of the car and examine the oil level. You observe that the oil is at an acceptable level, and you thus conclude that the oil level is not contributing to your car issue. To troubleshoot the issue further, you may devise a new hypothesis to test and then repeat the process again.

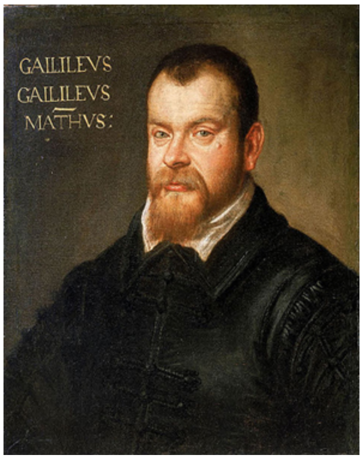
The Evolution of Natural Philosophy into Modern Physics

Physics was not always a separate and distinct discipline. It remains connected to other sciences to this day. The word *physics* comes from Greek, meaning nature. The study of nature came to be called “natural philosophy.” From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, physics, mathematics, and medicine. Over the last few centuries, the growth of knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. (See [link], [link], and [link].) Physics as it developed from the Renaissance to the end of the 19th century is called **classical physics**. It was transformed into modern physics by revolutionary discoveries made starting at the beginning of the 20th century.

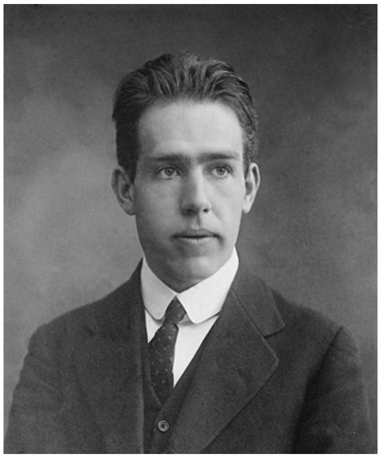


Over the centuries, natural philosophy has evolved into more specialized disciplines, as illustrated by the contributions of some of the greatest minds in history. The Greek philosopher **Aristotle** (384–322 B.C.) wrote on a broad range of topics including physics, animals, the soul, politics, and poetry. (credit: Jastrow

(2006)/Ludovisi Collection)



Galileo Galilei (1564–1642) laid the foundation of modern experimentation and made contributions in mathematics, physics, and astronomy. (credit: Domenico Tintoretto)



Niels Bohr (1885–1962) made fundamental contributions to the development of quantum mechanics, one part of modern physics. (credit: United States Library of Congress Prints and Photographs Division)

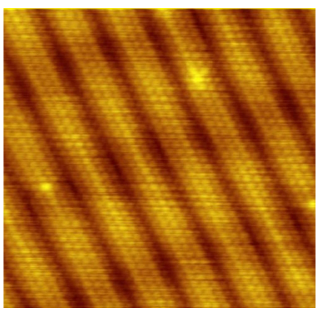
Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields, such as the field generated by the Earth, can be involved. Because humans live under such circumstances, classical physics seems intuitively reasonable, while many aspects of modern physics seem bizarre. This is why models are so useful in modern physics—they let us

conceptualize phenomena we do not ordinarily experience. We can relate to models in human terms and visualize what happens when objects move at high speeds or imagine what objects too small to observe with our senses might be like. For example, we can understand an atom's properties because we can picture it in our minds, although we have never seen an atom with our eyes. New tools, of course, allow us to better picture phenomena we cannot see. In fact, new instrumentation has allowed us in recent years to actually "picture" the atom.

Note:

Limits on the Laws of Classical Physics

For the laws of classical physics to apply, the following criteria must be met: Matter must be moving at speeds less than about 1% of the speed of light, the objects dealt with must be large enough to be seen with a microscope, and only weak gravitational fields (such as the field generated by the Earth) can be involved.



Using a scanning tunneling microscope (STM), scientists can see the individual atoms that

compose this sheet of gold. (credit: Erwinrossen)

Some of the most spectacular advances in science have been made in modern physics. Many of the laws of classical physics have been modified or rejected, and revolutionary changes in technology, society, and our view of the universe have resulted. Like science fiction, modern physics is filled with fascinating objects beyond our normal experiences, but it has the advantage over science fiction of being very real. Why, then, is the majority of this text devoted to topics of classical physics? There are two main reasons: Classical physics gives an extremely accurate description of the universe under a wide range of everyday circumstances, and knowledge of classical physics is necessary to understand modern physics.

Modern physics itself consists of the two revolutionary theories, relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. **Relativity** must be used whenever an object is traveling at greater than about 1% of the speed of light or experiences a strong gravitational field such as that near the Sun. **Quantum mechanics** must be used for objects smaller than can be seen with a microscope. The combination of these two theories is *relativistic quantum mechanics*, and it describes the behavior of small objects traveling at high speeds or experiencing a strong gravitational field. Relativistic quantum mechanics is the best universally applicable theory we have. Because of its mathematical complexity, it is used only when necessary, and the other theories are used whenever they will produce sufficiently accurate results.

Exercise:

Check Your Understanding

Problem:

A friend tells you he has learned about a new law of nature. What can you know about the information even before your friend describes the law? How would the information be different if your friend told you he had learned about a scientific theory rather than a law?

Solution:

Without knowing the details of the law, you can still infer that the information your friend has learned conforms to the requirements of all laws of nature: it will be a concise description of the universe around us; a statement of the underlying rules that all natural processes follow. If the information had been a theory, you would be able to infer that the information will be a large-scale, broadly applicable generalization.

Summary

- Science seeks to discover and describe the underlying order and simplicity in nature.
- Physics is the most basic of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Scientific laws and theories express the general truths of nature and the body of knowledge they encompass. These laws of nature are rules that all natural processes appear to follow.

Conceptual Questions

Exercise:

Problem:

Models are particularly useful in relativity and quantum mechanics, where conditions are outside those normally encountered by humans. What is a model?

Exercise:

Problem: How does a model differ from a theory?

Exercise:

Problem:

If two different theories describe experimental observations equally well, can one be said to be more valid than the other (assuming both use accepted rules of logic)?

Exercise:

Problem: What determines the validity of a theory?

Exercise:

Problem:

Certain criteria must be satisfied if a measurement or observation is to be believed. Will the criteria necessarily be as strict for an expected result as for an unexpected result?

Exercise:

Problem:

Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?

Exercise:

Problem:

Classical physics is a good approximation to modern physics under certain circumstances. What are they?

Exercise:

Problem: When is it *necessary* to use relativistic quantum mechanics?

Exercise:

Problem:

Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.

Glossary

classical physics
: physics that was developed from the Renaissance to the end of the 19th century

physics
: the science concerned with describing the interactions of energy, matter, space, and time; it is especially interested in what fundamental mechanisms underlie every phenomenon

model
: representation of something that is often too difficult (or impossible) to display directly

theory
: an explanation for patterns in nature that is supported by scientific evidence and verified multiple times by various groups of researchers

law
: a description, using concise language or a mathematical formula, a generalized pattern in nature that is supported by scientific evidence and repeated experiments

scientific method
: a method that typically begins with an observation and question that the scientist will research; next, the scientist typically performs some research about the topic and then devises a hypothesis; then, the scientist will test the hypothesis by performing an experiment; finally, the scientist analyzes the results of the experiment and draws a conclusion

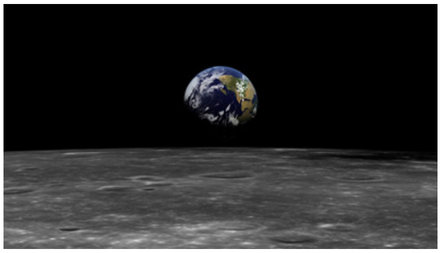
modern physics
: the study of relativity, quantum mechanics, or both

relativity
: the study of objects moving at speeds greater than about 1% of the speed of light, or of objects being affected by a strong gravitational field

quantum mechanics
: the study of objects smaller than can be seen with a microscope

Physical Quantities and Units

- Perform unit conversions both in the SI and English units.
- Explain the most common prefixes in the SI units and be able to write them in scientific notation.

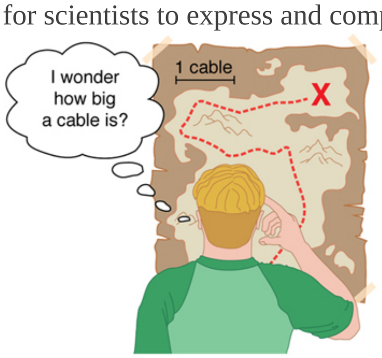


The distance from Earth to the Moon may seem immense, but it is just a tiny fraction of the distances from Earth to other celestial bodies. (credit: NASA)

The range of objects and phenomena studied in physics is immense. From the incredibly short lifetime of a nucleus to the age of the Earth, from the tiny sizes of sub-nuclear particles to the vast distance to the edges of the known universe, from the force exerted by a jumping flea to the force between Earth and the Sun, there are enough factors of 10 to challenge the imagination of even the most experienced scientist. Giving numerical values for physical quantities and equations for physical principles allows us to understand nature much more deeply than does qualitative description alone. To comprehend these vast ranges, we must also have accepted units in which to express them. And we shall find that (even in the potentially mundane discussion of meters, kilograms, and seconds) a profound simplicity of nature appears—all physical quantities can be expressed as combinations of only four fundamental physical quantities: length, mass, time, and electric current.

We define a **physical quantity** either by *specifying how it is measured* or by *stating how it is calculated* from other measurements. For example, we define distance and time by specifying methods for measuring them, whereas we define *average speed* by stating that it is calculated as distance traveled divided by time of travel.

Measurements of physical quantities are expressed in terms of **units**, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in units of meters (for sprinters) or kilometers (for distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way. (See [link].)



Distances given in unknown units are maddeningly useless.

There are two major systems of units used in the world: **SI units** (also known as the metric system) and **English units** (also known as the customary or imperial system). **English units** were historically used in nations once ruled by the British Empire and are still widely used in the United States. Virtually every other country in the world now uses SI units as the standard; the metric system is also the standard system agreed upon by scientists and mathematicians. The acronym “SI” is derived from the French *Système International*.

SI Units: Fundamental and Derived Units

[link] gives the fundamental SI units that are used throughout this textbook. This text uses non-SI units in a few applications where they are in very common use, such as the measurement of blood pressure in millimeters of mercury (mm Hg). Whenever non-SI units are discussed, they will be tied to SI units through conversions.

Length	**Mass**	**Time**	**Electric Current**
meter (m)	kilogram (kg)	second (s)	ampere (A)

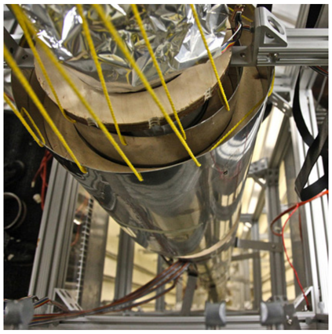
Fundamental SI Units

It is an intriguing fact that some physical quantities are more fundamental than others and that the most fundamental physical quantities can be defined *only* in terms of the procedure used to measure them. The units in which they are measured are thus called **fundamental units**. In this textbook, the fundamental physical quantities are taken to be length, mass, time, and electric current. (Note that electric current will not be introduced until much later in this text.) All other physical quantities, such as force and electric charge, can be expressed as algebraic combinations of length, mass, time, and current (for example, speed is length divided by time); these units are called **derived units**.

Units of Time, Length, and Mass: The Second, Meter, and Kilogram

The Second

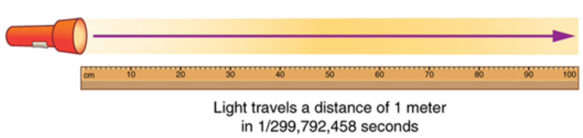
The SI unit for time, the **second**(abbreviated s), has a long history. For many years it was defined as 1/86,400 of a mean solar day. More recently, a new standard was adopted to gain greater accuracy and to define the second in terms of a non-varying, or constant, physical phenomenon (because the solar day is getting longer due to very gradual slowing of the Earth’s rotation). Cesium atoms can be made to vibrate in a very steady way, and these vibrations can be readily observed and counted. In 1967 the second was redefined as the time required for 9,192,631,770 of these vibrations. (See [link].) Accuracy in the fundamental units is essential, because all measurements are ultimately expressed in terms of fundamental units and can be no more accurate than are the fundamental units themselves.



An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of better than a microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic fountain nearly 30 feet tall! (credit: Steve Jurvetson/Flickr)

The Meter

The SI unit for length is the **meter** (abbreviated m); its definition has also changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar now kept near Paris. By 1960, it had become possible to define the meter even more accurately in terms of the wavelength of light, so it was again redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition (partly for greater accuracy) as the distance light travels in a vacuum in 1/299,792,458 of a second. (See [link].) This change defines the speed of light to be exactly 299,792,458 meters per second. The length of the meter will change if the speed of light is someday measured with greater accuracy.



The meter is defined to be the distance light travels in 1/299,792,458 of a second in a vacuum. Distance traveled is speed multiplied by time.

The Kilogram

The SI unit for mass is the **kilogram** (abbreviated kg); it is currently defined to be the mass of a platinum-iridium cylinder kept with the old meter standard at the International Bureau of Weights and Measures near Paris. Exact replicas of the standard kilogram are also kept at the United States' National Institute of Standards and Technology, or NIST, located in Gaithersburg, Maryland outside of Washington D.C., and at other locations around the world. The determination of all other masses can be ultimately traced to a comparison with the standard mass. At the November 2018 meeting of the International Committee for Weights and Measures, a proposed re-definition of kilogram was approved. This proposal (effective May 2019) re-defines the kilogram by precisely fixing the numerical value of Planck's constant (h) to be exactly $6.62607015 \times 10^{-34}\ \mathrm{kg\ \ m^2\ \ s^{-1}}$. This is similar to definition of the meter by fixing the speed of light (c), and we will see the importance of this re-definition in a later unit on quantum mechanics.

Electric current and its accompanying unit, the ampere, will be introduced when electricity and magnetism are covered. The initial modules in this textbook are concerned with mechanics, fluids, heat, and waves. In these subjects all pertinent physical quantities can be expressed in terms of the fundamental units of length, mass, and time.

Metric Prefixes

SI units are part of the **metric system**. The metric system is convenient for scientific and engineering calculations because the units are categorized by factors of 10. [link] gives metric prefixes and symbols used to denote various factors of 10.

Metric systems have the advantage that conversions of units involve only powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as the system of U.S. customary units, the relationships are not as simple—there are 12 inches in a foot, 5280 feet in a mile, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by using an appropriate metric prefix. For example, distances in meters are suitable in construction, while distances in kilometers are appropriate for air travel, and the tiny measure of nanometers are convenient in optical design. With the metric system there is no need to invent new units for particular applications.

The term **order of magnitude** refers to the scale of a value expressed in the metric system. Each power of 10 in the metric system represents a different order of magnitude. For example, 10^1, 10^2, 10^3, and so forth are all different orders of magnitude. All quantities that can be expressed as a product of a specific power of 10 are said to be of the *same* order of magnitude. For example, the number 800 can be written as 8×10^2, and the number 450 can be written as 4.5×10^2. Thus, the numbers 800 and 450 are of the same order of magnitude: 10^2. Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of $10^{-9}\ \mathrm{m}$, while the diameter of the Sun is on the order of $10^9\ \mathrm{m}$.

Prefix	Symbol	Value[footnote] See Appendix A for a discussion of powers of 10.	Example (some are approximate)			
exa	E	10^{18}	exameter	Em	10^{18} m	distance light travels in a century
peta	P	10^{15}	petasecond	Ps	10^{15} s	30 million years
tera	T	10^{12}	terawatt	TW	10^{12} W	powerful laser output
giga	G	10^{9}	gigahertz	GHz	10^{9} Hz	a microwave frequency
mega	M	10^{6}	megacurie	MCi	10^{6} Ci	high radioactivity
kilo	k	10^{3}	kilometer	km	10^{3} m	about 6/10 mile
hecto	h	10^{2}	hectoliter	hL	10^{2} L	26 gallons
deka	da	10^{1}	dekagram	dag	10^{1} g	teaspoon of butter
—	—	10^{0} (=1)				
deci	d	10^{-1}	deciliter	dL	10^{-1} L	less than half a soda
centi	c	10^{-2}	centimeter	cm	10^{-2} m	fingertip thickness
milli	m	10^{-3}	millimeter	mm	10^{-3} m	flea at its shoulders

Prefix	Symbol	Value[footnote] See Appendix A for a discussion of powers of 10.	Example (some are approximate)			
micro	µ	10^{-6}	micrometer	µm	10^{-6} m	detail in microscope
nano	n	10^{-9}	nanogram	ng	10^{-9} g	small speck of dust
pico	p	10^{-12}	picofarad	pF	10^{-12} F	small capacitor in radio
femto	f	10^{-15}	femtometer	fm	10^{-15} m	size of a proton
atto	a	10^{-18}	attosecond	as	10^{-18} s	time light crosses an atom

Metric Prefixes for Powers of 10 and their Symbols

Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook, some quantities may be expressed in units of liters and you need to convert them to cups. Or, perhaps you are reading walking directions from one location to another and you are interested in how many miles you will be walking. In this case, you will need to convert units of feet to miles.

Let us consider a simple example of how to convert units. Let us say that we want to convert 80 meters (m) to kilometers (km).

The first thing to do is to list the units that you have and the units that you want to convert to. In this case, we have units in *meters* and we want to convert to *kilometers*.

Next, we need to determine a **conversion factor** relating meters to kilometers. A conversion factor is a ratio expressing how many of one unit are equal to another unit. For example, there are 12 inches in 1 foot, 100 centimeters in 1 meter, 60 seconds in 1 minute, and so on. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor so that the units cancel out, as shown:
Equation:

$$80\,\cancel{\text{m}}\times\frac{1\text{ km}}{1000\,\cancel{\text{m}}}=0.080\text{ km}.$$

Note that the unwanted m unit cancels, leaving only the desired km unit. You can use this method to convert between any types of unit.

Example:
Unit Conversions: A Short Drive Home
Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note: Average speed is distance traveled divided by time of travel.)
Strategy
First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.
Solution for (a)
(1) Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,
Equation:

$$\text{average speed} = \frac{\text{distance}}{\text{time}}.$$

(2) Substitute the given values for distance and time.
Equation:

$$\text{average speed} = \frac{10.0\text{ km}}{20.0\text{ min}} = 0.500\frac{\text{km}}{\text{min}}.$$

(3) Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is 60 min/hr. Thus,
Equation:

$$\text{average speed} = 0.500\frac{\text{km}}{\text{min}} \times \frac{60\text{ min}}{1\text{ h}} = 30.0\frac{\text{km}}{\text{h}}.$$

Discussion for (a)
To check your answer, consider the following:
(1) Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows:
Equation:

$$\frac{\text{km}}{\text{min}} \times \frac{1\text{ hr}}{60\text{ min}} = \frac{1}{60}\frac{\text{km}\cdot\text{hr}}{\text{min}^2},$$

which are obviously not the desired units of km/h.
(2) Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.
(3) Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/hr does indeed have

three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 minutes, so the precision of the conversion factor is perfect.
(4) Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution for (b)

There are several ways to convert the average speed into meters per second.
(1) Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.
(2) Multiplying by these yields

Equation:

$$\text{Average speed} = 30.0\frac{\text{km}}{\text{h}} \times \frac{1\text{ h}}{3{,}600\text{ s}} \times \frac{1{,}000\text{ m}}{1\text{ km}},$$

Equation:

$$\text{Average speed} = 8.33\frac{\text{m}}{\text{s}}.$$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

Note:

Nonstandard Units

While there are numerous types of units that we are all familiar with, there are others that are much more obscure. For example, a **firkin** is a unit of volume that was once used to measure beer. One firkin equals about 34 liters. To learn more about nonstandard units, use a dictionary or encyclopedia to research different “weights and measures.” Take note of any unusual units, such as a barleycorn, that are not listed in the text. Think about how the unit is defined and state its relationship to SI units.

Exercise:

Check Your Understanding

Problem:

Some hummingbirds beat their wings more than 50 times per second. A scientist is measuring the time it takes for a hummingbird to beat its wings once. Which fundamental unit should the scientist use to describe the measurement? Which factor of 10 is the scientist likely to use to describe the motion precisely? Identify the metric prefix that corresponds to this factor of 10.

Solution:

The scientist will measure the time between each movement using the fundamental unit of seconds. Because the wings beat so fast, the scientist will probably need to measure in milliseconds, or 10^{-3} seconds. (50 beats per second corresponds to 20 milliseconds per beat.)

Exercise:

Check Your Understanding

Problem:

One cubic centimeter is equal to one milliliter. What does this tell you about the different units in the SI metric system?

Solution:

The fundamental unit of length (meter) is probably used to create the derived unit of volume (liter). The measure of a milliliter is dependent on the measure of a centimeter.

Summary

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- Units are standards for expressing and comparing the measurement of physical quantities. All units can be expressed as combinations of four fundamental units.
- The four fundamental units we will use in this text are the meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- The four fundamental units are abbreviated as follows: meter, m; kilogram, kg; second, s; and ampere, A. The metric system also uses a standard set of prefixes to denote each order of magnitude greater than or lesser than the fundamental unit itself.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.

Conceptual Questions

Exercise:

Problem: Identify some advantages of metric, or SI units.

Exercise:

Problem: Identify some advantages and disadvantages of English, or imperial units.

Exercise:

Problem:

How many fundamental units are there? What is the key difference between derived and fundamental units?

Glossary

physical quantity
: a characteristic or property of an object that can be measured or calculated from other measurements

units
: a standard used for expressing and comparing measurements

SI units
: the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams

English units
: system of measurement used in the United States; includes units of measurement such as feet, gallons, and pounds

fundamental units
: units that can only be expressed relative to the procedure used to measure them

derived units
: units that can be calculated using algebraic combinations of the fundamental units

second
: the SI unit for time, abbreviated (s)

meter
: the SI unit for length, abbreviated (m)

kilogram
: the SI unit for mass, abbreviated (kg)

metric system
: a system in which values can be calculated in factors of 10

order of magnitude
: refers to the size of a quantity as it relates to a power of 10

conversion factor
: a ratio expressing how many of one unit are equal to another unit

# Introduction to One-Dimensional Kinematics
class="introduction"

The motion of an American kestrel through the air can be described by the bird’s displacement , speed, velocity, and acceleration. When it flies in a straight line without any change in direction, its motion is said to be one dimensional. (credit: Vince Maidens, Wikimedia Commons)

Objects are in motion everywhere we look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. And even in inanimate objects, there is continuous motion in the vibrations of atoms and molecules. Questions about motion are interesting in and of themselves: *How long will it take for a space probe to get to Mars?* But an understanding of motion is also key to understanding other concepts in physics. An understanding of acceleration, for example, is crucial to the study of force.

Our formal study of physics begins with **kinematics** which is defined as the *study of motion without considering its causes*. The word “kinematics” comes from a Greek term meaning motion and is related to other English words such as “cinema” (movies) and “kinesiology” (the study of human motion). In one-dimensional kinematics we will study only the *motion* of a football, for example, without worrying about what forces cause or change its motion. Such considerations come in other chapters. In this chapter, we examine the simplest type of motion—namely, motion along a straight line, or one-dimensional motion.

Displacement

- Define position, displacement, distance, and distance traveled.
- Explain the relationship between position and displacement.
- Distinguish between displacement and distance traveled.
- Calculate displacement and distance given initial position, final position, and the path between the two.

These cyclists in Vietnam can be described by their position relative to buildings and a canal. Their motion can be described by their change in position, or displacement, in the frame of reference. (credit: Suzan Black, Fotopedia)

Position

In order to describe the motion of an object, you must first be able to describe its **position**—where it is at any particular time. More precisely, you need to specify its position relative to a convenient reference frame. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For

example, a rocket launch would be described in terms of the position of the rocket with respect to the Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. (See [link].) In other cases, we use reference frames that are not stationary but are in motion relative to the Earth. To describe the position of a person in an airplane, for example, we use the airplane, not the Earth, as the reference frame. (See [link].)

Displacement

If an object moves relative to a reference frame (for example, if a professor moves to the right relative to a white board or a passenger moves toward the rear of an airplane), then the object's position changes. This change in position is known as **displacement**. The word "displacement" implies that an object has moved, or has been displaced.

Note:
Displacement
Displacement is the *change in position* of an object:
Equation:

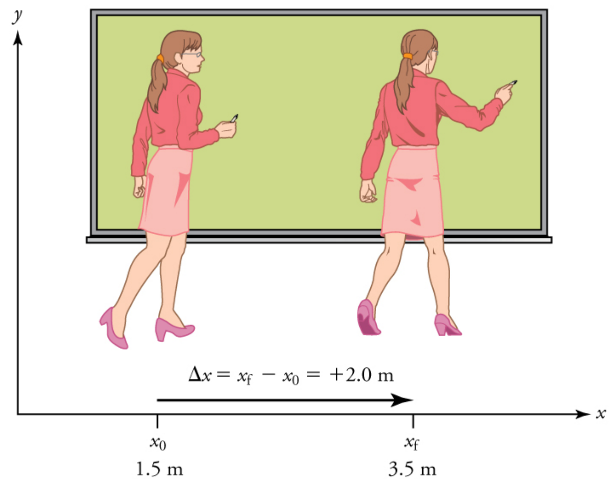
$$\Delta x = x_{\text{f}} - x_0,$$

where Δx is displacement, x_{f} is the final position, and x_0 is the initial position.

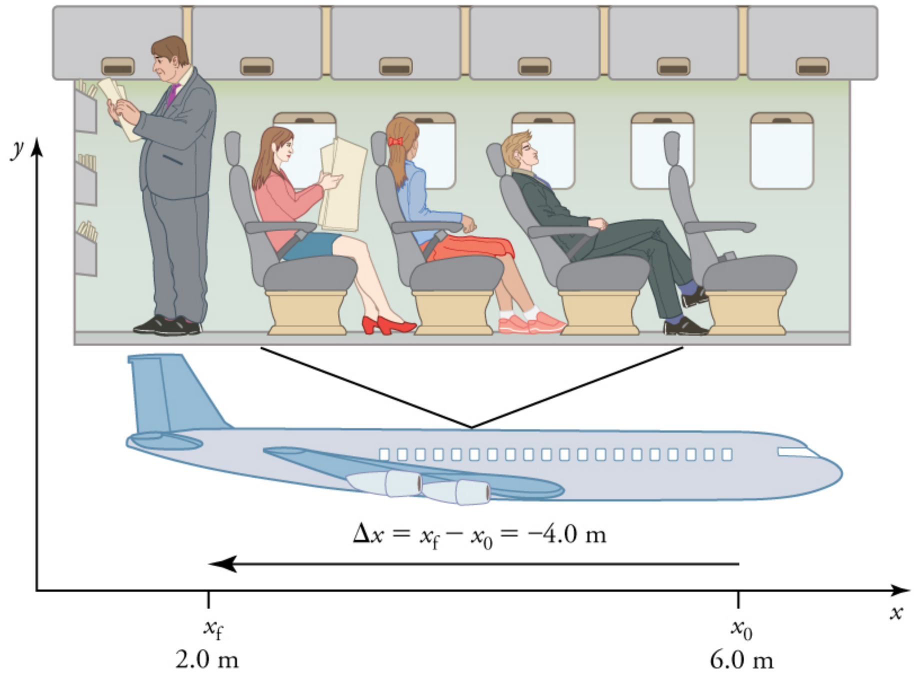
In this text the upper case Greek letter Δ (delta) always means "change in" whatever quantity follows it; thus, Δx means *change in position*. Always solve for displacement by subtracting initial position x_0 from final position x_{f}.

Note that the SI unit for displacement is the meter (m), but sometimes kilometers, miles, feet, and other units of length are used. Keep in mind that

when units other than the meter are used in a problem, you may need to convert them into meters to complete the calculation.



A professor paces left and right while lecturing. Her position relative to Earth is given by x. The $+2.0\text{ m}$ displacement of the professor relative to Earth is represented by an arrow pointing to the right.



A passenger moves from his seat to the back of the plane. His location relative to the airplane is given by x. The -4.0-m displacement of the passenger relative to the plane is represented by an arrow toward the rear of the plane. Notice that the arrow representing his displacement is twice as long as the arrow representing the displacement of the professor (he moves twice as far) in [link].

Note that displacement has a direction as well as a magnitude. The professor's displacement is 2.0 m to the right, and the airline passenger's displacement is 4.0 m toward the rear. In one-dimensional motion, direction can be specified with a plus or minus sign. When you begin a problem, you should select which direction is positive (usually that will be to the right or up, but you are free to select positive as being any direction). The professor's initial position is $x_0 = 1.5\ \text{m}$ and her final position is $x_\text{f} = 3.5\ \text{m}$. Thus her displacement is

Equation:

$$\Delta x = x_\text{f} - x_0 = 3.5\ \text{m} - 1.5\ \text{m} = +2.0\ \text{m}.$$

In this coordinate system, motion to the right is positive, whereas motion to the left is negative. Similarly, the airplane passenger's initial position is $x_0 = 6.0 \text{ m}$ and his final position is $x_\text{f} = 2.0 \text{ m}$, so his displacement is
Equation:

$$\Delta x = x_\text{f} - x_0 = 2.0 \text{ m} - 6.0 \text{ m} = -4.0 \text{ m}.$$

His displacement is negative because his motion is toward the rear of the plane, or in the negative x direction in our coordinate system.

Distance

Although displacement is described in terms of direction, distance is not. **Distance** is defined to be *the magnitude or size of displacement between two positions*. Note that the distance between two positions is not the same as the distance traveled between them. **Distance traveled** is *the total length of the path traveled between two positions*. Distance has no direction and, thus, no sign. For example, the distance the professor walks is 2.0 m. The distance the airplane passenger walks is 4.0 m.

Note:
Misconception Alert: Distance Traveled vs. Magnitude of Displacement
It is important to note that the *distance traveled*, however, can be greater than the magnitude of the displacement (by magnitude, we mean just the size of the displacement without regard to its direction; that is, just a number with a unit). For example, the professor could pace back and forth many times, perhaps walking a distance of 150 m during a lecture, yet still end up only 2.0 m to the right of her starting point. In this case her displacement would be +2.0 m, the magnitude of her displacement would be 2.0 m, but the distance she traveled would be 150 m. In kinematics we nearly always deal with displacement and magnitude of displacement, and almost never with distance traveled. One way to think about this is to assume you marked the start of the motion and the end of the motion. The

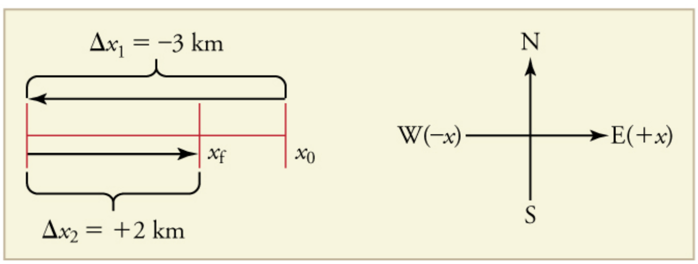
displacement is simply the difference in the position of the two marks and is independent of the path taken in traveling between the two marks. The distance traveled, however, is the total length of the path taken between the two marks.

Exercise:
Check Your Understanding

Problem:

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?

Solution:



(a) The rider's displacement is $\Delta x = x_{\text{f}} - x_0 = -1\ \text{km}$. (The displacement is negative because we take east to be positive and west to be negative.)

(b) The distance traveled is $3\ \text{km} + 2\ \text{km} = 5\ \text{km}$.

(c) The magnitude of the displacement is $1\ \text{km}$.

Section Summary

- Kinematics is the study of motion without considering its causes. In this chapter, it is limited to motion along a straight line, called one-dimensional motion.
- Displacement is the change in position of an object.

- In symbols, displacement Δx is defined to be
  Equation:

  $$\Delta x = x_{\text{f}} - x_0,$$

  where x_0 is the initial position and x_{f} is the final position. In this text, the Greek letter Δ (delta) always means "change in" whatever quantity follows it. The SI unit for displacement is the meter (m). Displacement has a direction as well as a magnitude.
- When you start a problem, assign which direction will be positive.
- Distance is the magnitude of displacement between two positions.
- Distance traveled is the total length of the path traveled between two positions.

Conceptual Questions

Exercise:

Problem:

Give an example in which there are clear distinctions among distance traveled, displacement, and magnitude of displacement. Specifically identify each quantity in your example.

Exercise:

Problem:

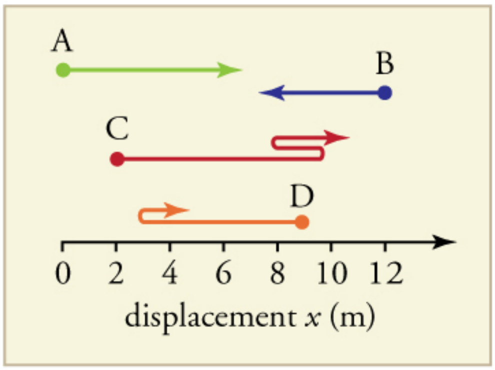
Under what circumstances does distance traveled equal magnitude of displacement? What is the only case in which magnitude of displacement and displacement are exactly the same?

Exercise:

Problem:

Bacteria move back and forth by using their flagella (structures that look like little tails). Speeds of up to $50\ \mu\text{m/s}$ $\left(50 \times 10^{-6}\ \text{m/s}\right)$ have been observed. The total distance traveled by a bacterium is large for its size, while its displacement is small. Why is this?

Problems & Exercises



Exercise:

Problem:

Find the following for path A in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Solution:

(a) 7 m

(b) 7 m

(c) $+7$ m

Exercise:

Problem:

Find the following for path B in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Exercise:

Problem:

Find the following for path C in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Solution:

(a) 13 m

(b) 9 m

(c) $+9$ m

Exercise:

Problem:

Find the following for path D in [link]: (a) The distance traveled. (b) The magnitude of the displacement from start to finish. (c) The displacement from start to finish.

Glossary

kinematics
: the study of motion without considering its causes

position
: the location of an object at a particular time

displacement
: the change in position of an object

distance
: the magnitude of displacement between two positions

distance traveled
: the total length of the path traveled between two positions

Vectors, Scalars, and Coordinate Systems

- Define and distinguish between scalar and vector quantities.
- Assign a coordinate system for a scenario involving one-dimensional motion.

The motion of this Eclipse Concept jet can be described in terms of the distance it has traveled (a scalar quantity) or its displacement in a specific direction (a vector quantity). In order to specify the direction of motion, its displacement must be described based on a coordinate system. In this case, it may be convenient to choose motion toward the left as positive motion (it is the forward direction for the plane), although in many cases, the x-coordinate runs from left to right, with motion to the right as positive and motion to the left as negative. (credit: Armchair Aviator, Flickr)

What is the difference between distance and displacement? Whereas displacement is defined by both direction and magnitude, distance is defined only by magnitude. Displacement is an example of a vector quantity. Distance is an example of a scalar quantity. A **vector** is any quantity with both *magnitude and direction*. Other examples of vectors include a velocity of 90 km/h east and a force of 500 newtons straight down.

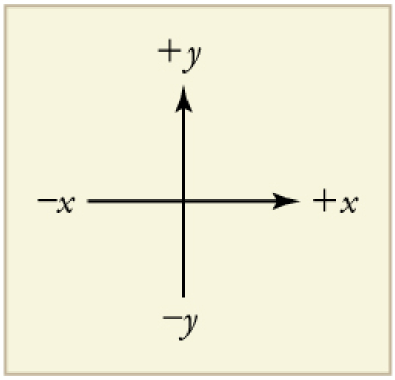
The direction of a vector in one-dimensional motion is given simply by a plus $(+)$ or minus $(-)$ sign. Vectors are represented graphically by arrows. An arrow used to represent a vector has a length proportional to the vector's magnitude (e.g., the larger the magnitude, the longer the length of the vector) and points in the same direction as the vector.

Some physical quantities, like distance, either have no direction or none is specified. A **scalar** is any quantity that has a magnitude, but no direction. For example, a 20°C temperature, the 250 kilocalories (250 Calories) of energy in a candy bar, a 90 km/h speed limit, a person's 1.8 m height, and a distance of 2.0 m are all scalars—quantities with no specified direction. Note, however, that a scalar can be negative, such as a -20°C temperature. In this case, the minus sign indicates a point on a scale rather than a direction. Scalars are never represented by arrows.

Coordinate Systems for One-Dimensional Motion

In order to describe the direction of a vector quantity, you must designate a coordinate system within the reference frame. For one-dimensional motion, this is a simple coordinate system consisting of a one-dimensional coordinate line. In general, when describing horizontal motion, motion to the right is usually considered positive, and motion to the left is considered negative. With vertical motion, motion up is usually positive and motion down is negative. In some cases, however, as with the jet in [link], it can be more convenient to switch the positive and negative directions. For example, if you are analyzing the motion of falling objects, it can be useful to define downwards as the positive direction. If people in a race are

running to the left, it is useful to define left as the positive direction. It does not matter as long as the system is clear and consistent. Once you assign a positive direction and start solving a problem, you cannot change it.



It is usually convenient to consider motion upward or to the right as positive $(+)$ and motion downward or to the left as negative $(-)$.

Exercise:
Check Your Understanding

Problem:

A person’s speed can stay the same as he or she rounds a corner and changes direction. Given this information, is speed a scalar or a vector quantity? Explain.

Solution:

Speed is a scalar quantity. It does not change at all with direction changes; therefore, it has magnitude only. If it were a vector quantity, it would change as direction changes (even if its magnitude remained constant).

Section Summary

- A vector is any quantity that has magnitude and direction.
- A scalar is any quantity that has magnitude but no direction.
- Displacement and velocity are vectors, whereas distance and speed are scalars.
- In one-dimensional motion, direction is specified by a plus or minus sign to signify left or right, up or down, and the like.

Conceptual Questions

Exercise:

Problem:

A student writes, "*A bird that is diving for prey has a speed of* $-10\ m/s$." What is wrong with the student's statement? What has the student actually described? Explain.

Exercise:

Problem: What is the speed of the bird in [link]?

Exercise:

Problem:

Acceleration is the change in velocity over time. Given this information, is acceleration a vector or a scalar quantity? Explain.

Exercise:

Problem:

A weather forecast states that the temperature is predicted to be $-5^{\circ}\mathrm{C}$ the following day. Is this temperature a vector or a scalar quantity? Explain.

Glossary

scalar
: a quantity that is described by magnitude, but not direction

vector
: a quantity that is described by both magnitude and direction

Time, Velocity, and Speed

- Explain the relationships between instantaneous velocity, average velocity, instantaneous speed, average speed, displacement, and time.
- Calculate velocity and speed given initial position, initial time, final position, and final time.
- Interpret graphs of position vs. time and velocity vs. time.

The motion of these racing snails can be described by their speeds and their velocities. (credit: tobitasflickr, Flickr)

There is more to motion than distance and displacement. Questions such as, “How long does a foot race take?” and “What was the runner’s speed?” cannot be answered without an understanding of other concepts. In this section we add definitions of time, velocity, and speed to expand our description of motion.

Time

The most fundamental physical quantities are defined by how they are measured. This is the case with time. Every measurement of time involves measuring a change in some physical quantity. It may be a number on a digital clock, a heartbeat, or the position of the Sun in the sky. In physics,

the definition of time is simple—**time** is *change*, or the interval over which change occurs. It is impossible to know that time has passed unless something changes.

The amount of time or change is calibrated by comparison with a standard. The SI unit for time is the second, abbreviated s. We might, for example, observe that a certain pendulum makes one full swing every 0.75 s. We could then use the pendulum to measure time by counting its swings or, of course, by connecting the pendulum to a clock mechanism that registers time on a dial. This allows us to not only measure the amount of time, but also to determine a sequence of events.

How does time relate to motion? We are usually interested in elapsed time for a particular motion, such as how long it takes an airplane passenger to get from his seat to the back of the plane. To find elapsed time, we note the time at the beginning and end of the motion and subtract the two. For example, a lecture may start at 11:00 A.M. and end at 11:50 A.M., so that the elapsed time would be 50 min. **Elapsed time** Δt is the difference between the ending time and beginning time,

Equation:

$$\Delta t = t_{\text{f}} - t_0,$$

where Δt is the change in time or elapsed time, t_{f} is the time at the end of the motion, and t_0 is the time at the beginning of the motion. (As usual, the delta symbol, Δ, means the change in the quantity that follows it.)

Velocity

Your notion of velocity is probably the same as its scientific definition. You know that if you have a large displacement in a small amount of time you have a large velocity, and that velocity has units of distance divided by time, such as miles per hour or kilometers per hour.

Note:

Average Velocity

Average velocity is *displacement (change in position) divided by the time of travel,*

Equation:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_\text{f} - x_0}{t_\text{f} - t_0},$$

where $\bar{v}$ is the *average* (indicated by the bar over the v) velocity, Δx is the change in position (or displacement), and x_f and x_0 are the final and beginning positions at times t_f and t_0, respectively.

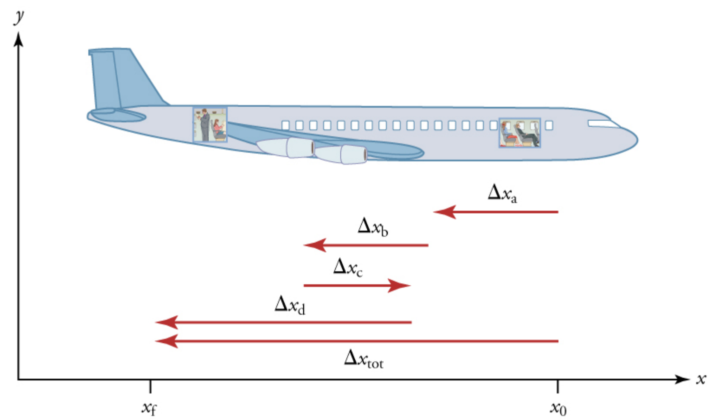
Notice that this definition indicates that *velocity is a vector because displacement is a vector*. It has both magnitude and direction. The SI unit for velocity is meters per second or m/s, but many other units, such as km/h, mi/h (also written as mph), and cm/s, are in common use. Suppose, for example, an airplane passenger took 5 seconds to move −4 m (the negative sign indicates that displacement is toward the back of the plane). His average velocity would be

Equation:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{-4\,\text{m}}{5\,\text{s}} = -0.8\,\text{m/s}.$$

The minus sign indicates the average velocity is also toward the rear of the plane.

The average velocity of an object does not tell us anything about what happens to it between the starting point and ending point, however. For example, we cannot tell from average velocity whether the airplane passenger stops momentarily or backs up before he goes to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals.



A more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

The smaller the time intervals considered in a motion, the more detailed the information. When we carry this process to its logical conclusion, we are left with an infinitesimally small interval. Over such an interval, the average velocity becomes the *instantaneous velocity* or the *velocity at a specific instant*. A car's speedometer, for example, shows the magnitude (but not the direction) of the instantaneous velocity of the car. (Police give tickets based on instantaneous velocity, but when calculating how long it will take to get from one place to another on a road trip, you need to use average velocity.) **Instantaneous velocity** v is the average velocity at a specific instant in time (or over an infinitesimally small time interval).

Mathematically, finding instantaneous velocity, v, at a precise instant t can involve taking a limit, a calculus operation beyond the scope of this text. However, under many circumstances, we can find precise values for instantaneous velocity without calculus.

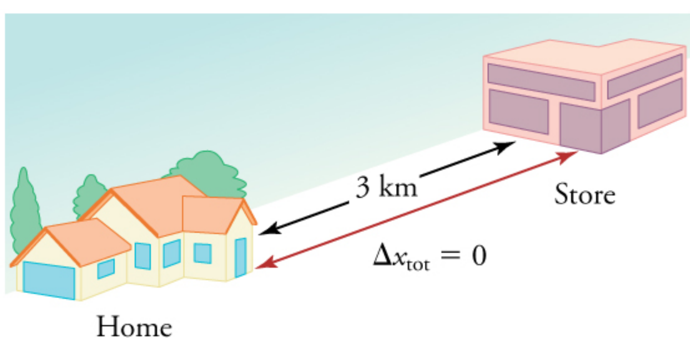
Speed

In everyday language, most people use the terms "speed" and "velocity" interchangeably. In physics, however, they do not have the same meaning

and they are distinct concepts. One major difference is that speed has no direction. Thus *speed is a scalar*. Just as we need to distinguish between instantaneous velocity and average velocity, we also need to distinguish between instantaneous speed and average speed.

Instantaneous speed is the magnitude of instantaneous velocity. For example, suppose the airplane passenger at one instant had an instantaneous velocity of −3.0 m/s (the minus meaning toward the rear of the plane). At that same time his instantaneous speed was 3.0 m/s. Or suppose that at one time during a shopping trip your instantaneous velocity is 40 km/h due north. Your instantaneous speed at that instant would be 40 km/h—the same magnitude but without a direction. Average speed, however, is very different from average velocity. **Average speed** is the distance traveled divided by elapsed time.

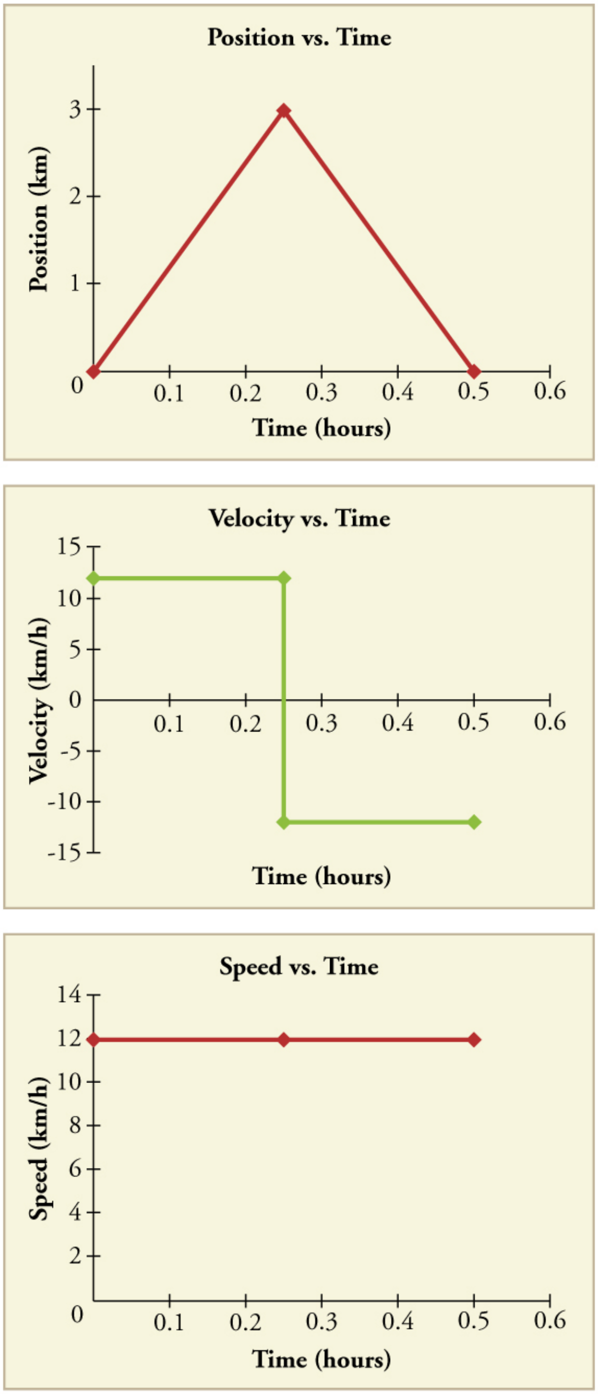
We have noted that distance traveled can be greater than displacement. So average speed can be greater than average velocity, which is displacement divided by time. For example, if you drive to a store and return home in half an hour, and your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero, because your displacement for the round trip is zero. (Displacement is change in position and, thus, is zero for a round trip.) Thus average speed is *not* simply the magnitude of average velocity.



During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h.

The displacement for the round trip is zero, since there was no net change in position. Thus the average velocity is zero.

Another way of visualizing the motion of an object is to use a graph. A plot of position or of velocity as a function of time can be very useful. For example, for this trip to the store, the position, velocity, and speed-vs.-time graphs are displayed in [link]. (Note that these graphs depict a very simplified **model** of the trip. We are assuming that speed is constant during the trip, which is unrealistic given that we'll probably stop at the store. But for simplicity's sake, we will model it with no stops or changes in speed. We are also assuming that the route between the store and the house is a perfectly straight line.)



Position vs. time, velocity vs. time, and speed vs. time on a trip. Note that the velocity for the return trip is negative.

Note:

Making Connections: Take-Home Investigation—Getting a Sense of Speed

If you have spent much time driving, you probably have a good sense of speeds between about 10 and 70 miles per hour. But what are these in meters per second? What do we mean when we say that something is moving at 10 m/s? To get a better sense of what these values really mean, do some observations and calculations on your own:

- calculate typical car speeds in meters per second
- estimate jogging and walking speed by timing yourself; convert the measurements into both m/s and mi/h
- determine the speed of an ant, snail, or falling leaf

Exercise:

Check Your Understanding

Problem:

A commuter train travels from Baltimore to Washington, DC, and back in 1 hour and 45 minutes. The distance between the two stations is approximately 40 miles. What is (a) the average velocity of the train, and (b) the average speed of the train in m/s?

Solution:

(a) The average velocity of the train is zero because $x_{\mathrm{f}} = x_0$; the train ends up at the same place it starts.

(b) The average speed of the train is calculated below. Note that the train travels 40 miles one way and 40 miles back, for a total distance of 80 miles.

Equation:

$$\frac{\text{distance}}{\text{time}} = \frac{80\text{ miles}}{105\text{ minutes}}$$

Equation:

$$\frac{80 \text{ miles}}{105 \text{ minutes}} \times \frac{5280 \text{ feet}}{1 \text{ mile}} \times \frac{1 \text{ meter}}{3.28 \text{ feet}} \times \frac{1 \text{ minute}}{60 \text{ seconds}} = 20 \text{ m/s}$$

Section Summary

- Time is measured in terms of change, and its SI unit is the second (s). Elapsed time for an event is
  Equation:

  $$\Delta t = t_{\text{f}} - t_0,$$

  where t_{f} is the final time and t_0 is the initial time.
- Average velocity $\bar{v}$ is defined as displacement divided by the travel time. In symbols, average velocity is
  Equation:

  $$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_{\text{f}} - x_0}{t_{\text{f}} - t_0}.$$

- The SI unit for velocity is m/s.
- Velocity is a vector and thus has a direction.
- Instantaneous velocity v is the velocity at a specific instant or the average velocity for an infinitesimal interval.
- Instantaneous speed is the magnitude of the instantaneous velocity.
- Instantaneous speed is a scalar quantity, as it has no direction specified.
- Average speed is the total distance traveled divided by the elapsed time. (Average speed is *not* the magnitude of the average velocity.) Speed is a scalar quantity; it has no direction associated with it.

Conceptual Questions

Exercise:

Problem:

Give an example (but not one from the text) of a device used to measure time and identify what change in that device indicates a change in time.

Exercise:

Problem:

There is a distinction between average speed and the magnitude of average velocity. Give an example that illustrates the difference between these two quantities.

Exercise:

Problem:

Does a car’s odometer measure position or displacement? Does its speedometer measure speed or velocity?

Exercise:

Problem:

If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity? Under what circumstances are these two quantities the same?

Exercise:

Problem:

How are instantaneous velocity and instantaneous speed related to one another? How do they differ?

Glossary

average speed
: distance traveled divided by time during which motion occurs

average velocity
: displacement divided by time over which displacement occurs

instantaneous velocity
: velocity at a specific instant, or the average velocity over an infinitesimal time interval

instantaneous speed
: magnitude of the instantaneous velocity

time
: change, or the interval over which change occurs

model
: simplified description that contains only those elements necessary to describe the physics of a physical situation

elapsed time
: the difference between the ending time and beginning time

Acceleration

- Define and distinguish between velocity and acceleration, and between instantaneous and average acceleration.
- Calculate acceleration given initial time, initial velocity, final time, and final velocity.

A plane slows down as it comes in for landing in St. Maarten. It is accelerating in a direction opposite to its velocity. (credit: Steve Conry, Flickr)

In everyday conversation, to accelerate means to speed up. The accelerator in a car can in fact cause it to speed up. The greater the **acceleration**, the greater the change in velocity over a given time. The formal definition of acceleration is consistent with these notions, but more inclusive.

Note:
Average Acceleration
Average Acceleration is *the rate at which velocity changes,*
Equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_{\mathrm{f}} - v_0}{t_{\mathrm{f}} - t_0},$$

where $\bar{a}$ is average acceleration, v is velocity, and t is time. (The bar over the a means *average* acceleration.)

Because acceleration is velocity in m/s divided by time in s, the SI units for acceleration are $\mathrm{m/s^2}$, meters per second squared or meters per second per second, which literally means by how many meters per second the velocity changes every second.

Recall that velocity is a vector—it has both magnitude and direction. This means that a change in velocity can be a change in magnitude (or speed), but it can also be a change in *direction*. For example, if a car turns a corner at constant speed, it is accelerating because its direction is changing. The quicker you turn, the greater the acceleration. So there is an acceleration when velocity changes either in magnitude (an increase or decrease in speed) or in direction, or both.

Note:
Acceleration as a Vector
Acceleration is a vector in the same direction as the *change* in velocity, Δv. Since velocity is a vector, it can change either in magnitude or in direction. Acceleration is therefore a change in either speed or direction, or both.

Keep in mind that although acceleration is in the direction of the *change* in velocity, it is not always in the direction of *motion*. If acceleration is in a direction opposite to the direction of motion, the object slows down.

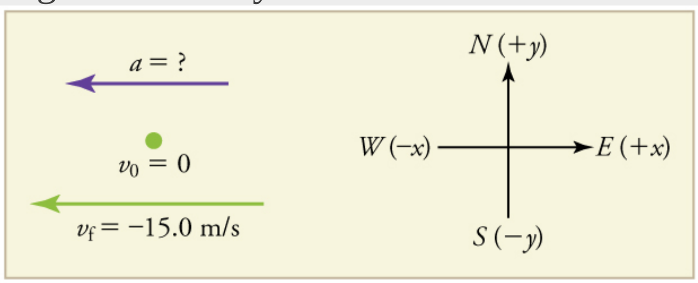
A subway train in Sao Paulo, Brazil, slows down as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki, Flickr)

Example:
Calculating Acceleration: A Racehorse Leaves the Gate
A racehorse coming out of the gate accelerates from rest to a velocity of 15.0 m/s due west in 1.80 s. What is its average acceleration?

(credit: Jon Sullivan, PD Photo.org)

Strategy

First we draw a sketch and assign a coordinate system to the problem. This is a simple problem, but it always helps to visualize it. Notice that we assign east as positive and west as negative. Thus, in this case, we have negative velocity.



We can solve this problem by identifying Δv and Δt from the given information and then calculating the average acceleration directly from the equation $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}$.

Solution

1. Identify the knowns. $v_0 = 0$, $v_f = -15.0\text{ m/s}$ (the negative sign indicates direction toward the west), $\Delta t = 1.80\text{ s}$.

2. Find the change in velocity. Since the horse is going from zero to -15.0 m/s, its change in velocity equals its final velocity: $\Delta v = v_f = -15.0\text{ m/s}$.

3. Plug in the known values (Δv and Δt) and solve for the unknown $\bar{a}$.

Equation:

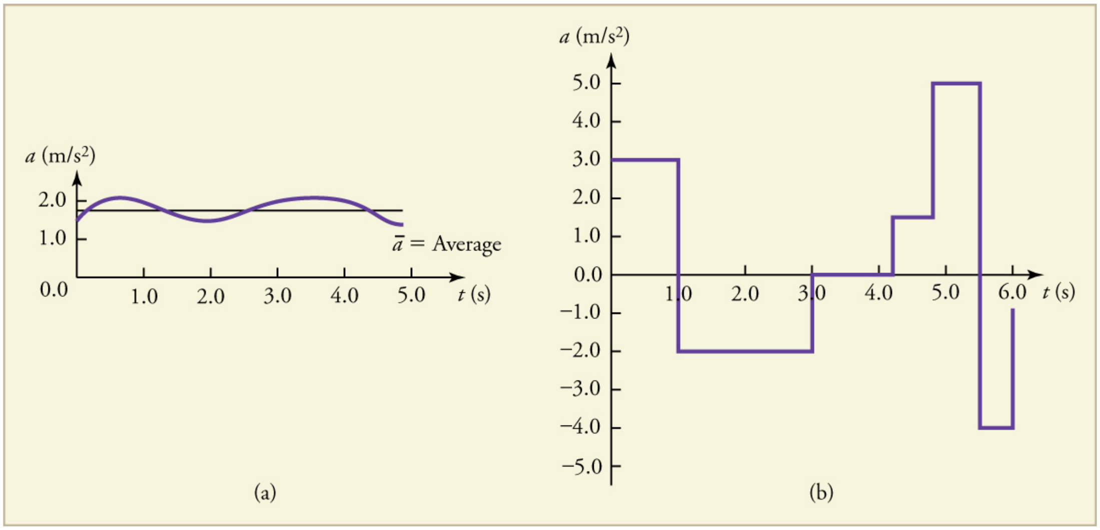
$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-15.0\text{ m/s}}{1.80\text{ s}} = -8.33\text{ m/s}^2.$$

Discussion

The negative sign for acceleration indicates that acceleration is toward the west. An acceleration of 8.33 m/s^2 due west means that the horse increases its velocity by 8.33 m/s due west each second, that is, 8.33 meters per second per second, which we write as 8.33 m/s^2. This is truly an average acceleration, because the ride is not smooth. We shall see later that an acceleration of this magnitude would require the rider to hang on with a force nearly equal to his weight.

Instantaneous Acceleration

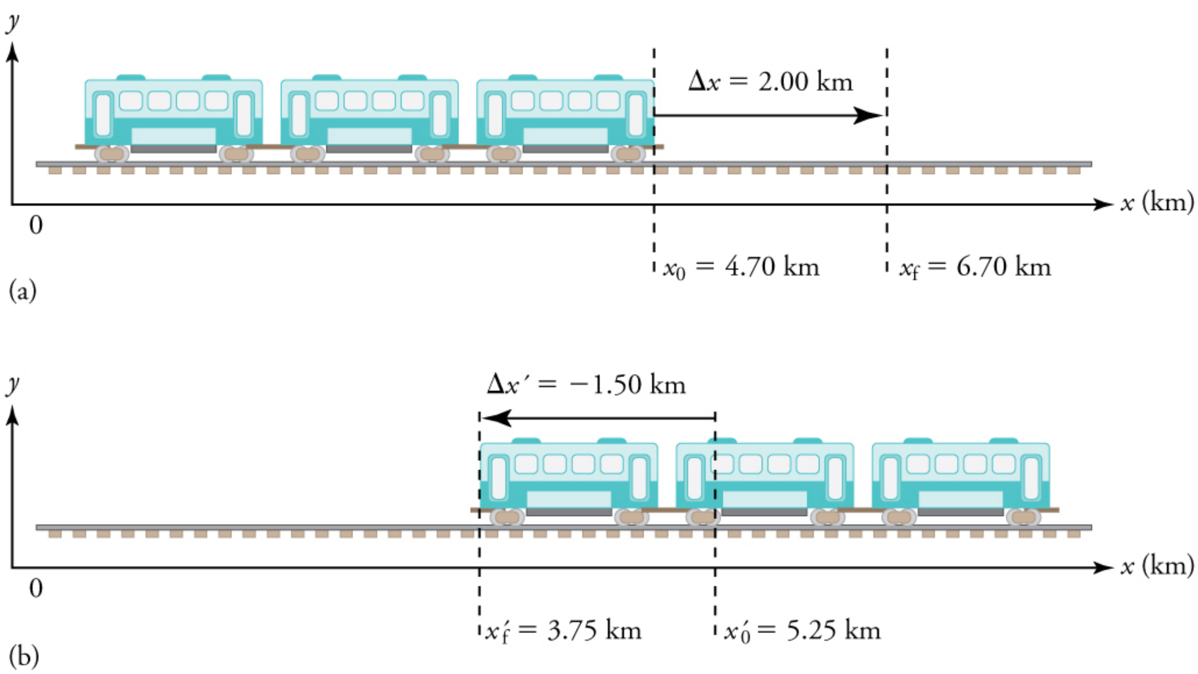
Instantaneous acceleration a, or the *acceleration at a specific instant in time*, is obtained by the same process as discussed for instantaneous velocity—that is, by considering an infinitesimally small interval of time. How do we find instantaneous acceleration using only algebra? The answer is that we choose an average acceleration that is representative of the motion. [link] shows graphs of instantaneous acceleration versus time for two very different motions. In [link](a), the acceleration varies slightly and the average over the entire interval is nearly the same as the instantaneous acceleration at any time. In this case, we should treat this motion as if it had a constant acceleration equal to the average (in this case about 1.8 m/s^2). In [link](b), the acceleration varies drastically over time. In such situations it is best to consider smaller time intervals and choose an average acceleration for each. For example, we could consider motion over the time intervals from 0 to 1.0 s and from 1.0 to 3.0 s as separate motions with accelerations of $+3.0\text{ m/s}^2$ and $–2.0\text{ m/s}^2$, respectively.



Graphs of instantaneous acceleration versus time for two different one-dimensional motions. (a) Here acceleration varies

only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time. (b) Here the acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0 to 1.0 s) with constant or nearly constant acceleration in such a situation.

The next several examples consider the motion of the subway train shown in [link]. In (a) the shuttle moves to the right, and in (b) it moves to the left. The examples are designed to further illustrate aspects of motion and to illustrate some of the reasoning that goes into solving problems.



One-dimensional motion of a subway train considered in [link], [link], [link], [link], [link], and [link]. Here we have chosen the x-axis so that + means to the right and − means to the left for displacements, velocities, and accelerations. (a) The subway train

moves to the right from x_0 to x_f. Its displacement Δx is +2.0 km. (b) The train moves to the left from $x\prime_0$ to $x\prime_\mathrm{f}$. Its displacement $\Delta x\prime$ is -1.5 km. (Note that the prime symbol (′) is used simply to distinguish between displacement in the two different situations. The distances of travel and the size of the cars are on different scales to fit everything into the diagram.)

Example:
Calculating Displacement: A Subway Train
What are the magnitude and sign of displacements for the motions of the subway train shown in parts (a) and (b) of [link]?
Strategy
A drawing with a coordinate system is already provided, so we don't need to make a sketch, but we should analyze it to make sure we understand what it is showing. Pay particular attention to the coordinate system. To find displacement, we use the equation $\Delta x = x_\mathrm{f} - x_0$. This is straightforward since the initial and final positions are given.
Solution
1. Identify the knowns. In the figure we see that $x_\mathrm{f} = 6.70\text{ km}$ and $x_0 = 4.70\text{ km}$ for part (a), and $x\prime_\mathrm{f} = 3.75\text{ km}$ and $x\prime_0 = 5.25\text{ km}$ for part (b).
2. Solve for displacement in part (a).
Equation:

$$\Delta x = x_\mathrm{f} - x_0 = 6.70\text{ km} - 4.70\text{ km} = +2.00\text{ km}$$

3. Solve for displacement in part (b).
Equation:

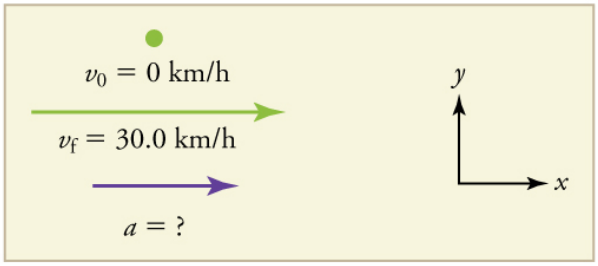
$$\Delta x\prime = x\prime_\mathrm{f} - x\prime_0 = 3.75\text{ km} - 5.25\text{ km} = -1.50\text{ km}$$

Discussion
The direction of the motion in (a) is to the right and therefore its displacement has a positive sign, whereas motion in (b) is to the left and

thus has a negative sign.

Example:
Comparing Distance Traveled with Displacement: A Subway Train
What are the distances traveled for the motions shown in parts (a) and (b) of the subway train in [link]?
Strategy
To answer this question, think about the definitions of distance and distance traveled, and how they are related to displacement. Distance between two positions is defined to be the magnitude of displacement, which was found in [link]. Distance traveled is the total length of the path traveled between the two positions. In the case of the subway train shown in [link], the distance traveled is the same as the distance between the initial and final positions of the train.
Solution
1. The displacement for part (a) was +2.00 km. Therefore, the distance between the initial and final positions was 2.00 km, and the distance traveled was 2.00 km.
2. The displacement for part (b) was -1.5 km. Therefore, the distance between the initial and final positions was 1.50 km, and the distance traveled was 1.50 km.
Discussion
Distance is a scalar. It has magnitude but no sign to indicate direction.

Example:
Calculating Acceleration: A Subway Train Speeding Up
Suppose the train in [link](a) accelerates from rest to 30.0 km/h in the first 20.0 s of its motion. What is its average acceleration during that time interval?
Strategy
It is worth it at this point to make a simple sketch:



This problem involves three steps. First we must determine the change in velocity, then we must determine the change in time, and finally we use these values to calculate the acceleration.

Solution

1. Identify the knowns. $v_0 = 0$ (the trains starts at rest), $v_\mathrm{f} = 30.0\ \mathrm{km/h}$, and $\Delta t = 20.0\ \mathrm{s}$.
2. Calculate Δv. Since the train starts from rest, its change in velocity is Δv= $+30.0\ \mathrm{km/h}$, where the plus sign means velocity to the right.
3. Plug in known values and solve for the unknown, $\bar{a}$.

Equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+30.0\ \mathrm{km/h}}{20.0\ \mathrm{s}}$$

4. Since the units are mixed (we have both hours and seconds for time), we need to convert everything into SI units of meters and seconds.

Equation:

$$\bar{a} = \left(\frac{+30\ \mathrm{km/h}}{20.0\ \mathrm{s}}\right)\left(\frac{10^3\ \mathrm{m}}{1\ \mathrm{km}}\right)\left(\frac{1\ \mathrm{h}}{3600\ \mathrm{s}}\right) = 0.417\ \mathrm{m/s^2}$$

Discussion

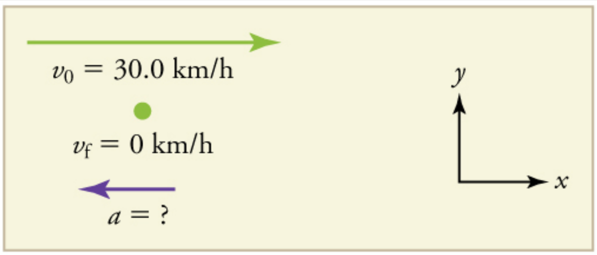
The plus sign means that acceleration is to the right. This is reasonable because the train starts from rest and ends up with a velocity to the right (also positive). So acceleration is in the same direction as the *change* in velocity, as is always the case.

Example:

Calculate Acceleration: A Subway Train Slowing Down

Now suppose that at the end of its trip, the train in [link](a) slows to a stop from a speed of 30.0 km/h in 8.00 s. What is its average acceleration while stopping?

Strategy



In this case, the train is slowing down and its acceleration is negative because it is toward the left. As in the previous example, we must find the change in velocity and the change in time and then solve for acceleration.

Solution

1. Identify the knowns. $v_0 = 30.0\ \text{km/h}$, $v_\text{f} = 0\ \text{km/h}$ (the train is stopped, so its velocity is 0), and $\Delta t = 8.00\ \text{s}$.

2. Solve for the change in velocity, Δv.

Equation:

$$\Delta v = v_\text{f} - v_0 = 0 - 30.0\ \text{km/h} = -30.0\ \text{km/h}$$

3. Plug in the knowns, Δv and Δt, and solve for $\bar{a}$.

Equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-30.0\ \text{km/h}}{8.00\ \text{s}}$$

4. Convert the units to meters and seconds.

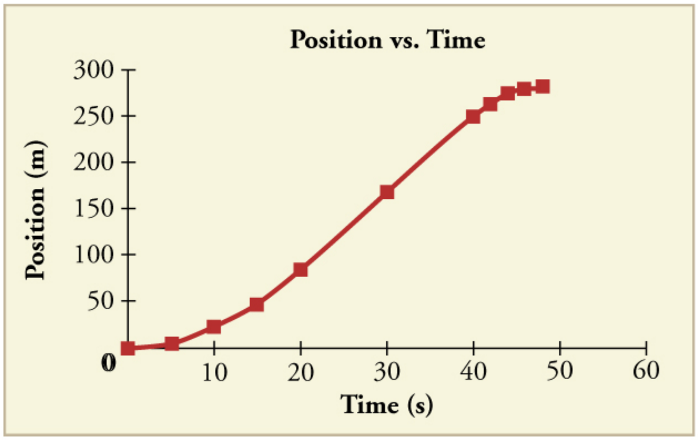
Equation:

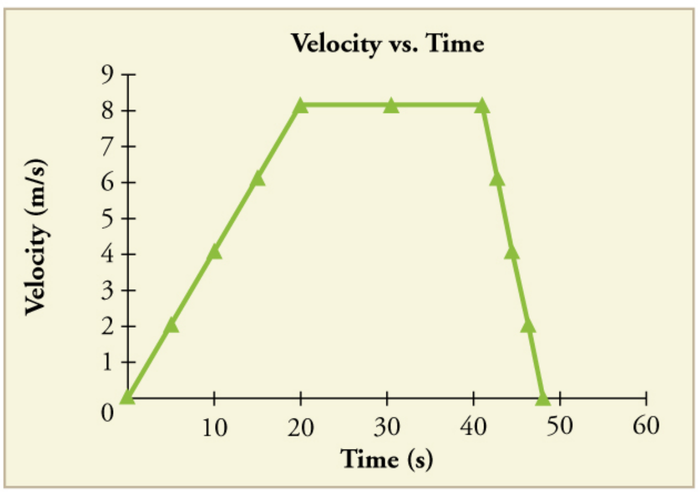
$$\bar{a} = \frac{\Delta v}{\Delta t} = \left(\frac{-30.0\ \text{km/h}}{8.00\ \text{s}}\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right) = -1.04\ \text{m/s}^2.$$

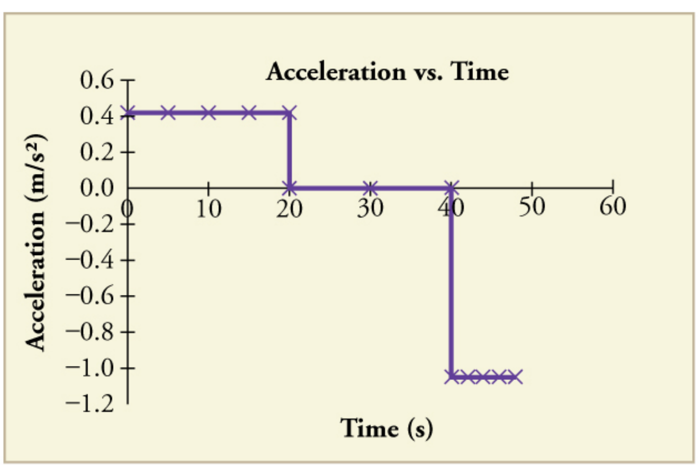
Discussion

The minus sign indicates that acceleration is to the left. This sign is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would oppose the motion.

The graphs of position, velocity, and acceleration vs. time for the trains in [link] and [link] are displayed in [link]. (We have taken the velocity to remain constant from 20 to 40 s, after which the train slows down.)

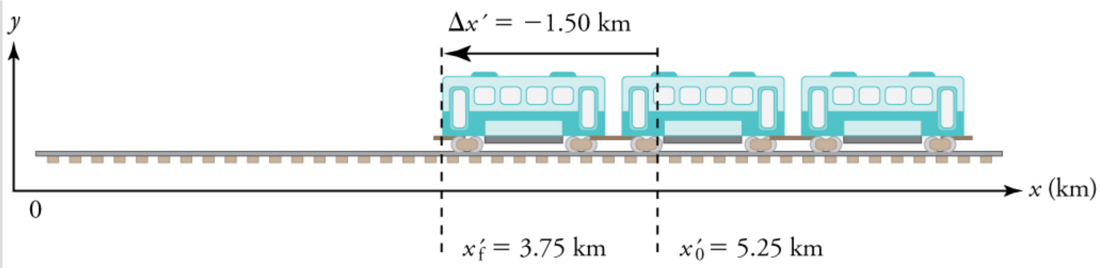






(a) Position of the train over time. Notice that the train's position changes slowly at the beginning of the journey, then more and more quickly as it picks up speed. Its position then changes more slowly as it slows down at the end of the journey. In the middle of the journey, while the velocity remains constant, the position changes at a constant rate. (b) Velocity of the train over time. The train's velocity increases as it accelerates at the beginning of the journey. It remains the same in the middle of the journey (where there is no acceleration). It decreases as the train slows down at the end of the journey. (c) The acceleration of the train over time. The train has positive acceleration as it speeds up at the beginning of the journey. It has no acceleration as it travels at constant velocity in the middle of the journey. Its acceleration is negative as it slows down at the end of the journey.

Example:
Calculating Average Velocity: The Subway Train
What is the average velocity of the train in part b of [link], and shown again below, if it takes 5.00 min to make its trip?



Strategy

Average velocity is displacement divided by time. It will be negative here, since the train moves to the left and has a negative displacement.

Solution

1. Identify the knowns. $x\prime_{\text{f}} = 3.75\text{ km}$, $x\prime_0 = 5.25\text{ km}$, $\Delta t = 5.00\text{ min}$.
2. Determine displacement, $\Delta x\prime$. We found $\Delta x\prime$ to be -1.5 km in [link].
3. Solve for average velocity.

Equation:

$$\bar{v} = \frac{\Delta x\prime}{\Delta t} = \frac{-1.50\text{ km}}{5.00\text{ min}}$$

4. Convert units.

Equation:

$$\bar{v} = \frac{\Delta x\prime}{\Delta t} = \left(\frac{-1.50\text{ km}}{5.00\text{ min}}\right)\left(\frac{60\text{ min}}{1\text{ h}}\right) = -18.0\text{ km/h}$$

Discussion

The negative velocity indicates motion to the left.

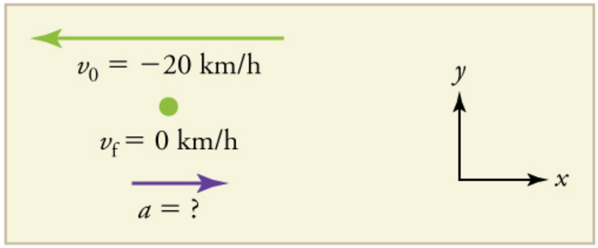
Example:

Calculating Deceleration: The Subway Train

Finally, suppose the train in [link] slows to a stop from a velocity of 20.0 km/h in 10.0 s. What is its average acceleration?

Strategy

Once again, let's draw a sketch:



As before, we must find the change in velocity and the change in time to calculate average acceleration.

Solution

1. Identify the knowns. $v_0 = -20 \text{ km/h}$, $v_f = 0 \text{ km/h}$, $\Delta t = 10.0 \text{ s}$.
2. Calculate Δv. The change in velocity here is actually positive, since

Equation:

$$\Delta v = v_f - v_0 = 0 - (-20 \text{ km/h}) = +20 \text{ km/h}.$$

3. Solve for $\bar{a}$.

Equation:

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{+20.0 \text{ km/h}}{10.0 \text{ s}}$$

4. Convert units.

Equation:

$$\bar{a} = \left(\frac{+20.0 \text{ km/h}}{10.0 \text{ s}}\right)\left(\frac{10^3 \text{ m}}{1 \text{ km}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = +0.556 \text{ m/s}^2$$

Discussion

The plus sign means that acceleration is to the right. This is reasonable because the train initially has a negative velocity (to the left) in this problem and a positive acceleration opposes the motion (and so it is to the right).

Sign and Direction

Perhaps the most important thing to note about these examples is the signs of the answers. In our chosen coordinate system, plus means the quantity is to the right and minus means it is to the left. This is easy to imagine for displacement and velocity. But it is a little less obvious for acceleration. Most people interpret negative acceleration as the slowing of an object. This was not the case in [link], where a positive acceleration slowed a negative velocity. The crucial distinction was that the acceleration was in the opposite direction from the velocity. In fact, a negative acceleration will *increase* a negative velocity. For example, the train moving to the left in [link] is sped up by an acceleration to the left. In that case, both v and a are negative. The plus and minus signs give the directions of the accelerations. If acceleration has the same sign as the change in velocity, the object is speeding up. If acceleration has the opposite sign of the change in velocity, the object is slowing down.

Exercise:

Check Your Understanding

Problem:

An airplane lands on a runway traveling east. Describe its acceleration.

Solution:

If we take east to be positive, then the airplane has negative acceleration, as it is accelerating toward the west. It is also slowing down: its acceleration is opposite in direction to its velocity.

Section Summary

- Acceleration is the rate at which velocity changes. In symbols, **average acceleration** $\bar{a}$ is
  Equation:

  $$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_{\mathrm{f}} - v_0}{t_{\mathrm{f}} - t_0}.$$

- The SI unit for acceleration is $\mathrm{m/s^2}$.

- Acceleration is a vector, and thus has a both a magnitude and direction.
- Acceleration can be caused by either a change in the magnitude or the direction of the velocity.
- Instantaneous acceleration a is the acceleration at a specific instant in time.
- When an acceleration is in a direction opposite to that of the velocity of an object, the object slows down.

Conceptual Questions

Exercise:

Problem:

Is it possible for speed to be constant while acceleration is not zero? Give an example of such a situation.

Exercise:

Problem:

Is it possible for velocity to be constant while acceleration is not zero? Explain.

Exercise:

Problem:

Give an example in which velocity is zero yet acceleration is not.

Exercise:

Problem:

If a subway train is moving to the left (has a negative velocity) and then comes to a stop, what is the direction of its acceleration? Is the acceleration positive or negative?

Exercise:

Problem:

Plus and minus signs are used in one-dimensional motion to indicate direction. What is the sign of an acceleration that reduces the magnitude of a negative velocity? Of a positive velocity?

Glossary

acceleration
: the rate of change in velocity; the change in velocity over time

average acceleration
: the change in velocity divided by the time over which it changes

instantaneous acceleration
: acceleration at a specific point in time

Motion Equations for Constant Acceleration in One Dimension

- Calculate displacement of an object that is not accelerating, given initial position and velocity.
- Calculate final velocity of an accelerating object, given initial velocity, acceleration, and time.
- Calculate displacement and final position of an accelerating object, given initial position, initial velocity, time, and acceleration.

Kinematic equations can help us describe and predict the motion of moving objects such as these kayaks racing in Newbury, England. (credit: Barry Skeates, Flickr)

We might know that the greater the acceleration of, say, a car moving away from a stop sign, the greater the displacement in a given time. But we have not developed a specific equation that relates acceleration and displacement. In this section, we develop some convenient equations for kinematic relationships, starting from the definitions of displacement, velocity, and acceleration already covered.

Notation: *t*, *x*, *v*, *a*

First, let us make some simplifications in notation. Taking the initial time to be zero, as if time is measured with a stopwatch, is a great simplification. Since elapsed time is $\Delta t = t_{\text{f}} - t_0$, taking $t_0 = 0$ means that $\Delta t = t_{\text{f}}$, the final time on the stopwatch. When initial time is taken to be zero, we use the subscript 0 to denote initial values of position and velocity. That is, x_0 *is the initial position* and v_0 *is the initial velocity*. We put no subscripts on the final values. That is, t *is the final time*, x *is the final position*, and v *is the final velocity*. This gives a simpler expression for elapsed time—now, $\Delta t = t$. It also simplifies the expression for displacement, which is now $\Delta x = x - x_0$. Also, it simplifies the expression for change in velocity, which is now $\Delta v = v - v_0$. To summarize, using the simplified notation, with the initial time taken to be zero,

Equation:

$$\left.\begin{array}{ccc}\Delta t & = & t \\ \Delta x & = & x - x_0 \\ \Delta v & = & v - v_0\end{array}\right\}$$

where *the subscript 0 denotes an initial value and the absence of a subscript denotes a final value* in whatever motion is under consideration.

We now make the important assumption that *acceleration is constant*. This assumption allows us to avoid using calculus to find instantaneous acceleration. Since acceleration is constant, the average and instantaneous accelerations are equal. That is,

Equation:

$$\bar{a} = a = \text{constant},$$

so we use the symbol a for acceleration at all times. Assuming acceleration to be constant does not seriously limit the situations we can study nor degrade the accuracy of our treatment. For one thing, acceleration *is* constant in a great number of situations. Furthermore, in many other situations we can accurately describe motion by assuming a constant acceleration equal to the average acceleration for that motion. Finally, in

motions where acceleration changes drastically, such as a car accelerating to top speed and then braking to a stop, the motion can be considered in separate parts, each of which has its own constant acceleration.

Note:
Solving for Displacement (Δx) and Final Position (x) from Average Velocity when Acceleration (a) is Constant
To get our first two new equations, we start with the definition of average velocity:
Equation:

$$\bar{v} = \frac{\Delta x}{\Delta t}.$$

Substituting the simplified notation for Δx and Δt yields
Equation:

$$\bar{v} = \frac{x - x_0}{t}.$$

Solving for x yields
Equation:

$$x = x_0 + \bar{v}t,$$

where the average velocity is
Equation:

$$\bar{v} = \frac{v_0 + v}{2} \text{ (constant } a\text{).}$$

The equation $\bar{v} = \frac{v_0+v}{2}$ reflects the fact that, when acceleration is constant, v is just the simple average of the initial and final velocities. For example, if

you steadily increase your velocity (that is, with constant acceleration) from 30 to 60 km/h, then your average velocity during this steady increase is 45 km/h. Using the equation $\bar{v} = \frac{v_0 + v}{2}$ to check this, we see that

Equation:

$$\bar{v} = \frac{v_0 + v}{2} = \frac{30\text{ km/h} + 60\text{ km/h}}{2} = 45\text{ km/h},$$

which seems reasonable.

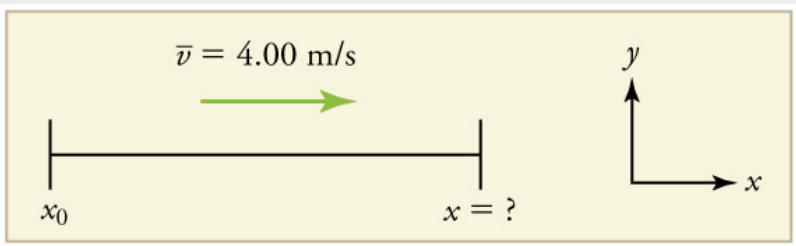
Example:

Calculating Displacement: How Far does the Jogger Run?

A jogger runs down a straight stretch of road with an average velocity of 4.00 m/s for 2.00 min. What is his final position, taking his initial position to be zero?

Strategy

Draw a sketch.



The final position x is given by the equation

Equation:

$$x = x_0 + \bar{v}t.$$

To find x, we identify the values of x_0, $\bar{v}$, and t from the statement of the problem and substitute them into the equation.

Solution

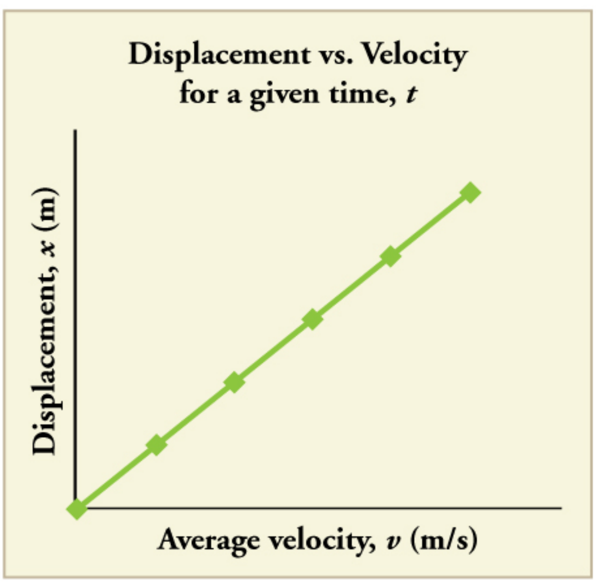
1. Identify the knowns. $\bar{v} = 4.00\text{ m/s}$, $\Delta t = 2.00\text{ min}$, and $x_0 = 0\text{ m}$.
2. Enter the known values into the equation.

Equation:

$$x = x_0 + \bar{v}t = 0 + (4.00\ \text{m/s})(120\ \text{s}) = 480\ \text{m}$$

Discussion
Velocity and final displacement are both positive, which means they are in the same direction.

The equation $x = x_0 + \bar{v}t$ gives insight into the relationship between displacement, average velocity, and time. It shows, for example, that displacement is a linear function of average velocity. (By linear function, we mean that displacement depends on $\bar{v}$ rather than on $\bar{v}$ raised to some other power, such as $\bar{v}^2$. When graphed, linear functions look like straight lines with a constant slope.) On a car trip, for example, we will get twice as far in a given time if we average 90 km/h than if we average 45 km/h.



There is a linear relationship between displacement and average velocity. For a given time t, an object moving twice as fast as another object will

move twice as far as the other object.

Note:

Solving for Final Velocity

We can derive another useful equation by manipulating the definition of acceleration.

Equation:

$$a = \frac{\Delta v}{\Delta t}$$

Substituting the simplified notation for Δv and Δt gives us

Equation:

$$a = \frac{v - v_0}{t} \text{ (constant } a\text{).}$$

Solving for v yields

Equation:

$$v = v_0 + \text{at (constant } a\text{).}$$

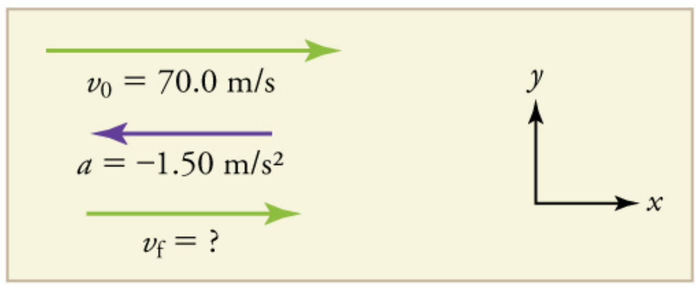
Example:

Calculating Final Velocity: An Airplane Slowing Down after Landing

An airplane lands with an initial velocity of 70.0 m/s and then slows down at 1.50 m/s^2 for 40.0 s. What is its final velocity?

Strategy

Draw a sketch. We draw the acceleration vector in the direction opposite the velocity vector because the plane is slowing down.



Solution

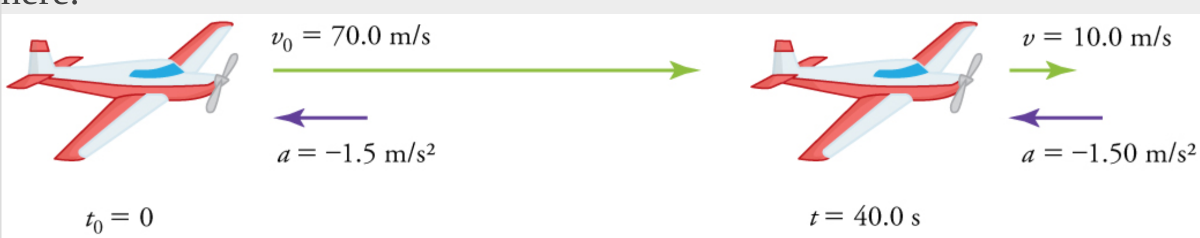
1. Identify the knowns. $v_0 = 70.0 \mathrm{m/s}$, $a = -1.50\ \mathrm{m/s}^2$, $t = 40.0$ s.
2. Identify the unknown. In this case, it is final velocity, v_{f}.
3. Determine which equation to use. We can calculate the final velocity using the equation $v = v_0 + \mathrm{at}$.
4. Plug in the known values and solve.

Equation:

$$v = v_0 + \mathrm{at} = 70.0\ \mathrm{m/s} + \left(-1.50\ \mathrm{m/s}^2\right)(40.0\ \mathrm{s}) = 10.0\ \mathrm{m/s}$$

Discussion

The final velocity is much less than the initial velocity, as desired when slowing down, but still positive. With jet engines, reverse thrust could be maintained long enough to stop the plane and start moving it backward. That would be indicated by a negative final velocity, which is not the case here.



The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note that the acceleration is negative because its direction is opposite to its velocity, which is positive.

In addition to being useful in problem solving, the equation $v = v_0 + at$ gives us insight into the relationships among velocity, acceleration, and time. From it we can see, for example, that

- final velocity depends on how large the acceleration is and how long it lasts
- if the acceleration is zero, then the final velocity equals the initial velocity $(v = v_0)$, as expected (i.e., velocity is constant)
- if a is negative, then the final velocity is less than the initial velocity

All of these observations fit our intuition, and it is always useful to examine basic equations in light of our intuition and experiences to check that they do indeed describe nature accurately.

Note:
Making Connections: Real-World Connection

The Space Shuttle *Endeavor* blasts off from the Kennedy Space Center in February 2010. (credit: Matthew Simantov, Flickr)

An intercontinental ballistic missile (ICBM) has a larger average acceleration than the Space Shuttle and achieves a greater velocity in the

first minute or two of flight (actual ICBM burn times are classified—short-burn-time missiles are more difficult for an enemy to destroy). But the Space Shuttle obtains a greater final velocity, so that it can orbit the earth rather than come directly back down as an ICBM does. The Space Shuttle does this by accelerating for a longer time.

Note:
Solving for Final Position When Velocity is Not Constant ($a \neq 0$)
We can combine the equations above to find a third equation that allows us to calculate the final position of an object experiencing constant acceleration. We start with
Equation:

$$v = v_0 + \text{at}.$$

Adding v_0 to each side of this equation and dividing by 2 gives
Equation:

$$\frac{v_0 + v}{2} = v_0 + \frac{1}{2}\text{at}.$$

Since $\frac{v_0+v}{2} = \bar{v}$ for constant acceleration, then
Equation:

$$\bar{v} = v_0 + \frac{1}{2}\text{at}.$$

Now we substitute this expression for $\bar{v}$ into the equation for displacement, $x = x_0 + \bar{v}t$, yielding
Equation:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 \text{ (constant } a\text{)}.$$

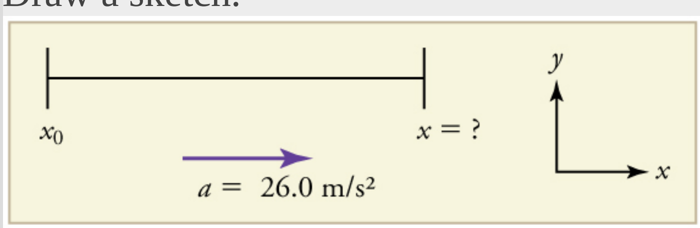
Example:
Calculating Displacement of an Accelerating Object: Dragsters

Dragsters can achieve average accelerations of $26.0\ \text{m/s}^2$. Suppose such a dragster accelerates from rest at this rate for 5.56 s. How far does it travel in this time?

U.S. Army Top Fuel pilot Tony “The Sarge” Schumacher begins a race with a controlled burnout. (credit: Lt. Col. William Thurmond. Photo Courtesy of U.S. Army.)

Strategy

Draw a sketch.



We are asked to find displacement, which is x if we take x_0 to be zero. (Think about it like the starting line of a race. It can be anywhere, but we call it 0 and measure all other positions relative to it.) We can use the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$once we identify v_0, a, and t from the statement of the problem.

Solution

1. Identify the knowns. Starting from rest means that $v_0 = 0$, a is given as 26.0 m/s^2 and t is given as 5.56 s.
2. Plug the known values into the equation to solve for the unknown x:
Equation:

$$x = x_0 + v_0 t + \frac{1}{2}at^2.$$

Since the initial position and velocity are both zero, this simplifies to
Equation:

$$x = \frac{1}{2}at^2.$$

Substituting the identified values of a and t gives
Equation:

$$x = \frac{1}{2}\left(26.0 \text{ m/s}^2\right)(5.56 \text{ s})^2,$$

yielding
Equation:

$$x = 402 \text{ m}.$$

Discussion
If we convert 402 m to miles, we find that the distance covered is very close to one quarter of a mile, the standard distance for drag racing. So the answer is reasonable. This is an impressive displacement in only 5.56 s, but top-notch dragsters can do a quarter mile in even less time than this.

What else can we learn by examining the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$? We see that:

- displacement depends on the square of the elapsed time when acceleration is not zero. In [link], the dragster covers only one fourth of the total distance in the first half of the elapsed time

- if acceleration is zero, then the initial velocity equals average velocity ($v_0 = \bar{v}$) and $x = x_0 + v_0 t + \frac{1}{2}at^2$ becomes $x = x_0 + v_0 t$

Note:

Solving for Final Velocity when Velocity Is Not Constant ($a \neq 0$)

A fourth useful equation can be obtained from another algebraic manipulation of previous equations.

If we solve $v = v_0 + \text{at}$ for t, we get

Equation:

$$t = \frac{v - v_0}{a}.$$

Substituting this and $\bar{v} = \frac{v_0 + v}{2}$ into $x = x_0 + \bar{v}t$, we get

Equation:

$$v^2 = v_0^2 + 2a(x - x_0) \ (\text{constant } a).$$

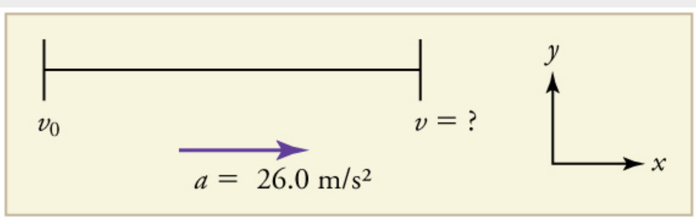
Example:

Calculating Final Velocity: Dragsters

Calculate the final velocity of the dragster in [link] without using information about time.

Strategy

Draw a sketch.



The equation $v^2 = v_0^2 + 2a(x - x_0)$ is ideally suited to this task because it relates velocities, acceleration, and displacement, and no time information is required.

Solution
1. Identify the known values. We know that $v_0 = 0$, since the dragster starts from rest. Then we note that $x - x_0 = 402 \text{ m}$ (this was the answer in [link]). Finally, the average acceleration was given to be $a = 26.0 \text{ m/s}^2$.
2. Plug the knowns into the equation $v^2 = v_0^2 + 2a(x - x_0)$ and solve for v.
Equation:

$$v^2 = 0 + 2\left(26.0 \text{ m/s}^2\right)(402 \text{ m}).$$

Thus
Equation:

$$v^2 = 2.09 \times 10^4 \text{ m}^2/\text{s}^2.$$

To get v, we take the square root:
Equation:

$$v = \sqrt{2.09 \times 10^4 \text{ m}^2/\text{s}^2} = 145 \text{ m/s}.$$

Discussion
145 m/s is about 522 km/h or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values; we took the positive value to indicate a velocity in the same direction as the acceleration.

An examination of the equation $v^2 = v_0^2 + 2a(x - x_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on how large the acceleration is and the distance over which it acts
- For a fixed magnitude of acceleration, a car that is going twice as fast doesn't simply stop in twice the distance—it takes much further to

stop. (This is why we have reduced speed zones near schools.)

Putting Equations Together

In the following examples, we further explore one-dimensional motion, but in situations requiring slightly more algebraic manipulation. The examples also give insight into problem-solving techniques. The box below provides easy reference to the equations needed.

Note:
Summary of Kinematic Equations (constant a)
Equation:

$$x = x_0 + \bar{v}t$$

Equation:

$$\bar{v} = \frac{v_0 + v}{2}$$

Equation:

$$v = v_0 + \text{at}$$

Equation:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Equation:

$$v^2 = v_0^2 + 2a(x - x_0)$$

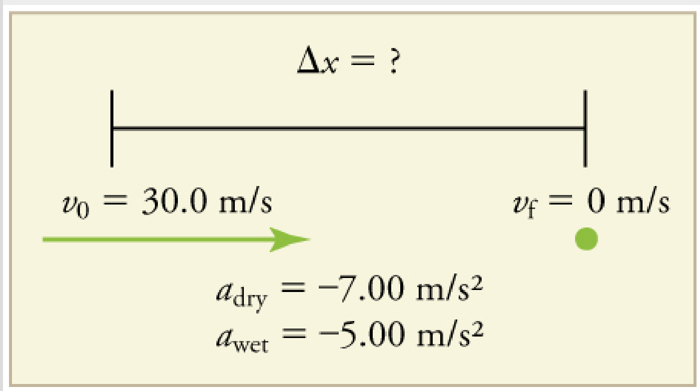
Example:

Calculating Displacement: How Far Does a Car Go When Coming to a Halt?

On dry concrete, suppose a car can slow down at a rate of $7.00\ \text{m/s}^2$, whereas on wet concrete the maximum magnitude of acceleration is only $5.00\ \text{m/s}^2$. Find the distances necessary to stop a car moving at 30.0 m/s (about 110 km/h) (a) on dry concrete and (b) on wet concrete. (c) Repeat both calculations, finding the displacement from the point where the driver sees a traffic light turn red, taking into account his reaction time of 0.500 s to get his foot on the brake.

Strategy

Draw a sketch.



In order to determine which equations are best to use, we need to list all of the known values and identify exactly what we need to solve for. We shall do this explicitly in the next several examples, using tables to set them off.

Solution for (a)

1. Identify the knowns and what we want to solve for. We know that $v_0 = 30.0\ \text{m/s}$; $v = 0$; $a = -7.00\ \text{m/s}^2$ (a is negative because it is in a direction opposite to velocity). We take x_0 to be 0. We are looking for displacement Δx, or $x - x_0$.

2. Identify the equation that will help up solve the problem. The best equation to use is

Equation:

$$v^2 = v_0^2 + 2a(x - x_0).$$

This equation is best because it includes only one unknown, x. We know the values of all the other variables in this equation. (There are other equations that would allow us to solve for x, but they require us to know

the stopping time, t, which we do not know. We could use them but it would entail additional calculations.)
3. Rearrange the equation to solve for x.
Equation:

$$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

4. Enter known values.
Equation:

$$x - 0 = \frac{0^2 - (30.0\ \mathrm{m/s})^2}{2\left(-7.00\ \mathrm{m/s}^2\right)}$$

Thus,
Equation:

$$x = 64.3\ \mathrm{m\ on\ dry\ concrete.}$$

Solution for (b)
This part can be solved in exactly the same manner as Part A. The only difference is that the acceleration is $-5.00\ \mathrm{m/s}^2$. The result is
Equation:

$$x_{\mathrm{wet}} = 90.0\ \mathrm{m\ on\ wet\ concrete.}$$

Solution for (c)
Once the driver reacts, the stopping distance is the same as it is in Parts A and B for dry and wet concrete. So to answer this question, we need to calculate how far the car travels during the reaction time, and then add that to the stopping time. It is reasonable to assume that the velocity remains constant during the driver's reaction time.
1. Identify the knowns and what we want to solve for. We know that $\bar{v} = 30.0\ \mathrm{m/s}$; $t_{\mathrm{reaction}} = 0.500\,\mathrm{s}$; $a_{\mathrm{reaction}} = 0$. We take $x_{0-\mathrm{reaction}}$ to be 0. We are looking for x_{reaction}.
2. Identify the best equation to use.

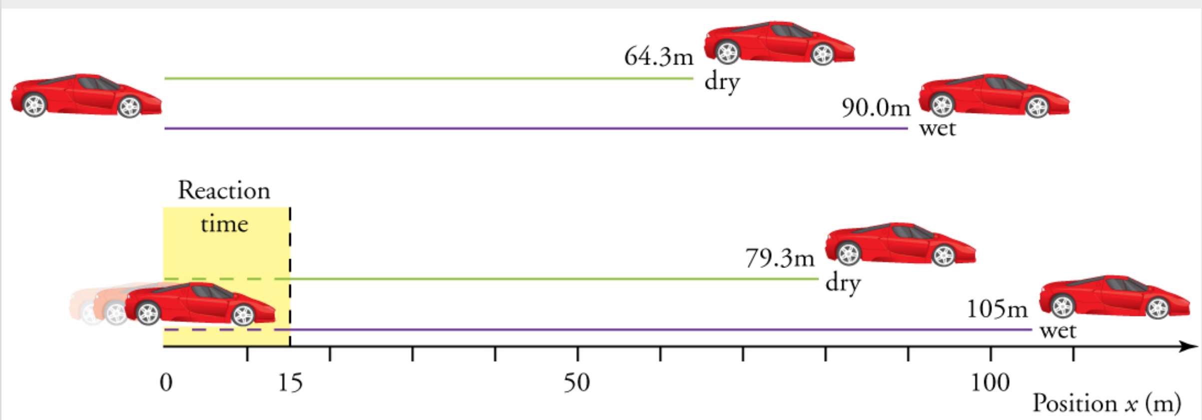
$x = x_0 + \bar{v}t$ works well because the only unknown value is x, which is what we want to solve for.
3. Plug in the knowns to solve the equation.
Equation:

$$x = 0 + (30.0 \text{ m/s})(0.500 \text{ s}) = 15.0 \text{ m}.$$

This means the car travels 15.0 m while the driver reacts, making the total displacements in the two cases of dry and wet concrete 15.0 m greater than if he reacted instantly.
4. Add the displacement during the reaction time to the displacement when braking.
Equation:

$$x_{\text{braking}} + x_{\text{reaction}} = x_{\text{total}}$$

a. 64.3 m + 15.0 m = 79.3 m when dry
b. 90.0 m + 15.0 m = 105 m when wet



The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car initially traveling at 30.0 m/s. Also shown are the total distances traveled from the point where the driver first sees a light turn red, assuming a 0.500 s reaction time.

Discussion

The displacements found in this example seem reasonable for stopping a fast-moving car. It should take longer to stop a car on wet rather than dry pavement. It is interesting that reaction time adds significantly to the displacements. But more important is the general approach to solving problems. We identify the knowns and the quantities to be determined and then find an appropriate equation. There is often more than one way to solve a problem. The various parts of this example can in fact be solved by other methods, but the solutions presented above are the shortest.

With the basics of kinematics established, we can go on to many other interesting examples and applications. In the process of developing kinematics, we have also glimpsed a general approach to problem solving that produces both correct answers and insights into physical relationships.

Note:

Making Connections: Take-Home Experiment—Breaking News

We have been using SI units of meters per second squared to describe some examples of acceleration of cars, runners, and trains. To achieve a better feel for these numbers, one can measure the braking acceleration of a car doing a slow (and safe) stop. Recall that, for average acceleration, $\bar{a} = \Delta v / \Delta t$. While traveling in a car, slowly apply the brakes as you come up to a stop sign. Have a passenger note the initial speed in miles per hour and the time taken (in seconds) to stop. From this, calculate the acceleration in miles per hour per second. Convert this to meters per second squared and compare with other accelerations mentioned in this chapter. Calculate the distance traveled in braking.

Exercise:

Check Your Understanding

Problem:

A manned rocket accelerates at a rate of 20 m/s^2 during launch. How long does it take the rocket reach a velocity of 400 m/s?

Solution:

To answer this, choose an equation that allows you to solve for time t, given only a, v_0, and v.
Equation:

$$v = v_0 + \text{at}$$

Rearrange to solve for t.
Equation:

$$t = \frac{v - v_0}{a} = \frac{400 \text{ m/s} - 0 \text{ m/s}}{20 \text{ m/s}^2} = 20 \text{ s}$$

Section Summary

- To simplify calculations we take acceleration to be constant, so that $\bar{a} = a$ at all times.
- We also take initial time to be zero.
- Initial position and velocity are given a subscript 0; final values have no subscript. Thus,
  Equation:

$$\left.\begin{array}{lcl} \Delta t & = & t \\ \Delta x & = & x - x_0 \\ \Delta v & = & v - v_0 \end{array}\right\}$$

- The following kinematic equations for motion with constant a are useful:

Equation:

$$x = x_0 + \bar{v}t$$

Equation:

$$\bar{v} = \frac{v_0 + v}{2}$$

Equation:

$$v = v_0 + \text{at}$$

Equation:

$$x = x_0 + v_0 t + \frac{1}{2}at^2$$

Equation:

$$v^2 = v_0^2 + 2a(x - x_0)$$

- In vertical motion, y is substituted for x.

Conceptual Questions

Exercise:

Problem:

In a constant-acceleration motion, how does the position change? How does the velocity change?

Exercise:

Problem:

Describe the relationship between the position and the velocity in a constant-acceleration motion.

Exercise:

Problem:

Describe the relationship between the velocity and the acceleration in a constant-acceleration motion.

Exercise:

Problem:

In a constant-acceleration motion starting from rest, how does the distance traveled change with the duration of motion? If the motion continues for double the time (total), how much does the distance traveled increase (total)?

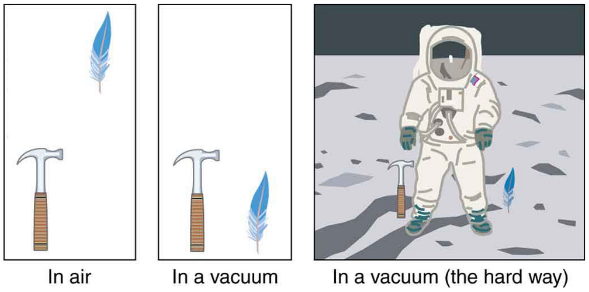
Falling Objects

- Describe the effects of gravity on objects in motion.
- Describe the motion of objects that are in free fall.
- Calculate the position and velocity of objects in free fall.

Falling objects form an interesting class of motion problems. For example, we can estimate the depth of a vertical mine shaft by dropping a rock into it and listening for the rock to hit the bottom. By applying the kinematics developed so far to falling objects, we can examine some interesting situations and learn much about gravity in the process.

Gravity

The most remarkable and unexpected fact about falling objects is that, if air resistance and friction are negligible, then in a given location all objects fall toward the center of Earth with the *same constant acceleration, independent of their mass*. This experimentally determined fact is unexpected, because we are so accustomed to the effects of air resistance and friction that we expect light objects to fall slower than heavy ones.



A hammer and a feather will fall with the same constant acceleration if air resistance is considered negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated on the Moon in 1971, where the

acceleration due to gravity is only 1.67 m/s^2.

In the real world, air resistance can cause a lighter object to fall slower than a heavier object of the same size. A tennis ball will reach the ground after a hard baseball dropped at the same time. (It might be difficult to observe the difference if the height is not large.) Air resistance opposes the motion of an object through the air, while friction between objects—such as between clothes and a laundry chute or between a stone and a pool into which it is dropped—also opposes motion between them. For the ideal situations of these first few chapters, an object *falling without air resistance or friction* is defined to be in **free-fall**.

The force of gravity causes objects to fall toward the center of Earth. The acceleration of free-falling objects is therefore called the **acceleration due to gravity**. The acceleration due to gravity is *constant*, which means we can apply the kinematics equations to any falling object where air resistance and friction are negligible. This opens a broad class of interesting situations to us. The acceleration due to gravity is so important that its magnitude is given its own symbol, *g*. It is constant at any given location on Earth and has the average value

Equation:

$$g = 9.80 \text{ m/s}^2.$$

Although g varies from 9.78 m/s^2 to 9.83 m/s^2, depending on latitude, altitude, underlying geological formations, and local topography, the average value of 9.80 m/s^2 will be used in this text unless otherwise specified. The direction of the acceleration due to gravity is *downward (towards the center of Earth)*. In fact, its direction *defines* what we call vertical. Note that whether the acceleration a in the kinematic equations has the value $+g$ or $-g$ depends on how we define our coordinate system. If we define the upward direction as positive, then $a = -g = -9.80 \text{ m/s}^2$, and if we define the downward direction as positive, then $a = g = 9.80 \text{ m/s}^2$. In certain examples and

homework exercises, slightly inaccurate but more convenient value of $g \simeq 10 \text{ m/s}^2$ will be used with appropriate specifications.

One-Dimensional Motion Involving Gravity

The best way to see the basic features of motion involving gravity is to start with the simplest situations and then progress toward more complex ones. So we start by considering straight up and down motion with no air resistance or friction. These assumptions mean that the velocity (if there is any) is vertical. If the object is dropped, we know the initial velocity is zero. Once the object has left contact with whatever held or threw it, the object is in free-fall. Under these circumstances, the motion is one-dimensional and has constant acceleration of magnitude g. We will also represent vertical displacement with the symbol y and use x for horizontal displacement.

Note:
Kinematic Equations for Objects in Free-Fall
Equation:

$$v = v_0 - \text{gt}$$

Equation:

$$y = y_0 + v_0 t - \frac{1}{2} g t^2$$

Equation:

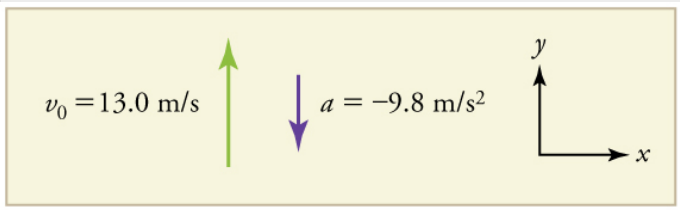
$$v^2 = v_0^2 - 2g(y - y_0)$$

Example:
Calculating Position and Velocity of a Falling Object: A Rock Thrown Upward

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity of 13.0 m/s. The rock misses the edge of the cliff as it falls back to earth. Calculate the position and velocity of the rock 1.00 s, 2.00 s, and 3.00 s after it is thrown, neglecting the effects of air resistance.

Strategy

Draw a sketch.



We are asked to determine the position y at various times. It is reasonable to take the initial position y_0 to be zero. This problem involves one-dimensional motion in the vertical direction. We use plus and minus signs to indicate direction, with up being positive and down negative. Since up is positive, and the rock is thrown upward, the initial velocity must be positive too. The acceleration due to gravity is downward, so a is negative. It is crucial that the initial velocity and the acceleration due to gravity have opposite signs. Opposite signs indicate that the acceleration due to gravity opposes the initial motion and will slow and eventually reverse it.

Since we are asked for values of position and velocity at three times, we will refer to these as y_1 and v_1; y_2 and v_2; and y_3 and v_3.

Solution for Position y_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\ \text{m/s}$; $a = -g = -9.80\ \text{m/s}^2$; and $t = 1.00\ \text{s}$.
2. Identify the best equation to use. We will use $y = y_0 + v_0 t + \frac{1}{2}at^2$ because it includes only one unknown, y (or y_1, here), which is the value we want to find.
3. Plug in the known values and solve for y_1.

Equation:

$$y_1 = 0 + (13.0\ \text{m/s})(1.00\ \text{s}) + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)(1.00\ \text{s})^2 = 8.10\ \text{m}$$

Discussion

The rock is 8.10 m above its starting point at $t = 1.00$ s, since $y_1 > y_0$. It could be *moving* up or down; the only way to tell is to calculate v_1 and find out if it is positive or negative.

Solution for Velocity v_1

1. Identify the knowns. We know that $y_0 = 0$; $v_0 = 13.0\ \text{m/s}$; $a = -g = -9.80\ \text{m/s}^2$; and $t = 1.00\ \text{s}$. We also know from the solution above that $y_1 = 8.10\ \text{m}$.
2. Identify the best equation to use. The most straightforward is $v = v_0 - \text{gt}$ (from $v = v_0 + \text{at}$, where $a = \text{gravitational acceleration} = -g$).
3. Plug in the knowns and solve.

Equation:

$$v_1 = v_0 - \text{gt} = 13.0\ \text{m/s} - \left(9.80\ \text{m/s}^2\right)(1.00\ \text{s}) = 3.20\ \text{m/s}$$

Discussion

The positive value for v_1 means that the rock is still heading upward at $t = 1.00\ \text{s}$. However, it has slowed from its original 13.0 m/s, as expected.

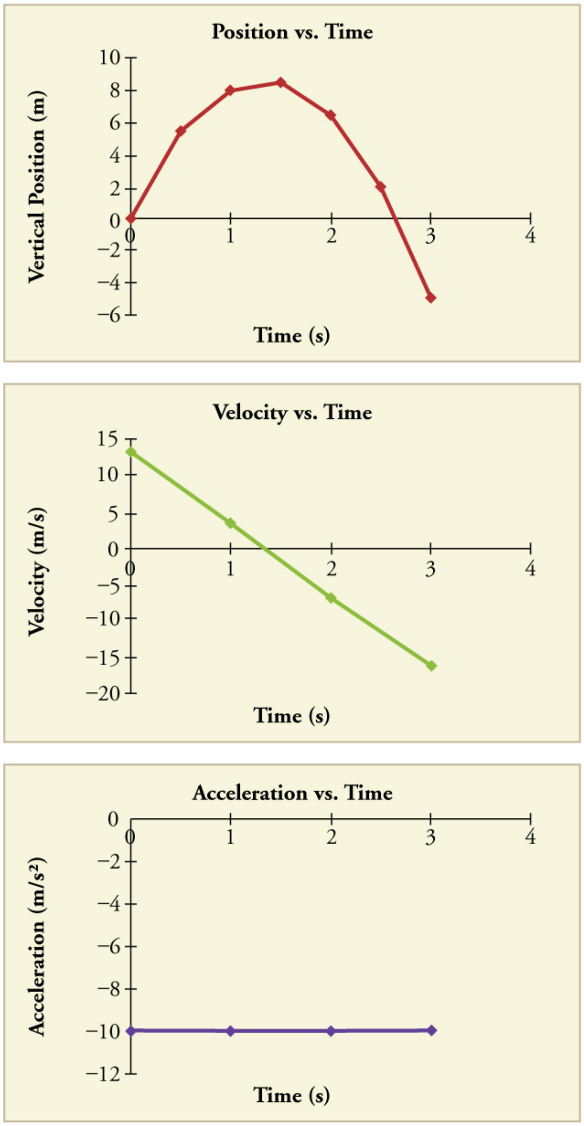
Solution for Remaining Times

The procedures for calculating the position and velocity at $t = 2.00\ \text{s}$ and $3.00\ \text{s}$ are the same as those above. The results are summarized in [link] and illustrated in [link].

Time, *t*	**Position, *y***	**Velocity, *v***	**Acceleration, *a***
$1.00\ \text{s}$	$8.10\ \text{m}$	$3.20\ \text{m/s}$	$-9.80\ \text{m/s}^2$
$2.00\ \text{s}$	$6.40\ \text{m}$	$-6.60\ \text{m/s}$	$-9.80\ \text{m/s}^2$
$3.00\ \text{s}$	$-5.10\ \text{m}$	$-16.4\ \text{m/s}$	$-9.80\ \text{m/s}^2$

Results

Graphing the data helps us understand it more clearly.



Vertical position, vertical velocity, and vertical acceleration vs. time for a rock thrown vertically up at the edge of a cliff. Notice that velocity changes linearly with time and that acceleration is constant. *Misconception Alert!* Notice that the position vs. time graph shows vertical position only. It is easy to get the impression that the graph shows some

horizontal motion—the shape of the graph looks like the path of a projectile. But this is not the case; the horizontal axis is *time*, not space. The actual path of the rock in space is straight up, and straight down.

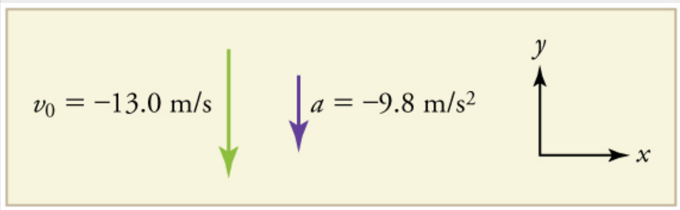
Discussion

The interpretation of these results is important. At 1.00 s the rock is above its starting point and heading upward, since y_1 and v_1 are both positive. At 2.00 s, the rock is still above its starting point, but the negative velocity means it is moving downward. At 3.00 s, both y_3 and v_3 are negative, meaning the rock is below its starting point and continuing to move downward. Notice that when the rock is at its highest point (at 1.5 s), its velocity is zero, but its acceleration is still $-9.80\ \text{m/s}^2$. Its acceleration is $-9.80\ \text{m/s}^2$ for the whole trip—while it is moving up and while it is moving down. Note that the values for y are the positions (or displacements) of the rock, not the total distances traveled. Finally, note that free-fall applies to upward motion as well as downward. Both have the same acceleration—the acceleration due to gravity, which remains constant the entire time. Astronauts training in the famous Vomit Comet, for example, experience free-fall while arcing up as well as down, as we will discuss in more detail later.

Note:

Making Connections: Take-Home Experiment—Reaction Time

A simple experiment can be done to determine your reaction time. Have a friend hold a ruler between your thumb and index finger, separated by about 1 cm. Note the mark on the ruler that is right between your fingers. Have your friend drop the ruler unexpectedly, and try to catch it between your two fingers. Note the new reading on the ruler. Assuming acceleration is that due to gravity, calculate your reaction time. How far would you travel in a car (moving at 30 m/s) if the time it took your foot to go from the gas pedal to the brake was twice this reaction time?

Example:
Calculating Velocity of a Falling Object: A Rock Thrown Down
What happens if the person on the cliff throws the rock straight down, instead of straight up? To explore this question, calculate the velocity of the rock when it is 5.10 m below the starting point, and has been thrown downward with an initial speed of 13.0 m/s.
Strategy
Draw a sketch.


Since up is positive, the final position of the rock will be negative because it finishes below the starting point at $y_0 = 0$. Similarly, the initial velocity is downward and therefore negative, as is the acceleration due to gravity. We expect the final velocity to be negative since the rock will continue to move downward.
Solution
1. Identify the knowns. $y_0 = 0$; $y_1 = -5.10\text{ m}$; $v_0 = -13.0\text{ m/s}$; $a = -g = -9.80\text{ m/s}^2$.
2. Choose the kinematic equation that makes it easiest to solve the problem. The equation $v^2 = v_0^2 + 2a(y - y_0)$ works well because the only unknown in it is v. (We will plug y_1 in for y.)
3. Enter the known values
Equation:

$$v^2 = (-13.0\text{ m/s})^2 + 2\left(-9.80\text{ m/s}^2\right)(-5.10\text{ m} - 0\text{ m}) = 268.96\text{ m}^2/\text{s}^2,$$

where we have retained extra significant figures because this is an intermediate result.
Taking the square root, and noting that a square root can be positive or negative, gives
Equation:

$$v = \pm 16.4\text{ m/s}.$$

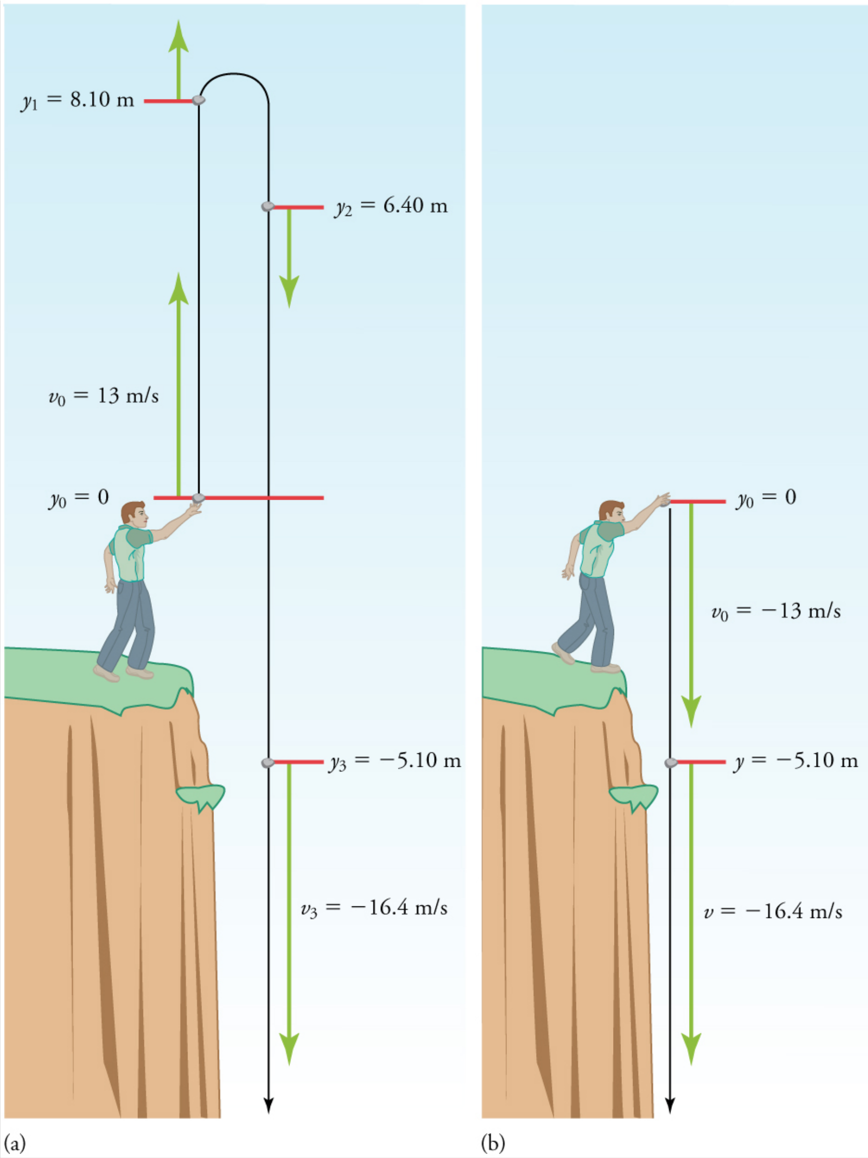
The negative root is chosen to indicate that the rock is still heading down. Thus,

Equation:

$$v = -16.4 \text{ m/s}.$$

Discussion

Note that *this is exactly the same velocity the rock had at this position when it was thrown straight upward with the same initial speed*. (See [link] and [link](a).) This is not a coincidental result. Because we only consider the acceleration due to gravity in this problem, the *speed* of a falling object depends only on its initial speed and its vertical position relative to the starting point. For example, if the velocity of the rock is calculated at a height of 8.10 m above the starting point (using the method from [link]) when the initial velocity is 13.0 m/s straight up, a result of $\pm 3.20 \text{ m/s}$ is obtained. Here both signs are meaningful; the positive value occurs when the rock is at 8.10 m and heading up, and the negative value occurs when the rock is at 8.10 m and heading back down. It has the same *speed* but the opposite direction.



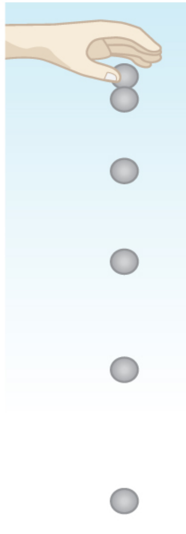
(a) A person throws a rock straight up, as explored in [link]. The arrows are velocity vectors at 0, 1.00, 2.00, and 3.00 s. (b) A person throws a rock straight down from a cliff with the same initial speed as before, as in [link]. Note that at the same distance below the point of release, the rock has the same velocity in both cases.

Another way to look at it is this: In [link], the rock is thrown up with an initial velocity of 13.0 m/s. It rises and then falls back down. When its

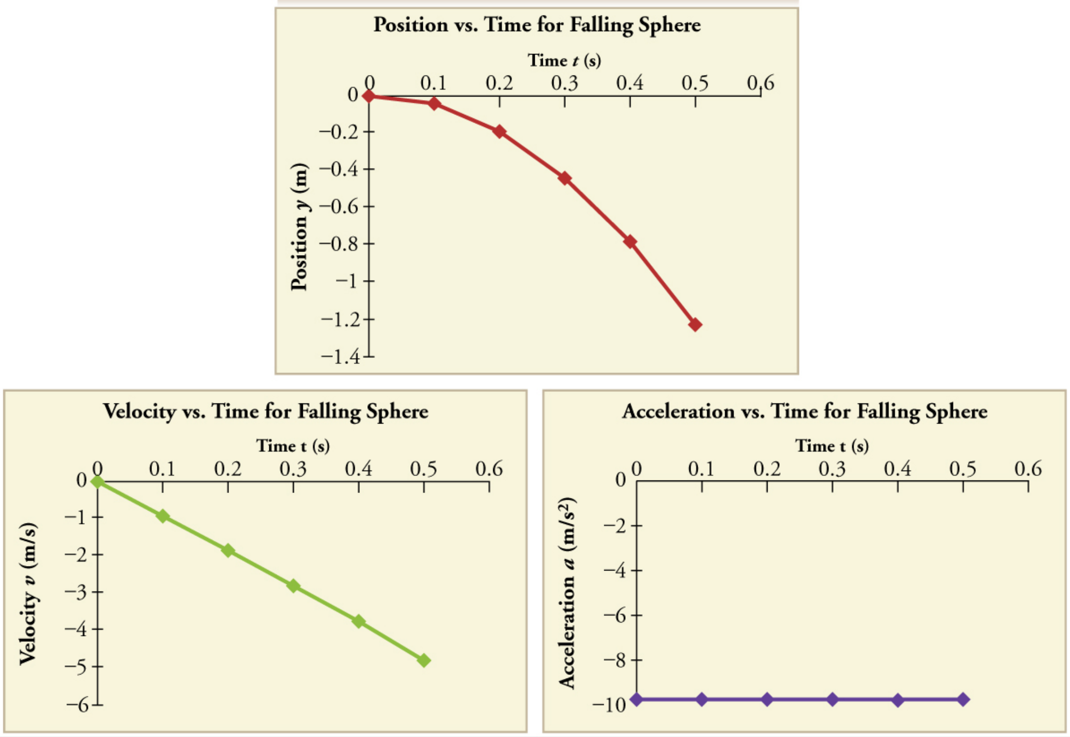
position is $y = 0$ on its way back down, its velocity is $-13.0\ \text{m/s}$. That is, it has the same speed on its way down as on its way up. We would then expect its velocity at a position of $y = -5.10\ \text{m}$ to be the same whether we have thrown it upwards at $+13.0\ \text{m/s}$ or thrown it downwards at $-13.0\ \text{m/s}$. The velocity of the rock on its way down from $y = 0$ is the same whether we have thrown it up or down to start with, as long as the speed with which it was initially thrown is the same.

Example:
Find *g* from Data on a Falling Object

The acceleration due to gravity on Earth differs slightly from place to place, depending on topography (e.g., whether you are on a hill or in a valley) and subsurface geology (whether there is dense rock like iron ore as opposed to light rock like salt beneath you.) The precise acceleration due to gravity can be calculated from data taken in an introductory physics laboratory course. An object, usually a metal ball for which air resistance is negligible, is dropped and the time it takes to fall a known distance is measured. See, for example, [link]. Very precise results can be produced with this method if sufficient care is taken in measuring the distance fallen and the elapsed time.



y (m)	v (m/s)	t (s)
0	0	0
−0.049	−0.98	0.1
−0.196	−1.96	0.2
−0.441	−2.94	0.3
−0.784	−3.92	0.4
−1.225	−4.90	0.5

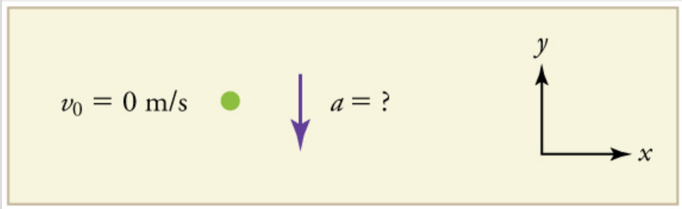


Positions and velocities of a metal ball released from rest when air resistance is negligible. Velocity is seen to increase linearly with time while displacement increases with time squared. Acceleration is a constant and is equal to gravitational acceleration.

Suppose the ball falls 1.0000 m in 0.45173 s. Assuming the ball is not affected by air resistance, what is the precise acceleration due to gravity at this location?

Strategy

Draw a sketch.



We need to solve for acceleration a. Note that in this case, displacement is downward and therefore negative, as is acceleration.

Solution

1. Identify the knowns. $y_0 = 0$; $y = –1.0000$ m; $t = 0.45173$; $v_0 = 0$.
2. Choose the equation that allows you to solve for a using the known values.

Equation:

$$y = y_0 + v_0 t + \frac{1}{2}at^2$$

3. Substitute 0 for v_0 and rearrange the equation to solve for a. Substituting 0 for v_0 yields

Equation:

$$y = y_0 + \frac{1}{2}at^2.$$

Solving for a gives

Equation:

$$a = \frac{2(y - y_0)}{t^2}.$$

4. Substitute known values yields

Equation:

$$a = \frac{2(-1.0000 \text{ m} - 0)}{(0.45173 \text{ s})^2} = -9.8010 \text{ m/s}^2,$$

so, because $a = -g$ with the directions we have chosen,
Equation:

$$g = 9.8010\ \text{m/s}^2.$$

Discussion
The negative value for a indicates that the gravitational acceleration is downward, as expected. We expect the value to be somewhere around the average value of $9.80\ \text{m/s}^2$, so $9.8010\ \text{m/s}^2$ makes sense. Since the data going into the calculation are relatively precise, this value for g is more precise than the average value of $9.80\ \text{m/s}^2$; it represents the local value for the acceleration due to gravity.

Exercise:
Check Your Understanding

Problem:

A chunk of ice breaks off a glacier and falls 30.0 meters before it hits the water. Assuming it falls freely (there is no air resistance), how long does it take to hit the water?

Solution:

We know that initial position $y_0 = 0$, final position $y = -30.0\ \text{m}$, and $a = -g = -9.80\ \text{m/s}^2$. We can then use the equation $y = y_0 + v_0 t + \frac{1}{2}at^2$ to solve for t. Inserting $a = -g$, we obtain
Equation:

$$\begin{aligned} y &= 0 + 0 - \tfrac{1}{2}gt^2 \\ t^2 &= \tfrac{2y}{-g} \\ t &= \pm\sqrt{\tfrac{2y}{-g}} = \pm\sqrt{\tfrac{2(-30.0\ \text{m})}{-9.80\ \text{m/s}^2}} = \pm\sqrt{6.12\ \text{s}^2} = 2.47\ \text{s} \approx 2.5\ \text{s} \end{aligned}$$

where we take the positive value as the physically relevant answer. Thus, it takes about 2.5 seconds for the piece of ice to hit the water.

Section Summary

- An object in free-fall experiences constant acceleration if air resistance is negligible.
- On Earth, all free-falling objects have an acceleration due to gravity g, which averages
  Equation:
  $$g = 9.80\ \text{m/s}^2.$$
- Whether the acceleration a should be taken as $+g$ or $-g$ is determined by your choice of coordinate system. If you choose the upward direction as positive, $a = -g = -9.80\ \text{m/s}^2$ is negative. In the opposite case, $a = +\text{g} = 9.80\ \text{m/s}^2$ is positive. Since acceleration is constant, the kinematic equations above can be applied with the appropriate $+g$ or $-g$ substituted for a.
- For objects in free-fall, up is normally taken as positive for displacement, velocity, and acceleration.

Conceptual Questions

Exercise:

Problem:

What is the acceleration of a rock thrown straight upward on the way up? At the top of its flight? On the way down?

Exercise:

Problem:

An object that is thrown straight up falls back to Earth. This is one-dimensional motion. (a) When is its velocity zero? (b) Does its velocity change direction? (c) Does the acceleration due to gravity have the same sign on the way up as on the way down?

Exercise:

Problem:

Suppose you throw a rock nearly straight up at a coconut in a palm tree, and the rock misses on the way up but hits the coconut on the way down. Neglecting air resistance, how does the speed of the rock when it hits the coconut on the way down compare with what it would have been if it had hit the coconut on the way up? Is it more likely to dislodge the coconut on the way up or down? Explain.

Exercise:

Problem:

If an object is thrown straight up and air resistance is negligible, then its speed when it returns to the starting point is the same as when it was released. If air resistance were not negligible, how would its speed upon return compare with its initial speed? How would the maximum height to which it rises be affected?

Exercise:

Problem:

The severity of a fall depends on your speed when you strike the ground. All factors but the acceleration due to gravity being the same, how many times higher could a safe fall on the Moon be than on Earth (gravitational acceleration on the Moon is about 1/6 that of the Earth)?

Exercise:

Problem:

How many times higher could an astronaut jump on the Moon than on Earth if his takeoff speed is the same in both locations (gravitational acceleration on the Moon is about 1/6 of g on Earth)?

Glossary

free-fall

the state of movement that results from gravitational force only

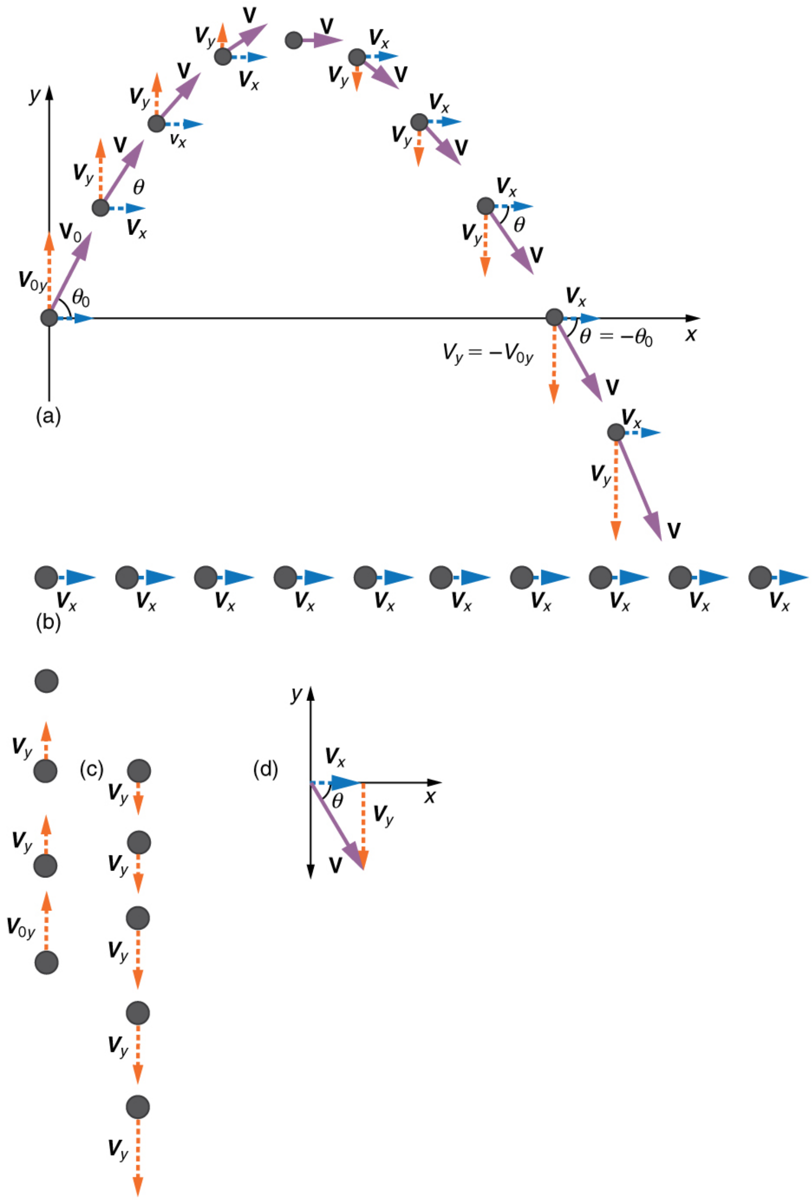
acceleration due to gravity
: acceleration an object undergoes while in free-fall

Projectile Motion

- Identify and explain the properties of a projectile, such as acceleration due to gravity, range, and trajectory.
- Apply the principle of independence of motion to solve projectile motion problems.

Projectile motion is the **motion** of an object thrown or projected into the air, subject to only the acceleration of gravity. The object is called a **projectile**, and its path is called its **trajectory**. The motion of falling objects is a simple one-dimensional type of projectile motion in which there is no horizontal movement. In this section, we consider two-dimensional projectile motion, such as that of a football or other object for which **air resistance** *is negligible*.

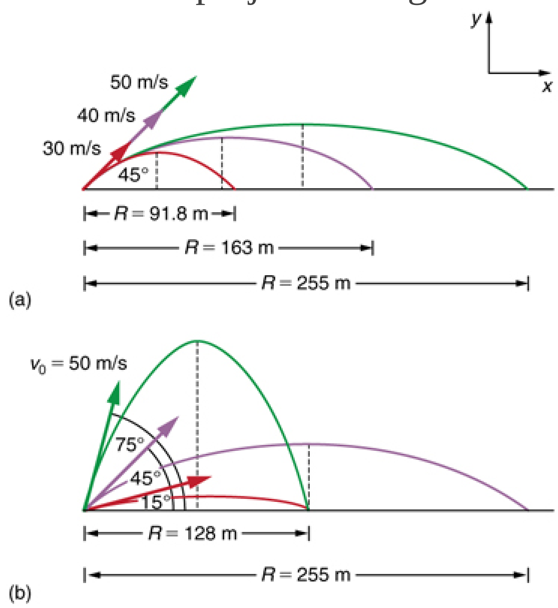
The most important fact to know here is that *motions along perpendicular axes are independent* and thus can be considered separately. In particular, it is often convenient to consider vertical and horizontal motions separately, or independently. The key to analyzing two-dimensional projectile motion is to break it into two motions, one along the horizontal axis and the other along the vertical. And we will assume all forces except gravity are negligible (that is, we ignore forces due to air resistance, wind, friction, etc.). The vertical component of acceleration is, then, $a_y = -g = -9.80 \text{ m/s}^2$, where we take upward direction as the positive y direction. And because there is no force in the horizontal direction, $a_x = 0$. [link] illustrates this approach to analyzing projectile motion.



(a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $a_x = 0$ and v_x is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in

magnitude. This is exactly same as in the case of falling objects (or objects thrown directly upward). (d) The x - and y -motions can be recombined to give the total velocity at any given point on the trajectory.

One of the conceptual aspects of projectile motion we can discuss without a detailed analysis is the range. On level ground, we define **range** to be the horizontal distance R traveled by a projectile. The range of projectiles describes everyday phenomena such as how far a football can be thrown or kicked. Also, investigating the range of projectiles can shed light on other interesting phenomena, such as the orbits of satellites around the Earth. Let us consider projectile range further.



Trajectories of projectiles on level ground. (a) The greater the initial speed v_0, the greater the range for a given initial angle. (b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that

the range is the same for 15° and 75°, although the maximum heights of those paths are different.

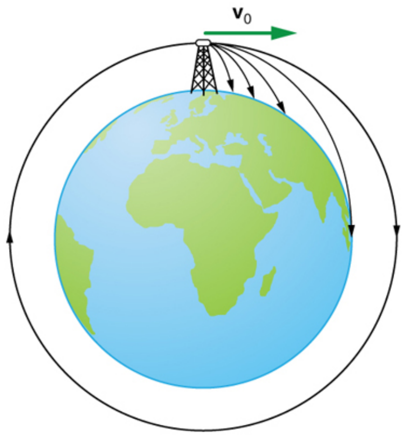
How does the initial velocity of a projectile affect its range? It is intuitive to guess that, the greater the initial speed v_0, the greater the range, as shown in [link](a). The initial angle θ_0 also has a dramatic effect on the range, as illustrated in [link](b). For a fixed initial speed, such as might be produced by a cannon, the maximum range is obtained with $\theta_0 = 45^{\circ}$. Interestingly, for every initial angle except 45°, there are two angles that give the same range. The range also depends on the value of the acceleration of gravity g. The lunar astronaut Alan Shepherd was able to drive a golf ball a great distance on the Moon because gravity is weaker there. The range R of a projectile on *level ground* for which air resistance is negligible is given by

Equation:

$$R = \frac{v_0^2 \sin 2\theta_0}{g},$$

where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. This formula (which can be derived using algebra and trigonometry) fits the major features of projectile range as described.

When we speak of the range of a projectile on level ground, we assume that R is very small compared with the circumference of the Earth. If, however, the range is large, the Earth curves away below the projectile and acceleration of gravity changes direction along the path. The range is larger than predicted by the range equation given above because the projectile has farther to fall than it would on level ground. (See [link].) If the initial speed is great enough, the projectile goes into orbit. This possibility was recognized centuries before it could be accomplished. When an object is in orbit, the Earth curves away from underneath the object at the same rate as it falls. The object thus falls continuously but never hits the surface.



Projectile to satellite. In each case shown here, a projectile is launched from a very high tower to avoid air resistance. With increasing initial speed, the range increases and becomes longer than it would be on level ground because the Earth curves away underneath its path. With a large enough initial speed, orbit is achieved.

Summary

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- The most important fact regarding projectile motion is that motions along vertical direction and the horizontal direction are independent.
- The maximum horizontal distance traveled by a projectile is called the **range**. The range R of a projectile on level ground launched at an angle θ_0 above the horizontal with initial speed v_0 is given by **Equation:**

$$R = \frac{v_0^2 \sin 2\theta_0}{g}.$$

Conceptual Questions

Exercise:

Problem:

Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0º nor 90º): (a) Is the velocity ever zero? (b) When is the velocity a minimum? A maximum? (c) Can the velocity ever be the same as the initial velocity at a time other than at $t = 0$? (d) Can the speed ever be the same as the initial speed at a time other than at $t = 0$?

Exercise:

Problem:

Answer the following questions for projectile motion on level ground assuming negligible air resistance (the initial angle being neither 0º nor 90º): (a) Is the acceleration ever zero? (b) Is the acceleration ever in the same direction as a component of velocity? (c) Is the acceleration ever opposite in direction to a component of velocity?

Exercise:

Problem:

For a fixed initial speed, the range of a projectile is determined by the angle at which it is fired. For all but the maximum, there are two angles that give the same range. Considering factors that might affect the ability of an archer to hit a target, such as wind, explain why the smaller angle (closer to the horizontal) is preferable. When would it be necessary for the archer to use the larger angle? Why does the punter in a football game use the higher trajectory?

Exercise:

Problem:

During a lecture demonstration, a professor places two coins on the edge of a table. She then flicks one of the coins horizontally off the table, simultaneously nudging the other over the edge. Describe the subsequent motion of the two coins, in particular discussing whether they hit the floor at the same time.

Glossary

air resistance
: a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

motion
: displacement of an object as a function of time

projectile
: an object that travels through the air and experiences only acceleration due to gravity

projectile motion
: the motion of an object that is subject only to the acceleration of gravity

range
: the maximum horizontal distance that a projectile travels

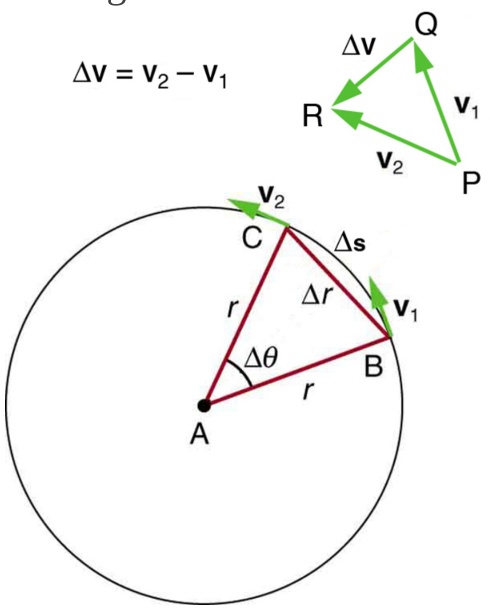
trajectory
: the path of a projectile through the air

Centripetal Acceleration

- Explain what centripetal acceleration is.
- Use the formula for centripetal acceleration in simple situations.

We defined acceleration as a change in velocity, either in its magnitude or in its direction, or both. When an object moves along a circular path, the direction of its velocity changes constantly, so there is always an associated acceleration, even if the speed of the object is constant. You experience this acceleration yourself when you turn a corner in your car. What you notice is a sideways acceleration because you and the car are changing direction. The sharper the curve and the greater your speed, the more noticeable this acceleration will become. In this section we briefly examine the direction and magnitude of that acceleration.

[link] shows an object moving in a circular path at constant speed, called **uniform circular motion**. The direction of the instantaneous velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity, which points directly toward the center of rotation (the center of the circular path). This pointing is shown with the vector diagram in the figure. We call the acceleration of an object moving in uniform circular motion (resulting from a net external force) the **centripetal acceleration**(a_c); centripetal means “toward the center” or “center seeking.”



The directions of the velocity of an object at two different points are shown, and the change in velocity $\Delta \mathbf{v}$ is seen to point directly toward the center of curvature. (See small inset.) Because $\mathbf{a}_{\text{c}} = \Delta \mathbf{v} / \Delta t$, the acceleration is also toward the center; $\mathbf{a}_c$ is called centripetal acceleration. (For small time differences, $\Delta \theta$ is very small, and the arc length Δs is approximately equal to the chord length Δr.)

The direction of centripetal acceleration is toward the center of curvature, but what is its magnitude? If we use the geometry shown in [link] along with some kinematics equations, we can obtain (detailed derivation skipped)

Equation:

$$a_{\text{c}} = \frac{v^2}{r},$$

which is the acceleration of an object in a circle of radius r at a speed v. Verify for yourself that a_{c} has unit of m/s^2, as expected for acceleration.

We see in [link] that centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you have noticed when driving a car. But it is a bit surprising that a_{c} is proportional to speed squared, implying, for

example, that it is four times as hard to take a curve at 100 km/h than at 50 km/h. A sharp corner has a small radius, so that a_{c} is greater for tighter turns, as you have probably noticed.

Section Summary

- Centripetal acceleration a_{c} is the acceleration experienced while in uniform circular motion. It always points toward the center of rotation. It is perpendicular to the linear velocity v and has the magnitude **Equation:**

$$a_{\text{c}} = \frac{v^2}{r}.$$

- The unit of centripetal acceleration is m/s^2.

Conceptual Questions

Exercise:

Problem:

Give examples of centripetal acceleration from everyday experience.

Exercise:

Problem:

A satellite in a circular orbit around the Earth is undergoing a centripetal acceleration. Explain how this description is consistent with the description that the satellite in free-fall.

Exercise:

Problem:

Can centripetal acceleration change the speed of circular motion? Explain.

Glossary

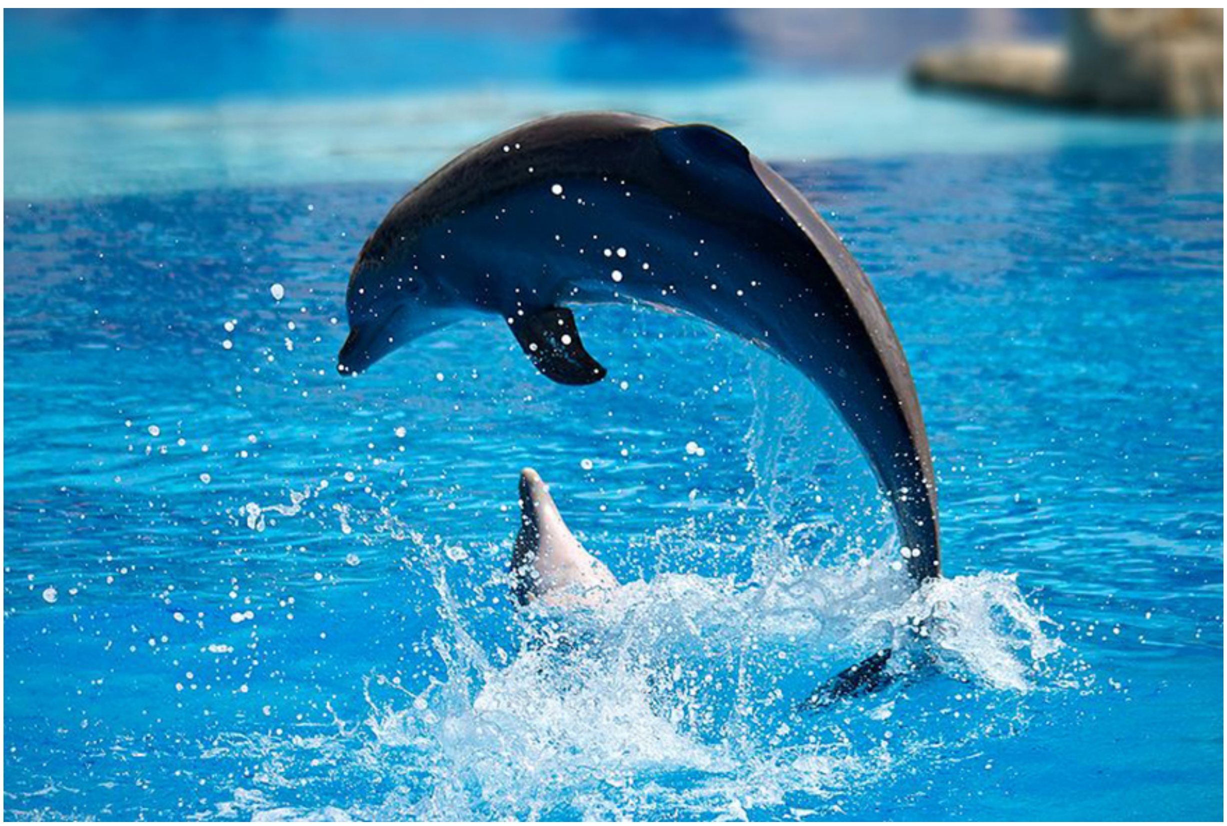
centripetal acceleration
: the acceleration of an object moving in a circle, directed toward the center

ultracentrifuge
: a centrifuge optimized for spinning a rotor at very high speeds

uniform circular motion
: an object moving in a circular path at constant speed

# Introduction to Dynamics: Newton’s Laws of Motion
class="introduction"

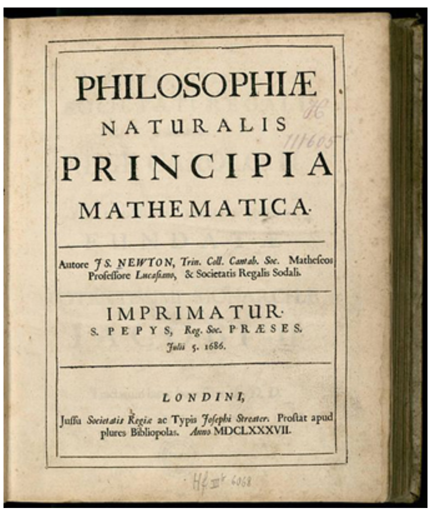
Newton’s laws of motion describe the motion of the dolphin’s path. (credit: Jin Jang)



Motion draws our attention. Motion itself can be beautiful, causing us to marvel at the forces needed to achieve spectacular motion, such as that of a

dolphin jumping out of the water, or a pole vaulter, or the flight of a bird, or the orbit of a satellite. The study of motion is kinematics, but kinematics only *describes* the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws provide an example of the breadth and simplicity of principles under which nature functions. They are also universal laws in that they apply to similar situations on Earth as well as in space.

Issac Newton's (1642–1727) laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance into the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. For many centuries natural philosophers had debated the nature of the universe based largely on certain rules of logic with great weight given to the thoughts of earlier classical philosophers such as Aristotle (384–322 BC). Among the many great thinkers who contributed to this change were Newton and Galileo.


PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore JS. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

Issac Newton's monumental work, *Philosophiae Naturalis Principia Mathematica*, was published in 1687. It proposed scientific

laws that are still used today to describe the motion of objects. (credit: Service commun de la documentation de l'Université de Strasbourg)

Galileo was instrumental in establishing *observation* as the absolute determinant of truth, rather than “logical” argument. Galileo’s use of the telescope was his most notable achievement in demonstrating the importance of observation. He discovered moons orbiting Jupiter and made other observations that were inconsistent with certain ancient ideas and religious dogma. For this reason, and because of the manner in which he dealt with those in authority, Galileo was tried by the Inquisition and punished. He spent the final years of his life under a form of house arrest. Because others before Galileo had also made discoveries by *observing* the nature of the universe, and because repeated observations verified those of Galileo, his work could not be suppressed or denied. After his death, his work was verified by others, and his ideas were eventually accepted by the church and scientific communities.

Galileo also contributed to the formation of what is now called Newton’s first law of motion. Newton made use of the work of his predecessors, which enabled him to develop laws of motion, discover the law of gravity, invent calculus, and make great contributions to the theories of light and color. It is amazing that many of these developments were made with Newton working alone, without the benefit of the usual interactions that take place among scientists today.

It was not until the advent of modern physics early in the 20th century that it was discovered that Newton’s laws of motion produce a good approximation to motion only when the objects are moving at speeds much, much less than the speed of light and when those objects are larger than the

size of most molecules (about 10^{-9} m in diameter). These constraints define the realm of classical mechanics. At the beginning of the 20th century, Albert Einstein (1879–1955) developed the theory of relativity and, along with many other scientists, developed quantum theory. This theory does not have the constraints present in classical physics. All of the situations we consider in this chapter are in the realm of classical physics.

Note:

Making Connections: Past and Present Philosophy

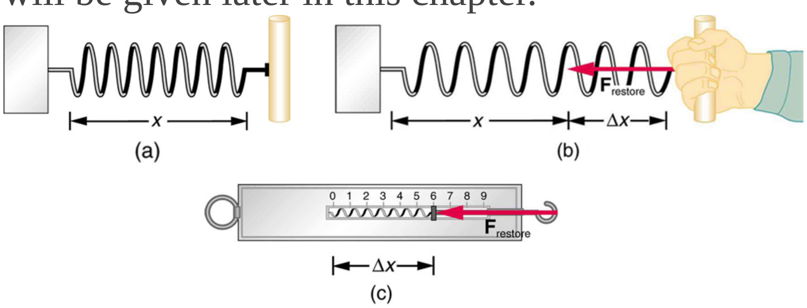
The importance of observation and the concept of *cause and effect* were not always so entrenched in human thinking. This realization was a part of the evolution of modern physics from natural philosophy. The achievements of Galileo, Newton, Einstein, and others were key milestones in the history of scientific thought. Most of the scientific theories that are described in this book descended from the work of these scientists.

Development of Force Concept

- Understand the definition of force.

Dynamics is the study of the forces that cause objects and systems to move. To understand this, we need a working definition of force. Our intuitive definition of **force**—that is, a push or a pull—is a good place to start. We know that a push or pull has both magnitude and direction (therefore, it is a vector quantity) and can vary considerably in each regard. For example, a cannon exerts a strong force on a cannonball that is launched into the air. In contrast, Earth exerts only a tiny downward pull on a flea.

A more quantitative definition of force can be based on some standard force, just as distance is measured in units relative to a standard distance. One possibility is to stretch a spring a certain fixed distance, as illustrated in [link], and use the force it exerts to pull itself back to its relaxed shape—called a *restoring force*—as a standard. The magnitude of all other forces can be stated as multiples of this standard unit of force. Many other possibilities exist for standard forces. Some alternative definitions of force will be given later in this chapter.



The force exerted by a stretched spring can be used as a standard unit of force. (a) This spring has a length x when undistorted. (b) When stretched a distance Δx, the spring exerts a restoring force, $\mathbf{F}_{\text{restore}}$, which is reproducible. (c) A spring scale is one device that uses a spring to measure force. The force $\mathbf{F}_{\text{restore}}$ is exerted on whatever is attached to the hook. Here $\mathbf{F}_{\text{restore}}$ has a

magnitude of 6 units in the force standard
being employed.

Note:
Take-Home Experiment: Force Standards
To investigate force standards and cause and effect, get two identical rubber bands. Hang one rubber band vertically on a hook. Find a small household item that could be attached to the rubber band using a paper clip, and use this item as a weight to investigate the stretch of the rubber band. Measure the amount of stretch produced in the rubber band with one, two, and four of these (identical) items suspended from the rubber band. What is the relationship between the number of items and the amount of stretch? How large a stretch would you expect for the same number of items suspended from two rubber bands? What happens to the amount of stretch of the rubber band (with the weights attached) if the weights are also pushed to the side with a pencil?

Section Summary

- **Dynamics** is the study of how forces affect the motion of objects.
- **Force** is a push or pull that can be defined in terms of various standards, and it is a vector having both magnitude and direction.

Conceptual Questions

Exercise:

Problem:

Propose a force standard different from the example of a stretched spring discussed in the text. Your standard must be capable of producing the same force repeatedly.

Exercise:

Problem:

What properties do forces have that allow us to classify them as vectors?

Glossary

dynamics
: the study of how forces affect the motion of objects and systems

force
: a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

Newton’s First Law of Motion: Inertia

- Define mass and inertia.
- Understand Newton's first law of motion.

Experience suggests that an object at rest will remain at rest if left alone, and that an object in motion tends to slow down and stop unless some effort is made to keep it moving. What **Newton’s first law of motion** states, however, is the following:

> **Note:**
> Newton’s First Law of Motion
> A body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force.

Note the repeated use of the verb “remains.” We can think of this law as preserving the status quo of motion.

Rather than contradicting our experience, **Newton’s first law of motion** states that there must be a *cause* (which is a net external force) *for there to be any change in velocity (either a change in magnitude or direction)*. We will define *net external force* in the next section. An object sliding across a table or floor slows down due to the net force of friction acting on the object. If friction disappeared, would the object still slow down?

The idea of cause and effect is crucial in accurately describing what happens in various situations. For example, consider what happens to an object sliding along a rough horizontal surface. The object quickly grinds to a halt. If we spray the surface with talcum powder to make the surface smoother, the object slides farther. If we make the surface even smoother by rubbing lubricating oil on it, the object slides farther yet. Extrapolating to a frictionless surface, we can imagine the object sliding in a straight line indefinitely. Friction is thus the *cause* of the slowing (consistent with Newton’s first law). The object would not slow down at all if friction were

completely eliminated. Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it creates a nearly frictionless surface, and the puck glides long distances without slowing down. Additionally, if we know enough about the friction, we can accurately predict how quickly the object will slow down. Friction is an external force.

Newton’s first law is completely general and can be applied to anything from an object sliding on a table to a satellite in orbit to blood pumped from the heart. Experiments have thoroughly verified that any change in velocity (speed or direction) must be caused by an external force. The idea of *generally applicable or universal laws* is important not only here—it is a basic feature of all laws of physics. Identifying these laws is like recognizing patterns in nature from which further patterns can be discovered. The genius of Galileo, who first developed the idea for the first law, and Newton, who clarified it, was to ask the fundamental question, “What is the cause?” Thinking in terms of cause and effect is a worldview fundamentally different from the typical ancient Greek approach when questions such as “Why does a tiger have stripes?” would have been answered in Aristotelian fashion, “That is the nature of the beast.” True perhaps, but not a useful insight.

Mass

The property of a body to remain at rest or to remain in motion with constant velocity is called **inertia**. Newton’s first law is often called the **law of inertia**. As we know from experience, some objects have more inertia than others. It is obviously more difficult to change the motion of a large boulder than that of a basketball, for example. The inertia of an object is measured by its **mass**. Roughly speaking, mass is a measure of the amount of “stuff” (or matter) in something. The quantity or amount of matter in an object is determined by the numbers of atoms and molecules of various types it contains. Unlike weight, mass does not vary with location. The mass of an object is the same on Earth, in orbit, or on the surface of the Moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so masses are not often determined in this

manner. Operationally, the masses of objects are determined by comparison with the standard kilogram.

Exercise:

Check Your Understanding

Problem:

Which has more mass: a kilogram of cotton balls or a kilogram of gold?

Solution:

Answer

They are equal. A kilogram of one substance is equal in mass to a kilogram of another substance. The quantities that might differ between them are volume and density.

Section Summary

- **Newton's first law of motion** states that a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force. This is also known as the **law of inertia**.
- **Inertia** is the tendency of an object to remain at rest or remain in motion. Inertia is related to an object's mass.
- **Mass** is the quantity of matter in a substance.

Conceptual Questions

Exercise:

Problem: How are inertia and mass related?

Exercise:

Problem:

What is the relationship between weight and mass? Which is an intrinsic, unchanging property of a body?

Glossary

inertia
: the tendency of an object to remain at rest or remain in motion

law of inertia
: see Newton's first law of motion

mass
: the quantity of matter in a substance; measured in kilograms

Newton's first law of motion
: a body at rest remains at rest, or, if in motion, remains in motion at a constant velocity unless acted on by a net external force; also known as the law of inertia

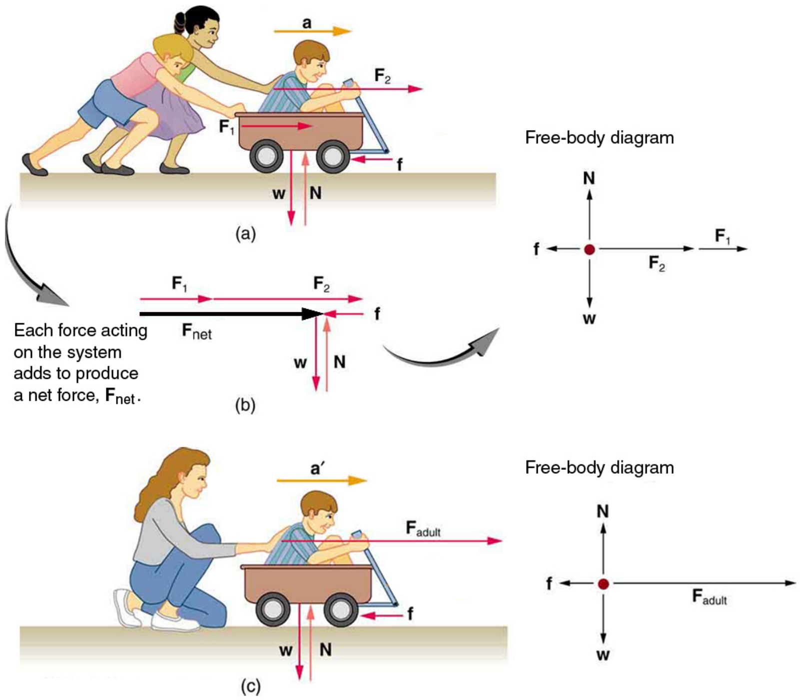
Newton's Second Law of Motion: Force and Acceleration

- Define net force, external force, and system.
- Understand Newton’s second law of motion.
- Apply Newton’s second law to determine the weight of an object.

Newton’s second law of motion is closely related to Newton’s first law of motion. It mathematically states the cause and effect relationship between force and changes in motion. Newton’s second law of motion is more quantitative and is used extensively to calculate what happens in situations involving a force. Before we can write down Newton’s second law as a simple equation giving the exact relationship of force, mass, and acceleration, we need to sharpen some ideas that have already been mentioned.

First, what do we mean by a change in motion? The answer is that a change in motion is equivalent to a change in velocity. A change in velocity means, by definition, that there is an **acceleration**. Newton’s first law says that a net external force causes a change in motion; thus, we see that a *net external force causes acceleration*.

Another question immediately arises. What do we mean by an external force? An intuitive notion of external is correct—an **external force** acts from outside the **system** of interest. For example, in [link](a) the system of interest is the wagon plus the child in it. The two forces exerted by the other children are external forces. An internal force acts between elements of the system. Again looking at [link](a), the force the child in the wagon exerts to hang onto the wagon is an internal force between elements of the system of interest. Only external forces affect the motion of a system, according to Newton’s first law. (The internal forces actually cancel, as we shall see in the next section.) *You must define the boundaries of the system before you can determine which forces are external*. Sometimes the system is obvious, whereas other times identifying the boundaries of a system is more subtle. The concept of a system is fundamental to many areas of physics, as is the correct application of Newton’s laws. This concept will be revisited many times on our journey through physics.



Different forces exerted on the same mass produce different accelerations. (a) Two children push a wagon with a child in it. Arrows representing all external forces are shown. The system of interest is the wagon and its rider. The weight $\boldsymbol{w}$ of the system and the support of the ground $\boldsymbol{N}$ are also shown for completeness and are assumed to cancel. The vector $\boldsymbol{f}$ represents the friction acting on the wagon, and it acts to the left, opposing the motion of the wagon. (b) All of the external forces acting on the system add together to produce a net force, $\boldsymbol{F}_{\text{net}}$. The free-body diagram shows all of the forces acting on the system of interest. The dot represents the center of mass of the system. Each force vector extends from this dot. Because there are two forces acting to the right, we draw the vectors collinearly. (c) A larger net external force produces a larger

acceleration ($|\boldsymbol{a}\prime| > |\boldsymbol{a}|$) when an adult pushes the child.

[link] is our first example of a **free-body diagram**, which is a technique used to illustrate all the external forces acting on a body. The body is represented by a single isolated point (or free body), and only those forces acting on the body from the outside (external forces) are shown. Free-body diagrams are very useful in analyzing forces acting on a system and are employed extensively in the study and application of Newton's laws of motion.

Now, it seems reasonable that acceleration should be directly proportional to and in the same direction as the net (total) external force acting on a system. This assumption has been verified experimentally and is illustrated in [link]. In part (a), a smaller force causes a smaller acceleration than the larger force illustrated in part (c). For completeness, the vertical forces are also shown; they are assumed to cancel since there is no acceleration in the vertical direction. The vertical forces are the weight $\boldsymbol{w}$ and the support of the ground $\boldsymbol{N}$, and the horizontal force $\boldsymbol{f}$ represents the force of friction. These will be discussed in more detail in later sections. For now, we will say that friction is a force that opposes the motion past each other of objects that are touching. [link](b) shows how vectors representing the external forces add together to produce a net force, $\boldsymbol{F}_{\text{net}}$.

To obtain an equation for Newton's second law, we first write the relationship of acceleration and net external force as the proportionality
Equation:

$$\boldsymbol{a} \propto \boldsymbol{F}_{\text{net}}$$

where the symbol $\propto$ means "proportional to," and $\boldsymbol{F}_{\text{net}}$ is the **net external force**, the vector sum of all external forces. This proportionality states what we have said in words—*acceleration is directly proportional to the net external force*. Once the system of interest is chosen, it is important to identify the external forces, so that the internal forces can be ignored (again,

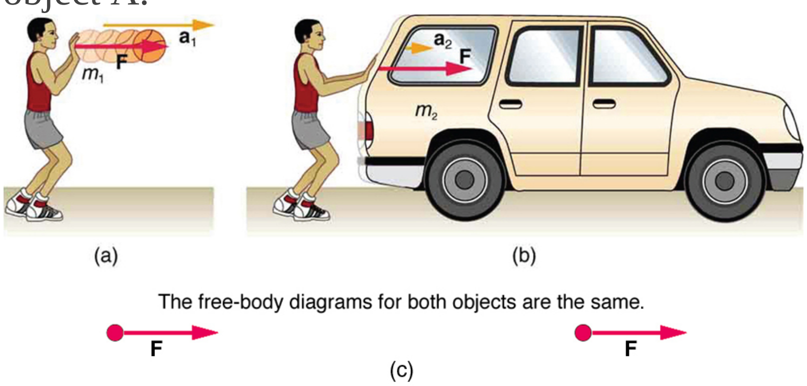
as we will see later, they exactly cancel out, allowing for a tremendous simplification).

Now, hopefully it also seems reasonable that acceleration should be inversely proportional to the mass of the system. That is, if the system has more “stuff”, then for the same net external force applied, the acceleration is less. And indeed, as illustrated in [link], the same net external force applied to a car produces a much smaller acceleration than when applied to a basketball. The proportionality is written as

Equation:

$$a \propto \frac{1}{m}$$

where m is the mass of the system. In fact, this is how we will *define* mass. That is, for an object B to have double the mass of an object A, the same net external force applied on object B produces half the acceleration it does on object A.



The same force exerted on systems of different masses produces different accelerations. (a) A basketball player pushes on a basketball to make a pass. (The effect of gravity on the ball is ignored.) (b) The same player exerts an identical force on a stalled SUV and produces a far smaller acceleration (even if friction is negligible). (c) The free-body diagrams are identical,

permitting direct comparison of the two situations. A series of patterns for the free-body diagram will emerge as you do more problems.

As it turns out, the acceleration of an object depends only on the net external force and the mass of the object. Combining the two proportionalities just given yields Newton's second law of motion.

Note:
Newton’s Second Law of Motion
The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
In equation form, Newton’s second law of motion is
Equation:

$$\boldsymbol{a} = \frac{\boldsymbol{F}_{\text{net}}}{m}$$

This is often written in the more familiar form
Equation:

$$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a},$$

with the vector notation indicating that the net external force is in the same direction as acceleration. When only the magnitude of force and acceleration are considered, this equation is simply (note the lack of vector notations)
Equation:

$$F_{\text{net}} = ma.$$

Although these equations are really the same, the first gives more insight into what Newton's second law means. The law is a *cause and effect relationship* among three quantities. That is, acceleration is *caused* by a net external force, not vice versa, as the second equation might mistakenly imply.

Units of Force

$F_{\text{net}} = ma$ is used to define the units of force in terms of the three basic units for mass, length, and time. The SI unit of force is called the **newton** (abbreviated N) and $1\ \text{N}$ is the force needed to accelerate a system of mass $1\ \text{kg}$ at the rate of $1\ \text{m/s}^2$. Putting these together,

Equation:

$$1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2.$$

While almost the entire world uses the newton for the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

Weight and the Gravitational Force

When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that a net force on an object is responsible for its acceleration. If air resistance is negligible, the net force on a falling object is the gravitational force, commonly called its **weight**, $\boldsymbol{w}$. Weight can be denoted as a vector $\boldsymbol{w}$ because it has a direction; *down* is, by definition, the direction of gravity, so $\boldsymbol{w}$ is directed downward. The magnitude of weight is denoted as w. Galileo was instrumental in showing that, in the absence of air resistance, all objects fall with the same acceleration g. Using Galileo's result and Newton's second law, we can derive an equation for the magnitude of weight.

Consider an object with mass m falling downward toward Earth. It experiences only the downward force of gravity, which has magnitude w.

Newton's second law states that the magnitude of the net external force on an object is $F_{\text{net}} = \text{ma}$.

Since the object experiences only the downward force of gravity, $F_{\text{net}} = w$. We know that the acceleration of an object due to gravity is g, or $a = g$. Substituting these into Newton's second law gives

Note:
Weight
This is the equation for *weight*—the gravitational force on a mass m:
Equation:

$$w = \text{mg}.$$

Since $g = 9.80 \text{ m/s}^2$ on Earth, the weight of a 1.0 kg object on Earth is 9.8 N, as we see:
Equation:

$$w = \text{mg} = (1.0 \text{ kg})(9.80 \text{ m/s}^2) = 9.8 \text{ N}.$$

When the net external force on an object is its weight, we say that it is in **free-fall**. That is, the only force acting on the object is the force of gravity. In the real world, when objects fall downward toward Earth, they are never truly in free-fall because there is always some upward force from the air acting on the object.

The acceleration due to gravity g varies slightly over the surface of Earth, so that the weight of an object depends on location and is not an intrinsic property of the object. Weight varies dramatically if one leaves Earth. On the Moon, for example, the acceleration due to gravity is only 1.67 m/s^2. A 1.0-kg mass thus has a weight of 9.8 N on Earth and only about 1.7 N on the Moon.

The broadest definition of weight in this sense is that *the weight of an object is the gravitational force on it from the nearest large body*, such as Earth, the Moon, the Sun, and so on. This is the most common and useful definition of weight in physics. It differs dramatically, however, from the definition of weight used by NASA and the popular media in relation to space travel and exploration. When they speak of “weightlessness” and “microgravity,” they are really referring to the phenomenon we call “free-fall” in physics. We shall use the above definition of weight, and we will make careful distinctions between free-fall and actual weightlessness.

It is important to be aware that weight and mass are very different physical quantities, although they are closely related. Mass is the quantity of matter (how much “stuff”) and does not vary, whereas weight is the gravitational force and does vary depending on gravity. It is tempting to equate the two, since most of our examples take place on Earth, where the weight of an object only varies a little with the location of the object. Furthermore, the terms *mass* and *weight* are used interchangeably in everyday language; for example, our medical records often show our “weight” in kilograms, but never in the correct units of newtons.

Note:
Common Misconceptions: Mass vs. Weight
Mass and weight are often used interchangeably in everyday language. However, in science, these terms are distinctly different from one another. Mass is a measure of how much matter is in an object. The typical measure of mass is the kilogram (or the “slug” in English units). Weight, on the other hand, is a measure of the force of gravity acting on an object. Weight is equal to the mass of an object (m) multiplied by the acceleration due to gravity (g). Like any other force, weight is measured in terms of newtons (or pounds in English units). This interchangeable use is why you might have heard an expression such as, “1 kilogram is 2.2 pounds.” The *correct* statement is “1 kilogram mass weighs 2.2 pounds on Earth.”
Assuming the mass of an object is kept intact, it will remain the same, regardless of its location. However, because weight depends on the acceleration due to gravity, the weight of an object *can change* when the

object enters into a region with stronger or weaker gravity. For example, the acceleration due to gravity on the Moon is 1.67 m/s^2 (which is much less than the acceleration due to gravity on Earth, 9.80 m/s^2). If you measured your weight on Earth and then measured your weight on the Moon, you would find that you "weigh" much less, even though you do not look any skinnier. This is because the force of gravity is weaker on the Moon. In fact, when people say that they are "losing weight," they really mean that they are losing "mass" (which in turn causes them to weigh less).

Note:

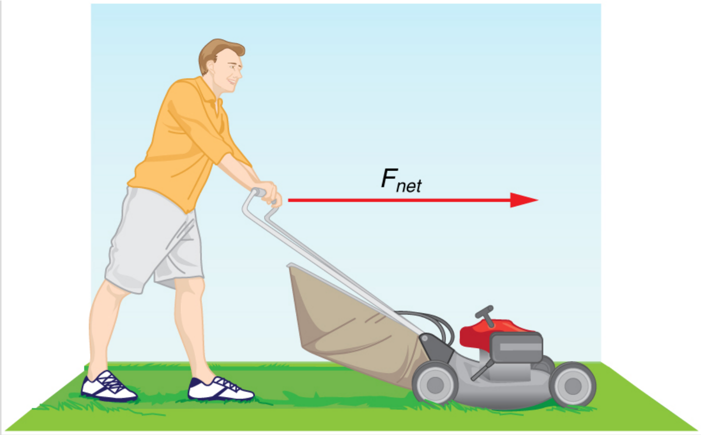
Take-Home Experiment: Mass and Weight

What do bathroom scales measure? When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled. The springs provide a measure of your weight (for an object which is not accelerating). This is a force in newtons (or pounds). In most countries, the measurement in newtons is divided by 9.80 to give a reading in mass units of kilograms. The scale measures weight but is calibrated to provide information about mass. While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why? Would your scale measure the same "mass" on Earth as on the Moon?

Example:

What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N (about 11 lb) parallel to the ground. The mass of the mower is 24 kg. What is its acceleration?



The net force on a lawn mower is 51 N to the right. At what rate does the lawn mower accelerate to the right?

Strategy
Since F_{net} and m are given, the acceleration can be calculated directly from Newton's second law as stated in $F_{\text{net}} = ma$.
Solution
The magnitude of the acceleration a is $a = \frac{F_{\text{net}}}{m}$. Entering known values gives
Equation:

$$a = \frac{51 \text{ N}}{24 \text{ kg}}$$

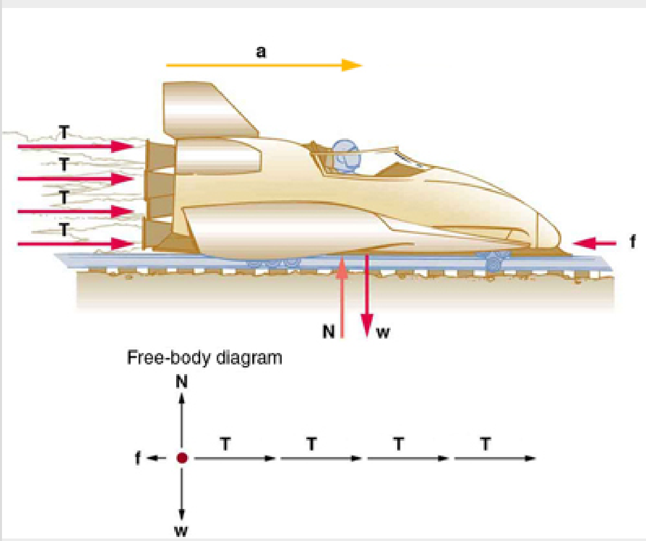
Substituting the units $\text{kg} \cdot \text{m/s}^2$ for N yields
Equation:

$$a = \frac{51 \text{ kg} \cdot \text{m/s}^2}{24 \text{ kg}} = 2.1 \text{ m/s}^2.$$

Discussion
The direction of the acceleration is the same direction as that of the net force, which is parallel to the ground. There is no information given in this example about the individual external forces acting on the system, but we

can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower forward must be greater than the friction opposing the motion (directed backward) to result in a forward net force, and the vertical forces must cancel if there is to be no acceleration in the vertical direction. The acceleration found is small enough to be reasonable for a person pushing a mower. When the person reaches the top speed, the acceleration will be zero (no change of velocity) and the force exerted by the person pushing the mower will be equal to the friction opposing the motion.

Example:
What Rocket Thrust Accelerates This Sled?
Prior to manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on human subjects at high speeds. They consisted of a platform that was mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust T, for the four-rocket propulsion system shown in [link]. The sled's initial acceleration is $49\ \text{m/s}^2$, the mass of the system is 2100 kg, and the force of friction opposing the motion is known to be 650 N.



A sled experiences a rocket thrust that accelerates it to the right. Each

rocket creates an identical thrust T. As in other situations where there is only horizontal acceleration, the vertical forces cancel. The ground exerts an upward force N on the system which balances out the weight force, w. The system here is the sled, its rockets, and rider, so none of the internal forces *between* these objects are considered. f represents the friction force (figure not drawn to scale).

Strategy

Although there are forces acting vertically and horizontally, we assume the vertical forces cancel since there is no vertical acceleration. This leaves us with only horizontal forces and a simpler one-dimensional problem. Directions are indicated with plus or minus signs, with right taken as the positive direction. See the free-body diagram in the figure.

Solution

Since acceleration, mass, and the force of friction are given, we start with Newton's second law and look for ways to find the thrust of the engines. Since we have defined the direction of the force and acceleration as acting "to the right," we need to consider only the magnitudes of these quantities in the calculations. Hence we begin with

Equation:

$$F_{\text{net}} = \text{ma},$$

where F_{net} is the net force along the horizontal direction. We can see from [link] that the engine thrusts add, while friction opposes the thrust. In equation form, the net external force is

Equation:

$$F_{\text{net}} = 4T - f.$$

Substituting this into Newton's second law gives

Equation:

$$F_{\text{net}} = \text{ma} = 4T - f.$$

Using a little algebra, we solve for the total thrust 4*T*:

Equation:

$$4T = \text{ma} + f.$$

Substituting known values yields

Equation:

$$4T = \text{ma} + f = (2100\ \text{kg})(49\ \text{m/s}^2) + 650\ \text{N}.$$

So the total thrust is

Equation:

$$4T = 1.04 \times 10^5\ \text{N},$$

and the individual thrusts are

Equation:

$$T = \frac{1.04 \times 10^5\ \text{N}}{4} = 2.6 \times 10^4\ \text{N}.$$

Discussion

The numbers are quite large, so to put the result in perspective, note that acceleration of 49 m/s^2 is approximately 5 times the gravitational acceleration on Earth. It's a large acceleration, which requires a large net external force to produce.

Experiments such as this were performed in the early 1960s to test the limits of human endurance and the setup designed to protect human subjects in jet fighter emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 *g*'s. (Recall that *g*, the acceleration due to gravity, is 9.80 m/s^2. When we say that an acceleration is 45 *g*'s, it is $45 \times 9.80\ \text{m/s}^2$, which is approximately 440 m/s^2.) While living subjects are not used any more, land speeds of 10,000 km/h have been obtained with rocket sleds. In this example, as in the preceding one, the system of

interest is obvious. We will see in later examples that choosing the system of interest is crucial—and the choice is not always obvious.
Newton's second law of motion gives a relationship among acceleration, force, and mass. It can help us make predictions. Each of those physical quantities can be defined independently, so the second law tells us something basic and universal about nature. The next section introduces the third and final law of motion.

Section Summary

- Acceleration, $\boldsymbol{a}$, is defined as a rate of change in velocity, resulting from a change in the magnitude and/or the direction of the velocity.
- An external force is one acting on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.
- In equation form, Newton's second law of motion is $\boldsymbol{a} = \frac{\boldsymbol{F}_{\text{net}}}{m}$, often written in the more familiar form: $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$.
- The weight $\boldsymbol{w}$ of an object is defined as the force of gravity acting on an object of mass m. Given acceleration due to gravity g, the magnitude of weight is:
  Equation:

  $$w = mg$$

Conceptual Questions

Exercise:

Problem:

Which statement is correct? (a) Net force causes motion. (b) Net force causes change in motion. Explain your answer and give an example.

Exercise:

Problem:

Why can we neglect forces such as those holding a body together when we apply Newton’s second law of motion?

Exercise:

Problem:

Explain how the choice of the “system of interest” affects which forces must be considered when applying Newton’s second law of motion.

Exercise:

Problem:

Describe a situation in which the net external force on a system is not zero, yet its speed remains constant.

Exercise:

Problem:

A system can have a nonzero velocity while the net external force on it *is* zero. Describe such a situation.

Exercise:

Problem:

A rock is thrown straight up. What is the net external force acting on the rock when it is at the top of its trajectory?

Exercise:

Problem:

(a) Give an example of different net external forces acting on the same system to produce different accelerations. (b) Give an example of the same net external force acting on systems of different masses, producing different accelerations. (c) What law accurately describes both effects? State it in words and as an equation.

Exercise:

Problem:

If the acceleration of a system is zero, are no external forces acting on it? What about internal forces? Explain your answers.

Exercise:

Problem:

If a constant, nonzero force is applied to an object, what can you say about the velocity and acceleration of the object?

Exercise:

Problem:

The gravitational force on the basketball in [link] is ignored. When gravity *is* taken into account, what is the direction of the net external force on the basketball—above horizontal, below horizontal, or still horizontal?

Glossary

acceleration
: the rate at which an object’s velocity changes over a period of time

free-fall
: a situation in which the only force acting on an object is the force due to gravity

external force
: a force acting on an object or system that originates outside of the object or system

net external force
: the vector sum of all external forces acting on an object or system; causes a mass to accelerate

free-body diagram
: a sketch showing all of the external forces acting on an object or system; the system is represented by a dot, and the forces are represented by vectors extending outward from the dot

Newton’s second law of motion
: The acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to its mass.

system
: the object or the group of objects under consideration

weight
: the force due to gravity; $w = mg$ for objects on Earth

Newton’s Third Law of Motion: Symmetry in Forces

- Understand Newton's third law of motion.
- Apply Newton's third law to define systems and solve problems of motion.

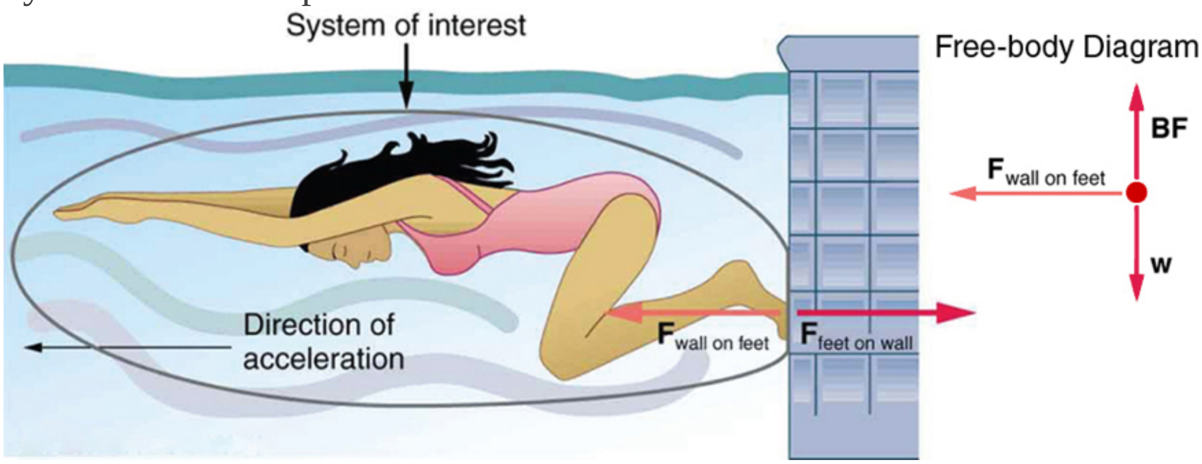
Whenever one body exerts a force on another—the first also experiences a force (equal in magnitude and opposite in direction). Numerous common experiences, such as stubbing a toe or throwing a ball, confirm this. It is precisely stated in **Newton’s third law of motion**.

Note:
Newton’s Third Law of Motion
Whenever one body exerts a force on a second body, the first body experiences a force by the second body that is equal in magnitude and opposite in direction to the force that it itself exerts.

This law represents a certain *symmetry in nature*: Forces always occur in pairs, and one body cannot exert a force on another without experiencing a force itself. We sometimes refer to this law loosely as “action-reaction,” where the force exerted is the action and the force experienced as a consequence is the reaction. Newton’s third law has practical uses in analyzing the origin of forces and understanding which forces are external to a system.

We can readily see Newton’s third law at work by taking a look at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in [link]. She pushes against the pool wall with her feet and accelerates in the direction *opposite* to that of her push. The wall has exerted an equal and opposite force back on the swimmer. You might think that two equal and opposite forces would cancel, but they do not *because they act on different bodies*. In this case, there are two bodies to consider: the swimmer and the wall. If we define the swimmer as the system of interest, as in the figure, then $\boldsymbol{F}_{\text{wall on feet}}$ is an external force on this system

and affects its motion. The swimmer moves in the direction of $\boldsymbol{F}_{\text{wall on feet}}$. In contrast, the force $\boldsymbol{F}_{\text{feet on wall}}$ acts on the wall and not on our system of interest. Thus $\boldsymbol{F}_{\text{feet on wall}}$ does not affect the motion of the system and does not counteract $\boldsymbol{F}_{\text{wall on feet}}$. Note that the swimmer pushes the wall in the direction opposite to that in which she wishes to move. The reaction force by the wall on her pushes her in the desired direction.



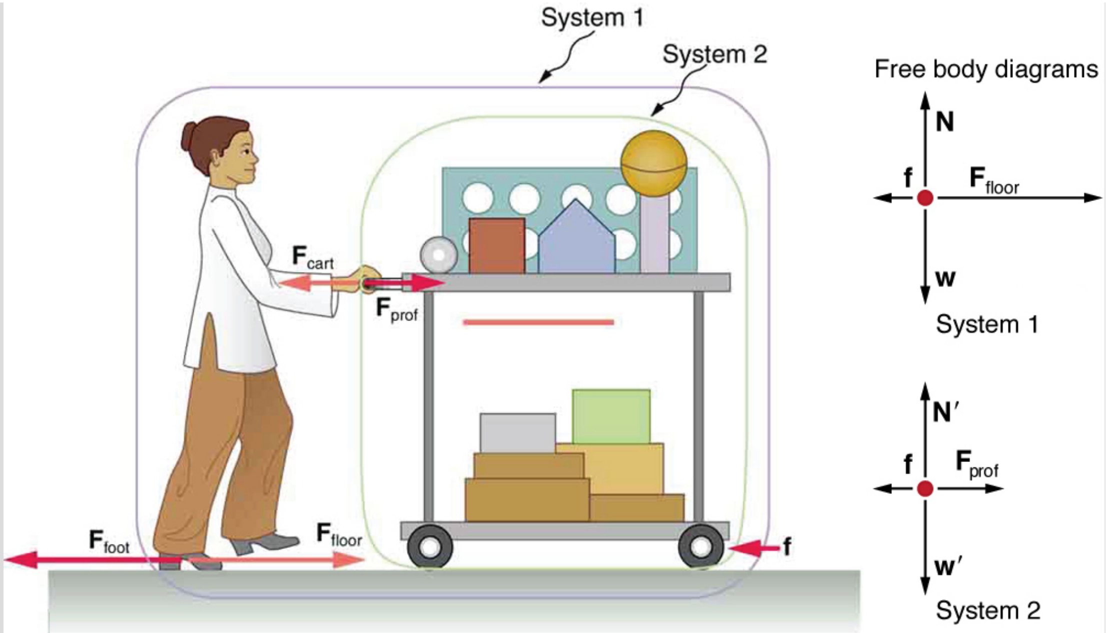
When the swimmer exerts a force $\boldsymbol{F}_{\text{feet on wall}}$ on the wall, she accelerates in the direction opposite to that of her push. This means the net external force on her is in the direction opposite to $\boldsymbol{F}_{\text{feet on wall}}$. This opposition occurs because, in accordance with Newton's third law of motion, the wall exerts a force $\boldsymbol{F}_{\text{wall on feet}}$ on her, equal in magnitude but in the direction opposite to the one she exerts on it. The line around the swimmer indicates the system of interest. Note that $\boldsymbol{F}_{\text{feet on wall}}$ does not act on this system (the swimmer) and, thus, does not cancel $\boldsymbol{F}_{\text{wall on feet}}$. Thus the free-body diagram shows only $\boldsymbol{F}_{\text{wall on feet}}$, $\boldsymbol{w}$, the gravitational force, and $\mathbf{BF}$, the buoyant force of the water supporting the swimmer's weight. The vertical forces $\boldsymbol{w}$ and $\mathbf{BF}$ cancel since there is no vertical motion.

Other examples of Newton's third law are easy to find. As a professor paces in front of a whiteboard, she exerts a force backward on the floor. The floor exerts a reaction force forward on the professor that causes her to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the drive wheels in reaction to the drive wheels pushing backward on the

ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward. In another example, rockets move forward by expelling gas backward at high velocity. This means the rocket exerts a large backward force on the gas in the rocket combustion chamber, and the gas therefore exerts a large reaction force forward on the rocket. This reaction force is called **thrust**. It is a common misconception that rockets propel themselves by pushing on the ground or on the air behind them. The rockets propel themselves by pushing their own exhaust gases, and that is how they work even in a vacuum. Helicopters create lift by pushing air down, thereby experiencing an upward reaction force. Birds and airplanes also fly by exerting force on air in a direction opposite to that of whatever force they need. For example, the wings of a bird force air downward and backward in order to get lift and move forward. An octopus propels itself in the water by ejecting water through a funnel from its body, similar to a jet ski. In a situation similar to Sancho's, professional cage fighters experience reaction forces when they punch, sometimes breaking their hand by hitting an opponent's body.

Example:
Getting Up To Speed: Choosing the Correct System

A physics professor pushes a cart of demonstration equipment to a lecture hall, as seen in [link]. Her mass is 65.0 kg, the cart's is 12.0 kg, and the equipment's is 7.0 kg. Calculate the acceleration produced when the professor exerts a backward force of 150 N on the floor. All forces opposing the motion, such as friction on the cart's wheels and air resistance, total 24.0 N.



A professor pushes a cart of demonstration equipment. The lengths of the arrows are proportional to the magnitudes of the forces (except for $\boldsymbol{f}$, since it is too small to draw to scale). Different questions are asked in each example; thus, the system of interest must be defined differently for each. System 1 is appropriate for this example, since it asks for the acceleration of the entire group of bodies. Only $\boldsymbol{F}_{\text{floor}}$ and $\boldsymbol{f}$ are external forces acting on System 1 along the line of motion. All other forces either cancel or act on the outside world. System 2 is appropriate for [link] so that $\boldsymbol{F}_{\text{prof}}$ will be an external force and enter into Newton's second law.

Strategy

Since they accelerate as a unit, we define the system to be the professor, cart, and equipment. This is System 1 in [link]. The professor pushes backward with a force $\boldsymbol{F}_{\text{foot}}$ of 150 N. According to Newton's third law, the floor exerts a forward reaction force $\boldsymbol{F}_{\text{floor}}$ of 150 N on System 1. Because all motion is horizontal, we can assume there is no net force in the vertical direction, and treat this as a one-dimensional problem in the horizontal direction. As noted, $\boldsymbol{f}$ opposes the motion and is thus in the opposite direction of $\boldsymbol{F}_{\text{floor}}$. Note that we do not include the forces $\boldsymbol{F}_{\text{prof}}$ or

$\boldsymbol{F}_{\text{cart}}$ because these are internal forces, and we do not include $\boldsymbol{F}_{\text{foot}}$ because it acts on the floor, not on the system. There are no other significant forces acting on System 1. If the net external force can be found from all this information, we can use Newton's second law to find the acceleration as requested. See the free-body diagram in the figure.

Solution

Newton's second law is given by

Equation:

$$a = \frac{F_{\text{net}}}{m}.$$

The net external force on System 1 is deduced from [link] and the discussion above to be

Equation:

$$F_{\text{net}} = F_{\text{floor}} - f = 150\text{ N} - 24.0\text{ N} = 126\text{ N}.$$

The mass of System 1 is

Equation:

$$m = (65.0 + 12.0 + 7.0)\text{ kg} = 84\text{ kg}.$$

These values of F_{net} and m produce an acceleration of

Equation:

$$\begin{aligned} a &= \tfrac{F_{\text{net}}}{m}, \\ a &= \tfrac{126\text{ N}}{84\text{ kg}} = 1.5\text{ m/s}^2. \end{aligned}$$

Discussion

None of the forces between components of System 1, such as between the professor's hands and the cart, contribute to the net external force because they are internal to System 1. Another way to look at this is to note that forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the professor on the cart results in an equal and opposite force back on her. In this case both forces act on the same system and, therefore, cancel. Thus

internal forces (between components of a system) cancel. Choosing System 1 was crucial to solving this problem.

Example:
Force on the Cart—Choosing a New System

Calculate the force the professor exerts on the cart in [link] using data from the previous example if needed.

Strategy

If we now define the system of interest to be the cart plus equipment (System 2 in [link]), then the net external force on System 2 is the force the professor exerts on the cart minus friction. The force she exerts on the cart, $\mathbf{F}_{\text{prof}}$, is an external force acting on System 2. $\mathbf{F}_{\text{prof}}$ was internal to System 1, but it is external to System 2 and will enter Newton's second law for System 2.

Solution

Newton's second law can be used to find $\mathbf{F}_{\text{prof}}$. Starting with

Equation:

$$a = \frac{F_{\text{net}}}{m}$$

and noting that the magnitude of the net external force on System 2 is

Equation:

$$F_{\text{net}} = F_{\text{prof}} - f,$$

we solve for F_{prof}, the desired quantity:

Equation:

$$F_{\text{prof}} = F_{\text{net}} + f.$$

The value of f is given, so we must calculate net F_{net}. That can be done since both the acceleration and mass of System 2 are known. Using Newton's second law we see that

Equation:

$$F_{\text{net}} = \text{ma},$$

where the mass of System 2 is 19.0 kg (m= 12.0 kg + 7.0 kg) and its acceleration was found to be $a = 1.5 \text{ m/s}^2$ in the previous example. Thus,

Equation:

$$F_{\text{net}} = \text{ma},$$

Equation:

$$F_{\text{net}} = (19.0 \text{ kg})(1.5 \text{ m/s}^2) = 29 \text{ N}.$$

Now we can find the desired force:

Equation:

$$F_{\text{prof}} = F_{\text{net}} + f,$$

Equation:

$$F_{\text{prof}} = 29 \text{ N} + 24.0 \text{ N} = 53 \text{ N}.$$

Discussion

It is interesting that this force is significantly less than the 150-N force the professor exerted backward on the floor. Not all of that 150-N force is transmitted to the cart; some of it accelerates the professor.

The choice of a system is an important analytical step both in solving problems and in thoroughly understanding the physics of the situation (which is not necessarily the same thing).

Note:

Common Misconception: Reaction vs. Result

The phrases "action" and "reaction" in reference to the pair of forces described by Newton's third law often leads to unfortunate misunderstandings regarding the nature of the reaction force. Because of the common English usage of the word "reaction" (root word: react)—for example, in film, "reaction shot" means a shot of actors' faces as they react

to an event that just happened—many students intuitively guess that there is a cause-and-effect relationship between "action force" and "reaction force."

There is no such relationship. Newton's third law describes *symmetry* in forces. Action force and reaction force are symmetric—they occur simultaneously; they are two sides of one and the same interaction. Action force does not cause reaction force, no more than the head of a coin causes the tail of the coin (they merely occur together, always). Here's a quick check to see if you understood the pairing of action force and reaction force correctly: try swapping the labels. That is, if you originally called A "action force" and B "reaction force", try calling B "action force" and A "reaction force." Does this re-labeling make some sense? If not, chances are, you have mis-identified the pair of action force and reaction force. (Here, a little weirdness is O.K., but if it feels like complete nonsense to swap the labels—like reversal of a cause-and-effect relationship—that tells you you used "reaction" in the common English usage, not how the word is used in Newton's third law.)

Section Summary

- **Newton’s third law of motion** represents a basic symmetry in nature. It states: Whenever one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that the first body exerts.
- A **thrust** is a reaction force that pushes a body forward in response to a backward force. Rockets, airplanes, and cars are pushed forward by a thrust reaction force.

Conceptual Questions

Exercise:

Problem:

When you take off in a jet aircraft, there is a sensation of being pushed back into the seat. Explain why you move backward in the seat—is there really a force backward on you? (The same reasoning explains whiplash injuries, in which the head is apparently thrown backward.)

Exercise:

Problem:

A device used since the 1940s to measure the kick or recoil of the body due to heart beats is the “ballistocardiograph.” What physics principle(s) are involved here to measure the force of cardiac contraction? How might we construct such a device?

Exercise:

Problem:

Describe a situation in which one system exerts a force on another and, as a consequence, experiences a force that is equal in magnitude and opposite in direction. Which of Newton’s laws of motion apply?

Exercise:

Problem:

Why does an ordinary rifle recoil (kick backward) when fired? The barrel of a recoilless rifle is open at both ends. Describe how Newton’s third law applies when one is fired. Can you safely stand close behind one when it is fired?

Exercise:

Problem:

An American football lineman reasons that it is senseless to try to out-push the opposing player, since no matter how hard he pushes he will experience an equal and opposite force from the other player. Use Newton's laws and draw a free-body diagram of an appropriate system to explain how he can still out-push the opposition if he is strong enough.

Exercise:

Problem:

Newton's third law of motion tells us that forces always occur in pairs of equal and opposite magnitude. Explain how the choice of the "system of interest" affects whether one such pair of forces cancels.

Glossary

Newton's third law of motion
: Whenever one body exerts a force on a second body, the first body experiences a force by the second body that is equal in magnitude and opposite in direction to the force that it itself exerts.

thrust
: a reaction force that pushes a body forward; rockets, airplanes, and cars are pushed forward by a thrust, a reaction force to propellants pushed backward

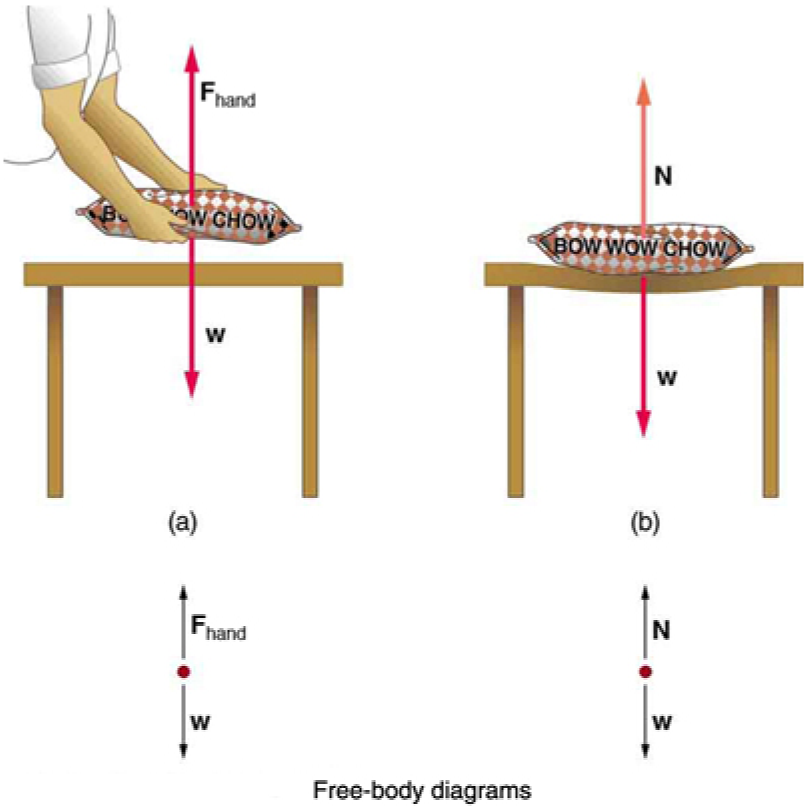
Normal Force and Tension

- Define normal force.
- Explain the relationship between normal force and apparent weight.
- Describe properties of tension force.
- Apply Newton's laws of motion to analyze forces.

Forces are given many names, such as push, pull, thrust, lift, weight, friction, and tension. Traditionally, forces have been grouped into several categories and given names relating to their source, how they are transmitted, or their effects. The most important of these categories are discussed in this section, together with some interesting applications. Further examples of forces are discussed later in this text.

Normal Force

Weight (also called force of gravity) is a pervasive force that acts at all times and must be counteracted to keep an object from falling. You definitely notice that you must support the weight of a heavy object by pushing up on it when you hold it stationary, as illustrated in [link](a). But how do inanimate objects like a table support the weight of a mass placed on them, such as shown in [link](b)? When the bag of dog food is placed on the table, the table actually sags slightly under the load. This would be noticeable if the load were placed on a card table, but even rigid objects deform when a force is applied to them. Unless the object is deformed beyond its limit, it will exert a restoring force much like a deformed spring (or trampoline or diving board). The greater the deformation, the greater the restoring force. So when the load is placed on the table, the table sags until the restoring force becomes as large as the weight of the load. At this point the net external force on the load is zero. That is the situation when the load is stationary on the table. The table sags quickly, and the sag is slight so we do not notice it. But it is similar to the sagging of a trampoline when you climb onto it.



(a) The person holding the bag of dog food must supply an upward force $\boldsymbol{F}_{\text{hand}}$ equal in magnitude and opposite in direction to the weight of the food $\boldsymbol{w}$. (b) The card table sags when the dog food is placed on it, much like a stiff trampoline. Elastic restoring forces in the table grow as it sags until they supply a force $\boldsymbol{N}$ equal in magnitude and opposite in direction to the weight of the load.

We must conclude that whatever supports a load, be it animate or not, must supply an upward force equal to the weight of the load, as we assumed in a few of the previous examples. The force supporting a load is perpendicular to the surface of contact between the load and its support, and this force is called a **normal force**, often indicated with symbol N (Please do not confuse this with the abbreviation for newton, the unit of force, N). The

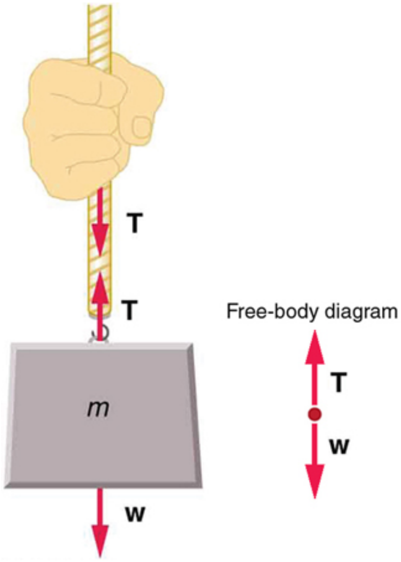
word *normal* means perpendicular to a surface. The normal force is not always equal to the object's weight, if there are other forces acting on the object, or if the object is accelerating, so that the net force is not zero.

Normal force also goes by another name, **apparent weight**. This is because all the forces you feel, especially your sensation of weight, are directly attributed to contact forces, such as the normal force. When NASA astronauts are weightlessly floating in the International Space Station, their actual weight (gravitational force on them by the Earth) is not zero, but their apparent weight (sensation of weight due to contact forces) is zero.

Tension

A **tension** is a force along the length of a medium, especially a force carried by a flexible medium, such as a rope or cable. The word “tension” comes from a Latin word meaning “to stretch.” Not coincidentally, the flexible cords that carry muscle forces to other parts of the body are called *tendons*. Any flexible connector, such as a string, rope, chain, wire, or cable, can exert pulls only parallel to its length; thus, a force carried by a flexible connector is a tension with direction parallel to the connector. It is important to understand that tension is a pull in a connector. In contrast, consider the phrase: “You can’t push a rope.” The tension force pulls outward along the two ends of a rope.

Consider a person holding a mass on a rope as shown in [link].



When a perfectly flexible connector (one requiring no force to bend it) such as this rope transmits a force $\boldsymbol{T}$, that force must be parallel to the length of the rope, as shown. The pull such a flexible connector exerts is a tension. Note that the rope pulls with equal force but in opposite directions on the hand and the supported mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that carries

the equal and opposite forces between the two objects. The tension anywhere in the rope between the hand and the mass is equal. Once you have determined the tension in one location, you have determined the tension at all locations along the rope.

Tension in the rope must equal the weight of the supported mass, as we can prove using Newton's second law. If the 5.00-kg mass in the figure is stationary, then its acceleration is zero, and thus $\boldsymbol{F}_{\text{net}} = 0$. The only external forces acting on the mass are its weight $\boldsymbol{w}$ and the tension $\boldsymbol{T}$ supplied by the rope. Thus,

Equation:

$$F_{\text{net}} = T - w = 0,$$

where T and w are the magnitudes of the tension and weight and their signs indicate direction, with up being positive here. Thus, just as you would expect, the tension equals the weight of the supported mass:

Equation:

$$T = w = \text{mg}.$$

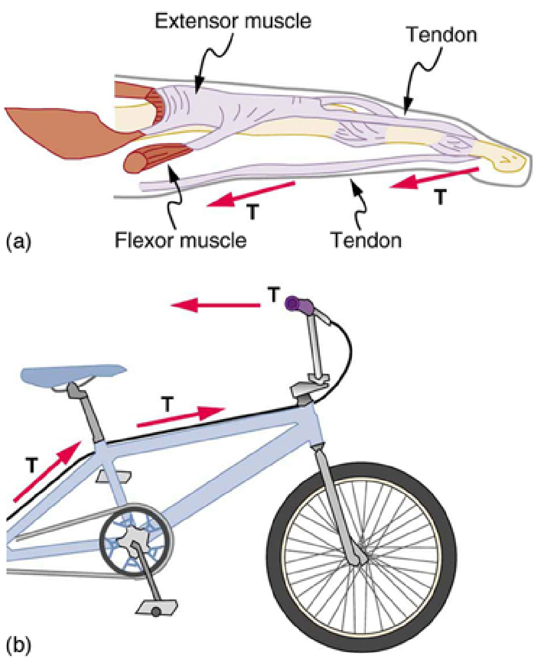
For a 5.00-kg mass, then (neglecting the mass of the rope) we see that

Equation:

$$T = \text{mg} = (5.00 \text{ kg})(9.80 \text{ m/s}^2) = 49.0 \text{ N}.$$

If we cut the rope and insert a spring, the spring would extend a length corresponding to a force of 49.0 N, providing a direct observation and measure of the tension force in the rope.

Flexible connectors are often used to transmit forces around corners, such as in a hospital traction system, a finger joint, or a bicycle brake cable. If there is no friction, the tension is transmitted undiminished. Only its direction changes, and it is always parallel to the flexible connector. This is illustrated in [link] (a) and (b).



(a) Tendons in the finger carry force $\boldsymbol{T}$ from the muscles to other parts of the finger, usually changing the force's direction, but not its magnitude (the

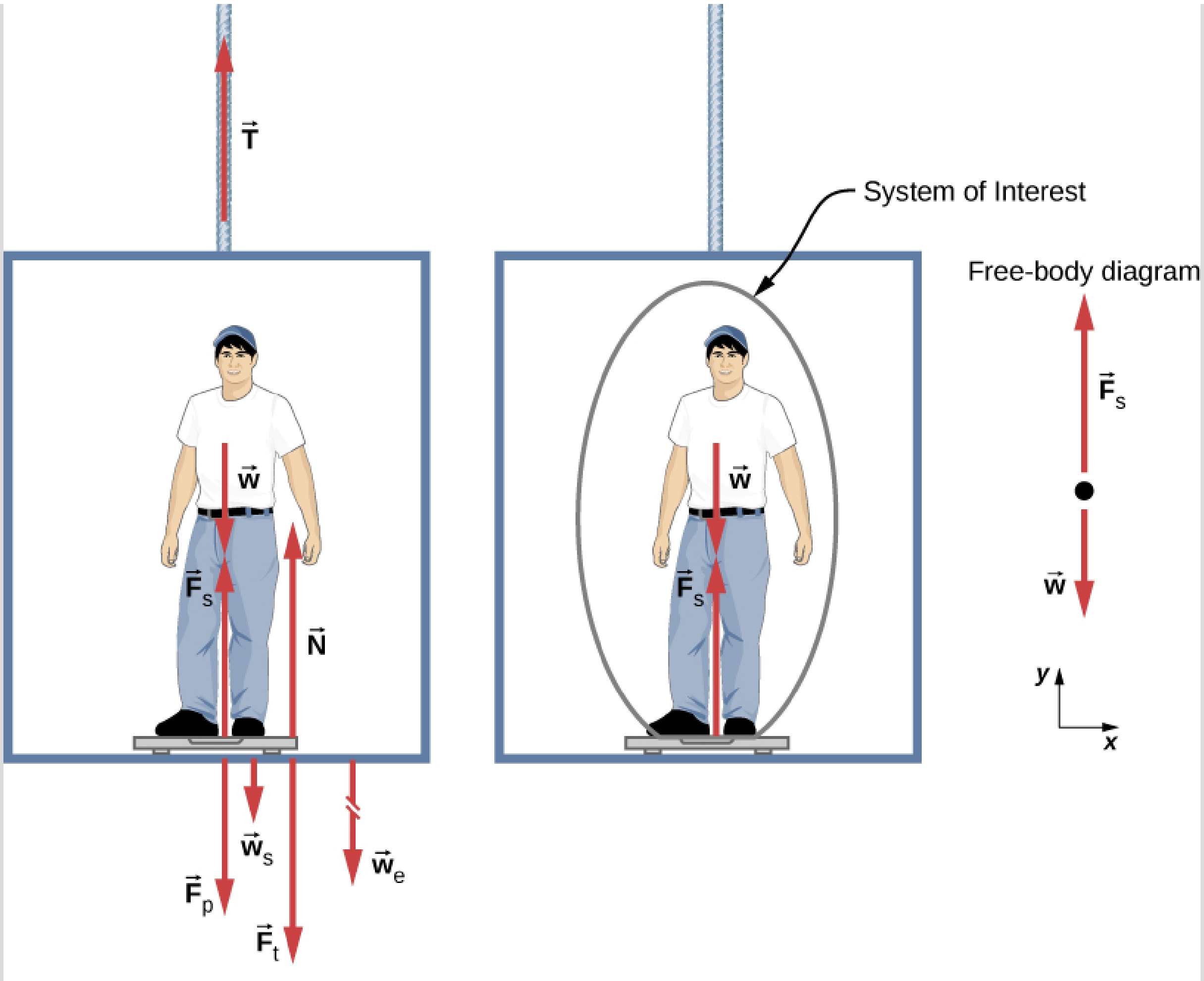
tendons are relatively friction free). (b) The brake cable on a bicycle carries the tension $\boldsymbol{T}$ from the handlebars to the brake mechanism. Again, the direction but not the magnitude of $\boldsymbol{T}$ is changed.

Unless an infinite tension is exerted, any flexible connector—such as the chain at the bottom of the picture—will sag under its own weight, giving a characteristic curve when the weight is evenly distributed along the length, known as a catenary curve. Suspension bridges—such as the Golden Gate Bridge shown in this image—are essentially very heavy flexible connectors. The weight of the bridge is evenly distributed along

the length of flexible connectors, usually cables, which take on the characteristic shape. (credit: Leaflet, Wikimedia Commons)

Normal force and tension are examples of forces that are determined not by a specific formula but by enforcing a constraint on motion state of a body. In the example of the bag on a table above, the normal force is equal to weight, because that is the value needed to ensure that the acceleration of the bag is zero. What happens if the body experiences a non-zero acceleration? We will consider example below of a person standing on a scale (which measures his apparent weight) while riding in an elevator.

Example:
What Does the Bathroom Scale Read in an Elevator?
[link] shows a 75.0-kg man (weight of about 165 lb.) standing on a bathroom scale in an elevator. Calculate the scale reading: (a) if the elevator accelerates upward at a rate of $1.20\ \text{m/s}^2$, and (b) if the elevator moves upward at a constant speed of 1 m/s.



(a) The various forces acting when a person stands on a bathroom scale in an elevator. The arrows are approximately correct for when the elevator is accelerating upward—broken arrows represent forces too large to be drawn to scale. $\vec{\mathbf{T}}$ is the tension in the supporting cable, $\vec{\mathbf{w}}$ is the weight of the person, $\vec{\mathbf{w}}_s$ is the weight of the scale, $\vec{\mathbf{w}}_e$ is the weight of the elevator, $\vec{\mathbf{F}}_s$ is the force of the scale on the person, $\vec{\mathbf{F}}_p$ is the force of the person on the scale, $\vec{\mathbf{F}}_t$ is the force of the scale on the floor of the elevator, and $\vec{\mathbf{N}}$ is the force of the floor upward on the scale. (b) The free-body diagram shows only the external forces acting *on* the designated system of interest—the person—and is the diagram we use for the solution of the problem.

Strategy

If the scale at rest is accurate, its reading equals $\vec{\mathbf{F}}_\text{p}$, the magnitude of the force the person exerts downward on it. [link](a) shows the numerous forces acting on the elevator, scale, and person. It makes this one-dimensional problem look much more formidable than if the person is chosen to be the system of interest and a free-body diagram is drawn, as in [link](b). Analysis of the free-body diagram using Newton's laws can produce answers to both [link](a) and (b) of this example, as well as some other questions that might arise. The only forces acting on the person are his weight $\vec{\mathbf{w}}$ and the upward force of the scale $\vec{\mathbf{F}}_\text{s}$. According to Newton's third law, $\vec{\mathbf{F}}_\text{p}$ and $\vec{\mathbf{F}}_\text{s}$ are equal in magnitude and opposite in direction, so that we need to find F_s in order to find what the scale reads. We can do this, as usual, by applying Newton's second law,

Equation:

$$\vec{\mathbf{F}}_\text{net} = m\vec{\mathbf{a}}.$$

From the free-body diagram, we see that the net force $\vec{\mathbf{F}}_\text{net}$ is sum of the scale force $\vec{\mathbf{F}}_s$ and the weight $\vec{\mathbf{w}}$. Since the scale force and the weight are pointing in opposite directions, in terms of the magnitudes of the vectors,

Equation:

$$F_\text{net} = F_s - w = ma.$$

Solving for F_s gives us an equation with only one unknown:

Equation:

$$F_\text{s} = ma + w,$$

or, because $w = mg$, simply

Equation:

$$F_\text{s} = ma + mg.$$

No assumptions were made about the acceleration, so this solution should be valid for a variety of accelerations in addition to those in this situation. (*Note:* We are considering the case when the elevator is accelerating

upward. If the elevator is accelerating downward, Newton's second law becomes $F_s - w = -ma$.)

Solution

a. We have $a = 1.20\,\mathrm{m/s^2}$, so that
Equation:

$$F_\mathrm{s} = (75.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) + (75.0\,\mathrm{kg})(1.20\,\mathrm{m/s^2})$$

yielding
Equation:

$$F_\mathrm{s} = 825\,\mathrm{N}.$$

b. Now, what happens when the elevator reaches a constant upward velocity? Will the scale still read more than his weight? For any constant velocity—up, down, or stationary—acceleration is zero because $a = \frac{\Delta v}{\Delta t}$ and $\Delta v = 0$. Thus,
Equation:

$$F_\mathrm{s} = ma + mg = 0 + mg$$

or
Equation:

$$F_\mathrm{s} = (75.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}),$$

which gives
Equation:

$$F_\mathrm{s} = 735\,\mathrm{N}.$$

Significance

The scale reading in [link](a) is about 185 lb. What would the scale have read if he were stationary? Since his acceleration would be zero, the force of the scale would be equal to his weight:

Equation:

$$F_{\text{net}} = ma = 0 = F_{\text{s}} - w$$

Equation:

$$F_{\text{s}} = w = mg$$

Equation:

$$F_{\text{s}} = (75.0\ \text{kg})(9.80\ \text{m/s}^2) = 735\ \text{N}.$$

Thus, the scale reading in the elevator is greater than his 735-N (165-lb.) weight. This means that the scale is pushing up on the person with a force greater than his weight, as it must in order to accelerate him upward. Clearly, the greater the acceleration of the elevator, the greater the scale reading, consistent with what you feel in rapidly accelerating versus slowly accelerating elevators. In [link](b), the scale reading is 735 N, which equals the person's weight. This is the case whenever the elevator has a constant velocity—moving up, moving down, or stationary.

Section Summary

- When objects rest on a surface, the surface applies a force to the object that supports the weight of the object. This supporting force acts perpendicular to and away from the surface. It is called a normal force, and it also goes by the name "apparent weight."
- *When objects rest on a non-accelerating horizontal surface*, the magnitude of the normal force is equal to the weight of the object:
  Equation:

  $$N = \text{mg}.$$

- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension, $\mathbf{T}$. When a rope supports the weight of an object that is at rest, the tension in the rope is equal to the weight of the object:
  Equation:

$$T = \text{mg}.$$

Conceptual Questions

Exercise:

Problem:

Find at least one example of each case from everyday experiences (find an example that is not in the textbook!): (a) a situation where normal force on a person is equal to the person's weight, (b) a situation where normal force on a person is greater than the person's weight, and (c) a situation where normal force on a person is less than the person's weight. Also, can you find an example (that is not in the textbook!) which covers each of all three cases, depending on changing circumstances?

Exercise:

Problem:

As noted in the text, the normal force and the tension force can only act in one direction: the normal force can only push, and the tension force can only pull. The numerical value of the force in these given directions is determined by what force is necessary to keep a body in the observed state of motion (either not accelerating, in presence of other forces, or accelerating at a particular rate). Under certain conditions, the normal force would go to zero (but not negative, as direction can't reverse), as would the tension force.

Explain qualitatively what you would see as: (a) the normal force goes to zero, and (b) the tension force goes to zero.

Glossary

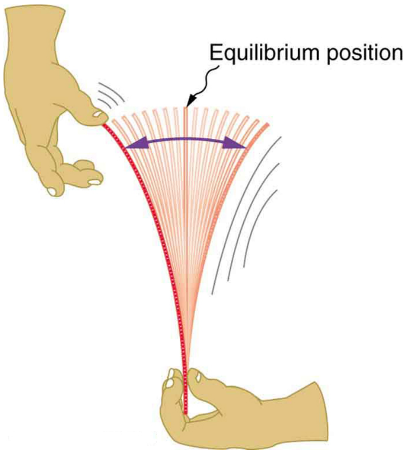
normal force
: the force that a surface applies to an object to support the weight of the object; acts perpendicular to the surface on which the object rests

apparent weight
: sensation of weight due to contact forces; usually the same as normal force

tension
: the pulling force that acts along a medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force on the object due to the rope is called a tension force

Spring Force: Hooke’s Law

- Describe the restoring force and displacement.
- Explain oscillatory motion under a spring force.



When displaced from its vertical equilibrium position, this plastic ruler oscillates back and forth because of the restoring force opposing displacement. When the ruler is on the left, there is a force to the right, and vice versa.

Newton’s first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line at a constant speed rather than oscillate. Consider, for example, plucking a plastic ruler to the left as shown in [link]. The deformation of the ruler creates a force in the opposite direction, known as a **restoring force**. Once

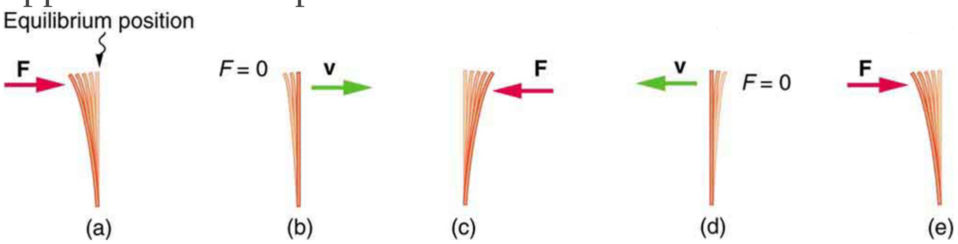
released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until dissipative forces dampen the motion. These forces remove mechanical energy from the system, gradually reducing the motion until the ruler comes to rest.

The simplest oscillations occur when the restoring force is directly proportional to displacement. This is called Hooke’s law force, or spring force:

Equation:

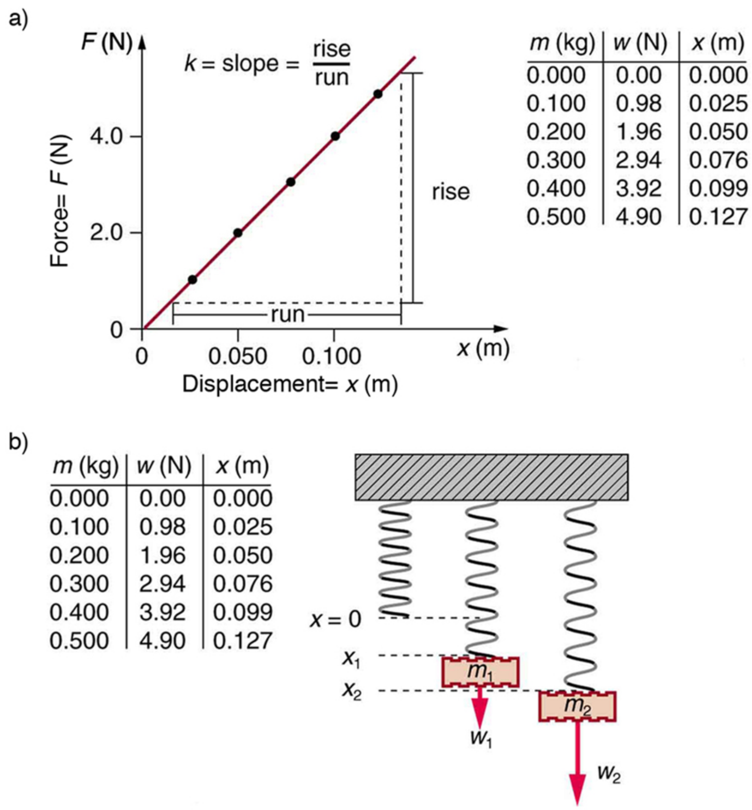
$$F = -\mathrm{kx}.$$

Here, F is the restoring force, x is the displacement from equilibrium or **deformation**, and k is a constant related to the difficulty in deforming the system. The minus sign indicates the restoring force is in the direction opposite to the displacement.



(a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping (caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself.

The **spring constant** k is related to the rigidity (or stiffness) of a system—the larger the spring constant, the greater the restoring force, and the stiffer the system. The units of k are newtons per meter (N/m). [link] shows a graph of the absolute value of the restoring force versus the displacement for a system that can be described by Hooke's law—a simple spring in this case. The slope of the graph equals the spring constant k in newtons per meter.



(a) A graph of absolute value of the restoring force versus displacement is displayed. The fact that the graph is a straight line means that the system obeys Hooke's law. The slope of the graph is the spring constant k. (b) The data in the graph were generated by measuring the displacement of a spring from equilibrium while supporting various weights. The restoring force equals the

weight supported, if the mass is stationary.

Example:
How Stiff Are Car Springs?

The mass of a car increases due to the introduction of a passenger. This affects the displacement of the car on its suspension system. (credit: exfordy on Flickr)

What is the spring constant for the suspension system of a car that settles 1.20 cm when an 80.0-kg person gets in?

Strategy

Consider the car to be in its equilibrium position $x=0$ before the person gets in. The car then settles down 1.20 cm, which means it is displaced to a position $x=-1.20\times 10^{-2}$ m. At that point, the springs supply a restoring force F equal to the person's weight $w=\text{mg}=(80.0\text{ kg})\left(9.80\text{ m/s}^2\right)=784\text{ N}$. We take this force to be F in Hooke's law. Knowing F and x, we can then solve the spring constant k.

Solution

1. Solve Hooke's law, $F=-\text{kx}$, for k:
   Equation:
   $$k=-\frac{F}{x}.$$
   Substitute known values and solve k:
   Equation:
   $$\begin{aligned} k &= -\frac{784\text{ N}}{-1.20\times 10^{-2}\text{ m}} \\ &= 6.53\times 10^{4}\text{ N/m.} \end{aligned}$$

Discussion

Note that F and x have opposite signs because they are in opposite directions—the restoring force is up, and the displacement is down. Also, note that the car would oscillate up and down when the person got in if it were not for damping (due to frictional forces) provided by shock absorbers. Bouncing cars are a sure sign of bad shock absorbers.

Section Summary

- An oscillation is a back and forth motion of an object between two points of deformation.
- An oscillation may create a wave, which is a disturbance that propagates from where it was created.
- The simplest type of oscillations and waves are related to systems that can be described by Hooke’s law:
  Equation:

  $$F = -\text{kx},$$

  where F is the restoring force, x is the displacement from equilibrium or deformation, and k is the spring constant of the system.

Conceptual Questions

Exercise:

Problem:

Describe a system which undergoes an oscillation under a Hooke's law force.

Exercise:

Problem:

Imagine an object under a hypothetical force behaving opposite to Hooke's law force. That is, instead of restoring force ($F = -kx$, with the negative sign indicating that the force acts in the opposite direction of displacement), the force reinforces an initial displacement: $F = +kx$. What would happen to this object if it is placed at the equilibrium ($x = 0$) and then very slightly disturbed?

Glossary

deformation
: displacement from equilibrium

spring constant

a constant related to the rigidity of a system: the larger the spring constant, the more rigid the system; the spring constant is represented by k

restoring force

force acting in opposition to the force caused by a deformation

Friction

- Discuss the general characteristics of friction.
- Describe the various types of friction.
- Explain effect of friction on motion.

Friction is a force that is around us all the time that opposes relative motion between systems in contact but also allows us to move (which you have discovered if you have ever tried to walk on ice). While a common force, the behavior of friction is actually very complicated and is still not completely understood. We have to rely heavily on observations for whatever understandings we can gain. However, we can still deal with its more elementary general characteristics and understand the circumstances in which it behaves.

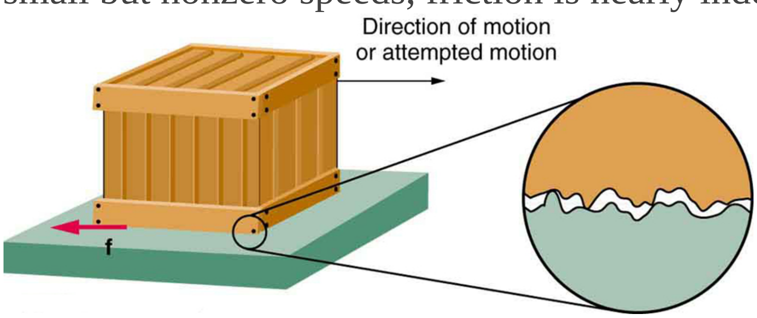
Note:
Friction
Friction is a force that opposes relative motion between systems in contact.

One of the simpler characteristics of friction is that it is parallel to the contact surface between systems and always in a direction that opposes motion or attempted motion of the systems relative to each other. If two systems are in contact and moving relative to one another, then the friction between them is called **kinetic friction**. For example, friction slows a hockey puck sliding on ice. But when objects are stationary, **static friction** can act between them; the static friction is usually greater than the kinetic friction between the objects.

Note:
Kinetic Friction
If two systems are in contact and moving relative to one another, then the friction between them is called kinetic friction.

Imagine, for example, trying to slide a heavy crate across a concrete floor—you may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion it is easier to keep it in motion than it was to get it started, indicating that the kinetic friction force is less than the static friction force. If you add mass to the crate, say by placing a box on top of it, you need to push even harder to get it started and also to keep it moving. Furthermore, if you oiled the concrete you would find it to be easier to get the crate started and keep it going (as you might expect).

[link] is a crude pictorial representation of how friction occurs at the interface between two objects. Close-up inspection of these surfaces shows them to be rough. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. A considerable force can be resisted by friction with no apparent motion. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them. Part of the friction is due to adhesive forces between the surface molecules of the two objects, which explain the dependence of friction on the nature of the substances. Adhesion varies with substances in contact and is a complicated aspect of surface physics. Once an object is moving, there are fewer points of contact (fewer molecules adhering), so less force is required to keep the object moving. At small but nonzero speeds, friction is nearly independent of speed.



Frictional forces, such as f, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of

the surfaces in contact, as seen in the expanded view. In order for the object to move, it must rise to where the peaks can skip along the bottom surface. Thus a force is required just to set the object in motion. Some of the peaks will be broken off, also requiring a force to maintain motion. Much of the friction is actually due to attractive forces between molecules making up the two objects, so that even perfectly smooth surfaces are not friction-free. Such adhesive forces also depend on the substances the surfaces are made of, explaining, for example, why rubber-soled shoes slip less than those with leather soles.

The magnitude of the frictional force has two forms: one for static situations (static friction), the other for when there is motion (kinetic friction).

When there is no motion between the objects, the **magnitude of static friction $\mathbf{f}_\mathrm{s}$** is

Equation:

$$f_\mathrm{s} \leq \mu_\mathrm{s} N,$$

where μ_s is the coefficient of static friction and N is the magnitude of the normal force (the force perpendicular to the surface).

Note:

Magnitude of Static Friction

Magnitude of static friction f_s is

Equation:

$$f_{\mathrm{s}} \leq \mu_{\mathrm{s}} N,$$

where μ_{s} is the coefficient of static friction and N is the magnitude of the normal force.

The symbol $\leq$ means *less than or equal to*, implying that static friction can have a minimum and a maximum value of $\mu_{\mathrm{s}} N$. Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds $f_{\mathrm{s(max)}}$, the object will move. Thus

Equation:

$$f_{\mathrm{s(max)}} = \mu_{\mathrm{s}} N.$$

Once an object is moving, the **magnitude of kinetic friction $\mathbf{f}_{\mathrm{k}}$** is given by

Equation:

$$f_{\mathrm{k}} = \mu_{\mathrm{k}} N,$$

where μ_{k} is the coefficient of kinetic friction. A system in which $f_{\mathrm{k}} = \mu_{\mathrm{k}} N$ is described as a system in which *friction behaves simply*.

Note:

Magnitude of Kinetic Friction

The magnitude of kinetic friction f_{k} is given by

Equation:

$$f_{\mathrm{k}} = \mu_{\mathrm{k}} N,$$

where μ_{k} is the coefficient of kinetic friction.

As seen in [link], the coefficients of kinetic friction are less than their static counterparts. That values of μ in [link] are stated to only one or, at most, two digits is an indication of the approximate description of friction given by the above two equations.

System	**Static friction** μ_s	**Kinetic friction** μ_k
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7

System	Static friction μ_s	Kinetic friction μ_k
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.4	0.02

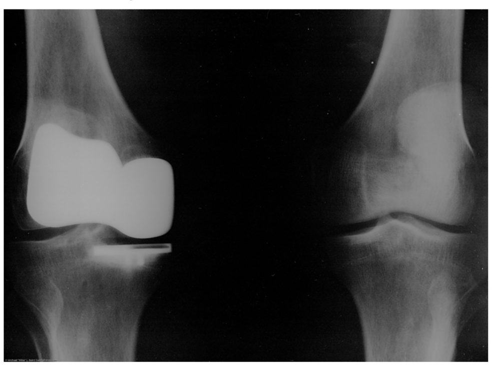
Coefficients of Static and Kinetic Friction

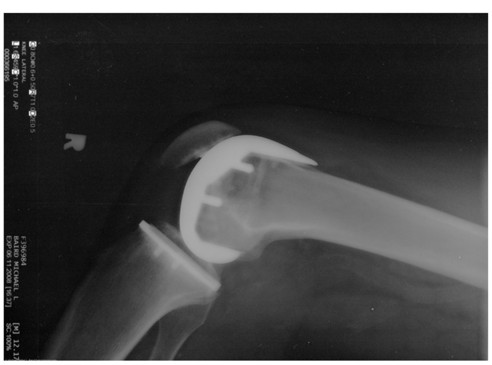
The equations given earlier include the dependence of friction on materials and the normal force. The direction of friction is always opposite that of motion, parallel to the surface between objects, and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight, $W = mg = (100 \text{ kg})(9.80 \text{ m/s}^2) = 980 \text{ N}$, perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than $f_{\text{s(max)}} = \mu_{\text{s}} N = (0.45)(980 \text{ N}) = 440 \text{ N}$ to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N ($f_{\text{k}} = \mu_{\text{k}} N = (0.30)(980 \text{ N}) = 290 \text{ N}$) would keep it moving at a constant speed. If the floor is lubricated, both coefficients are considerably less than they would be without lubrication. Coefficient of friction is a unit less quantity with a magnitude usually between 0 and 1.0. The coefficient of the friction depends on the two surfaces that are in contact.

Note:
Take-Home Experiment
Find a small plastic object (such as a food container) and slide it on a kitchen table by giving it a gentle tap. Now spray water on the table,

simulating a light shower of rain. What happens now when you give the object the same-sized tap? Now add a few drops of (vegetable or olive) oil on the surface of the water and give the same tap. What happens now? This latter situation is particularly important for drivers to note, especially after a light rain shower. Why?

Many people have experienced the slipperiness of walking on ice. However, many parts of the body, especially the joints, have much smaller coefficients of friction—often three or four times less than ice. A joint is formed by the ends of two bones, which are connected by thick tissues. The knee joint is formed by the lower leg bone (the tibia) and the thighbone (the femur). The hip is a ball (at the end of the femur) and socket (part of the pelvis) joint. The ends of the bones in the joint are covered by cartilage, which provides a smooth, almost glassy surface. The joints also produce a fluid (synovial fluid) that reduces friction and wear. A damaged or arthritic joint can be replaced by an artificial joint ([link]). These replacements can be made of metals (stainless steel or titanium) or plastic (polyethylene), also with very small coefficients of friction.





Artificial knee replacement is a procedure that has been performed for more than 20 years. In this figure, we see the post-op X-rays of the right knee joint replacement. (credit: Mike Baird, Flickr)

Other natural lubricants include saliva produced in our mouths to aid in the swallowing process, and the slippery mucus found between organs in the body, allowing them to move freely past each other during heartbeats, during breathing, and when a person moves. Artificial lubricants are also common in hospitals and doctor's clinics. For example, when ultrasonic imaging is carried out, the gel that couples the transducer to the skin also serves to to lubricate the surface between the transducer and the skin—thereby reducing the coefficient of friction between the two surfaces. This allows the transducer to mover freely over the skin.

We have discussed that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Furthermore, simple friction is always proportional to the normal force.

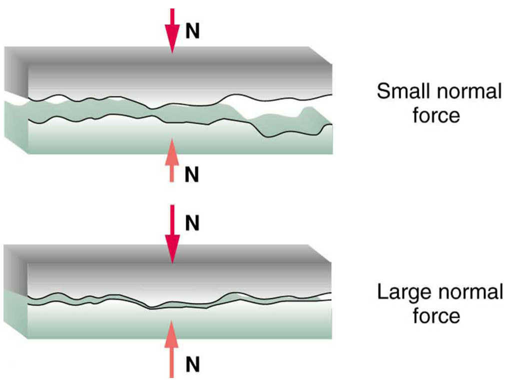
Note:

Making Connections: Submicroscopic Explanations of Friction

The simpler aspects of friction dealt with so far are its macroscopic (large-scale) characteristics. Great strides have been made in the atomic-scale explanation of friction during the past several decades. Researchers are finding that the atomic nature of friction seems to have several fundamental characteristics. These characteristics not only explain some of the simpler aspects of friction—they also hold the potential for the development of nearly friction-free environments that could save hundreds

of billions of dollars in energy which is currently being converted (unnecessarily) to heat.

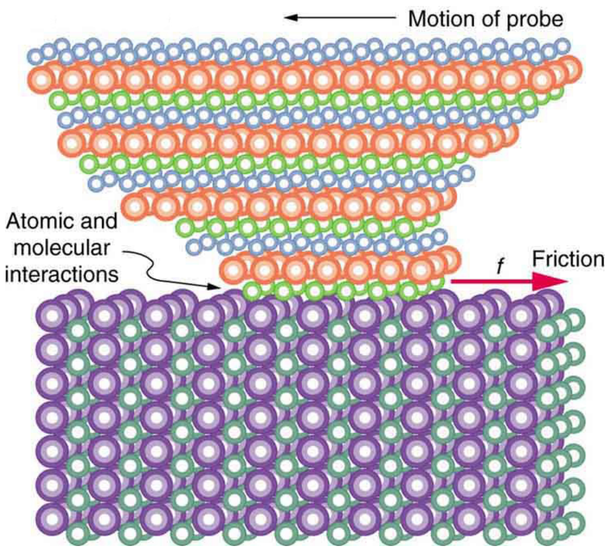
[link] illustrates one macroscopic characteristic of friction that is explained by microscopic (small-scale) research. We have noted that friction is proportional to the normal force, but not to the area in contact, a somewhat counterintuitive notion. When two rough surfaces are in contact, the actual contact area is a tiny fraction of the total area since only high spots touch. When a greater normal force is exerted, the actual contact area increases, and it is found that the friction is proportional to this area.



Two rough surfaces in contact have a much smaller area of actual contact than their total area. When there is a greater normal force as a result of a greater applied force, the area of actual contact increases as does friction.

But the atomic-scale view promises to explain far more than the simpler features of friction. The mechanism for how heat is generated is now being determined. In other words, why do surfaces get warmer when rubbed?

Essentially, atoms are linked with one another to form lattices. When surfaces rub, the surface atoms adhere and cause atomic lattices to vibrate—essentially creating sound waves that penetrate the material. The sound waves diminish with distance and their energy is converted into heat. Chemical reactions that are related to frictional wear can also occur between atoms and molecules on the surfaces. [link] shows how the tip of a probe drawn across another material is deformed by atomic-scale friction. The force needed to drag the tip can be measured and is found to be related to shear stress, which will be discussed later in this chapter. The variation in shear stress is remarkable (more than a factor of 10^{12}) and difficult to predict theoretically, but shear stress is yielding a fundamental understanding of a large-scale phenomenon known since ancient times—friction.



The tip of a probe is deformed sideways by frictional force as the probe is dragged across a surface. Measurements of how the force varies for different materials are yielding fundamental insights into the atomic nature of friction.

Section Summary

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force N pushing the systems together. (A normal force is always perpendicular to the contact surface between systems.) Friction depends on both of the materials involved. The magnitude of static friction f_s between systems stationary relative to one another is given by
  Equation:
  $$f_s \leq \mu_s N,$$
  where μ_s is the coefficient of static friction, which depends on both of the materials.
- The kinetic friction force f_k between systems moving relative to one another is given by
  Equation:
  $$f_k = \mu_k N,$$
  where μ_k is the coefficient of kinetic friction, which also depends on both materials.

Conceptual Questions

Exercise:

Problem:

Define normal force. What is its relationship to friction when friction behaves simply?

Exercise:

Problem:

The glue on a piece of tape can exert forces. Can these forces be a type of simple friction? Explain, considering especially that tape can stick to vertical walls and even to ceilings.

Exercise:

Problem:

When you learn to drive, you discover that you need to let up slightly on the brake pedal as you come to a stop or the car will stop with a jerk. Explain this in terms of the relationship between static and kinetic friction.

Exercise:

Problem:

When you push a piece of chalk across a chalkboard, it sometimes screeches because it rapidly alternates between slipping and sticking to the board. Describe this process in more detail, in particular explaining how it is related to the fact that kinetic friction is less than static friction. (The same slip-grab process occurs when tires screech on pavement.)

Glossary

friction
: a force that opposes relative motion or attempts at motion between systems in contact

kinetic friction
: a force that opposes the motion of two systems that are in contact and moving relative to one another

static friction
: a force that opposes the motion of two systems that are in contact and are not moving relative to one another

magnitude of static friction

$f_{\mathrm{s}} \leq \mu_{\mathrm{s}}\, N$, where μ_{s} is the coefficient of static friction and N is the magnitude of the normal force

magnitude of kinetic friction

$f_{\mathrm{k}} = \mu_{\mathrm{k}} N$, where μ_{k} is the coefficient of kinetic friction

Newton’s Universal Law of Gravitation

- Explain Earth’s gravitational force.
- Describe the gravitational effect of the Moon on Earth.
- Explain sensation of weightlessness in space.

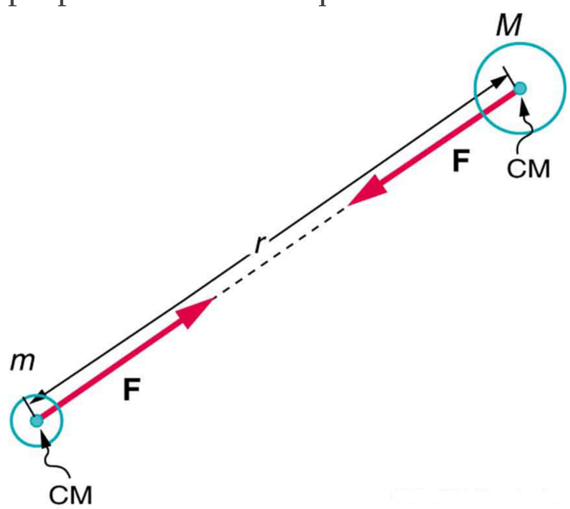
What do aching feet, a falling apple, and the orbit of the Moon have in common? Each is caused by the gravitational force. Our feet are strained by supporting our weight—the force of Earth’s gravity on us. An apple falls from a tree because of the same force acting a few meters above Earth’s surface. And the Moon orbits Earth because gravity is able to supply the necessary centripetal force at a distance of hundreds of millions of meters. In fact, the same force causes planets to orbit the Sun, stars to orbit the center of the galaxy, and galaxies to cluster together. Gravity is another example of underlying simplicity in nature. It is the weakest of the four basic forces found in nature, and in some ways the least understood. It is a force that acts at a distance, without physical contact, and is expressed by a formula that is valid everywhere in the universe, for masses and distances that vary from the tiny to the immense.

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See [link]. But Newton was not the first to suspect that the same force caused both our weight and the motion of planets. His forerunner Galileo Galilei had contended that falling bodies and planetary motions had the same cause. Some of Newton’s contemporaries, such as Robert Hooke, Christopher Wren, and Edmund Halley, had also made some progress toward understanding gravitation. But Newton was the first to propose an exact mathematical form and to use that form to show that the motion of heavenly bodies should be conic sections—circles, ellipses, parabolas, and hyperbolas. This theoretical prediction was a major triumph—it had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose a mechanism that caused them to follow these paths and not others.

According to early accounts, Newton was inspired to make the connection between falling bodies and astronomical motions when he saw an apple fall from a tree and realized that if the gravitational force could extend above the ground to a tree, it might also reach the Sun. The inspiration of Newton's apple is a part of worldwide folklore and may even be based in fact. Great importance is attached to it because Newton's universal law of gravitation and his laws of motion

answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature. Scientists still expect underlying simplicity to emerge from their ongoing inquiries into nature.

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them. Stated in modern language, **Newton's universal law of gravitation** states that every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.



Gravitational attraction is along a line joining the centers of mass of these two bodies. The magnitude of the force is the same on each, consistent with Newton's third law.

Note:
Misconception Alert
The magnitude of the force on each object (one has larger mass than the other) is the same, consistent with Newton's third law.

The bodies we are dealing with tend to be large. To simplify the situation we assume that the body acts as if its entire mass is concentrated at one specific point called the **center of mass** (CM). For two bodies having masses m and M with a distance r between their centers of mass, the equation for Newton's universal law of gravitation is
Equation:

$$F = G\frac{\text{mM}}{r^2},$$

where F is the magnitude of the gravitational force and G is a proportionality factor called the **gravitational constant**. G is a universal gravitational constant—that is, it is thought to be the same everywhere in the universe. It has been measured experimentally to be
Equation:

$$G = 6.673 \times 10^{-11}\,\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$$

in SI units. Note that the units of G are such that a force in newtons is obtained from $F = G\frac{\text{mM}}{r^2}$, when considering masses in kilograms and distance in meters. For example, two 1.000 kg masses separated by 1.000 m will experience a gravitational attraction of 6.673×10^{-11} N. This is an extraordinarily small force. The small magnitude of the gravitational force is consistent with everyday experience. We are unaware that even large objects like mountains exert gravitational forces on us. In fact, our body weight is the force of attraction of the *entire Earth* on us with a mass of 5.98×10^{24} kg.

Recall that the acceleration due to gravity g is about $9.80\ \text{m/s}^2$ on Earth. We can now determine why this is so. The weight of an object *mg* is the gravitational force between it and Earth. Substituting *mg* for F in Newton's universal law of gravitation gives

Equation:

$$\text{mg} = G\frac{\text{mM}}{r^2},$$

where m is the mass of the object, M is the mass of Earth, and r is the distance to the center of Earth (the distance between the centers of mass of the object and Earth). See [link]. The mass m of the object cancels, leaving an equation for g:

Equation:

$$g = G\frac{M}{r^2}.$$

Substituting known values for Earth's mass and radius (to three significant figures),

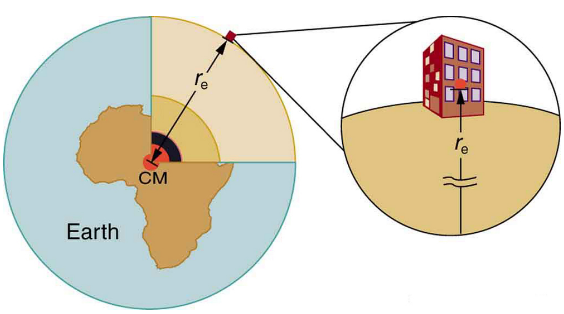
Equation:

$$g = \left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) \times \frac{5.98 \times 10^{24}\ \text{kg}}{(6.38 \times 10^6\ \text{m})^2},$$

and we obtain a value for the acceleration of a falling body:

Equation:

$$g = 9.80\ \text{m/s}^2.$$



The distance between the

centers of mass of Earth and an object on its surface is very nearly the same as the radius of Earth, because Earth is so much larger than the object.

This is the expected value *and is independent of the body’s mass*. Newton’s law of gravitation takes Galileo’s observation that all masses fall with the same acceleration a step further, explaining the observation in terms of a force that causes objects to fall—in fact, in terms of a universally existing force of attraction between masses.

Note:
Take-Home Experiment
Take a marble, a ball, and a spoon and drop them from the same height. Do they hit the floor at the same time? If you drop a piece of paper as well, does it behave like the other objects? Explain your observations.

Example:
Earth’s Gravitational Force on a Mass
(a) Determine the weight of a 5.00 kg rock when on Earth's surface.
(b) Determine the weight of a 5.00 kg rock when 3620 km above the surface of Earth.
Strategy for (a)
Use acceleration due to gravity near Earth's surface and Newton's second law.
Solution for (a)
Equation:

$$F = m\,g = 5.00\,\mathrm{kg} \times 9.80\,\mathrm{m/s^2} = 49.0\,\mathrm{N}$$

Strategy for (b)
Use Newton’s universal law of gravitation. Remember that distance is from the center of Earth. In this case the distance is 6380 km + 3620 km = 10,000 km = 1.00×10^7m.

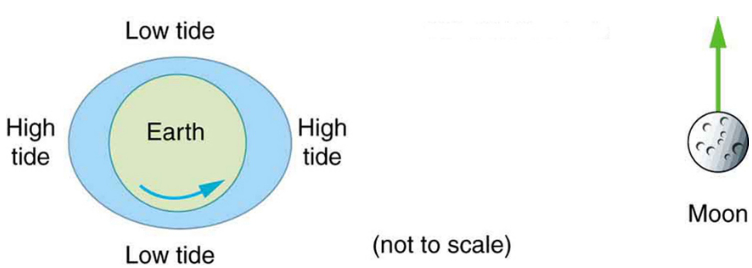
Solution for (b)

Equation:

$$F = G\frac{mM}{r^2} = \left(6.673 \times 10^{-11}\frac{\mathrm{N}\cdot\mathrm{m}^2}{\mathrm{kg}^2}\right)\frac{(5.00\mathrm{kg})\left(5.98 \times 10^{24}\mathrm{kg}\right)}{\left(1.00 \times 10^{7}\mathrm{m}\right)^2} = 20.0\mathrm{N}$$

Tides

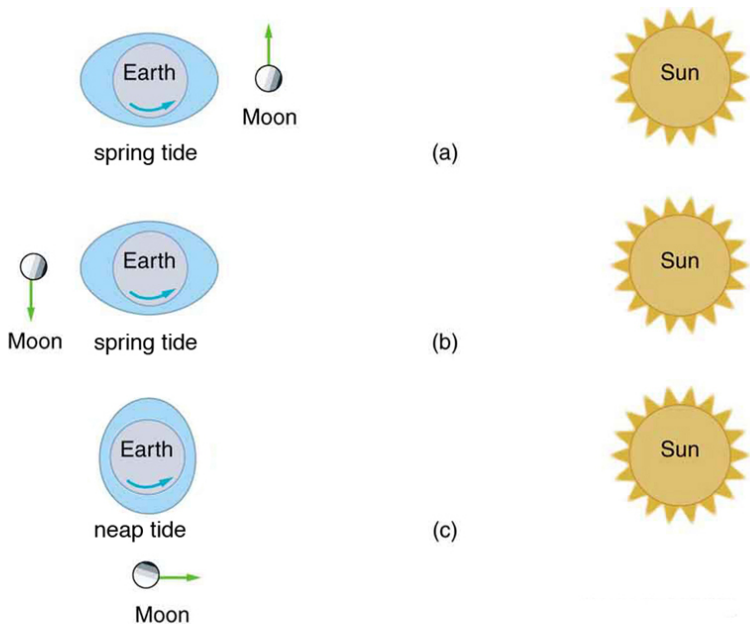
Ocean tides are one very observable result of the Moon's gravity acting on Earth. [link] is a simplified drawing of the Moon's position relative to the tides. Because water easily flows on Earth's surface, a high tide is created on the side of Earth nearest to the Moon, where the Moon's gravitational pull is strongest. Why is there also a high tide on the opposite side of Earth? The answer is that Earth is pulled toward the Moon more than the water on the far side, because Earth is closer to the Moon. So the water on the side of Earth closest to the Moon is pulled away from Earth, and Earth is pulled away from water on the far side. As Earth rotates, the tidal bulge (an effect of the tidal forces between an orbiting natural satellite and the primary planet that it orbits) keeps its orientation with the Moon. Thus there are two tides per day (the actual tidal period is about 12 hours and 25.2 minutes), because the Moon moves in its orbit each day as well).



The Moon causes ocean tides by attracting the water on the near side more than Earth, and by attracting Earth more than the water on the far side. The distances and sizes are not to scale. For this simplified representation of the Earth-Moon system, there are two high and two low tides per day

at any location, because Earth rotates under the tidal bulge.

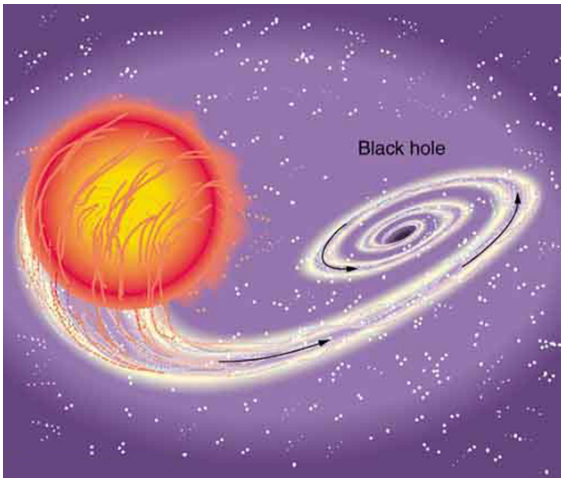
The Sun also affects tides, although it has about half the effect of the Moon. However, the largest tides, called spring tides, occur when Earth, the Moon, and the Sun are aligned. The smallest tides, called neap tides, occur when the Sun is at a 90º angle to the Earth-Moon alignment.



(a, b) Spring tides: The highest tides occur when Earth, the Moon, and the Sun are aligned. (c) Neap tide: The lowest tides occur when the Sun lies at 90º to the Earth-Moon alignment. Note that this figure is not drawn to scale.

Tides are not unique to Earth but occur in many astronomical systems. The most extreme tides occur where the gravitational force is the strongest and varies most rapidly, such as near black holes (see [link]). A few likely candidates for black holes have been observed in our galaxy. These have masses greater than the Sun

but have diameters only a few kilometers across. The tidal forces near them are so great that they can actually tear matter from a companion star.

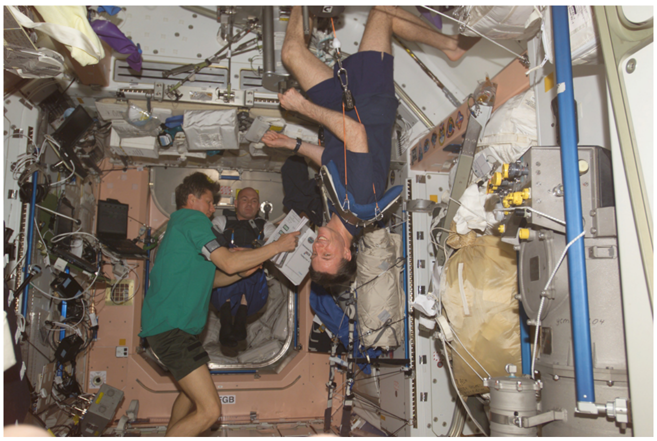


A black hole is an object with such strong gravity that not even light can escape it. This black hole was created by the supernova of one star in a two-star system. The tidal forces created by the black hole are so great that it tears matter from the companion star. This matter is compressed and heated as it is sucked into the black hole, creating light and X-rays observable from Earth.

”Weightlessness” and Microgravity

You may have seen images of astronauts in International Space Station, floating in their environment. What is the effect of “weightlessness” upon an astronaut who is in orbit for months? Or what about the effect of weightlessness upon plant growth? Weightlessness doesn’t mean that an astronaut is not being acted upon by the gravitational force. There is no “zero gravity” in an astronaut’s orbit. The term

just means that the astronaut is in free-fall, accelerating with the acceleration due to gravity. If an elevator cable breaks, the passengers inside will be in free fall and will experience weightlessness. You can experience short periods of weightlessness in some rides in amusement parks, and the commercial “zero-G” airplane rides simulate this experience of weightlessness about 20 seconds at a time.



Astronauts experiencing weightlessness on board the International Space Station. (credit: NASA)

Microgravity refers to an environment in which the acceleration of a body due to non-gravitational forces is small compared with that produced by Earth on its surface. Many interesting biology and physics topics have been studied over the past three decades in the presence of microgravity. Of immediate concern is the effect on astronauts of extended times in outer space, such as at the International Space Station. Researchers have observed that muscles will atrophy (waste away) in this environment. There is also a corresponding loss of bone mass. Study continues on cardiovascular adaptation to space flight. On Earth, blood pressure is usually higher in the feet than in the head, because the higher column of blood exerts a downward force on it, due to gravity. When standing, 70% of your blood is below the level of the heart, while in a horizontal position, just the opposite occurs. What difference does the absence of this pressure differential have upon the heart?

Some findings in human physiology in space can be clinically important to the management of diseases back on Earth. On a somewhat negative note, spaceflight is known to affect the human immune system, possibly making the crew members more vulnerable to infectious diseases. Experiments flown in space also have shown that some bacteria grow faster in microgravity than they do on Earth. However, on a positive note, studies indicate that microbial antibiotic production can increase by a factor of two in space-grown cultures. One hopes to be able to understand these mechanisms so that similar successes can be achieved on the ground. In another area of physics space research, inorganic crystals and protein crystals have been grown in outer space that have much higher quality than any grown on Earth, so crystallography studies on their structure can yield much better results.

Plants have evolved with the stimulus of gravity and with gravity sensors. Roots grow downward and shoots grow upward. Plants might be able to provide a life support system for long duration space missions by regenerating the atmosphere, purifying water, and producing food. Some studies have indicated that plant growth and development are not affected by gravity, but there is still uncertainty about structural changes in plants grown in a microgravity environment.

Section Summary

- Newton's universal law of gravitation: Every particle in the universe attracts every other particle with a force along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. In equation form, this is
  Equation:

  $$F = G\frac{\mathrm{mM}}{r^2},$$

  where F is the magnitude of the gravitational force. G is the gravitational constant, given by $G = 6.673 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$.
- Newton's law of gravitation applies universally.

Conceptual Questions

Exercise:

Problem:

Action at a distance, such as is the case for gravity, was once thought to be illogical and therefore untrue. What is the ultimate determinant of the truth in physics, and why was this action ultimately accepted?

Exercise:

Problem:

Two friends are having a conversation. Anna says a satellite in orbit is in freefall because the satellite keeps falling toward Earth. Tom says a satellite in orbit is not in freefall because the acceleration due to gravity is not $9.80\ \mathrm{m/s^2}$. Who do you agree with and why?

Exercise:

Problem:

Newton's laws of motion and gravity were among the first to convincingly demonstrate the underlying simplicity and unity in nature. Many other examples have since been discovered, and we now expect to find such underlying order in complex situations. Is there proof that such order will always be found in new explorations?

Glossary

gravitational constant, *G*
: a proportionality factor used in the equation for Newton's universal law of gravitation; it is a universal constant—that is, it is thought to be the same everywhere in the universe

center of mass
: the point where the entire mass of an object can be thought to be concentrated

microgravity
: an environment in which the acceleration of a body due to non-gravitational forces is small compared with that produced by Earth on its surface

Newton's universal law of gravitation

every particle in the universe attracts every other particle with a force along a line joining them; the force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them

Centripetal Force

- Explain the role of centripetal force in a uniform circular motion.
- Calculate centripetal force and acceleration for simple situations.

Any force or combination of forces can cause a centripetal or radial acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth’s gravity on the Moon, friction between roller skates and a rink floor, a banked roadway’s force on a car, and forces on the tube of a spinning centrifuge.

Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of curvature, the same as the direction of centripetal acceleration. According to Newton’s second law of motion, net force is mass times acceleration: $F_{\text{net}} = ma$. For uniform circular motion, the acceleration is the centripetal acceleration—$a = a_c$. Thus, the magnitude of centripetal force F_{c} is

Equation:

$$F_{\text{c}} = ma_{\text{c}}.$$

By using the expressions for centripetal acceleration a_c from $a_c = \frac{v^2}{r}$, we get an expression for the centripetal force $\mathrm{F_c}$ in terms of mass, velocity, and radius of curvature:

Equation:

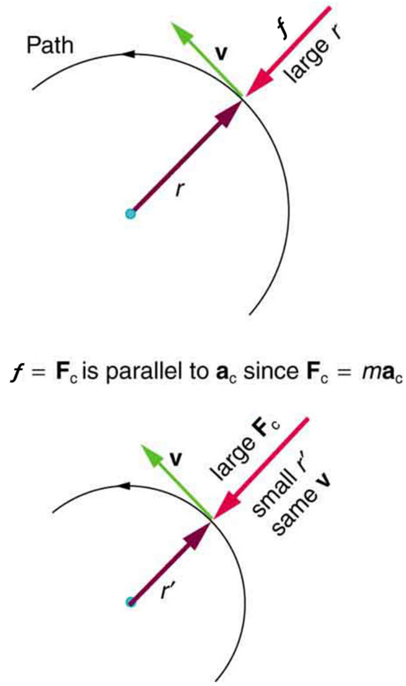
$$F_c = m\frac{v^2}{r}.$$

Note that if you solve the first expression for r, you get

Equation:

$$r = \frac{mv^2}{F_c}.$$

This implies that for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve.



The frictional force supplies the centripetal force and is numerically equal to it. Centripetal force is perpendicular to velocity and causes uniform circular motion. The larger the F_c, the smaller the radius of curvature r and the sharper the curve. The second curve has the same v, but a larger F_c produces a smaller r'.

Example:

What Coefficient of Friction Do Car Tires Need on a Flat Curve?

(a) Calculate the centripetal force exerted on a 900 kg car that negotiates a 500 m radius curve at 25.0 m/s.

(b) Assuming an unbanked curve, find the minimum static coefficient of friction, between the tires and the road, static friction being the reason that keeps the car from slipping (see [link]).

Strategy and Solution for (a)

We know that $F_{\text{c}} = \frac{mv^2}{r}$. Thus,

Equation:

$$F_{\text{c}} = \frac{mv^2}{r} = \frac{(900\ \text{kg})(25.0\ \text{m/s})^2}{(500\ \text{m})} = 1125\ \text{N}.$$

Strategy for (b)

[link] shows the forces acting on the car on an unbanked (level ground) curve. Friction is to the left, keeping the car from slipping, and because it is the only horizontal force acting on the car, the friction is the centripetal force in this case. We know that the maximum static friction (at which the tires roll but do not slip) is $\mu_{\text{s}}N$, where μ_{s} is the static coefficient of friction and N is the normal force. The normal force equals the car's weight on level ground, so that $N = mg$. Thus the centripetal force in this situation is

Equation:

$$F_{\text{c}} = f = \mu_{\text{s}}N = \mu_{\text{s}}\text{mg}.$$

Now we have a relationship between centripetal force and the coefficient of friction. Using the expression for F_{c}

Equation:

$$F_c = m\frac{v^2}{r},$$

Equation:

$$m\frac{v^2}{r} = \mu_s \text{mg}.$$

We solve this for μ_s, noting that mass cancels, and obtain

Equation:

$$\mu_s = \frac{v^2}{\text{rg}}.$$

Solution for (b)

Substituting the knowns,

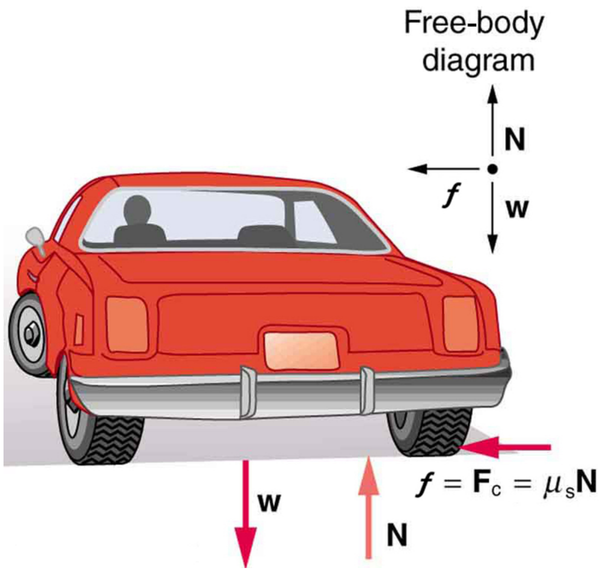
Equation:

$$\mu_s = \frac{(25.0 \text{ m/s})^2}{(500 \text{ m})(9.80 \text{ m/s}^2)} = 0.13.$$

(Because coefficients of friction are approximate, the answer is given to only two digits.)

Discussion

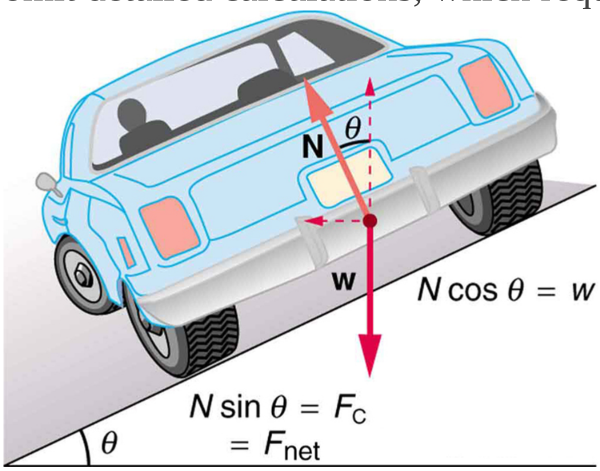
The coefficient of friction found in part (b) is much smaller than is typically found between tires and roads. The car will still negotiate the curve if the coefficient is greater than 0.13, because static friction is a responsive force, being able to assume a value less than but no more than $\mu_s N$. A higher coefficient would also allow the car to negotiate the curve at a higher speed, but if the coefficient of friction is less, the safe speed would be less than 25 m/s. Note that mass cancels, implying that in this example, it does not matter how heavily loaded the car is to negotiate the turn. Mass cancels because friction is assumed proportional to the normal force, which in turn is proportional to mass. If the surface of the road were banked, the normal force would be less as will be discussed below.



This car on level ground is moving away and turning to the left. The centripetal force causing the car to turn in a circular path is due to friction between the tires and the road. A minimum coefficient of friction is needed, or the car will move in a larger-radius curve and leave the roadway.

In the case of **banked curves**, where the slope of the road helps you negotiate the curve, some or all of the necessary centripetal force is provided by the normal force. See [link]. The greater the angle θ, the faster you can take the curve. Race tracks for bikes as well as cars, for example, often have steeply banked curves. In an “ideally banked curve,” the angle θ is such that you can negotiate the curve at a certain speed without the aid of friction between the tires and the road. Conceptually, for **ideal banking**, the net external force equals the horizontal centripetal force in the absence of friction. The components of the normal force $\mathbf{N}$ in the horizontal and vertical directions must equal the centripetal force and the weight of the car, respectively.

[link] shows a free body diagram for a car on a frictionless banked curve. If the angle θ is ideal for the speed and radius, then the net external force will equal the necessary centripetal force. The only two external forces acting on the car are its weight $\mathbf{w}$ and the normal force of the road $\mathbf{N}$. (A frictionless surface can only exert a force perpendicular to the surface—that is, a normal force.) These two forces must add to give a net external force that is horizontal toward the center of curvature and has magnitude mv^2/r. We omit detailed calculations, which require trigonometry.



The car on this banked curve is moving away and turning to the left.

Section Summary

- Centripetal force $\mathbf{F}_\mathrm{c}$ is any force causing uniform circular motion. It is a “center-seeking” force that always points toward the center of rotation. It is perpendicular to linear velocity v and has magnitude
  Equation:

  $$F_\mathrm{c} = ma_\mathrm{c},$$

  which can also be expressed as
  Equation:

$$F_c = m\frac{v^2}{r}.$$

Conceptual Questions

Exercise:

Problem:

If you wish to reduce the stress (which is related to centripetal force) on high-speed tires, would you use large- or small-diameter tires? Explain.

Exercise:

Problem:

Define centripetal force. Can any type of force (for example, tension, gravitational force, friction, and so on) be a centripetal force? Can any combination of forces be a centripetal force?

Exercise:

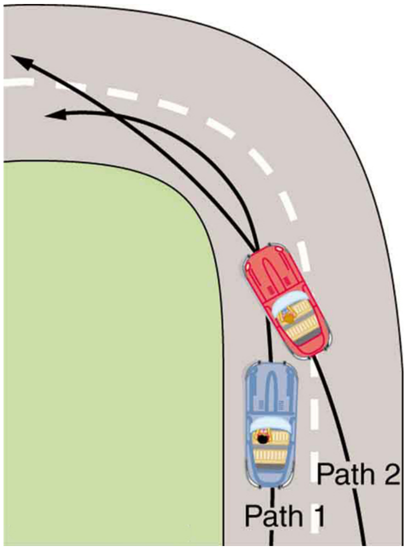
Problem:

If centripetal force is directed toward the center, why do you feel that you are 'thrown' away from the center as a car goes around a curve? Explain.

Exercise:

Problem:

Race car drivers routinely cut corners as shown in [link]. Explain how this allows the curve to be taken at the greatest speed.



Two paths around a race track curve are shown. Race car drivers will take the inside path (called cutting the corner) whenever possible because it allows them to take the curve at the highest speed.

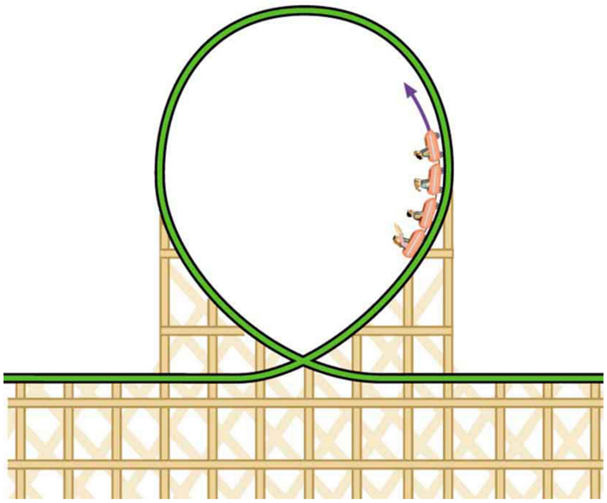
Exercise:

Problem:

A number of amusement parks have rides that make vertical loops like the one shown in [link]. For safety, the cars are attached to the rails in such a way that they cannot fall off. If the car goes over the top at just the right speed, gravity alone will supply the centripetal force. What other force acts and what is its direction if:

(a) The car goes over the top at faster than this speed?

(b)The car goes over the top at slower than this speed?



Amusement rides with a vertical loop are an example of a form of curved motion.

Exercise:

Problem:

What is the direction of the force exerted by the car on the passenger as the car goes over the top of the amusement ride pictured in [link] under the following circumstances:

(a) The car goes over the top at such a speed that the gravitational force is the only force acting?

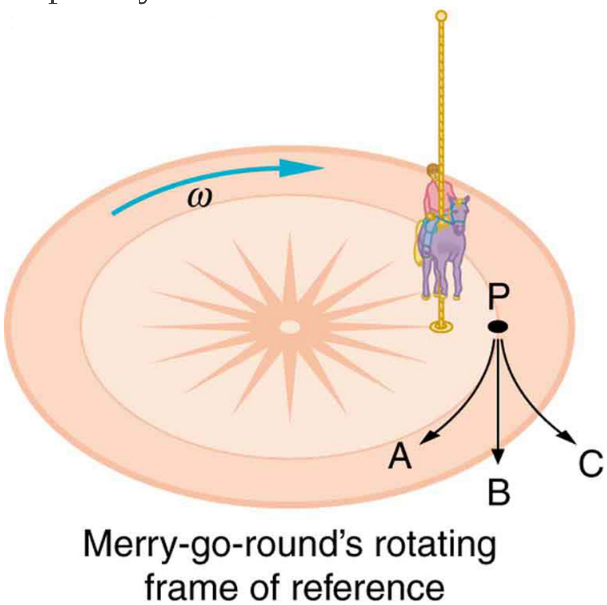
(b) The car goes over the top faster than this speed?

(c) The car goes over the top slower than this speed?

Exercise:

Problem:

Suppose a child is riding on a merry-go-round at a distance about halfway between its center and edge. She has a lunch box resting on wax paper, so that there is very little friction between it and the merry-go-round. Which path shown in [link] will the lunch box take when she lets go? The lunch box leaves a trail in the dust on the merry-go-round. Is that trail straight, curved to the left, or curved to the right? Explain your answer.



A child riding on a merry-go-round releases her lunch box at point P. This is a view from above the clockwise rotation. Assuming it slides with negligible friction, will it follow path A, B, or C, as viewed from Earth's frame of reference? What will be the shape of the path it leaves in the dust on the merry-go-round?

Exercise:

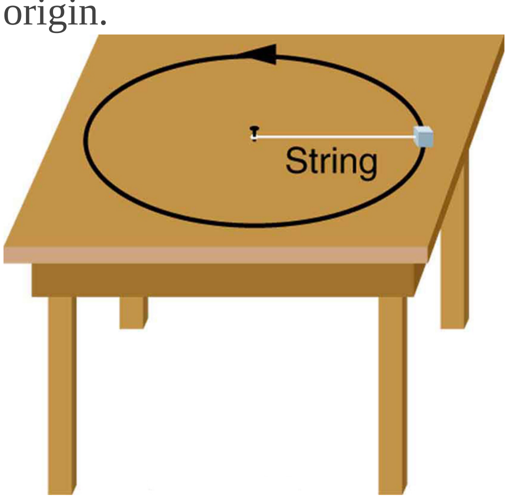
Problem:

Do you feel yourself thrown to either side when you negotiate a curve that is ideally banked for your car's speed? What is the direction of the force exerted on you by the car seat?

Exercise:

Problem:

Suppose a mass is moving in a circular path on a frictionless table as shown in figure. In the Earth's frame of reference, there is no centrifugal force pulling the mass away from the centre of rotation, yet there is a very real force stretching the string attaching the mass to the nail. Using concepts related to centripetal force and Newton's third law, explain what force stretches the string, identifying its physical origin.



A mass attached to a nail on a frictionless table moves in a circular path. The force stretching the string is real and not fictional. What is the physical origin of the force on the string?

Glossary

centripetal force
: any net force causing uniform circular motion

ideal banking
: the sloping of a curve in a road, where the angle of the slope allows the vehicle to negotiate the curve at a certain speed without the aid of friction between the tires and the road; the net external force on the vehicle equals the horizontal centripetal force in the absence of friction

banked curve
: the curve in a road that is sloping in a manner that helps a vehicle negotiate the curve

Introduction to Work and Energy

class="introduction"

How many forms of energy can you identify in this photograph of a wind farm in Iowa? (credit: Jürgen from Sandesneben, Germany, Wikimedia Commons)

Energy plays an essential role both in everyday events and in scientific phenomena. You can no doubt name many forms of energy, from that provided by our foods, to the energy we use to run our cars, to the sunlight that warms us on the beach. You can also cite examples of what people call energy that may not be scientific, such as someone having an energetic personality. Not only does energy have many interesting forms, it is

involved in almost all phenomena, and is one of the most important concepts of physics. What makes it even more important is that the total amount of energy in the universe is constant. Energy can change forms, but it cannot appear from nothing or disappear without a trace. Energy is thus one of a handful of physical quantities that we say is *conserved*.

Conservation of energy (as physicists like to call the principle that energy can neither be created nor destroyed) is based on experiment. Even as scientists discovered new forms of energy, conservation of energy has always been found to apply. Perhaps the most dramatic example of this was supplied by Einstein when he suggested that mass is equivalent to energy (his famous equation $E = \mathrm{mc}^2$).

From a societal viewpoint, energy is one of the major building blocks of modern civilization. Energy resources are key limiting factors to economic growth. The world use of energy resources, especially oil, continues to grow, with ominous consequences economically, socially, politically, and environmentally. We will briefly examine the world's energy use patterns at the end of this chapter.

There is no simple, yet accurate, scientific definition for energy. Energy is characterized by its many forms and the fact that it is conserved. We can loosely define **energy** as the ability to do work, admitting that in some circumstances not all energy is available to do work. Because of the association of energy with work, we begin the chapter with a discussion of work. Work is intimately related to energy and how energy moves from one system to another or changes form.

Work: The Scientific Definition

- Explain how an object must be displaced for a force on it to do work.
- Explain how relative directions of force and displacement determine whether the work done is positive, negative, or zero.

What It Means to Do Work

The scientific definition of work differs in some ways from its everyday meaning. Certain things we think of as hard work, such as writing an exam or carrying a heavy load on level ground, are not work as defined by a scientist. The scientific definition of work reveals its relationship to energy—whenever work is done, energy is transferred.

For work, in the scientific sense, to be done, a force must be exerted and there must be motion or displacement in the direction of the force.

Formally, the **work** done on a system by a constant force is defined to be *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as
Equation:

$$W = F_{||}d$$

Where W is work, d is the distance the force acts, and $F_{||}$ is the force parallel to the direction of motion. We can also write this more simply as
Equation:

$$W = Fd$$

as long as one keeps in mind that the force is in the same direction as the distance.

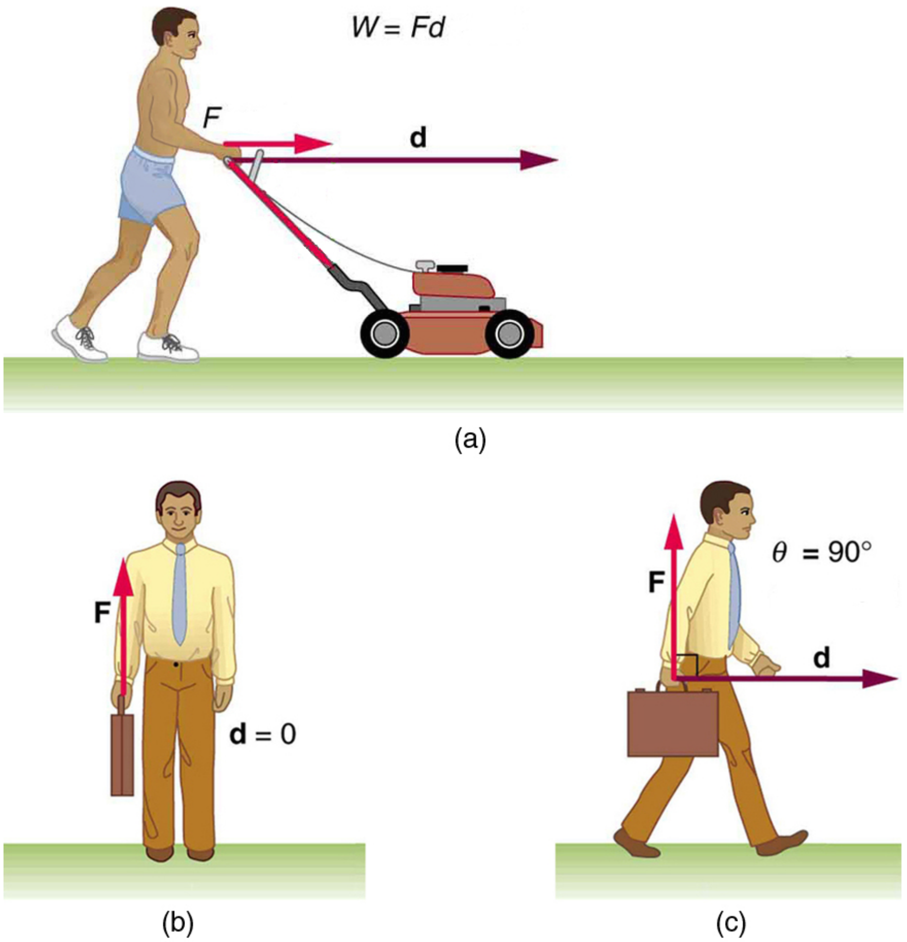
To find the work done on a system that undergoes motion that is not one-way or that is in two or three dimensions, we would divide the motion into

one-way one-dimensional segments and add up the work done over each segment.

Note:
What is Work?
The work done on a system by a constant force is *the product of the component of the force in the direction of motion times the distance through which the force acts*. For one-way motion in one dimension, this is expressed in equation form as
Equation:

$$W = Fd$$

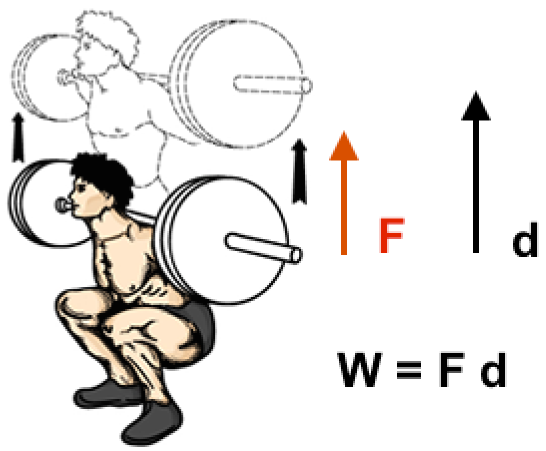
Where W is work, d is the distance the force acts, F is the force parallel to the direction of motion.



Examples of work. (a) Note that the force is in the direction of motion. (b) A person holding a briefcase does no work on it, because there is no motion. No energy is transferred to or from the briefcase. (c) The person moving the briefcase horizontally at a constant speed does no work on it, and transfers no energy to it. The force acting on the briefcase (against gravity) is perpendicular to the direction of motion.

To examine what the definition of work means, let us consider the other situations shown in [link]. The person holding the briefcase in [link](b) does no work, for example. Here $d = 0$, so $W = 0$. Why is it you get tired

just holding a load? The answer is that your muscles are doing work against one another, *but they are doing no work on the system of interest* (the “briefcase-Earth system”). There must be motion for work to be done, and there must be a component of the force in the direction of the motion. For example, the person carrying the briefcase on level ground in [link](c) does no work on it, because the force is perpendicular to the motion.



Work done when lifting a mass. Work is positive on the way up because force and displacement point in the same direction. On the way down work is negative due to force and displacement pointing in opposite directions (force up and displacement down).

Work can be positive or negative. In [link] work done lifting the mass is positive because both force and displacement are in the same direction. Likewise, when the mass is lowered the work done is negative because the force and displacement are in *opposite* directions. We will soon see that

positive work adds energy to the system and negative work removes energy from a system.

Calculating Work

Work and energy have the same units. From the definition of work, we see that those units are force times distance. Thus, in SI units, work and energy are measured in **newton-meters**. A newton-meter is given the special name **joule** (J), and $1\ \mathrm{J} = 1\ \mathrm{N} \cdot \mathrm{m} = 1\ \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2$. One joule is not a large amount of energy; it would lift a small 100-gram apple a distance of about 1 meter.

Example:
Calculating the Work You Do to Push a Lawn Mower Across a Large Lawn

How much work is done on the lawn mower by the person in [link](a) if he exerts a constant force of 75.0 N and pushes the mower 25.0 m on level ground? Compare it with this person's average daily intake of 10,000 kJ (about 2400 kcal) of food energy.

Strategy

We can solve this problem by substituting the given values into the definition of work done on a system, stated in the equation $W = Fd$. The force and displacement are given, so that only the work W is unknown. Note that force and displacement are in the same direction.

Solution

The equation for the work is

Equation:

$$W = Fd.$$

Substituting the known values gives

Equation:

$$W = (75.0\,\mathrm{N})(25.0\,\mathrm{m}) = 1875\,\mathrm{J} = 1.88 \times 10^3\,\mathrm{J}.$$

Equation: The ratio of the work done to the daily consumption is

$$\frac{W}{1.00 \times 10^{4}\,\mathrm{kJ}} = \frac{1.88 \times 10^{3}\ \mathrm{J}}{1.00 \times 10^{7}\ \mathrm{J}} = 1.88 \times 10^{-4}.$$

Discussion

This ratio is a tiny fraction of what the person consumes, but it is typical. Very little of the energy released in the consumption of food is used to do work. Even when we “work” all day long, less than 10% of our food energy intake is used to do work and more than 90% is converted to thermal energy or stored as chemical energy in fat.

Example:

Calculating the Work Done Lifting a Mass

The 60.0 kg mass shown in [link] is raised at a constant speed through a vertical distance of 0.800 m. What is the force that must be exerted? How much work is done lifting the mass?

Strategy

Constant speed tells us that the upward force must have the same magnitude as the downward force of gravity $F = mg$ (to have a zero net force). Using that information, along with the given values for mass and displacement, we can determine work using $W = Fd$.

Solution

The force is given by

$$F = mg = (60.0\,\mathrm{kg})(9.80\,\mathrm{m/s^2}) = 588\,\mathrm{N}.$$

The work done is

$$W = Fd = (588\,\mathrm{N})(0.800\,\mathrm{m}) = 470\,\mathrm{J}.$$

Discussion

Note that the work done lowering the mass back to its starting position would be $-470\ \mathrm{J}$ because force and displacement point in opposite directions. The total work done raising and lowering the mass is $470\ \mathrm{J} - 470\ \mathrm{J} = 0\ \mathrm{J}.$

Section Summary

- Work is the transfer of energy by a force acting on an object as it is displaced.
- The work W that a force $\mathbf{F}$ does on an object is the product of the magnitude F of the force parallel to the motion, times the magnitude d of the displacement. In symbols,

$$W = Fd.$$

- The SI unit for work and energy is the joule (J), where $1\text{ J} = 1\text{ N}\cdot\text{m} = 1\text{ kg}\cdot\text{m}^2/\text{s}^2$.
- The work done by a force is zero if the displacement is either zero or perpendicular to the force.
- The work done is positive if the force and displacement have the same direction, and negative if they have opposite direction.

Conceptual Questions

Exercise:

Problem:

Give an example of something we think of as work in everyday circumstances that is not work in the scientific sense. Is energy transferred or changed in form in your example? If so, explain how this is accomplished without doing work.

Exercise:

Problem:

Give an example of a situation in which there is a force and a displacement, but the force does no work. Explain why it does no work.

Exercise:

Problem:

Describe a situation in which a force is exerted for a long time but does no work. Explain.

Glossary

energy
: the ability to do work

work
: the transfer of energy by a force that causes an object to be displaced; the product of the component of the force in the direction of the displacement and the magnitude of the displacement

joule
: SI unit of work and energy, equal to one newton-meter

Kinetic Energy and the Work-Energy Theorem

- Explain work as a transfer of energy and net work as the work done by the net force.
- Explain and apply the work-energy theorem.

Work Transfers Energy

What happens to the work done on a system? Energy is transferred into the system, but in what form? Does it remain in the system or move on? The answers depend on the situation. For example, if a lawn mower is pushed just hard enough to keep it going at a constant speed, then energy put into the mower by the person is removed continuously by friction, and eventually leaves the system in the form of heat transfer. In contrast, work done on a briefcase by a person carrying it up stairs is stored in the briefcase-Earth system and can be recovered at any time. In fact, the building of the pyramids in ancient Egypt is an example of storing energy in a system by doing work on the system. Some of the energy imparted to the stone blocks in lifting them during construction of the pyramids remains in the stone-Earth system and has the potential to do work.

In this section we begin the study of various types of work and forms of energy. We will find that some types of work leave the energy of a system constant, for example, whereas others change the system in some way, such as making it move. We will also develop definitions of important forms of energy, such as the energy of motion.

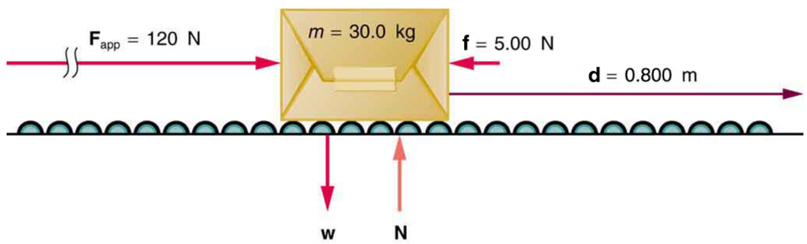
Net Work and the Work-Energy Theorem

We know from the study of Newton's laws that net force causes acceleration. We will see in this section that work done by the net force gives a system energy of motion, and in the process we will also find an expression for the energy of motion.

Let us start by considering the total, or net, work done on a system. Net work is defined to be the sum of work done by all external forces—that is,

net work is the work done by the net external force $\mathbf{F_{net}}$. In equation form, this is $W_{\text{net}} = F_{\text{net}}d$.

Consider a one-dimensional situation where a force is used to accelerate an object in a direction parallel to its initial velocity. Such a situation occurs for the package on the roller belt conveyor system shown in [link].



A package on a roller belt is pushed horizontally through a distance $\mathbf{d}$.

The force of gravity and the normal force acting on the package are perpendicular to the displacement and do no work. Moreover, they are also equal in magnitude and opposite in direction so they cancel in calculating the net force. The net force arises solely from the horizontal applied force $\mathbf{F}_{\text{app}}$ and the horizontal friction force $\mathbf{f}$. Thus, as expected, the net force is parallel to the displacement and the net work is given by

Equation:

$$W_{\text{net}} = F_{\text{net}}d.$$

The effect of the net force $\mathbf{F}_{\text{net}}$ is to accelerate the package from v_0 to v. By using Newton's second law, and doing some algebra, we can reach an interesting conclusion. Substituting $F_{\text{net}} = \text{ma}$ from Newton's second law gives

Equation:

$$W_{\text{net}} = \text{mad}.$$

To get a relationship between net work and the speed given to a system by the net force acting on it, we take $d = x - x_0$ and use the equation studied in "Motion Equations for Constant Acceleration in One Dimension" for the change in speed over a distance d if the acceleration has the constant value a; namely, $v^2 = {v_0}^2 + 2\text{ad}$ (note that a appears in the expression for the net work). Solving for acceleration gives $a = \frac{v^2 - {v_0}^2}{2d}$. When a is substituted into the preceding expression for W_{net}, we obtain

Equation:

$$W_{\text{net}} = m\left(\frac{v^2 - {v_0}^2}{2d}\right)d.$$

The d cancels, and we rearrange this to obtain

Equation:

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

This expression is called the **work-energy theorem**, and it actually applies *in general* (even for forces that vary in direction and magnitude), although we have derived it for the special case of a constant force parallel to the displacement. The theorem implies that the net work on a system equals the change in the quantity $\frac{1}{2}mv^2$. This quantity is our first example of a form of energy.

Note:

The Work-Energy Theorem

The net work on a system equals the change in the quantity $\frac{1}{2}mv^2$.

Equation:

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}\text{mv}_0^2$$

The quantity $\frac{1}{2}mv^2$ in the work-energy theorem is defined to be the translational **kinetic energy** (KE) of a mass m moving at a speed v. In equation form, the translational kinetic energy,

Equation:

$$\text{KE} = \frac{1}{2}mv^2,$$

is the energy associated with translational motion. Kinetic energy is a form of energy associated with the motion of a particle, single body, or system of objects moving together. The work-energy theorem can be compactly written as,

$$W_{\text{net}} = \Delta\text{KE}$$

where ΔKE is understood to mean "change in KE."

We are aware that it takes energy to get an object, like a car or the package in [link], up to speed, but it may be a bit surprising that kinetic energy is proportional to speed squared. This proportionality means, for example, that a car traveling at 40 m/s has four times the kinetic energy it has at 20 m/s, helping to explain why high-speed collisions are so devastating. We will now consider a series of examples to illustrate various aspects of work and energy.

Example:
Calculating the Kinetic Energy of a Package

Suppose a 30.0-kg package on the roller belt conveyor system in [link] is moving at 0.500 m/s. What is its kinetic energy?

Strategy

Because the mass m and speed v are given, the kinetic energy can be calculated from its definition as given in the equation $\text{KE} = \frac{1}{2}mv^2$.

Solution
The kinetic energy is given by
Equation:

$$\mathrm{KE} = \frac{1}{2}mv^2.$$

Entering known values gives
Equation:

$$\mathrm{KE} = 0.5(30.0\ \mathrm{kg})(0.500\ \mathrm{m/s})^2,$$

which yields
Equation:

$$\mathrm{KE} = 3.75\ \mathrm{kg \cdot m^2/s^2} = 3.75\ \mathrm{J}.$$

Discussion
Note that the unit of kinetic energy is the joule, the same as the unit of work, as mentioned when work was first defined. It is also interesting that, although this is a fairly massive package, its kinetic energy is not large at this relatively low speed. This fact is consistent with the observation that people can move packages like this without exhausting themselves.

Example:
Determining the Work to Accelerate a Package
Suppose that you push on the 30.0-kg package in [link] with a constant force of 120 N through a distance of 0.800 m, and that the opposing friction force averages 5.00 N.
(a) Calculate the net work done on the package. (b) Solve the same problem as in part (a), this time by finding the work done by each force that contributes to the net force.
Strategy and Concept for (a)
This is a motion in one dimension problem, because the downward force (from the weight of the package) and the normal force have equal magnitude and opposite direction, so that they cancel in calculating the net

force, while the applied force, friction, and the displacement are all horizontal. (See [link].) As expected, the net work is the net force times distance.

Solution for (a)

The net force is the push force minus friction, or $F_{\text{net}} = 120\text{ N} – 5.00\text{ N} = 115\text{ N}$. Thus the net work is

Equation:

$$\begin{aligned} W_{\text{net}} &= F_{\text{net}}d = (115\text{ N})(0.800\text{ m}) \\ &= 92.0\text{ N}\cdot\text{m} = 92.0\text{ J}. \end{aligned}$$

Discussion for (a)

This value is the net work done on the package. The person actually does more work than this, because friction opposes the motion. Friction does negative work and removes some of the energy the person expends and converts it to thermal energy. The net work equals the sum of the work done by each individual force.

Strategy and Concept for (b)

The forces acting on the package are gravity, the normal force, the force of friction, and the applied force. The normal force and force of gravity are each perpendicular to the displacement, and therefore do no work.

Solution for (b)

The applied force does work.

Equation:

$$W_{\text{app}} = F_{\text{app}}d = (120\,\text{N})(0.800\,\text{m}) = 96.0\,\text{J}$$

The friction force and displacement are in opposite directions. The work done by friction is therefore negative.

Equation:

$$W_{\text{fr}} = -F_{\text{fr}}d = -(5.00\,\text{N})(0.800\,\text{m}) = -4.00\,\text{J}$$

So the amounts of work done by gravity, by the normal force, by the applied force, and by friction are, respectively,

Equation:

$$\begin{array}{lcl} W_{\text{gr}} & = & 0, \\ W_{\text{N}} & = & 0, \\ W_{\text{app}} & = & 96.0\text{ J}, \\ W_{\text{fr}} & = & -4.00\text{ J}. \end{array}$$

The total work done as the sum of the work done by each force is then seen to be

Equation:

$$W_{\text{total}} = W_{\text{gr}} + W_{\text{N}} + W_{\text{app}} + W_{\text{fr}} = 92.0\text{ J}.$$

Discussion for (b)

The calculated total work W_{total} as the sum of the work by each force agrees, as expected, with the work W_{net} done by the net force. The work done by a collection of forces acting on an object can be calculated by either approach.

Example:

Determining Speed from Work and Energy

Find the speed of the package in [link] at the end of the push, using work and energy concepts.

Strategy

Here the work-energy theorem can be used, because we have just calculated the net work, W_{net}, and the initial kinetic energy, $\frac{1}{2}m{v_0}^2$. These calculations allow us to find the final kinetic energy, $\frac{1}{2}mv^2$, and thus the final speed v.

Solution

The work-energy theorem in equation form is

Equation:

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}m{v_0}^2.$$

Solving for $\frac{1}{2}mv^2$ gives

Equation:

$$\frac{1}{2}\mathrm{mv}^2 = W_{\text{net}} + \frac{1}{2}m{v_0}^2.$$

Thus,

Equation:

$$\frac{1}{2}mv^2 = 92.0\ \text{J} + 3.75\ \text{J} = 95.75\ \text{J}.$$

Solving for the final speed as requested and entering known values gives

Equation:

$$\begin{aligned} v &= \sqrt{\frac{2(95.75\ \text{J})}{m}} = \sqrt{\frac{191.5\ \text{kg}\cdot\text{m}^2/\text{s}^2}{30.0\ \text{kg}}} \\ &= 2.53\ \text{m/s}. \end{aligned}$$

Discussion

Using work and energy, we not only arrive at an answer, we see that the final kinetic energy is the sum of the initial kinetic energy and the net work done on the package. This means that the work indeed adds to the energy of the package.

Example:

Work and Energy Can Reveal Distance, Too

How far does the package in [link] coast after the push, assuming friction remains constant? Use work and energy considerations.

Strategy

We know that once the person stops pushing, friction will bring the package to rest. In terms of energy, friction does negative work until it has removed all of the package’s kinetic energy. The work done by friction can be expressed as the force of friction times the distance traveled *and* as the change in kinetic energy. Equating both expressions for work gives us a way of finding the distance traveled.

Solution

The normal force and force of gravity cancel in calculating the net force. The horizontal friction force is then the net force, and it acts opposite to the

displacement. To reduce the kinetic energy of the package to zero, the work W_{fr} by friction must be equal to the change in kinetic energy.

Equation:

$$W_{\text{fr}} = -F_{\text{fr}}d = \Delta\text{KE}$$

Equation:

$$-F_{\text{fr}}d = \Delta\text{KE}$$

and so

Equation:

$$-F_{\text{fr}}d = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

Initial speed is the speed at the instant the push stops (the result determined in the previous example). The final speed is zero for this case and leads to the expression below.

Equation:

$$F_{\text{fr}}d = \frac{1}{2}mv_0^2$$

Solving for distance, we obtain the result shown below.

Equation:

$$d = \frac{95.75\,\text{J}}{5.00\,\text{N}} = 19.2\,\text{m}$$

Discussion

This is a reasonable distance for a package to coast on a relatively friction-free conveyor system. Note that the work done by friction is negative (the force is in the opposite direction of motion), so it removes the kinetic energy.

Some of the examples in this section can be solved without considering energy, but at the expense of missing out on gaining insights about what work and energy are doing in this situation. On the whole, solutions involving energy are generally shorter and easier than those using kinematics and dynamics alone.

Section Summary

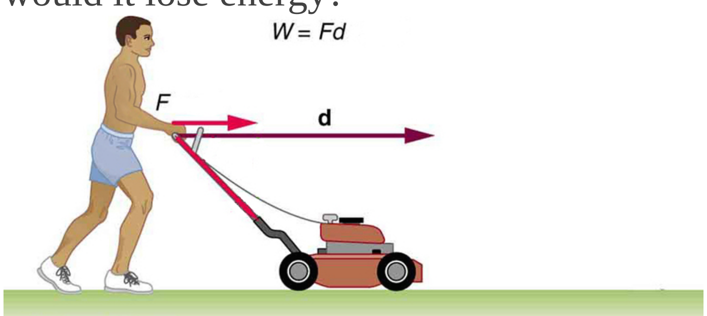
- The net work W_{net} is the work done by the net force acting on an object.
- Work done on an object transfers energy to the object.
- The translational kinetic energy of an object of mass m moving at speed v is $\text{KE} = \frac{1}{2}mv^2$.
- The work-energy theorem states that the net work W_{net} on a system changes its kinetic energy, $W_{\text{net}} = \Delta\text{KE} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$.

Conceptual Questions

Exercise:

Problem:

The person in [link] does work on the lawn mower. Under what conditions would the mower gain energy? Under what conditions would it lose energy?



Exercise:

Problem:

Work done on a system puts energy into it. Work done by a system removes energy from it. Give an example for each statement.

Exercise:

Problem:

When solving for speed in [link], we kept only the positive root. Why?

Glossary

net work
: work done by the net force, or vector sum of all the forces, acting on an object

work-energy theorem
: the result, based on Newton's laws, that the net work done on an object is equal to its change in kinetic energy

kinetic energy
: the energy an object has by reason of its motion, equal to $\frac{1}{2}mv^2$ for the translational (i.e., non-rotational) motion of an object of mass m moving at speed v

Gravitational Potential Energy

- Explain gravitational potential energy in terms of work done against gravity.
- Show that the gravitational potential energy of an object near surface of Earth is given by product of its mass, height, and gravitational acceleration.
- Show how knowledge of the potential energy as a function of position can be used to simplify calculations and explain physical phenomena.

Work Done Against Gravity

Consider lifting a weight (of, say, 5 kg, which weighs about 50 N) from the floor to a table surface at a height of 1 meter from the floor. In lifting this weight, you have done work, both in the usual sense (if you do this lifting of weight over and over, you will eventually get tired) and in the scientific sense we defined in the earlier section ($W = Fd$, and you applied an upward force as you displaced the weight upward by 1 meter). One thing is different in this scenario, however. Earlier, in describing work-kinetic energy theorem, we said that net work done on an object changes the kinetic energy of the object ($\Delta\mathrm{KE} = W_{\mathrm{net}}$). But in this case, if you imagine lifting the weight very slowly up to the height of the table, you will find that the kinetic energy of the weight does not change. It starts at zero and remains at zero throughout this very slow motion. What is going on here?

The key difference is that, for this weight, there are now two forces acting on it as it is being lifted. The gravitational force pulls downward, while the force you apply is directed upward. If you are lifting the weight very slowly, then its acceleration is very close to zero, so the net force on it is zero ($F_{\mathrm{net}} = 0$), which means the *net work* done on the weight is also zero ($W_{\mathrm{net}} = F_{\mathrm{net}}d$; the applied upward force does positive work, while the downward gravitational force does negative work).

While this is a correct description (the negative work done by the gravity cancels out the positive work done by the applied force), this description misses something. Specifically, you can see that this weight that has been lifted by 1 meter *does* have more energy than a weight that was resting on the floor. This is how you can see it: if you let go of the weight (after removing the table), as the weight originally at 1-meter height reaches the floor, it gains a substantial kinetic energy as it continues to be pulled by gravity. Rather than saying that this kinetic energy simply appeared as the weight fell through the 1-meter height, we opt for the description that this weight already had this energy, when it was at rest at the 1-meter height, just in a form that was not kinetic energy. We call this energy associated with the height of objects the **gravitational potential energy**, PE_g. And in this example, the energy originally came from the work done by the applied force—rather than considering positive

work done by the applied force to be cancelled out by the negative work done by the gravity, we say that this positive work done is *stored* as positive PE_g.

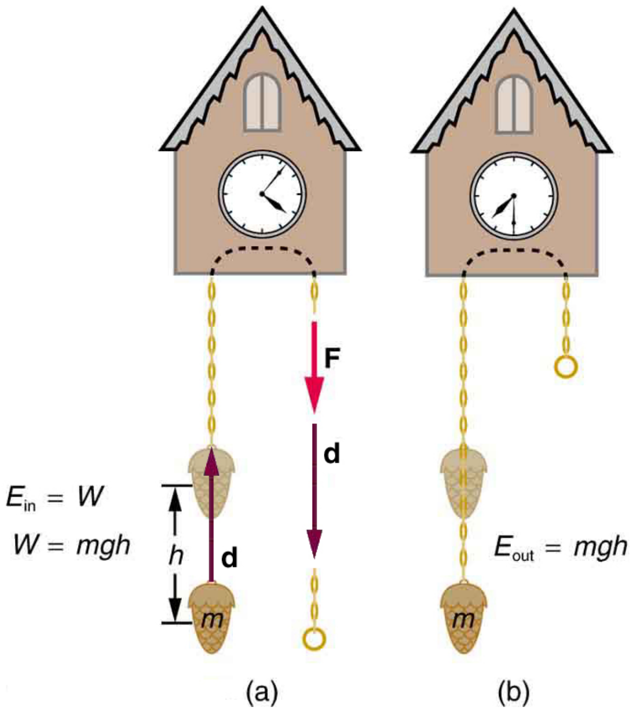
This phenomenon of potential energy turns out to be quite general and goes beyond the gravitational force. There are other forces in nature where negative work done by these forces turns into positive potential energy. We call these forces **conservative forces**, which will be the subject of the next section. For the purpose of this section, following is the quick derivation of formula for PE_g (everything in terms of symbols now). In lifting the weight of mass m through a height h slowly, you have to apply a force equal to its weight, mg. So the work done is $W = Fd = mgh$, and since it's this work done that is stored as gravitational potential energy, the gravitational potential energy must be:

Equation:

$$\mathrm{PE}_g = mgh.$$

Converting Between Potential Energy and Kinetic Energy

Gravitational potential energy may be converted to other forms of energy, such as kinetic energy. If we release the mass, gravitational force will do an amount of work equal to mgh on it, thereby increasing its kinetic energy by that same amount (by the work-energy theorem). We will find it more useful to consider just the conversion of $\mathrm{PE_g}$ to KE without explicitly considering the intermediate step of work. (See [link].) This shortcut makes it is easier to solve problems using energy (if possible) rather than explicitly using forces.



(a) The work done to lift the weight is stored in the mass-Earth system as gravitational potential energy. (b) As the weight moves downward, this gravitational potential energy is transferred to the cuckoo clock.

More precisely, we define the *change* in gravitational potential energy ΔPE_g to be

Equation:

$$\Delta\text{PE}_\text{g} = \text{PE}_\text{gf} - \text{PE}_\text{gi} = mgh_\text{f} - mgh_\text{i} = mg(\,h_\text{f} - h_\text{i}) = mg\Delta h$$

Note that Δh is positive when the final height is greater than the initial height, and vice versa. For example, if a 0.500-kg mass hung from a cuckoo clock is raised 1.00 m, then its change in gravitational potential energy is

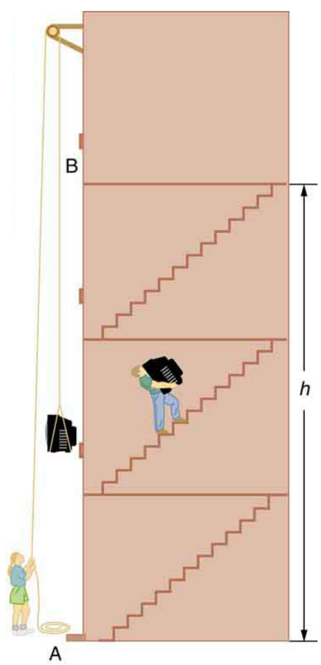
Equation:

$$\begin{aligned} mg\Delta h &= (0.500\text{ kg})\left(9.80\text{ m/s}^2\right)(1.00\text{ m}) \\ &= 4.90\text{ kg}\cdot\text{m}^2/\text{s}^2 = 4.90\text{ J}. \end{aligned}$$

Note that the units of gravitational potential energy turn out to be joules, the same as for work and other forms of energy. As the clock runs, the mass is lowered. We can think of the mass as gradually giving up its 4.90 J of gravitational potential energy, *without directly considering the force of gravity that does the work*.

Using Potential Energy to Simplify Calculations

The equation $\Delta \mathrm{PE_g} = \mathrm{mg}\Delta \mathrm{h}$ applies for any path that has a change in height of $\Delta \mathrm{h}$, not just when the mass is lifted straight up. (See [link].) It is much easier to calculate $\mathrm{mg}\Delta \mathrm{h}$ (a simple multiplication) than it is to calculate the work done along a complicated path. The idea of gravitational potential energy has the double advantage that it is very broadly applicable and it makes calculations easier. From now on, we will consider that any change in vertical position $\Delta \mathrm{h}$ of a mass m is accompanied by a change in gravitational potential energy $\mathrm{mg}\Delta \mathrm{h}$, and we will avoid the equivalent but more difficult task of calculating work done by or against the gravitational force.



The change in gravitational potential energy $(\Delta \mathrm{PE_g})$ between points

A and B is independent of the path. $\Delta PE_g = mg\Delta h$ for any path between the two points. Gravity is one of a small class of forces where the work done by or against the force depends only on the starting and ending points, not on the path between them.

Example:

The Force to Stop Falling

A 60.0-kg person jumps onto the floor from a height of 3.00 m. If he lands stiffly (with his knee joints compressing by 0.500 cm), calculate the force on the knee joints.

Strategy

This person's energy is brought to zero in this situation by the work done on him by the floor as he stops. The initial PE_g is transformed into KE as he falls. The work done by the floor reduces this kinetic energy to zero.

Solution

The work done on the person by the floor as he stops is given by

Equation:

$$W = -Fd,$$

with a minus sign because the displacement while stopping and the force from floor are in opposite directions. The floor removes energy from the system, so it does negative work.

The kinetic energy the person has upon reaching the floor is the amount of potential energy lost by falling through height Δh:

Equation:

$$\mathrm{KE} = -\Delta\mathrm{PE_g} = -\mathrm{mg}\Delta\mathrm{h},$$

The distance d that the person’s knees bend is much smaller than the height $\Delta\mathrm{h}$ of the fall, so the additional change in gravitational potential energy during the knee bend is ignored.

The work W done by the floor on the person stops the person and brings the person’s kinetic energy to zero:

Equation:

$$W = -\mathrm{KE} = \mathrm{mg}\Delta\mathrm{h}.$$

Combining this equation with the expression for W gives

Equation:

$$-\mathrm{Fd} = \mathrm{mg}\Delta\mathrm{h}.$$

Recalling that $\Delta\mathrm{h}$ is negative because the person fell *down*, the force on the knee joints is given by

Equation:

$$F = -\frac{\mathrm{mg}\Delta\mathrm{h}}{d} = -\frac{(60.0\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)(-3.00\ \mathrm{m})}{5.00\times10^{-3}\ \mathrm{m}} = 3.53\times10^{5}\ \mathrm{N}.$$

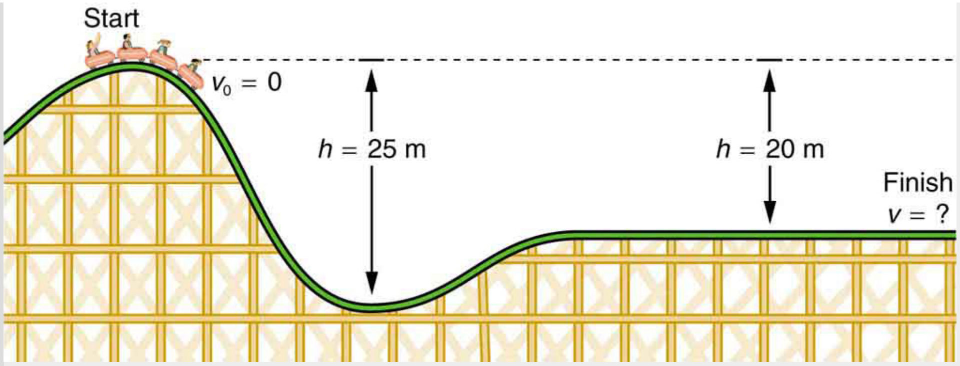
Discussion

Such a large force (500 times more than the person’s weight) over the short impact time is enough to break bones. A much better way to cushion the shock is by bending the legs or rolling on the ground, increasing the time over which the force acts. A bending motion of 0.5 m this way yields a force 100 times smaller than in the example. A kangaroo's hopping shows this method in action. The kangaroo is the only large animal to use hopping for locomotion, but the shock in hopping is cushioned by the bending of its hind legs in each jump.(See [link].)

The work done by the ground upon the kangaroo reduces its kinetic energy to zero as it lands. However, by applying the force of the ground on the hind legs over a longer distance, the impact on the bones is reduced. (credit: Chris Samuel, Flickr)

Example:
Finding the Speed of a Roller Coaster from its Height

(a) What is the final speed of the roller coaster shown in [link] if it starts from rest at the top of the 20.0 m hill and work done by frictional forces is negligible? (b) What is its final speed (again assuming negligible friction) if its initial speed is 5.00 m/s?



The speed of a roller coaster increases as gravity pulls it downhill and is greatest at its lowest point. Viewed in terms of energy, the roller-coaster-Earth system's gravitational potential energy is converted to kinetic energy. If work done by friction is negligible, all $\Delta\mathrm{PE_g}$ is converted to KE.

Strategy
The roller coaster loses potential energy as it goes downhill. We neglect friction, so that the remaining force exerted by the track is the normal force, which is perpendicular to the direction of motion and does no work. The net work on the roller coaster is then done by gravity alone. The *loss* of gravitational potential energy from moving *downward* through a distance h equals the *gain* in kinetic energy. This can be written in equation form as $-\Delta\mathrm{PE_g} = \Delta\mathrm{KE}$. Using the equations for $\mathrm{PE_g}$ and KE, we can solve for the final speed v, which is the desired quantity.
Solution for (a)
Equation:

$$-\Delta\mathrm{PE_g} = \Delta\mathrm{KE}$$

becomes
Equation:

$$-\left(\mathrm{PE_{gf}} - \mathrm{PE_{gi}}\right) = \frac{1}{2}mv_\mathrm{f}^2 - \frac{1}{2}mv_\mathrm{i}^2$$

Noting that the initial speed is zero, and using the formula for gravitational potential energy, we have
Equation:

$$-(mgh_{\mathrm{f}} - mgh_{\mathrm{i}}) = \frac{1}{2}mv_{\mathrm{f}}^2$$

Taking the initial height as zero gives
Equation:

$$-mgh_{\mathrm{f}} = \frac{1}{2}mv_{\mathrm{f}}^2$$

Solving for the final speed, we find that mass cancels and that
Equation:

$$v_{\mathrm{f}} = \sqrt{-2g\,h_{\mathrm{f}}}$$

Substituting known values,
Equation:

$$v_{\mathrm{f}} = \sqrt{-2g\,h_{\mathrm{f}}} = \sqrt{(-2)(9.80\,\mathrm{m/s^2})(-20.0\,\mathrm{m})} = 19.8\,\mathrm{m/s}$$

Please note that the height used above was negative. This is because we chose the initial height to be our reference level for gravitational potential energy. Our final height is 20 meters *below* that reference level.
Solution for (b)
Again $-\Delta\mathrm{PE_g} = \Delta\mathrm{KE}$. Thus,
Equation:

$$-(\,\mathrm{PE_{gf}} - \mathrm{PE_{gi}}) = \frac{1}{2}mv_{\mathrm{f}}^2 - \frac{1}{2}mv_{\mathrm{i}}^2$$

Taking the initial height as zero leads to
Equation:

$$-mgh_{\mathrm{f}} = \frac{1}{2}mv_{\mathrm{f}}^2 - \frac{1}{2}mv_{\mathrm{i}}^2$$

Rearranging gives
Equation:

$$\frac{1}{2}mv_{\mathrm{f}}^2 = -mgh_{\mathrm{f}} + \frac{1}{2}mv_{\mathrm{i}}^2$$

This means that the final kinetic energy is the sum of the initial kinetic energy and the gravitational potential energy. Mass again cancels, and

Equation:

$$v_{\text{f}} = \sqrt{-2gh_{\text{f}} + v_{\text{i}}^2}$$

Now, substituting known values gives

Equation:

$$v_{\text{f}} = \sqrt{-2gh_{\text{f}} + v_{\text{i}}^2} = \sqrt{(-2)(9.80\,\text{m/s}^2)(-20.0\,\text{m}) + (5.00\,\text{m/s})^2} = 20.4\,\text{m/s}.$$

Discussion and Implications

First, note that mass cancels. This is quite consistent with observations that all objects fall at the same rate if friction is negligible. Second, only the speed of the roller coaster is considered; there is no information about its direction at any point. This reveals another general truth. When friction is negligible, the speed of a falling body depends only on its initial speed and height, and not on its mass or the path taken. For example, the roller coaster will have the same final speed whether it falls 20.0 m straight down or takes a more complicated path like the one in the figure. Third, and perhaps unexpectedly, the final speed in part (b) is greater than in part (a), but by far less than 5.00 m/s. Finally, note that speed can be found at *any* height along the way by simply using the appropriate value of h at the point of interest.

We have seen that work done by or against the gravitational force depends only on the starting and ending points, and not on the path between, allowing us to define the simplifying concept of gravitational potential energy. We can do the same thing for a few other forces, and we will see that this leads to a formal definition of the law of conservation of energy.

Section Summary

- Work done against gravity in lifting an object becomes potential energy of the object-Earth system.
- The change in gravitational potential energy, $\Delta\text{PE}_{\text{g}}$, is $\Delta\text{PE}_{\text{g}} = \text{mg}\Delta\text{h}$, with Δh being the change in height and g the acceleration due to gravity.
- The gravitational potential energy of an object near Earth's surface is due to its position in the mass-Earth system. Only differences in gravitational potential energy, $\Delta\text{PE}_{\text{g}}$, have physical significance.
- As an object descends without friction, its gravitational potential energy changes into kinetic energy corresponding to increasing speed, so that

$$\Delta \mathrm{KE} = -\Delta \mathrm{PE}_{\mathrm{g}}.$$

Conceptual Questions

Exercise:

Problem:

In [link], we calculated the final speed of a roller coaster that descended 20 m in height and had an initial speed of 5 m/s downhill. Suppose the roller coaster had had an initial speed of 5 m/s *uphill* instead, and it coasted uphill, stopped, and then rolled back down to a final point 20 m below the start. We would find in that case that it had the same final speed. Explain in terms of conservation of energy.

Exercise:

Problem:

Does the work you do on a book when you lift it onto a shelf depend on the path taken? On the time taken? On the height of the shelf? On the mass of the book?

Glossary

gravitational potential energy
: energy associated with height of objects on the Earth

Conservative Forces, Potential Energy, and Conservation of Energy

- Define conservative force, potential energy, and mechanical energy.
- Apply conservation of mechanical energy to simple physical situations.
- Explain the law of the conservation of energy.
- Describe some of the many forms of energy.

Potential Energy and Conservative Forces

Work is done by a force, and some forces, such as weight, have special characteristics. A **conservative force** is one, like the gravitational force, for which work done by or against it depends only on the starting and ending points of a motion and not on the path taken. This happens if the force is a function of position alone and not time, velocity, or other parameters. We can define a **potential energy** (PE) for any conservative force, just as we did for the gravitational force. For example, when you wind up a toy, an egg timer, or an old-fashioned watch, you do work against its spring and store energy in it. (We treat these springs as ideal, in that we assume there is no friction and no production of thermal energy.) This stored energy is recoverable as work, and it is useful to think of it as potential energy contained in the spring. Indeed, the reason that the spring has this characteristic is that its force is *conservative*. That is, a conservative force results in stored or potential energy. Gravitational potential energy is one example, as is the energy stored in a spring. We will also see how conservative forces are related to the conservation of energy.

Note:
Potential Energy and Conservative Forces
Potential energy is the energy a system has due to position, shape, or configuration. It is stored energy that is completely recoverable.
A conservative force is a force that is a function of position alone, with the result that the work done by the force depends only on the starting and ending points of a motion and not on the particular path taken.

Change in potential energy comes from the work done against a conservative force. Or flipping this around, potential energy increases as a conservative force does a negative work. That is,
Equation:

$$\Delta\mathrm{PE} = -W_c,$$

where W_c is work done by a conservative force.

Conservation of Mechanical Energy

Consider an object in a system. If it gains any kinetic energy, this is a result of a net work done on the object, according to the work-kinetic energy theorem. If only conservative forces, such as the gravitational force or a spring force, do work in this system, then as kinetic energy increases with the net work done by the conservative forces, the system loses potential energy. That is, $\Delta\mathrm{KE} = W_c = -\Delta\mathrm{PE}$. In other words,
Equation:

$$\Delta\mathrm{KE} + \Delta\mathrm{PE} = 0.$$

This equation means that the total kinetic and potential energy is constant for any process involving only conservative forces. That is,
Equation:

$$\left.\begin{array}{cc} & \mathrm{KE} + \mathrm{PE} = \text{constant} \\ \text{or} & \\ & \mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f} \end{array}\right\} (\text{conservative forces only}),$$

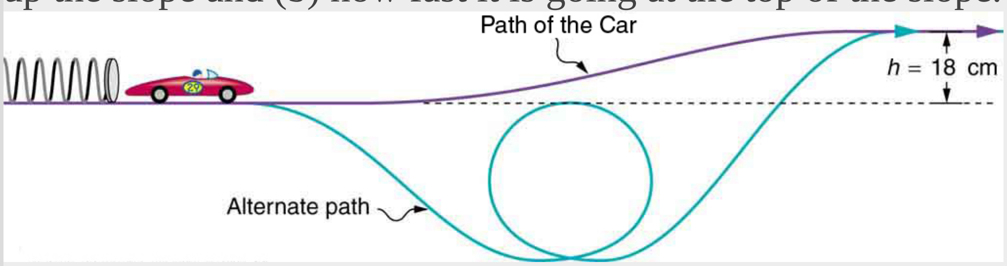
where i and f denote initial and final values. This equation is a form of the work-energy theorem for conservative forces; it is known as the **conservation of mechanical energy** principle. Remember that this applies to the extent that all the forces are conservative, so that friction is negligible. The total kinetic plus potential energy of a system is defined to

be its **mechanical energy**, $(\mathrm{KE} + \mathrm{PE})$. In a system that experiences only conservative forces, there is a potential energy associated with each force, and the energy only changes form between KE and the various types of PE, with the total energy remaining constant.

Example:

Using Conservation of Mechanical Energy to Calculate the Speed of a Toy Car

A 0.100-kg toy car is propelled by a compressed spring, as shown in [link]. The car follows a track that rises 0.180 m above the starting point. The compressed spring has a potential energy of 0.200 J. Assuming work done by friction to be negligible, find (a) how fast the car is going before it starts up the slope and (b) how fast it is going at the top of the slope.



A toy car is pushed by a compressed spring and coasts up a slope. Assuming negligible friction, the potential energy in the spring is first completely converted to kinetic energy, and then to a combination of kinetic and gravitational potential energy as the car rises. The details of the path are unimportant because all forces are conservative—the car would have the same final speed if it took the alternate path shown.

Strategy

The spring force and the gravitational force are conservative forces, so conservation of mechanical energy can be used. Thus,

Equation:

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}$$

or

Equation:

$$\frac{1}{2}mv_{\mathrm{i}}^2 + mgh_{\mathrm{i}} + \mathrm{PE}_{\mathrm{si}} = \frac{1}{2}mv_{\mathrm{f}}^2 + mgh_{\mathrm{f}} + \mathrm{PE}_{\mathrm{sf}}$$

where h is the height (vertical position) and PE_{s} is the potential energy of the spring. This general statement looks complex but becomes much simpler when we start considering specific situations. First, we must identify the initial and final conditions in a problem; then, we enter them into the last equation to solve for an unknown.

Solution for (a)

This part of the problem is limited to conditions just before the car is released and just after it leaves the spring. Take the initial height to be zero, so that both h_{i} and h_{f} are zero. Furthermore, the initial speed v_{i} is zero and the final compression of the spring is zero, and so several terms in the conservation of mechanical energy equation are zero and it simplifies to

Equation:

$$\mathrm{PE}_{\mathrm{si}} = \frac{1}{2}mv_{\mathrm{f}}^2.$$

In other words, the initial potential energy in the spring is converted completely to kinetic energy in the absence of friction. Solving for the final speed and entering known values yields

Equation:

$$v_{\mathrm{f}} = \sqrt{\frac{2\,\mathrm{PE}_{\mathrm{si}}}{m}} = \sqrt{\frac{(2)(0.200\,\mathrm{J})}{0.100\,\mathrm{kg}}} = 2.00\,\mathrm{m/s}.$$

Solution for (b)

One method of finding the speed at the top of the slope is to consider conditions just before the car is released and just after it reaches the top of the slope, completely ignoring everything in between. Doing the same type of analysis to find which terms are zero, the conservation of mechanical energy becomes

Equation:

$$\mathrm{PE}_{\mathrm{si}} = \frac{1}{2}mv_{\mathrm{f}}^2 + mgh_{\mathrm{f}}.$$

This form of the equation means that the spring’s initial potential energy is converted partly to gravitational potential energy and partly to kinetic energy. The final speed at the top of the slope will be less than at the bottom. Solving for v_{f} and substituting known values gives

Equation:

$$v_{\mathrm{f}} = \sqrt{\frac{(2)(\mathrm{PE}_{\mathrm{si}} - mgh_{\mathrm{f}})}{m}} = \sqrt{\frac{0.0472\,\mathrm{J}}{0.100\,\mathrm{kg}}} = 0.687\,\mathrm{m/s}$$

Discussion

Another way to solve this problem is to realize that the car’s kinetic energy before it goes up the slope is converted partly to potential energy—that is, to take the final conditions in part (a) to be the initial conditions in part (b).

Note that, for conservative forces, we do not directly calculate the work they do; rather, we consider their effects through their corresponding potential energies, just as we did in [link]. Note also that we do not consider details of the path taken—only the starting and ending points are important (as long as the path is not impossible). This assumption is usually a tremendous simplification, because the path may be complicated and forces may vary along the way.

Conservation of Total Energy

Energy, as we have noted, is conserved, making it one of the most important physical quantities in nature. The **law of conservation of energy** can be stated as follows:

Total energy is constant in any process. It may change in form or be transferred from one system to another, but the total remains the same.

We have explored some forms of energy and some ways it can be transferred from one system to another, in the case of work done by conservative forces transforming between potential energy and kinetic energy. Together, these make up the mechanical energy $(\text{KE}+\text{PE})$, and the mechanical energy is not always conserved, because it can be transformed into other forms of energy through work done by non-conservative forces. Here, we list some of the many forms energy can take.

Some of the Many Forms of Energy

Here are some of the many forms of energy. You probably have heard of some of these before; many of these will be covered in later chapters, but let us detail a few here. **Electrical energy** is a common form that is converted to many other forms and does work in a wide range of practical situations. Fuels, such as gasoline and food, carry **chemical energy** that can be transferred to a system through oxidation. Chemical fuel can also produce electrical energy, such as in batteries. Batteries can in turn produce light, which is a very pure form of energy. Most energy sources on Earth are in fact stored energy from the energy we receive from the Sun. We sometimes refer to this as **radiant energy**, or electromagnetic radiation, which includes visible light, infrared, and ultraviolet radiation. **Nuclear energy** comes from processes that convert measurable amounts of mass into energy. Nuclear energy is transformed into the energy of sunlight, into electrical energy in power plants, and into the energy of the heat transfer and blast in weapons. Atoms and molecules inside all objects are in random motion. This internal mechanical energy from the random motions is called **thermal energy**, because it is related to the temperature of the object. These and all other forms of energy can be converted into one another and can do work.

Note:

Problem-Solving Strategies for Energy Problems

You will find the following problem-solving strategies useful whenever you deal with energy. The strategies help in organizing and reinforcing energy concepts. In fact, they are used in the examples presented in this chapter. The familiar general problem-solving strategies presented earlier

—involving identifying physical principles, knowns, and unknowns, checking units, and so on—continue to be relevant here.

Step 1. Determine the system of interest and identify what information is given and what quantity is to be calculated. A sketch will help.

Step 2. Examine all the forces involved and determine whether you know or are given the potential energy from the work done by the forces. Then use step 3 or step 4.

Step 3. If you know the potential energies for the forces that enter into the problem, then forces are all conservative, and you can apply conservation of mechanical energy simply in terms of potential and kinetic energy. Use $\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}$. or $\Delta\mathrm{KE} + \Delta\mathrm{PE} = 0$ as starting point and use formulas for kinetic energy and potential energies.

Step 4. If you know that the total mechanical energy changes (usually there will be some hint of work done by non-conservative forces, energy input from other forms of energy, or even an explicit statement of how much work is done on the system, changing its total mechanical energy), then you can use the information given in the problem to say that, the change of total mechanical energy is due to these non-conservative forces: $\Delta\mathrm{KE} + \Delta\mathrm{PE} = W_{\mathrm{nc}}$. Make sure W_{nc} does not include work done by conservative forces, since those are already accounted for in the potential energy.

Step 5. You have already identified the types of work and energy involved (in step 2). Before solving for the unknown, *eliminate terms wherever possible* to simplify the algebra. For example, choose $h = 0$ at either the initial or final point, so that $\mathrm{PE_g}$ is zero there. Then solve for the unknown in the customary manner.

Step 6. *Check the answer to see if it is reasonable*. Once you have solved a problem, reexamine the forms of work and energy to see if you have set up the conservation of energy equation correctly. For example, work done against friction should be negative, potential energy at the bottom of a hill should be less than that at the top, and so on. Also check to see that the numerical value obtained is reasonable. For example, the final speed of a skateboarder who coasts down a 3-m-high ramp could reasonably be 20 km/h, but *not* 80 km/h.

Transformation of Energy

The transformation of energy from one form into others is happening all the time. The chemical energy in food is converted into thermal energy through metabolism; light energy is converted into chemical energy through photosynthesis. In a larger example, the chemical energy contained in coal is converted into thermal energy as it burns to turn water into steam in a boiler. This thermal energy in the steam in turn is converted to mechanical energy as it spins a turbine, which is connected to a generator to produce electrical energy. (In all of these examples, not all of the initial energy is converted into the forms mentioned. This important point is discussed later in this section.)

Another example of energy conversion occurs in a solar cell. Sunlight impinging on a solar cell (see [link]) produces electricity, which in turn can be used to run an electric motor. Energy is converted from the primary source of solar energy into electrical energy and then into mechanical energy.

Solar energy is converted into electrical energy by solar cells, which is used to run a motor in this solar-power aircraft. (credit: NASA)

Section Summary

- A conservative force is one for which work depends only on the starting and ending points of a motion, not on the path taken.
- We can define potential energy (PE) for any conservative force, just as we defined PE_{g} for the gravitational force.
- Mechanical energy is defined to be $\mathrm{KE} + \mathrm{PE}$ for a conservative force.
- When only conservative forces act on and within a system, the total mechanical energy is conserved, $\Delta\mathrm{KE} + \Delta\mathrm{PE} = 0$.
- The law of conservation of energy states that the total energy, including the mechanical energy and other forms of energy, is constant in any process. Energy may change in form or be transferred from one system to another, but the total remains the same.
- Commonly encountered forms of energy include electric energy, chemical energy, radiant energy, nuclear energy, and thermal energy.

Conceptual Questions

Exercise:

Problem: What is a conservative force?

Exercise:

Problem:

The force exerted by a diving board is conservative, provided the internal friction is negligible. Assuming friction is negligible, describe changes in the potential energy of a diving board as a swimmer dives from it, starting just before the swimmer steps on the board until just after his feet leave it.

Exercise:

Problem:

Define mechanical energy. What is the relationship of mechanical energy to nonconservative forces? What happens to mechanical energy if only conservative forces act?

Exercise:

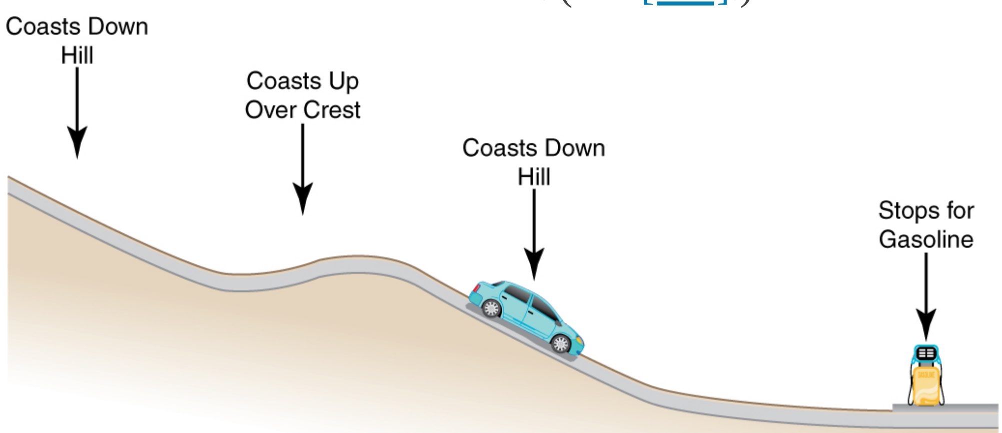
Problem:

What is the relationship of potential energy to conservative force?

Exercise:

Problem:

Consider the following scenario. A car for which friction is *not* negligible accelerates from rest down a hill, running out of gasoline after a short distance. The driver lets the car coast farther down the hill, then up and over a small crest. He then coasts down that hill into a gas station, where he brakes to a stop and fills the tank with gasoline. Identify the forms of energy the car has, and how they are changed and transferred in this series of events. (See [link].)



A car experiencing non-negligible friction coasts down a hill, over a small crest, then downhill again, and comes to a stop at a gas station.

Exercise:

Problem:

Describe the energy transfers and transformations for a javelin, starting from the point at which an athlete picks up the javelin and ending when the javelin is stuck into the ground after being thrown.

Exercise:

Problem:

List four different forms or types of energy. Give one example of a conversion from each of these forms to another form.

Exercise:

Problem:

List the energy conversions that occur when riding a bicycle.

Glossary

conservative force
: a force that is a function of position alone, with the result that the work done by the force depends only on the starting and ending points of a motion and not on the particular path taken

potential energy
: energy due to position, shape, or configuration

conservation of mechanical energy
: the rule that the sum of the kinetic energies and potential energies remains constant if only conservative forces act on and within a system

mechanical energy
: the sum of kinetic energy and potential energy

law of conservation of energy
: the general law that total energy is constant in any process; energy may change in form or be transferred from one system to another, but the total remains the same

electrical energy
: the energy carried by a flow of charge

chemical energy

the energy in a substance stored in the bonds between atoms and molecules that can be released in a chemical reaction

radiant energy
: the energy carried by electromagnetic waves

nuclear energy
: energy released by changes within atomic nuclei, such as the fusion of two light nuclei or the fission of a heavy nucleus

thermal energy
: the energy within an object due to the random motion of its atoms and molecules that accounts for the object's temperature

Spring Potential Energy

- Explain the work done in deforming a spring.
- Describe the potential energy stored in a deformed spring.

Hooke's Law, $F = -kx$, describes force exerted by a spring being deformed. Here, F is the restoring force, x is the displacement from equilibrium or **deformation**, and k is a constant related to the difficulty in deforming the system. The minus sign indicates the restoring force is in the direction opposite to the displacement.

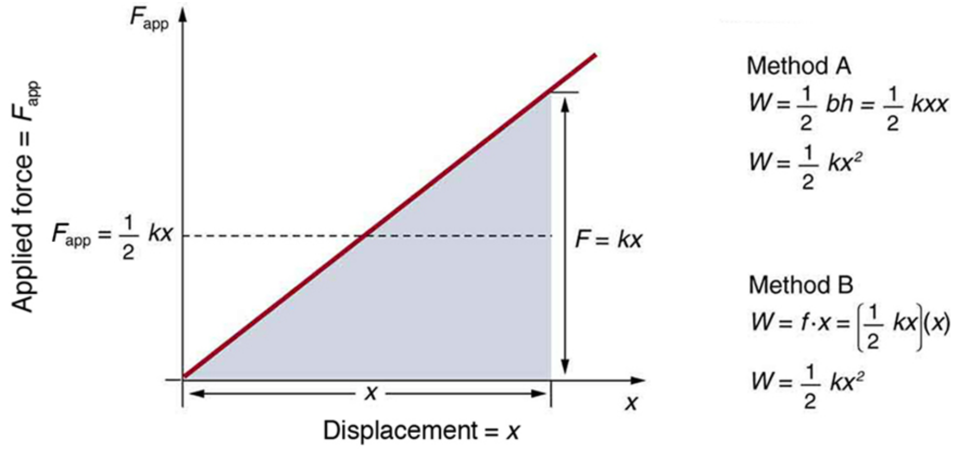
In order to produce a deformation, work must be done. That is, a force must be exerted through a distance, whether you pluck a guitar string or compress a car spring. If the only result is deformation, and no work goes into thermal, sound, or kinetic energy, then all the work is initially stored in the deformed object as some form of potential energy. The potential energy stored in a spring is $\mathrm{PE_{el}} = \frac{1}{2}\mathrm{kx}^2$. Here, we generalize the idea to elastic potential energy for a deformation of any system that can be described by Hooke's law. Hence,
Equation:

$$\mathrm{PE_{el}} = \frac{1}{2}\mathrm{kx}^2,$$

where $\mathrm{PE_{el}}$ is the **elastic potential energy** stored in any deformed system that obeys Hooke's law and has a displacement x from equilibrium and a force constant k.

It is possible to find the work done in deforming a system in order to find the energy stored. This work is performed by an applied force F_{app}. The applied force is exactly opposite to the restoring force (action-reaction), and so $F_{\text{app}} = \mathrm{kx}$. [link] shows a graph of the applied force versus deformation x for a system that can be described by Hooke's law. Work done on the system is force multiplied by distance, which equals the area under the curve or $(1/2)\mathrm{kx}^2$(Method A in the figure). Another way to determine the work is to note that the force increases linearly from 0 to kx, so that the

average force is $(1/2)\,\mathrm{kx}$, the distance moved is x, and thus $W = F_{\text{app}}d = [(1/2)\mathrm{kx}](x) = (1/2)\mathrm{kx}^2$ (Method B in the figure).

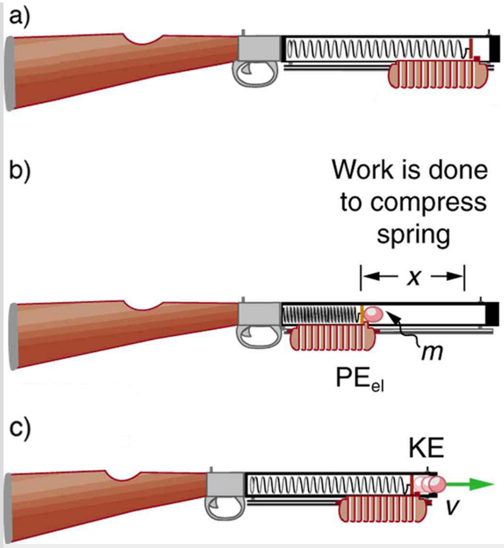


A graph of applied force versus distance for the deformation of a system that can be described by Hooke's law is displayed. The work done on the system equals the area under the graph or the area of the triangle, which is half its base multiplied by its height, or $W = (1/2)\mathrm{kx}^2$.

Example:

Calculating Stored Energy: A Tranquilizer Gun Spring

We can use a toy gun's spring mechanism to ask and answer two simple questions: (a) How much energy is stored in the spring of a tranquilizer gun that has a force constant of 50.0 N/m and is compressed 0.150 m? (b) If you neglect friction and the mass of the spring, at what speed will a 2.00-g projectile be ejected from the gun?



(a) In this image of the gun, the spring is uncompressed before being cocked. (b) The spring has been compressed a distance x, and the projectile is in place. (c) When released, the spring converts elastic potential energy PE_{el} into kinetic energy.

Strategy for a
(a): The energy stored in the spring can be found directly from elastic potential energy equation, because k and x are given.
Solution for a
Entering the given values for k and x yields
Equation:

$$\begin{aligned}\text{PE}_{\text{el}} &= \frac{1}{2}\text{kx}^2 = \frac{1}{2}(50.0\ \text{N/m})(0.150\ \text{m})^2 = 0.563\ \text{N}\cdot\text{m} \\ &= 0.563\ \text{J}\end{aligned}$$

Strategy for b

Because there is no friction, the potential energy is converted entirely into kinetic energy. The expression for kinetic energy can be solved for the projectile's speed.

Solution for b

1. Identify known quantities:
   Equation:

$$\mathrm{KE_f} = \mathrm{PE_{el}} \text{ or } 1/2mv^2 = (1/2)kx^2 = \mathrm{PE_{el}} = 0.563 \text{ J}$$

2. Solve for v:
   Equation:

$$v = \left[\frac{2\mathrm{PE_{el}}}{m}\right]^{1/2} = \left[\frac{2(0.563 \text{ J})}{0.002 \text{ kg}}\right]^{1/2} = 23.7(\text{J/kg})^{1/2}$$

3. Convert units: 23.7 m/s

Discussion

(a) and (b): This projectile speed is impressive for a tranquilizer gun (more than 80 km/h). The numbers in this problem seem reasonable. The force needed to compress the spring is small enough for an adult to manage, and the energy imparted to the dart is small enough to limit the damage it might do. Yet, the speed of the dart is great enough for it to travel an acceptable distance.

Exercise:

Check your Understanding

Problem:

Envision holding the end of a ruler with one hand and deforming it with the other. When you let go, you can see the oscillations of the ruler. In what way could you modify this simple experiment to increase the rigidity of the system?

Solution:
Answer

You could hold the ruler at its midpoint so that the part of the ruler that oscillates is half as long as in the original experiment.

Exercise:
Check your Understanding

Problem:

If you apply a deforming force on an object and let it come to equilibrium, what happened to the work you did on the system?

Solution:
Answer

It was stored in the object as potential energy.

Section Summary

- Hooke's law describes force exerted by a spring being deformed,
  Equation:

  $$F = -\mathrm{kx},$$

  where F is the restoring force, x is the displacement from equilibrium or deformation, and k is the force constant of the system.
- Elastic potential energy $\mathrm{PE_{el}}$ stored in the deformation of a system that can be described by Hooke's law is given by
  Equation:

  $$\mathrm{PE_{el}} = (1/2)kx^2.$$

Conceptual Questions

Exercise:

Problem:

Describe a system in which elastic potential energy is stored.

Glossary

deformation
: displacement from equilibrium

elastic potential energy
: potential energy stored as a result of deformation of an elastic object, such as the stretching of a spring

Power

- Understand the relationship among power, energy, and work done.
- Examine power consumption and calculations of the cost of energy consumed.

What is Power?

Power—the word conjures up many images: a professional football player muscling aside his opponent, a dragster roaring away from the starting line, a volcano blowing its lava into the atmosphere, or a rocket blasting off, as in [link].

This powerful rocket on the Space Shuttle *Endeavor* did work and consumed energy at a very high rate. (credit: NASA)

These images of power have in common the rapid performance of work, consistent with the scientific definition of **power** (P) as the rate at which work is done.

Note:
Power
Power is the rate at which work is done.
Equation:

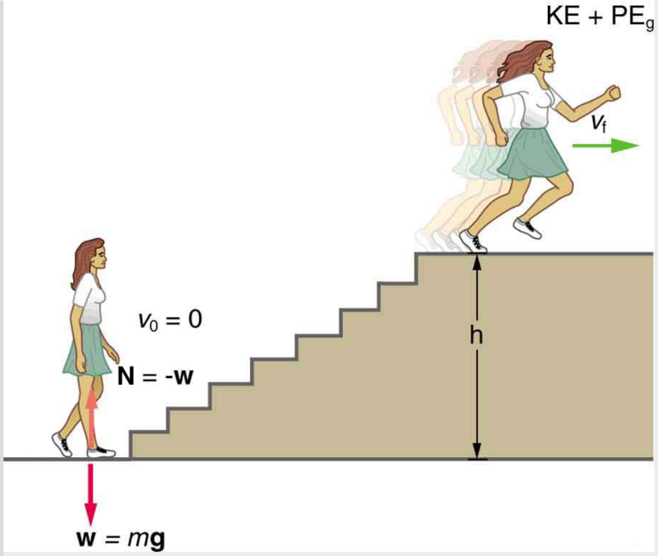
$$P = \frac{W}{t}$$

The SI unit for power is the **watt** (W), where 1 watt equals 1 joule/second $(1 \text{ W} = 1 \text{ J/s})$.

Because work is energy transfer, power is also the rate at which energy is expended. A 60-W light bulb, for example, expends 60 J of energy per second. Great power means a large amount of work or energy developed in a short time. For example, when a powerful car accelerates rapidly, it does a large amount of work and consumes a large amount of fuel in a short time.

Calculating Power from Energy

Example:
Calculating the Power to Climb Stairs
What is the power output for a 60.0-kg woman who runs up a 3.00 m high flight of stairs in 3.50 s, starting from rest but having a final speed of 2.00 m/s? (See [link].)



When this woman runs upstairs starting from rest, she converts the chemical energy originally from food into kinetic energy and gravitational potential energy. Her power output depends on how fast she does this.

Strategy and Concept
The work going into mechanical energy is $W = \text{KE} + \text{PE}$. At the bottom of the stairs, we take both KE and PE_g as initially zero; thus, $W = \text{KE}_\text{f} + \text{PE}_\text{g} = \frac{1}{2}mv_\text{f}^2 + mgh$, where h is the vertical height of the stairs. Because all terms are given, we can calculate W and then divide it by time to get power.
Solution
Substituting the expression for W into the definition of power given in the previous equation, $P = W/t$ yields
Equation:

$$P = \frac{W}{t} = \frac{\frac{1}{2}mv_\text{f}^2 + mgh}{t}.$$

Entering known values yields

Equation:

$$\begin{aligned} P &= \frac{0.5(60.0\text{ kg})(2.00\text{ m/s})^2+(60.0\text{ kg})\left(9.80\text{ m/s}^2\right)(3.00\text{ m})}{3.50\text{ s}} \\ &= \frac{120\text{ J}+1764\text{ J}}{3.50\text{ s}} \\ &= 538\text{ W}. \end{aligned}$$

Discussion

The woman does 1764 J of work to move up the stairs compared with only 120 J to increase her kinetic energy; thus, most of her power output is required for climbing rather than accelerating.

It is impressive that this woman's useful power output is slightly less than 1 **horsepower** $(1\text{ hp} = 746\text{ W})$! People can generate more than a horsepower with their leg muscles for short periods of time by rapidly converting available blood sugar and oxygen into work output. (A horse can put out 1 hp for hours on end.) Once oxygen is depleted, power output decreases and the person begins to breathe rapidly to obtain oxygen to metabolize more food—this is known as the *aerobic* stage of exercise. If the woman climbed the stairs slowly, then her power output would be much less, although the amount of work done would be the same.

Note:

Making Connections: Take-Home Investigation—Measure Your Power Rating

Determine your own power rating by measuring the time it takes you to climb a flight of stairs. We will ignore the gain in kinetic energy, as the above example showed that it was a small portion of the energy gain. Don't expect that your output will be more than about 0.5 hp.

Examples of Power

Examples of power are limited only by the imagination, because there are as many types as there are forms of work and energy. (See [link] for some examples.) Sunlight reaching Earth's surface carries a maximum power of about 1.3 kilowatts per square meter $(\mathrm{kW/m^2})$. A tiny fraction of this is retained by Earth over the long term. Our consumption rate of fossil fuels is far greater than the rate at which they are stored, so it is inevitable that they will be depleted. Power implies that energy is transferred, perhaps changing form. It is never possible to change one form completely into another without losing some of it as thermal energy. For example, a 60-W incandescent bulb converts only 5 W of electrical power to light, with 55 W dissipating into thermal energy. Furthermore, the typical electric power plant converts only 35 to 40% of its fuel into electricity. The remainder becomes a huge amount of thermal energy that must be dispersed as heat transfer, as rapidly as it is created. A coal-fired power plant may produce 1000 megawatts; 1 megawatt (MW) is 10^6 W of electric power. But the power plant consumes chemical energy at a rate of about 2500 MW, creating heat transfer to the surroundings at a rate of 1500 MW. (See [link].)

Tremendous amounts of electric power are generated by coal-fired power plants such as this one in China, but an even larger amount of power goes into heat transfer to the surroundings.

The large cooling towers here are needed to transfer heat as rapidly as it is produced. The transfer of heat is not unique to coal plants but is an unavoidable consequence of generating electric power from any fuel—nuclear, coal, oil, natural gas, or the like. (credit: Kleinolive, Wikimedia Commons)

Object or Phenomenon	Power in Watts
Supernova (at peak)	5×10^{37}
Milky Way galaxy	10^{37}
Crab Nebula pulsar	10^{28}
The Sun	4×10^{26}

Object or Phenomenon	Power in Watts
Volcanic eruption (maximum)	4×10^{15}
Lightning bolt	2×10^{12}
Nuclear power plant (total electric and heat transfer)	3×10^{9}
Aircraft carrier (total useful and heat transfer)	10^{8}
Dragster (total useful and heat transfer)	2×10^{6}
Car (total useful and heat transfer)	8×10^{4}
Football player (total useful and heat transfer)	5×10^{3}
Clothes dryer	4×10^{3}
Person at rest (all heat transfer)	100

Object or Phenomenon	**Power in Watts**
Typical incandescent light bulb (total useful and heat transfer)	60
Heart, person at rest (total useful and heat transfer)	8
Electric clock	3
Pocket calculator	10^{-3}

Power Output or Consumption

Power and Energy Consumption

We usually have to pay for the energy we use. It is interesting and easy to estimate the cost of energy for an electrical appliance if its power consumption rate and time used are known. The higher the power consumption rate and the longer the appliance is used, the greater the cost of that appliance. The power consumption rate is $P = W/t = E/t$, where E is the energy supplied by the electricity company. So the energy consumed over a time t is

Equation:

$$E = \mathrm{Pt}.$$

Electricity bills state the energy used in units of **kilowatt-hours** $(\mathrm{kW} \cdot \mathrm{h})$, which is the product of power in kilowatts and time in hours. This unit is convenient because electrical power consumption at the kilowatt level for hours at a time is typical.

Example:
Calculating Energy Costs
What is the cost of running a 0.200-kW computer 6.00 h per day for 30.0 d if the cost of electricity is $0.120 per $\mathrm{kW \cdot h}$?
Strategy
Cost is based on energy consumed; thus, we must find E from $E = \mathrm{Pt}$ and then calculate the cost. Because electrical energy is expressed in $\mathrm{kW \cdot h}$, at the start of a problem such as this it is convenient to convert the units into kW and hours.
Solution
The energy consumed in $\mathrm{kW \cdot h}$ is
Equation:

$$\begin{aligned} E &= \mathrm{Pt} = (0.200\,\mathrm{kW})(6.00\,\mathrm{h/d})(30.0\,\mathrm{d}) \\ &= 36.0\,\mathrm{kW \cdot h}, \end{aligned}$$

and the cost is simply given by
Equation:

$$\text{cost} = (36.0\ \mathrm{kW \cdot h})(\$0.120\ \text{per}\ \mathrm{kW \cdot h}) = \$4.32\ \text{per month.}$$

Discussion
The cost of using the computer in this example is neither exorbitant nor negligible. It is clear that the cost is a combination of power and time. When both are high, such as for an air conditioner in the summer, the cost is high.

The motivation to save energy has become more compelling with its ever-increasing price. Armed with the knowledge that energy consumed is the product of power and time, you can estimate costs for yourself and make the necessary value judgments about where to save energy. Either power or time must be reduced. It is most cost-effective to limit the use of high-power devices that normally operate for long periods of time, such as water heaters and air conditioners. This would not include relatively high power devices like toasters, because they are on only a few minutes per day. It would also not include electric clocks, in spite of their 24-hour-per-day

usage, because they are very low power devices. It is sometimes possible to use devices that have greater efficiencies—that is, devices that consume less power to accomplish the same task. One example is the compact fluorescent light bulb, which produces over four times more light per watt of power consumed than its incandescent cousin.

Modern civilization depends on energy, but current levels of energy consumption and production are not sustainable. The likelihood of a link between global warming and fossil fuel use (with its concomitant production of carbon dioxide), has made reduction in energy use as well as a shift to non-fossil fuels of the utmost importance. Even though energy in an isolated system is a conserved quantity, the final result of most energy transformations is waste heat transfer to the environment, which is no longer useful for doing work. As we will discuss in more detail later, the potential for energy to produce useful work has been “degraded” in the energy transformation.

Section Summary

- Power is the rate at which work is done, or in equation form, for the average power P for work W done over a time t, $P = W/t$.
- The SI unit for power is the watt (W), where $1\ \text{W} = 1\ \text{J/s}$.
- The power of many devices such as electric motors is also often expressed in horsepower (hp), where $1\ \text{hp} = 746\ \text{W}$.

Conceptual Questions

Exercise:

Problem:

Most electrical appliances are rated in watts. Does this rating depend on how long the appliance is on? (When off, it is a zero-watt device.) Explain in terms of the definition of power.

Exercise:

Problem:

Explain, in terms of the definition of power, why energy consumption is sometimes listed in kilowatt-hours rather than joules. What is the relationship between these two energy units?

Exercise:

Problem:

A spark of static electricity, such as that you might receive from a doorknob on a cold dry day, may carry a few hundred watts of power. Explain why you are not injured by such a spark.

Glossary

power
: the rate at which work is done

watt
: (W) SI unit of power, with $1\text{ W}=1\text{ J/s}$

horsepower
: an older non-SI unit of power, with $1\text{ hp}=746\text{ W}$

kilowatt-hour
: $(\text{kW}\cdot\text{h})$ unit used primarily for electrical energy provided by electric utility companies

# Introduction to Linear Momentum and Collisions
class="introduction"

Each rugby player has great momentum, which will affect the outcome of their collisions with each other and the ground. (credit: ozzzie, Flickr)

We use the term momentum in various ways in everyday language, and most of these ways are consistent with its precise scientific definition. We speak of sports teams or politicians gaining and maintaining the momentum to win. We also recognize that momentum has something to do with collisions. For example, looking at the rugby players in the photograph colliding and falling to the ground, we expect their momenta to have great effects in the resulting collisions. Generally, momentum implies a tendency to continue on course—to move in the same direction—and is associated with great mass and speed.

Momentum, like energy, is important because it is conserved. Only a few physical quantities are conserved in nature, and studying them yields fundamental insight into how nature works, as we shall see in our study of momentum.
https://www.youtube.com/embed/hxMaoFcYSrw

Linear Momentum and Force

- Define linear momentum.
- Explain the relationship between momentum and force.
- State Newton's second law of motion in terms of momentum.
- Calculate momentum given mass and velocity.

Linear Momentum

The scientific definition of linear momentum is consistent with most people's intuitive understanding of momentum: a large, fast-moving object has greater momentum than a smaller, slower object. **Linear momentum** is defined as the product of a system's mass multiplied by its velocity. In symbols, linear momentum is expressed as

Equation:

$$\boldsymbol{p} = m\boldsymbol{v}$$

Momentum is directly proportional to the object's mass and also its velocity. Thus the greater an object's mass or the greater its velocity, the greater its momentum. Momentum $\boldsymbol{p}$ is a vector having the same direction as the velocity $\boldsymbol{v}$. The SI unit for momentum is $\text{kg} \cdot \text{m/s}$.

Note:

Linear Momentum

Linear momentum is defined as the product of a system's mass multiplied by its velocity:

Equation:

$$\boldsymbol{p} = m\boldsymbol{v}.$$

Example:

Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110-kg football player running at 8.00 m/s. (b) Compare the player's momentum with the momentum of a hard-thrown 0.410-kg football that has a speed of 25.0 m/s.

Strategy

No information is given regarding direction, and so we can calculate only the magnitude of the momentum, p. (As usual, a symbol that is in italics is a magnitude, whereas one that is italicized and is boldfaced or has an arrow is a vector.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum given in the equation, which becomes

Equation:

$$p = \mathrm{mv}$$

when only magnitudes are considered.

Solution for (a)

To determine the momentum of the player, substitute the known values for the player's mass and speed into the equation.

Equation:

$$p_{\text{player}} = (110\ \mathrm{kg})(8.00\ \mathrm{m/s}) = 880\ \mathrm{kg \cdot m/s}$$

Solution for (b)

To determine the momentum of the ball, substitute the known values for the ball's mass and speed into the equation.

Equation:

$$p_{\text{ball}} = (0.410\ \mathrm{kg})(25.0\ \mathrm{m/s}) = 10.3\ \mathrm{kg \cdot m/s}$$

The ratio of the player's momentum to that of the ball is

Equation:

$$\frac{p_{\text{player}}}{p_{\text{ball}}} = \frac{880}{10.3} = 85.9.$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Thus the momentum of the player is much greater than the momentum of the football, as you might guess. As a result, the player's motion is only slightly affected if he catches the ball. We shall quantify what happens in such collisions in terms of momentum in later sections.

Momentum and Newton's Second Law

The importance of momentum, unlike the importance of energy, was recognized early in the development of classical physics. Momentum was deemed so important that it was called the "quantity of motion." Newton actually stated his **second law of motion** in terms of momentum: The net external force equals the change in momentum of a system divided by the time over which it changes. Using symbols, this law is

Equation:

$$\boldsymbol{F}_{\text{net}} = \frac{\Delta \boldsymbol{p}}{\Delta t},$$

where $\boldsymbol{F}_{\text{net}}$ is the net external force, $\Delta \boldsymbol{p}$ is the change in momentum, and Δt is the change in time.

Note:

Newton's Second Law of Motion in Terms of Momentum

The net external force equals the change in momentum of a system divided by the time over which it changes: $\boldsymbol{F}_{\text{net}} = \Delta \boldsymbol{p}/\Delta t$.

Note:

Making Connections: Force and Momentum

Force and momentum are intimately related. Force acting over time can change momentum, and Newton's second law of motion, can be stated in

its most broadly applicable form in terms of momentum. Momentum continues to be a key concept in the study of atomic and subatomic particles in quantum mechanics.

This statement of Newton's second law of motion includes the more familiar $\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$ as a special case. We can derive this form as follows. First, note that the change in momentum $\boldsymbol{\Delta p}$ is given by

Equation:

$$\Delta \boldsymbol{p} = \Delta(m\boldsymbol{v}).$$

If the mass of the system is constant, then

Equation:

$$\Delta(m\boldsymbol{v}) = m\Delta\boldsymbol{v}.$$

So that for constant mass, Newton's second law of motion becomes

Equation:

$$\boldsymbol{F}_{\text{net}} = \frac{\Delta \boldsymbol{p}}{\Delta t} = m\frac{\Delta \boldsymbol{v}}{\Delta t}$$

Because $\frac{\Delta \boldsymbol{v}}{\Delta t} = \boldsymbol{a}$, we get the familiar equation

Equation:

$$\boldsymbol{F}_{\text{net}} = m\boldsymbol{a}$$

when the mass of the system is constant.

Newton's second law of motion stated in terms of momentum is more generally applicable because it can be applied to systems where the mass is changing, such as rockets, as well as to systems of constant mass. We will consider systems with varying mass in some detail**;** however, the

relationship between momentum and force remains useful when mass is constant, such as in the following example.

Example:

Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet, assuming that the ball's speed just after impact is 58 m/s, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms (milliseconds)?

Strategy

This problem involves only one dimension because the ball starts from having no horizontal velocity component before impact. Newton's second law stated in terms of momentum is then written as $\boldsymbol{F}_{\text{net}} = \Delta\boldsymbol{p}/\Delta t$.

As noted above, when mass is constant, the change in momentum is given by

Equation:

$$\Delta p = m\Delta v = m(v_{\text{f}} - v_{\text{i}}).$$

In this example, the velocity just after impact and the change in time are given; thus, once Δp is calculated, $F_{\text{net}} = \frac{\Delta p}{\Delta t}$ can be used to find the force.

Solution

To determine the change in momentum, substitute the values for the initial and final velocities into the equation above.

Equation:

$$\begin{aligned}\Delta p &= m(v_{\text{f}} – v_{\text{i}}) \\ &= (0.057\ \text{kg})(58\ \text{m/s} – 0\ \text{m/s}) \\ &= 3.306\ \text{kg}\cdot\text{m/s} \approx 3.3\ \text{kg}\cdot\text{m/s}\end{aligned}$$

Now the magnitude of the net external force can determined by using $F_{\text{net}} = \frac{\Delta p}{\Delta t}$:
Equation:

$$\begin{aligned} F_{\text{net}} &= \frac{\Delta p}{\Delta t} = \frac{3.306 \text{ kg}\cdot\text{m/s}}{5.0\times10^{-3}\text{ s}} \\ &= 661 \text{ N} \approx 660 \text{ N}, \end{aligned}$$

where we have retained only two significant figures in the final step.
Discussion
This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact (note that the ball also experienced the 0.56-N force of gravity, but that force was not due to the racquet). This problem could also be solved by first finding the acceleration and then using $F_{\text{net}} = \text{ma}$, but one additional step would be required compared with the strategy used in this example.

Section Summary

- Linear momentum (*momentum* for brevity) is defined as the product of a system's mass multiplied by its velocity.
- In symbols, linear momentum $\boldsymbol{p}$ is defined to be
  Equation:

  $$\boldsymbol{p} = m\boldsymbol{v},$$

  where m is the mass of the system and $\boldsymbol{v}$ is its velocity.
- The SI unit for momentum is $\text{kg} \cdot \text{m/s}$.
- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes.
- In symbols, Newton's second law of motion is defined to be
  Equation:

  $$\boldsymbol{F}_{\text{net}} = \frac{\Delta \boldsymbol{p}}{\Delta t},$$

where $\boldsymbol{F}_{\text{net}}$ is the net external force, $\Delta\boldsymbol{p}$ is the change in momentum, and Δt is the change time.

Conceptual Questions

Exercise:

Problem:

An object that has a small mass and an object that has a large mass have the same momentum. Which object has the largest kinetic energy?

Exercise:

Problem:

An object that has a small mass and an object that has a large mass have the same kinetic energy. Which mass has the largest momentum?

Exercise:

Problem: Professional Application

Football coaches advise players to block, hit, and tackle with their feet on the ground rather than by leaping through the air. Using the concepts of momentum, work, and energy, explain how a football player can be more effective with his feet on the ground.

Exercise:

Problem:

How can a small force impart the same momentum to an object as a large force?

Glossary

linear momentum

the product of mass and velocity

second law of motion
physical law that states that the net external force equals the change in momentum of a system divided by the time over which it changes

Impulse

- Define impulse.
- Describe effects of impulses in everyday life.
- Calculate average force and impulse given mass, velocity, and time.

The effect of a force on an object depends on how long it acts, as well as how great the force is. In previous examples, a very large force acting for a short time had a great effect on the momentum of the tennis ball. A small force could cause the same **change in momentum**, but it would have to act for a much longer time. For example, if the ball were thrown upward, the gravitational force (which is much smaller than the tennis racquet's force) would eventually reverse the momentum of the ball. Quantitatively, the effect we are talking about is the change in momentum $\Delta\boldsymbol{p}$.

By rearranging the equation $\boldsymbol{F}_{\text{net}} = \frac{\Delta\boldsymbol{p}}{\Delta t}$ to be
Equation:

$$\Delta\boldsymbol{p} = \boldsymbol{F}_{\text{net}}\Delta t,$$

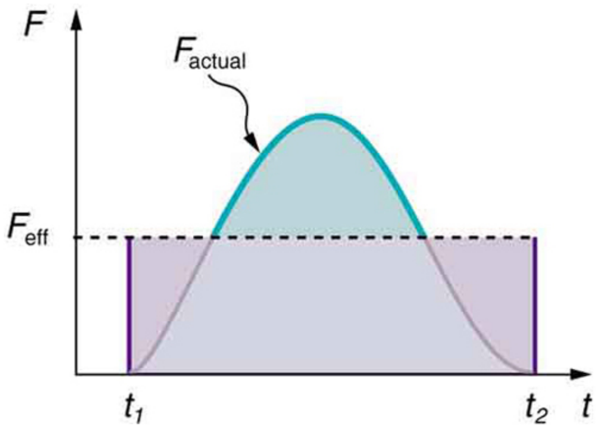
we can see how the change in momentum equals the average net external force multiplied by the time this force acts. The quantity $\boldsymbol{F}_{\text{net}}\Delta t$, force times duration, is given the name **impulse**. Net impulse on an object gives its change in momentum.

Note:
Impulse: Change in Momentum
Change in momentum equals the average net external force multiplied by the time this force acts: $\Delta\boldsymbol{p} = \boldsymbol{F}_{\text{net}}\Delta t$.
The quantity $\boldsymbol{F}_{\text{net}}\Delta t$ is given the name impulse.
There are many ways in which an understanding of impulse can save lives, or at least limbs. The dashboard padding in a car, and certainly the airbags, allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant, whether an air bag is deployed or not, but the force (to bring

the occupant to a stop) will be much less if it acts over a larger time. Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

Bones in a body will fracture if the force on them is too large. If you jump onto the floor from a table, the force on your legs can be immense if you land stiff-legged on a hard surface. Rolling on the ground after jumping from the table, or landing with a parachute, extends the time over which the force (on you from the ground) acts.

Our definition of impulse includes an assumption that the force is constant over the time interval Δt. *Forces are usually not constant*. Forces vary considerably even during the brief time intervals considered. It is, however, possible to find an average effective force F_{eff} that produces the same result as the corresponding time-varying force. [link] shows a graph of what an actual force looks like as a function of time for a ball bouncing off the floor. The area under the curve has units of momentum and is equal to the impulse or change in momentum between times t_1 and t_2. That area is equal to the area inside the rectangle bounded by F_{eff}, t_1, and t_2. Thus the impulses and their effects are the same for both the actual and effective forces.



A graph of force versus time with time along the x-axis and force along the y-axis for an actual force and an equivalent effective force. The areas under the two curves are equal.

Note:
Making Connections: Take-Home Investigation—Hand Movement and Impulse
Try catching a ball while “giving” with the ball, pulling your hands toward your body. Then, try catching a ball while keeping your hands still. Hit water in a tub with your full palm. After the water has settled, hit the water again by diving your hand with your fingers first into the water. (Your full palm represents a swimmer doing a belly flop and your diving hand represents a swimmer doing a dive.) Explain what happens in each case and why. Which orientations would you advise people to avoid and why?

Note:
Making Connections: Constant Force and Constant Acceleration
The assumption of a constant force in the definition of impulse is analogous to the assumption of a constant acceleration in kinematics. In both cases, nature is adequately described without the use of calculus.

Section Summary

- Impulse is force times duration, and the net impulse, equal to the net external force multiplied by duration, gives the change in momentum:
  Equation:

$$\Delta \boldsymbol{p} = \boldsymbol{F}_{\text{net}} \Delta t.$$

- For non-constant net external force, you need to use time-averaged net external force to calculate the impulse over the duration.

Conceptual Questions

Exercise:

Problem: Professional Application

Explain in terms of impulse how padding reduces forces in a collision. State this in terms of a real example, such as the advantages of a carpeted vs. tile floor for a day care center.

Exercise:

Problem:

While jumping on a trampoline, sometimes you land on your back and other times on your feet. In which case can you reach a greater height and why?

Exercise:

Problem: Professional Application

Tennis racquets have “sweet spots.” If the ball hits a sweet spot then the player's arm is not jarred as much as it would be otherwise. Explain why this is the case.

Glossary

change in momentum
: the difference between the final and initial momentum; the mass times the change in velocity

impulse

the average net external force times the time it acts; equal to the change in momentum

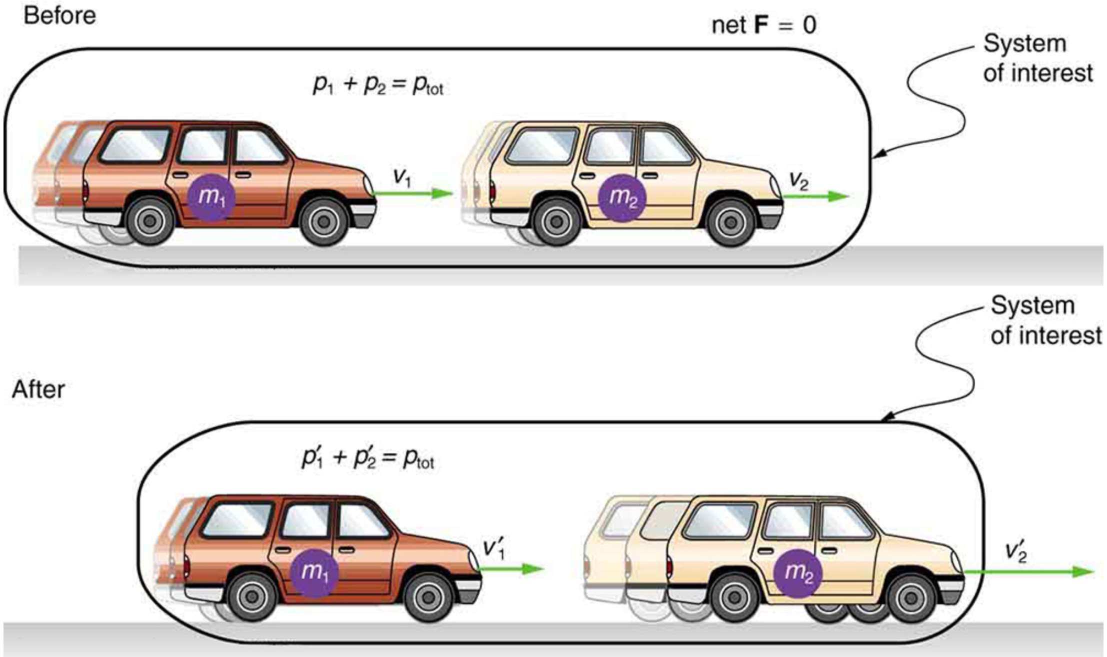
Conservation of Momentum

- Describe the principle of conservation of momentum.
- Explain conservation of momentum with examples.

In the previous sections in this chapter, changes in momentum for an object were considered in terms of impulse and force. But for certain conditions, the total momentum is conserved (stays constant). What are these conditions?

The answer to this question entails considering a sufficiently large system. It is always possible to find a larger system in which total momentum is constant, even if momentum changes for components of the system. If a football player runs into the goalpost in the end zone, there will be a force on him that causes him to bounce backward. However, the Earth also recoils —conserving momentum—because of the force applied to it through the goalpost. Because Earth is many orders of magnitude more massive than the player, its recoil is immeasurably small and can be neglected in any practical sense, but it is real nevertheless.

Consider what happens if the masses of two colliding objects are more similar (i.e, both objects are about the same mass) than the masses of a football player and Earth—for example, one car bumping into another, as shown in [link]. Both cars are coasting in the same direction when the lead car (labeled m_2) is bumped by the trailing car (labeled m_1). The only unbalanced force on each car is the force of the collision. (Assume that the effects due to friction are negligible.) Car 1 slows down as a result of the collision, losing some momentum, while car 2 speeds up and gains some momentum. We shall now show that the total momentum of the two-car system remains constant.



A car of mass m_1 moving with a velocity of v_1 bumps into another car of mass m_2 and velocity v_2 that it is following. As a result, the first car slows down to a velocity of $v\prime_1$ and the second speeds up to a velocity of $v\prime_2$. The momentum of each car is changed, but the total momentum p_{tot} of the two cars is the same before and after the collision (if you assume friction is negligible).

Using the definition of impulse, the change in momentum of car 1 is given by

Equation:

$$\Delta p_1 = F_1 \Delta t,$$

where F_1 is the force on car 1 due to car 2, and Δt is the time the force acts (the duration of the collision). Similarly, the change in momentum of car 2 is

Equation:

$$\Delta p_2 = F_2 \Delta t,$$

where F_2 is the force on car 2 due to car 1, and, intuitively, the duration of the collision Δt is the same for both cars. We know from Newton’s third law that $F_2 = -F_1$, and so

Equation:

$$\Delta p_2 = -F_1 \Delta t = -\Delta p_1.$$

Thus, the changes in momentum cancel out, and

Equation:

$$\Delta p_1 + \Delta p_2 = 0.$$

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

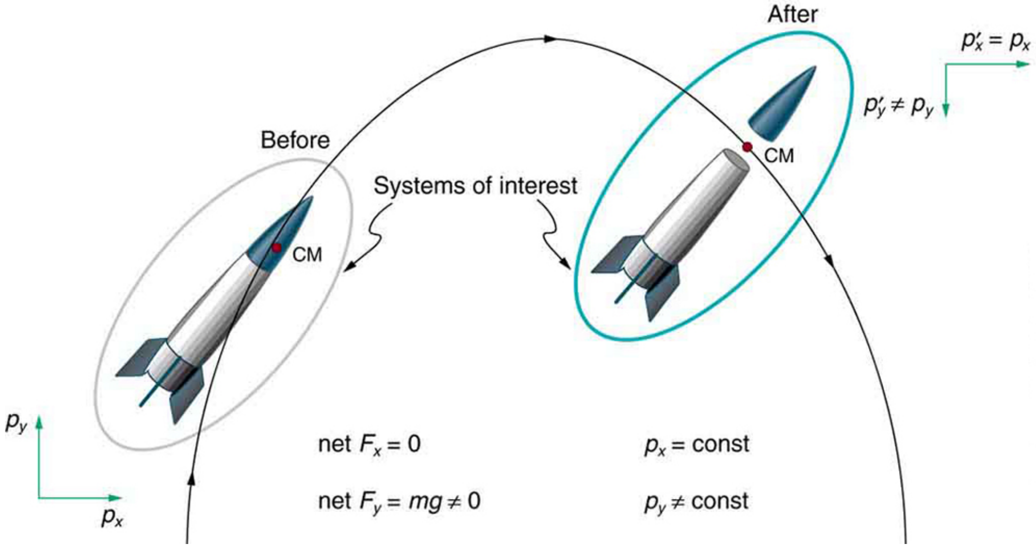
Equation:

$$p_1 + p_2 = \text{constant}.$$

Because of Newton's third law, when two objects in a system interact with each other, it does not change the total momentum of the system. In the absence of an external force (that is, force due to an object outside of the system, so that we can ignore the impulse due to the reaction force), the total momentum of the system is constant, or conserved. We call this the **conservation of momentum principle**.

We have noted that the motion in perpendicular directions—x, y, and z—are independent. Momentum conservation follows this paradigm, and components of total momentum along each direction are conserved separately. For example, during projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero and momentum is unchanged. But along the vertical direction, the net vertical force is not zero and the momentum of the projectile is not conserved. (See [link].) However, if the momentum of the

projectile-Earth system is considered in the vertical direction, we find that the total momentum is conserved.



The horizontal component of a projectile's momentum is conserved if air resistance is negligible, even in this case where a space probe separates. The forces causing the separation are internal to the system, so that the net external horizontal force $F_{x\text{–net}}$ is still zero. The vertical component of the momentum is not conserved, because the net vertical force $F_{y\text{–net}}$ is not zero. In the vertical direction, the space probe-Earth system needs to be considered and we find that the total momentum is conserved. The center of mass of the space probe takes the same path it would if the separation did not occur.

The conservation of momentum principle can be applied to systems as different as a comet striking Earth and a gas containing huge numbers of atoms and molecules. Conservation of momentum is violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

Note:

Making Connections: Take-Home Investigation—Drop of Tennis Ball and a Basketball

Hold a tennis ball side by side and in contact with a basketball. Drop the balls together. (Be careful!) What happens? Explain your observations. Now hold the tennis ball above and in contact with the basketball. What happened? Explain your observations. What do you think will happen if the basketball ball is held above and in contact with the tennis ball?

Note:

Making Connections: Take-Home Investigation—Two Tennis Balls in a Ballistic Trajectory

Tie two tennis balls together with a string about a foot long. Hold one ball and let the other hang down and throw it in a ballistic trajectory. Explain your observations. Now mark the center of the string with bright ink or attach a brightly colored sticker to it and throw again. What happened? Explain your observations.

Some aquatic animals such as jellyfish move around based on the principles of conservation of momentum. A jellyfish fills its umbrella section with water and then pushes the water out resulting in motion in the opposite direction to that of the jet of water. Squids propel themselves in a similar manner but, in contrast with jellyfish, are able to control the direction in which they move by aiming their nozzle forward or backward. Typical squids can move at speeds of 8 to 12 km/h.

The ballistocardiograph (BCG) was a diagnostic tool used in the second half of the 20th century to study the strength of the heart. About once a second, your heart beats, forcing blood into the aorta. A force in the opposite direction is exerted on the rest of your body (recall Newton’s third law). A ballistocardiograph is a device that can measure this reaction force. This measurement is done by using a sensor (resting on the person) or by using a moving table suspended from the ceiling. This technique can gather information on the strength of the heart beat and the volume of blood passing from the heart. However, the electrocardiogram (ECG or EKG) and the echocardiogram (cardiac ECHO or ECHO; a technique that uses

ultrasound to see an image of the heart) are more widely used in the practice of cardiology.

Note:
Making Connections: Conservation of Momentum and Collision
Conservation of momentum is quite useful in describing collisions. Momentum is crucial to our understanding of atomic and subatomic particles because much of what we know about these particles comes from collision experiments.

Section Summary

- The conservation of momentum principle says that the total momentum is conserved, or constant, in the absence of a net external force, as a direct consequence of Newton's third law.
- During projectile motion and where air resistance is negligible, momentum is conserved in the horizontal direction because horizontal forces are zero.

Conceptual Questions

Exercise:

Problem: Professional Application

If you dive into water, you reach greater depths than if you do a belly flop. Explain this difference in depth using the concept of conservation of energy. Explain this difference in depth using what you have learned in this chapter.

Exercise:

Problem: Under what circumstances is momentum conserved?

Exercise:

Problem:

Can momentum be conserved for a system if there are external forces acting on the system? If so, under what conditions? If not, why not?

Exercise:

Problem:

Momentum for a system can be conserved in one direction while not being conserved in another. What is the angle between the directions? Give an example.

Exercise:

Problem: Professional Application

Explain in terms of momentum and Newton's laws how a car's air resistance is due in part to the fact that it pushes air in its direction of motion.

Exercise:

Problem:

Can objects in a system have momentum while the momentum of the system is zero? Explain your answer.

Exercise:

Problem:

Must the total energy of a system be conserved whenever its momentum is conserved? Explain why or why not.

Glossary

conservation of momentum principle

when the net external force is zero, the total momentum of the system is conserved or constant

Elastic Collisions in One Dimension

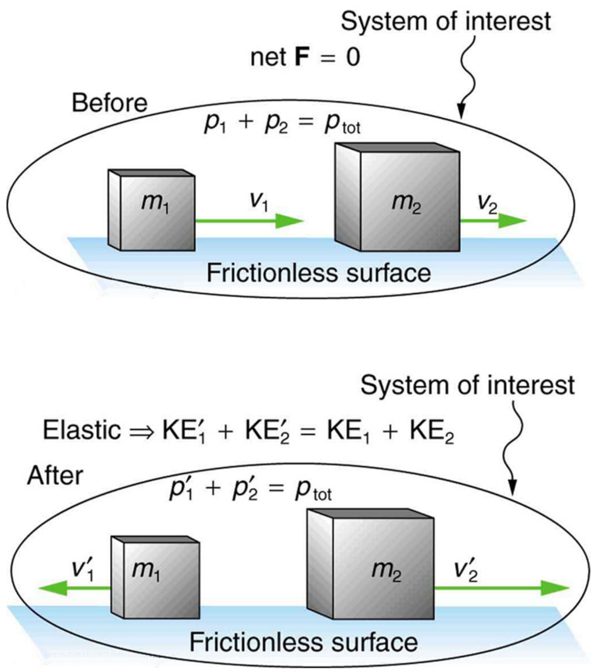
- Describe an elastic collision of two objects in one dimension.
- Determine the final velocities in an elastic collision given masses and initial velocities.

Let us consider various types of two-object collisions. These collisions are the easiest to analyze, and they illustrate many of the physical principles involved in collisions. The conservation of momentum principle is very useful here, and it can be used whenever the net external force on a system is zero.

We start with the elastic collision of two objects moving along the same line—a one-dimensional problem. An **elastic collision** is one that also conserves total kinetic energy, in addition to the total momentum. [link] illustrates an elastic collision in which kinetic energy and momentum are conserved.

Truly elastic collisions can only be achieved with subatomic particles, such as electrons striking nuclei. Macroscopic collisions can be very nearly, but not quite, elastic—some kinetic energy is always converted into other forms of energy such as heat transfer due to friction and sound. One macroscopic collision that is nearly elastic is that of two steel blocks on ice. Another nearly elastic collision is that between two carts with spring bumpers on an air track. Icy surfaces and air tracks are nearly frictionless, more readily allowing nearly elastic collisions on them.

Note:
Elastic Collision
An **elastic collision** is one that conserves total kinetic energy.



An elastic one-dimensional two-object collision. Momentum and internal kinetic energy are conserved.

Now, to solve problems involving one-dimensional elastic collisions between two objects we can use the equations for conservation of momentum and conservation of internal kinetic energy. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

Equation:

$$p_1 + p_2 = p'_1 + p'_2 \quad (F_{\text{net}} = 0)$$

or

Equation:

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2 \quad (F_{\text{net}} = 0),$$

where the primes (') indicate values after the collision. By definition, an elastic collision conserves total kinetic energy, and so the sum of kinetic energies before the collision equals the sum after the collision. Thus,

Equation:

$$\frac{1}{2}m_1{v_1}^2+\frac{1}{2}m_2{v_2}^2=\frac{1}{2}m_1{v\prime_1}^2+\frac{1}{2}m_2{v\prime_2}^2 \text{ (two-object elastic collision)}$$

expresses the equation for conservation of total kinetic energy in a one-dimensional collision.

Example:

Calculating Velocities Following an Elastic Collision

Calculate the velocities of two objects following an elastic collision, given that

Equation:

$$m_1=0.500\text{ kg},\ m_2=3.50\text{ kg},\ v_1=4.00\text{ m/s, and }\ v_2=0.$$

Strategy and Concept

First, visualize what the initial conditions mean—a small object strikes a larger object that is initially at rest. This situation is slightly simpler than the situation shown in [link] where both objects are initially moving. We are asked to find two unknowns (the final velocities $v\prime_1$ and $v\prime_2$). To find two unknowns, we must use two independent equations. Because this collision is elastic, we can use the above two equations. Both can be simplified by the fact that object 2 is initially at rest, and thus $v_2=0$. Once we simplify these equations, we combine them algebraically to solve for the unknowns.

Solution

For this problem, note that $v_2=0$ and use conservation of momentum. Thus,

Equation:

$$p_1=p\prime_1+p\prime_2$$

or

Equation:

$$m_1 v_1 = m_1 v\prime_1 + m_2 v\prime_2.$$

Using conservation of internal kinetic energy and that $v_2 = 0$,
Equation:

$$\frac{1}{2}m_1{v_1}^2 = \frac{1}{2}m_1{v\prime_1}^2 + \frac{1}{2}m_2{v\prime_2}^2.$$

Solving the first equation (momentum equation) for $v\prime_2$, we obtain
Equation:

$$v\prime_2 = \frac{m_1}{m_2}(v_1 - v\prime_1).$$

Substituting this expression into the second equation (internal kinetic energy equation) eliminates the variable $v\prime_2$, leaving only $v\prime_1$ as an unknown (the algebra is left as an exercise for the reader). There are two solutions to any quadratic equation; in this example, they are
Equation:

$$v\prime_1 = 4.00 \text{ m/s}$$

and
Equation:

$$v\prime_1 = -3.00 \text{ m/s}.$$

As noted when quadratic equations were encountered in earlier chapters, both solutions may or may not be meaningful. In this case, the first solution is the same as the initial condition. The first solution thus represents the situation before the collision and is discarded. The second solution $(v\prime_1 = -3.00 \text{ m/s})$ is negative, meaning that the first object bounces backward. When this negative value of $v\prime_1$ is used to find the velocity of the second object after the collision, we get
Equation:

$$v\prime_2 = \frac{m_1}{m_2}(v_1 - v\prime_1) = \frac{0.500 \text{ kg}}{3.50 \text{ kg}}\left[4.00 - (-3.00)\right] \text{ m/s}$$

or

Equation:

$$v\prime_2 = 1.00 \text{ m/s}.$$

Discussion

The result of this example is intuitively reasonable. A small object strikes a larger one at rest and bounces backward. The larger one is knocked forward, but with a low speed. (This is like a compact car bouncing backward off a full-size SUV that is initially at rest.) As a check, try calculating the internal kinetic energy before and after the collision. You will see that the internal kinetic energy is unchanged at 4.00 J. Also check the total momentum before and after the collision; you will find it, too, is unchanged.

The equations for conservation of momentum and internal kinetic energy as written above can be used to describe any one-dimensional elastic collision of two objects. These equations can be extended to more objects if needed.

Note:

Making Connections: Take-Home Investigation—Ice Cubes and Elastic Collision

Find a few ice cubes which are about the same size and a smooth kitchen tabletop or a table with a glass top. Place the ice cubes on the surface several centimeters away from each other. Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes. Have you created approximately elastic collisions? Explain the speeds and directions of the ice cubes using momentum.

Section Summary

- An elastic collision is one that conserves total kinetic energy.
- Conservation of kinetic energy and momentum together allow the final velocities to be calculated in terms of initial velocities and masses in one dimensional two-body collisions.

Conceptual Questions

Exercise:

Problem: What physical quantities are conserved in an elastic collision?

Exercise:

Problem:

Give examples of elastic (or nearly elastic) collisions from everyday experiences.

Glossary

elastic collision
: a collision that also conserves total kinetic energy, in addition to the total momentum

Inelastic Collisions in One Dimension

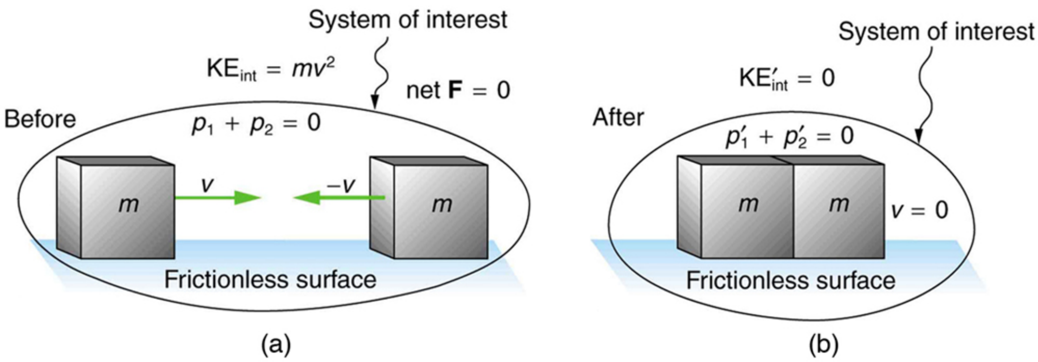
- Define inelastic collision.
- Explain perfectly inelastic collision.
- Apply an understanding of collisions to everyday situations.
- Determine recoil velocity and loss in kinetic energy given mass and initial velocity.

We have seen that in an elastic collision, total kinetic energy is conserved. An **inelastic collision** is one in which the total kinetic energy changes (it is not conserved). This lack of conservation means that the forces between colliding objects may remove or add total kinetic energy. Work done by internal forces may change the forms of energy within a system. For inelastic collisions, such as when colliding objects stick together, this internal work may transform some kinetic energy into thermal energy. Or it may convert stored energy into total kinetic energy, such as when exploding bolts separate a satellite from its launch vehicle.

Note:
Inelastic Collision
An inelastic collision is one in which the total kinetic energy changes (it is not conserved).

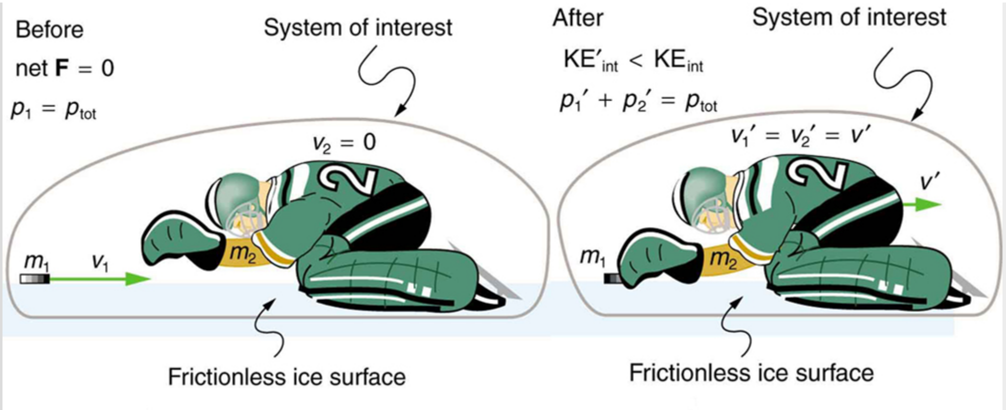
[link] shows an example of an inelastic collision. Two objects that have equal masses head toward one another at equal speeds and then stick together. Their total kinetic energy is initially $\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$. The two objects come to rest after sticking together, conserving total momentum. And the total kinetic energy is zero after this collision. A collision in which the objects stick together is called a **perfectly inelastic collision** because it reduces the total kinetic energy as much as possible, while conserving total momentum.

Note:
Perfectly Inelastic Collision
A collision in which the objects stick together is sometimes called "perfectly inelastic."



An inelastic one-dimensional two-object collision. Momentum is conserved, but total kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward one another at the same speed. (b) The objects stick together (a perfectly inelastic collision), and so their final velocity is zero.

Example:
Calculating Velocity and Change in Kinetic Energy: Inelastic Collision of a Puck and a Goalie
(a) Find the recoil velocity of a 70.0-kg ice hockey goalie, originally at rest, who catches a 0.150-kg hockey puck slapped at him at a velocity of 35.0 m/s. (b) How much kinetic energy is lost during the collision? Assume friction between the ice and the puck-goalie system is negligible. (See [link])



An ice hockey goalie catches a hockey puck and recoils backward. The initial kinetic energy of the puck is almost entirely converted to thermal energy and sound in this inelastic collision.

Strategy
Momentum is conserved because the net external force on the puck-goalie system is zero. We can thus use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same. Once the final velocity is found, the kinetic energies can be calculated before and after the collision and compared as requested.
Solution for (a)
Momentum is conserved because the net external force on the puck-goalie system is zero.
Conservation of momentum is
Equation:

$$p_1 + p_2 = p\prime_1 + p\prime_2$$

or
Equation:

$$m_1 v_1 + m_2 v_2 = m_1 v\prime_1 + m_2 v\prime_2.$$

Because the goalie is initially at rest, we know $v_2 = 0$. Because the goalie catches the puck, the final velocities are equal, or $v\prime_1 = v\prime_2 = v\prime$. Thus, the

conservation of momentum equation simplifies to

Equation:

$$m_1 v_1 = (m_1 + m_2)v\prime.$$

Solving for $v\prime$ yields

Equation:

$$v\prime = \frac{m_1}{m_1 + m_2} v_1.$$

Entering known values in this equation, we get

Equation:

$$v\prime = \left(\frac{0.150 \text{ kg}}{0.150 \text{ kg} + 70.0 \text{ kg}}\right)(35.0 \text{ m/s}) = 7.48 \times 10^{-2} \text{ m/s}.$$

Discussion for (a)

This recoil velocity is small and in the same direction as the puck's original velocity, as we might expect.

Solution for (b)

Before the collision, the total kinetic energy KE_{int} of the system is that of the hockey puck, because the goalie is initially at rest. Therefore, KE_{int} is initially

Equation:

$$\begin{aligned} \text{KE}_{\text{int}} &= \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.150 \text{ kg})(35.0 \text{ m/s})^2 \\ &= 91.9 \text{ J}. \end{aligned}$$

After the collision, the total kinetic energy is

Equation:

$$\begin{aligned} \text{KE}\prime_{\text{int}} &= \tfrac{1}{2}(m + M)v^2 = \tfrac{1}{2}(70.15 \text{ kg})\left(7.48 \times 10^{-2} \text{ m/s}\right)^2 \\ &= 0.196 \text{ J}. \end{aligned}$$

The change in total kinetic energy is thus

Equation:

$$\begin{aligned}\mathrm{KE}\prime_{\mathrm{int}} - \mathrm{KE}_{\mathrm{int}} &= 0.196\ \mathrm{J} - 91.9\ \mathrm{J} \\ &= -91.7\ \mathrm{J}\end{aligned}$$

where the minus sign indicates that the energy was lost.

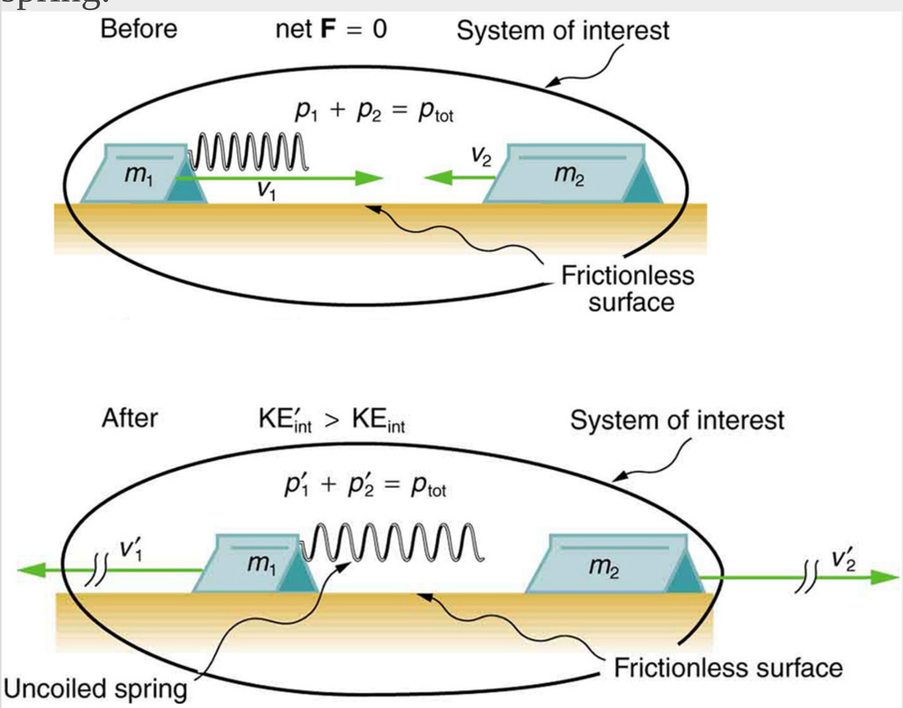
Discussion for (b)

Nearly all of the initial total kinetic energy is lost in this perfectly inelastic collision. $\mathrm{KE}_{\mathrm{int}}$ is mostly converted to thermal energy and sound. Note that in this case, the total kinetic energy does not decrease all the way down to zero, because doing so would violate conservation of momentum, because initial total momentum is not zero. This is the most amount of energy that can be converted from the initial total kinetic energy without violating conservation of momentum.

Example:

Calculating Final Velocity and Energy Release: Two Carts Collide

During some collisions stored energy may be converted into kinetic energy during a collision. [link] shows a one-dimensional example in which two carts on an air track collide, releasing potential energy from a compressed spring.



An air track is nearly frictionless, so that momentum is conserved. Motion is one-dimensional. In this collision, examined in [link], the potential energy of a compressed spring is released during the collision and is converted to internal kinetic energy.

In the collision pictured in [link], two carts collide inelastically. Cart 1 (denoted m_1 carries a spring which is initially compressed. During the collision, the spring releases its potential energy and converts it to total kinetic energy. The mass of cart 1 and the spring is 0.350 kg, and the cart and the spring together have an initial velocity of 2.00 m/s. Cart 2 (denoted m_2 in [link]) has a mass of 0.500 kg and an initial velocity of -0.500 m/s. After the collision, cart 1 is observed to recoil with a velocity of -4.00 m/s. (a) What is the final velocity of cart 2? (b) How much energy was released by the spring (assuming all of it was converted into internal kinetic energy)?

Strategy

We can use conservation of momentum to find the final velocity of cart 2, because $F_{\text{net}} = 0$ (the track is frictionless and the force of the spring is internal). Once this velocity is determined, we can compare the total kinetic energy before and after the collision to see how much energy was released by the spring.

Solution for (a)

As before, the equation for conservation of momentum in a two-object system is

Equation:

$$m_1v_1 + m_2v_2 = m_1v\prime_1 + m_2v\prime_2.$$

The only unknown in this equation is $v\prime_2$. Solving for $v\prime_2$ and substituting known values into the previous equation yields

Equation:

$$\begin{aligned} v'_2 &= \frac{m_1 v_1 + m_2 v_2 - m_1 v'_1}{m_2} \\ &= \frac{(0.350\text{ kg})(2.00\text{ m/s}) + (0.500\text{ kg})(-0.500\text{ m/s})}{0.500\text{ kg}} - \frac{(0.350\text{ kg})(-4.00\text{ m/s})}{0.500\text{ kg}} \\ &= 3.70\text{ m/s.} \end{aligned}$$

Solution for (b)
The total kinetic energy before the collision is
Equation:

$$\begin{aligned} \text{KE}_{\text{int}} &= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \\ &= \frac{1}{2}(0.350\text{ kg})(2.00\text{ m/s})^2 + \frac{1}{2}(0.500\text{ kg})(-0.500\text{ m/s})^2 \\ &= 0.763\text{ J.} \end{aligned}$$

After the collision, the total kinetic energy is
Equation:

$$\begin{aligned} \text{KE}'_{\text{int}} &= \frac{1}{2} m_1 v'^2_1 + \frac{1}{2} m_2 v'^2_2 \\ &= \frac{1}{2}(0.350\text{ kg})(-4.00\text{ m/s})^2 + \frac{1}{2}(0.500\text{ kg})(3.70\text{ m/s})^2 \\ &= 6.22\text{ J.} \end{aligned}$$

The change in total kinetic energy is thus
Equation:

$$\begin{aligned} \text{KE}'_{\text{int}} - \text{KE}_{\text{int}} &= 6.22\text{ J} - 0.763\text{ J} \\ &= 5.46\text{ J.} \end{aligned}$$

Discussion
The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision. The total kinetic energy in this collision increases by 5.46 J. That energy was released by the spring.

Section Summary

- An inelastic collision is one in which the total kinetic energy changes (it is not conserved).
- A collision in which the objects stick together is sometimes called perfectly inelastic because it reduces total kinetic energy as much as possible while conserving total momentum.

Conceptual Questions

Exercise:

Problem:

What is an inelastic collision? What is a perfectly inelastic collision?

Exercise:

Problem:

Mixed-pair ice skaters performing in a show are standing motionless at arms length just before starting a routine. They reach out, clasp hands, and pull themselves together by only using their arms. Assuming there is no friction between the blades of their skates and the ice, what is their velocity after their bodies meet?

Exercise:

Problem:

A small pickup truck that has a camper shell slowly coasts toward a red light with negligible friction. Two dogs in the back of the truck are moving and making various inelastic collisions with each other and the walls. What is the effect of the dogs on the motion of the center of mass of the system (truck plus entire load)? What is their effect on the motion of the truck?

Glossary

inelastic collision
: a collision in which total kinetic energy is not conserved

perfectly inelastic collision

a collision in which the colliding objects stick together

Introduction to Oscillatory Motion and Waves class="introduction"

There are at least four types of waves in this picture—only the water waves are evident. There are also sound waves, light waves, and waves on the guitar strings. (credit: John Norton)

What do an ocean buoy, a child in a swing, the cone inside a speaker, a guitar, atoms in a crystal, the motion of chest cavities, and the beating of hearts all have in common? They all **oscillate**—-that is, they move back and forth between two points. Many systems oscillate, and they have certain characteristics in common. All oscillations involve force and energy. You push a child in a swing to get the motion started. The energy of atoms vibrating in a crystal can be increased with heat. You put energy into a guitar string when you pluck it.

Some oscillations create **waves**. A guitar creates sound waves. You can make water waves in a swimming pool by slapping the water with your hand. You can no doubt think of other types of waves. Some, such as water waves, are visible. Some, such as sound waves, are not. But *every wave is a disturbance that moves from its source and carries energy*. Other examples of waves include earthquakes and visible light. Even subatomic particles, such as electrons, can behave like waves.

By studying oscillatory motion and waves, we shall find that a small number of underlying principles describe all of them and that wave phenomena are more common than you have ever imagined.

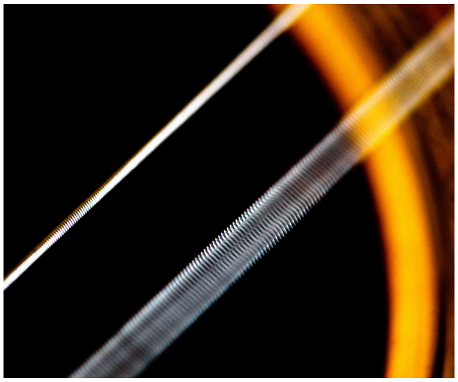
Glossary

oscillate
: moving back and forth regularly between two points

wave
: a disturbance that moves from its source and carries energy

Period and Frequency in Oscillations

- Understand the relationship between the frequency and the period of oscillations.
- Determine the frequency of oscillations.



The strings on this guitar vibrate at regular time intervals. (credit: JAR)

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each successive vibration of the string takes the same time as the previous one. We define **periodic motion** to be a motion that repeats itself at regular time intervals, such as exhibited by the guitar string or by an object on a spring moving up and down. The time to complete one oscillation remains constant and is called the **period** T. Its units are usually seconds, but may be any convenient unit of time. The word period refers to the time for some event whether repetitive or not; but we shall be primarily interested in periodic motion, which is by definition repetitive. A concept closely related to period is the frequency of an event. For example, if you get a paycheck twice a month, the frequency of payment is two per month and the period between checks is half a month. **Frequency** f is defined to be the number of events per unit time. For periodic motion, frequency is the number of oscillations per unit time. The relationship between frequency and period is

Equation:

$$f = \frac{1}{T}.$$

The SI unit for frequency is the *cycle per second*, which is defined to be a *hertz* (Hz):

Equation:

$$1\text{ Hz} = 1\frac{\text{cycle}}{\text{sec}}\text{ or }1\text{ Hz} = \frac{1}{\text{s}}$$

A cycle is one complete oscillation. Note that a vibration can be a single or multiple event, whereas oscillations are usually repetitive for a significant number of cycles.

Example:

Determine the Frequency of Two Oscillations: Medical Ultrasound and the Period of Middle C

We can use the formulas presented in this module to determine both the frequency based on known oscillations and the oscillation based on a known frequency. Let's try one example of each. (a) A medical imaging device produces ultrasound by oscillating with a period of 0.400 µs. What is the frequency of this oscillation? (b) The frequency of middle C on a typical musical instrument is 264 Hz. What is the time for one complete oscillation?

Strategy

Both questions (a) and (b) can be answered using the relationship between period and frequency. In question (a), the period T is given and we are asked to find frequency f. In question (b), the frequency f is given and we are asked to find the period T.

Solution a

1. Substitute 0.400 µs for T in $f = \frac{1}{T}$:

Equation:

$$f = \frac{1}{T} = \frac{1}{0.400 \times 10^{-6}\ \text{s}}.$$

Solve to find
Equation:

$$f = 2.50 \times 10^{6}\ \text{Hz}.$$

Discussion a
The frequency of sound found in (a) is much higher than the highest frequency that humans can hear and, therefore, is called ultrasound. Appropriate oscillations at this frequency generate ultrasound used for noninvasive medical diagnoses, such as observations of a fetus in the womb.
Solution b

1. Identify the known values:

   The time for one complete oscillation is the period T:
   Equation:

   $$f = \frac{1}{T}.$$

2. Solve for T:
   Equation:

   $$T = \frac{1}{f}.$$

3. Substitute the given value for the frequency into the resulting expression:
   Equation:

$$T = \frac{1}{f} = \frac{1}{264\ \text{Hz}} = \frac{1}{264\ \text{cycles/s}} = 3.79 \times 10^{-3}\ \text{s} = 3.79\ \text{ms}.$$

Discussion

The period found in (b) is the time per cycle, but this value is often quoted as simply the time in convenient units (ms or milliseconds in this case).

Exercise:
Check your Understanding

Problem:

Identify an event in your life (such as receiving a paycheck) that occurs regularly. Identify both the period and frequency of this event.

Solution:

I visit my parents for dinner every other Sunday. The frequency of my visits is 26 per calendar year. The period is two weeks.

Section Summary

- Periodic motion is a repetitious oscillation.
- The time for one oscillation is the period T.
- The number of oscillations per unit time is the frequency f.
- These quantities are related by
  Equation:

$$f = \frac{1}{T}.$$

Conceptual Questions

Exercise:

Problem:

If an oscillation occurs more rapidly, how does the period of oscillation change? How does the frequency change?

Exercise:

Problem:

Give examples of natural phenomena that occur with constant frequency (or period) over a length of time.

Glossary

period
: time it takes to complete one oscillation

periodic motion
: motion that repeats itself at regular time intervals

frequency
: number of events per unit of time

Simple Harmonic Motion: A Special Periodic Motion

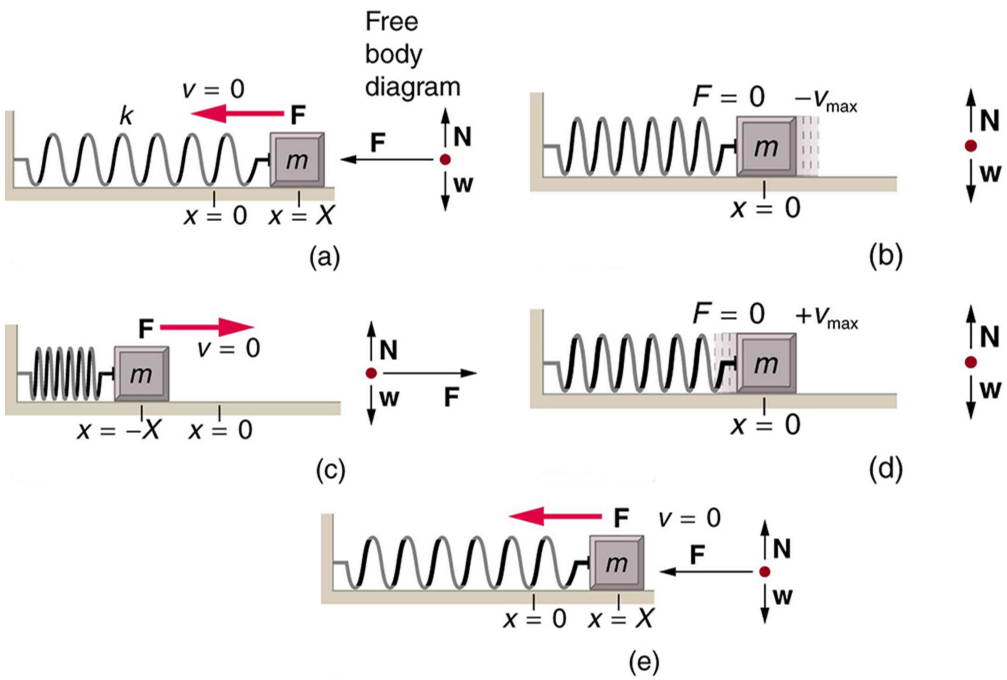
- Describe a simple harmonic oscillator.
- Explain the link between simple harmonic motion and waves.

The oscillations of a system in which the net force can be described by Hooke's law are of special importance, because they are very common. They are also the simplest oscillatory systems. **Simple Harmonic Motion** (SHM) is the name given to oscillatory motion for a system where the net force can be described by Hooke's law, and such a system is called a **simple harmonic oscillator**. If the net force can be described by Hooke's law and there is no *damping* (by friction or other non-conservative forces), then a simple harmonic oscillator will oscillate with equal displacement on either side of the equilibrium position, as shown for an object on a spring in [link]. The maximum displacement from equilibrium is called the **amplitude** X. The units for amplitude and displacement are the same, but depend on the type of oscillation. For the object on the spring, the units of amplitude and displacement are meters; whereas for sound oscillations, they have units of pressure (and other types of oscillations have yet other units). Because amplitude is the maximum displacement, it is related to the energy in the oscillation.

Note:

Take-Home Experiment: SHM and the Marble

Find a bowl or basin that is shaped like a hemisphere on the inside. Place a marble inside the bowl and tilt the bowl periodically so the marble rolls from the bottom of the bowl to equally high points on the sides of the bowl. Get a feel for the force required to maintain this periodic motion. What is the restoring force and what role does the force you apply play in the simple harmonic motion (SHM) of the marble?



An object attached to a spring sliding on a frictionless surface is an uncomplicated simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude X and a period T. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T. The greater the mass of the object is, the greater the period T.

What is so significant about simple harmonic motion? One special thing is that the period T and frequency f of a simple harmonic oscillator are independent of amplitude. The string of a guitar, for example, will oscillate with the same frequency whether plucked gently or hard. Because the period is constant, a simple harmonic oscillator can be used as a clock.

Two important factors do affect the period of a simple harmonic oscillator. The period is related to how stiff the system is. A very stiff object has a large spring constant k, which causes the system to have a smaller period. For example, you can adjust a diving board's stiffness—the stiffer it is, the

faster it vibrates, and the shorter its period. Period also depends on the mass of the oscillating system. The more massive the system is, the longer the period. For example, a heavy person on a diving board bounces up and down more slowly than a light one.

In fact, the mass m and the spring constant k are the *only* factors that affect the period and frequency of simple harmonic motion of a mass on a spring.

Note:
Period of Simple Harmonic Oscillator
The *period of a simple harmonic oscillator* is given by
Equation:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

and, because $f = 1/T$, the *frequency of a simple harmonic oscillator* is
Equation:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

Note that neither T nor f has any dependence on amplitude.

Note:
Take-Home Experiment: Mass and Ruler Oscillations
Find two identical wooden or plastic rulers. Tape one end of each ruler firmly to the edge of a table so that the length of each ruler that protrudes from the table is the same. On the free end of one ruler tape a heavy object such as a few large coins. Pluck the ends of the rulers at the same time and observe which one undergoes more cycles in a time period, and measure the period of oscillation of each of the rulers.

Example:

Calculate the Frequency and Period of Oscillations: Bad Shock Absorbers in a Car

If the shock absorbers in a car go bad, then the car will oscillate at the least provocation, such as when going over bumps in the road and after stopping (See [link]). Calculate the frequency and period of these oscillations for such a car if the car's mass (including its load) is 900 kg and the spring constant (k) of the suspension system is $6.53 \times 10^4 \text{ N/m}$.

Strategy

The frequency of the car's oscillations will be that of a simple harmonic oscillator as given in the equation $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. The mass and the spring constant are both given.

Solution

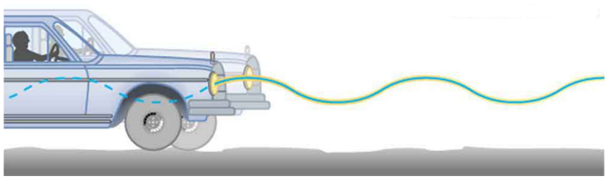
1. Enter the known values of k and m:
   Equation:
   $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{6.53 \times 10^4 \text{ N/m}}{900 \text{ kg}}}.$$
2. Calculate the frequency:
   Equation:
   $$\frac{1}{2\pi}\sqrt{72.6/\text{s}^{-2}} = 1.3656/\text{s}^{-1} \approx 1.36/\text{s}^{-1} = 1.36 \text{ Hz}.$$
3. You could use $T = 2\pi\sqrt{\frac{m}{k}}$ to calculate the period, but it is simpler to use the relationship $T = 1/f$ and substitute the value just found for f:
   Equation:
   $$T = \frac{1}{f} = \frac{1}{1.356 \text{ Hz}} = 0.738 \text{ s}.$$

Discussion

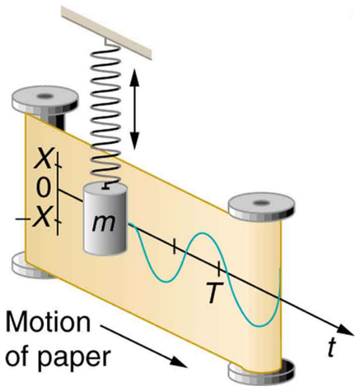
The values of T and f both seem about right for a bouncing car. You can observe these oscillations if you push down hard on the end of a car and let go.

The Link between Simple Harmonic Motion and Waves

If a time-exposure photograph of the bouncing car were taken as it drove by, the headlight would make a wavelike streak, as shown in [link]. Similarly, [link] shows an object bouncing on a spring as it leaves a wavelike "trace of its position on a moving strip of paper. Both waves are sine functions. All simple harmonic motion is intimately related to sine and cosine waves.



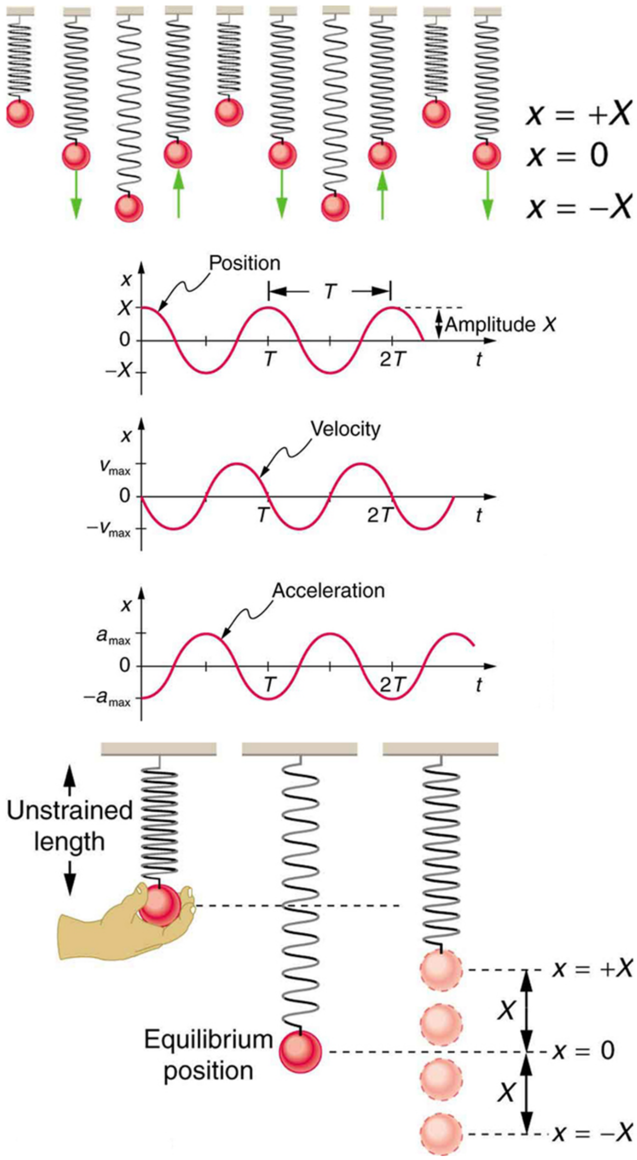
The bouncing car makes a wavelike motion. If the restoring force in the suspension system can be described only by Hooke's law, then the wave is a sine function. (The wave is the trace produced by the headlight as the car moves to the right.)



The vertical position of an object bouncing on a spring is recorded on a strip of moving paper, leaving a sine wave.

The displacement $x(t)$, the velocity $v(t)$, and the acceleration $a(t)$ of simple harmonic motion sketches out sinusoidal functions as a function of time. That is, if you sketch out $\sin\,(t)$ and $\cos\,(t)$, with t in seconds, for example, what you graph will appear similar to the actual graphs of $x(t)$, $v(t)$, and $a(t)$. You just need to scale them horizontally and vertically, so that they have correct period and amplitude, respectively.

[link] shows the simple harmonic motion of an object on a spring and presents graphs of $x(t)$,$v(t\ )$, and $a(t)$ versus time.



Graphs of $x(t)$, $v(t)$, and $a(t)$ versus t for the motion of an object on a spring. The net force on the object can be described by Hooke's law, and so the object undergoes simple harmonic motion. Note that the initial position has the vertical displacement at its maximum value X; v is initially zero and then negative as the object moves down; and the initial acceleration

is negative, back toward the equilibrium position and becomes zero at that point.

The most important point here is that these relationships are valid for all simple harmonic motion. They are very useful in visualizing waves associated with simple harmonic motion, including visualizing how waves add with one another.

Exercise:

Check Your Understanding

Problem:

Suppose you pluck a banjo string. You hear a single note that starts out loud and slowly quiets over time. Describe what happens to the sound waves in terms of period, frequency and amplitude as the sound decreases in volume.

Solution:

Frequency and period remain essentially unchanged. Only amplitude decreases as volume decreases.

Exercise:

Check Your Understanding

Problem:

A babysitter is pushing a child on a swing. At the point where the swing reaches x, where would the corresponding point on a wave of this motion be located?

Solution:

x is the maximum deformation, which corresponds to the amplitude of the wave. The point on the wave would either be at the very top or the very bottom of the curve.

Section Summary

- Simple harmonic motion is oscillatory motion for a system that can be described only by Hooke’s law. Such a system is also called a simple harmonic oscillator.
- Maximum displacement is the amplitude X. The period T and frequency f of a simple harmonic oscillator are given by

  $T = 2\pi\sqrt{\frac{m}{k}}$ and $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where m is the mass of the system.
- Displacement, velocity, and acceleration in simple harmonic motion as a function of time are represented by sinusoidal functions in time, that is functions that are similar to $\sin\,(t)$ and $\cos\,(t)$.

Conceptual Questions

Exercise:

Problem:

What conditions must be met to produce simple harmonic motion?

Exercise:

Problem:

(a) If frequency is not constant for some oscillation, can the oscillation be simple harmonic motion?

(b) Can you think of any examples of harmonic motion where the frequency may depend on the amplitude?

Exercise:

Problem:

Give an example of a simple harmonic oscillator, specifically noting how its frequency is independent of amplitude.

Exercise:

Problem:

Explain why you expect an object made of a stiff material to vibrate at a higher frequency than a similar object made of a spongy material.

Exercise:

Problem:

As you pass a freight truck with a trailer on a highway, you notice that its trailer is bouncing up and down slowly. Is it more likely that the trailer is heavily loaded or nearly empty? Explain your answer.

Exercise:

Problem:

Some people modify cars to be much closer to the ground than when manufactured. Should they install stiffer springs? Explain your answer.

Glossary

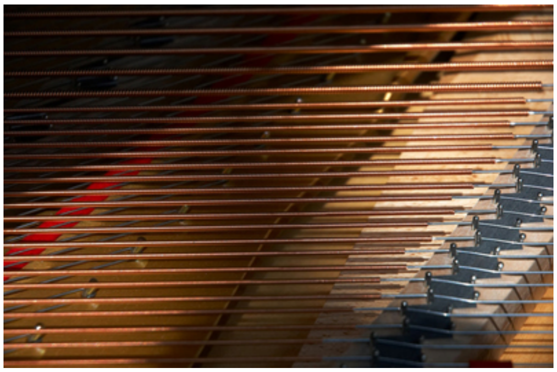
amplitude
: the maximum displacement from the equilibrium position of an object oscillating around the equilibrium position

simple harmonic motion
: the oscillatory motion in a system where the net force can be described by Hooke’s law

simple harmonic oscillator
: a device that implements Hooke’s law, such as a mass that is attached to a spring, with the other end of the spring being connected to a rigid support such as a wall

Forced Oscillations and Resonance

- Observe the resonance phenomena in several examples.
- Observe amplitude of a damped harmonic oscillator.
- Understand the origin of damping of resonance.

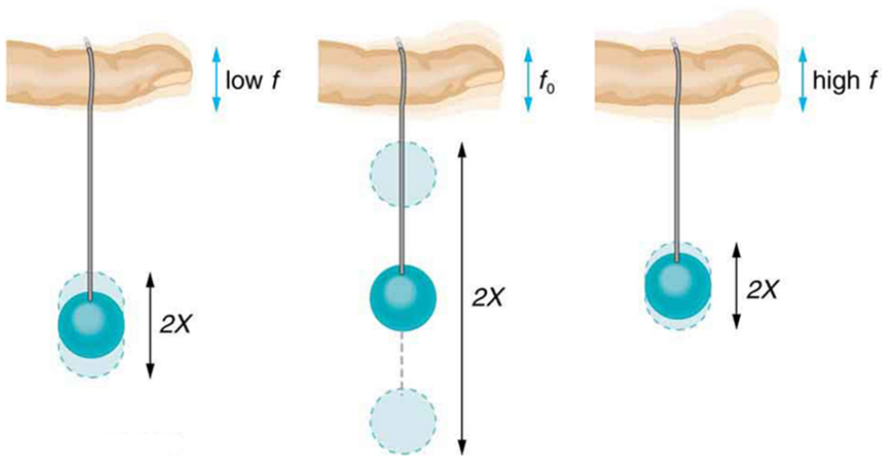


You can cause the strings in a piano to vibrate simply by producing sound waves from your voice. (credit: Matt Billings, Flickr)

Sit in front of a piano sometime and sing a loud brief note at it with the dampers off its strings. It will sing the same note back at you—the strings, having the same frequencies as your voice, are resonating in response to the forces from the sound waves that you sent to them. Your voice and a piano's strings is a good example of the fact that objects—in this case, piano strings—can be forced to oscillate but oscillate best at their natural frequency. In this section, we shall briefly explore applying a *periodic driving force* acting on a simple harmonic oscillator. The driving force puts energy into the system at a certain frequency, not necessarily the same as the natural frequency of the system. The **natural frequency** is the frequency at which a simple harmonic oscillator oscillates, if there is no damping force, and other than an initial kick to get the motion started, there is no driving force. A **damping force** is a frictional force (friction, air resistance, and other

forces that lead to decrease in mechanical energy) which converts the mechanical energy of the oscillatory motion into thermal energy.

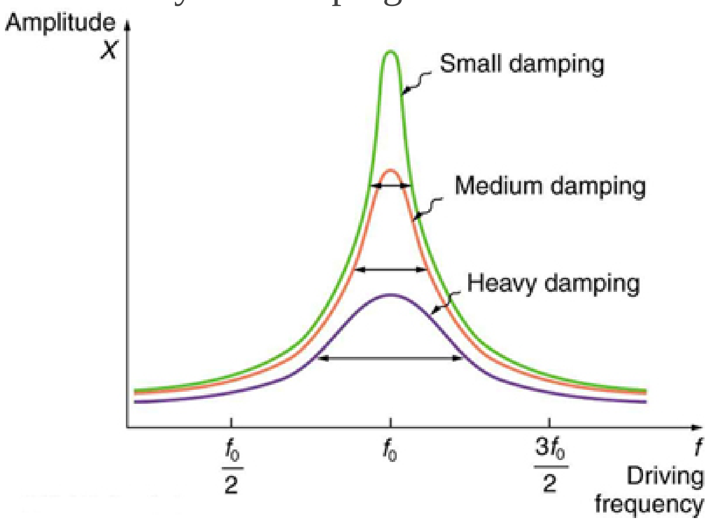
Most of us have played with toys involving an object supported on an elastic band, something like the paddle ball suspended from a finger in [link]. Imagine the finger in the figure is your finger. At first you hold your finger steady, and the ball bounces up and down with a small amount of damping. If you move your finger up and down slowly, the ball will follow along without bouncing much on its own. As you increase the frequency at which you move your finger up and down, the ball will respond by oscillating with increasing amplitude. When you drive the ball at its natural frequency, the ball's oscillations increase in amplitude with each oscillation for as long as you drive it. The phenomenon of driving a system with a frequency equal to its natural frequency is called **resonance**. A system being driven at its natural frequency is said to **resonate**. As the driving frequency gets progressively higher than the resonant or natural frequency, the amplitude of the oscillations becomes smaller, until the oscillations nearly disappear and your finger simply moves up and down with little effect on the ball.



The paddle ball on its rubber band moves in response to the finger supporting it. If the finger moves with the natural frequency f_0 of the ball on the rubber band, then a resonance is achieved, and the amplitude of the ball's oscillations increases dramatically. At higher and lower driving frequencies, energy is transferred to the ball less

efficiently, and it responds with lower-amplitude oscillations.

[link] shows a graph of the amplitude of a damped harmonic oscillator as a function of the frequency of the periodic force driving it. There are three curves on the graph, each representing a different amount of damping. All three curves peak at the point where the frequency of the driving force equals the natural frequency of the harmonic oscillator. The highest peak, or greatest response, is for the least amount of damping, because less energy is removed by the damping force.

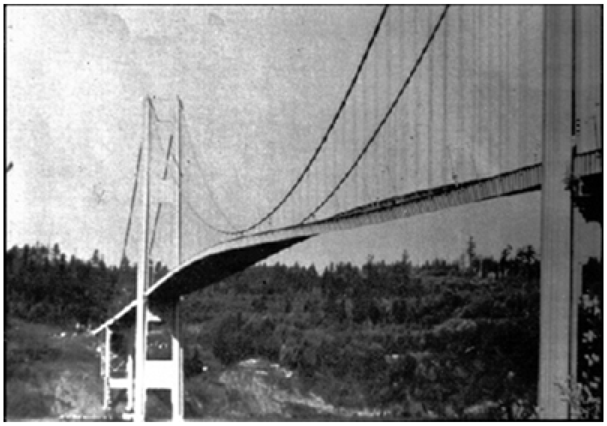


Amplitude of a harmonic oscillator as a function of the frequency of the driving force. The curves represent the same oscillator with the same natural frequency but with different amounts of damping. Resonance occurs when the driving frequency equals the natural frequency, and the greatest response is for the least amount of damping. The narrowest response is also for the least damping.

It is interesting that the widths of the resonance curves shown in [link] depend on damping: the less the damping, the narrower the resonance. The message is that if you want a driven oscillator to resonate at a very specific frequency, you need as little damping as possible. Little damping is the case for piano strings and many other musical instruments. Conversely, if you want small-amplitude oscillations, such as in a car’s suspension system, then you want heavy damping. Heavy damping reduces the amplitude, but the tradeoff is that the system responds at more frequencies.

These features of driven harmonic oscillators apply to a huge variety of systems. When you tune a radio, for example, you are adjusting its resonant frequency so that it only oscillates to the desired station’s broadcast (driving) frequency. The more selective the radio is in discriminating between stations, the smaller its damping. Magnetic resonance imaging (MRI) is a widely used medical diagnostic tool in which atomic nuclei (mostly hydrogen nuclei) are made to resonate by incoming radio waves (on the order of 100 MHz). A child on a swing is driven by a parent at the swing’s natural frequency to achieve maximum amplitude. In all of these cases, the efficiency of energy transfer from the driving force into the oscillator is best at resonance. Speed bumps and gravel roads prove that even a car’s suspension system is not immune to resonance. In spite of finely engineered shock absorbers, which ordinarily convert mechanical energy to thermal energy almost as fast as it comes in, speed bumps still cause a large-amplitude oscillation. On gravel roads that are corrugated, you may have noticed that if you travel at the “wrong” speed, the bumps are very noticeable whereas at other speeds you may hardly feel the bumps at all. [link] shows a photograph of a famous example (the Tacoma Narrows Bridge) of the destructive effects of a driven harmonic oscillation. The Millennium Bridge in London was closed for a short period of time for the same reason while inspections were carried out.

In our bodies, the chest cavity is a clear example of a system at resonance. The diaphragm and chest wall drive the oscillations of the chest cavity which result in the lungs inflating and deflating. The system is critically damped and the muscular diaphragm oscillates at the resonant value for the system, making it highly efficient.



In 1940, the Tacoma Narrows Bridge in Washington state collapsed. Heavy cross winds drove the bridge into oscillations at its resonant frequency. Damping decreased when support cables broke loose and started to slip over the towers, allowing increasingly greater amplitudes until the structure failed (credit: PRI's *Studio 360*, via Flickr)

Exercise:
Check Your Understanding

Problem:

A famous magic trick involves a performer singing a note toward a crystal glass until the glass shatters. Explain why the trick works in terms of resonance and natural frequency.

Solution:

The performer must be singing a note that corresponds to the natural frequency of the glass. As the sound wave is directed at the glass, the glass responds by resonating at the same frequency as the sound wave.

With enough energy introduced into the system, the glass begins to vibrate and eventually shatters.

Section Summary

- A system's natural frequency is the frequency at which the system will oscillate if not affected by driving or damping forces.
- A periodic force driving a harmonic oscillator at its natural frequency produces resonance. The system is said to resonate.
- The less damping a system has, the higher the amplitude of the forced oscillations near resonance. The more damping a system has, the broader response it has to varying driving frequencies.

Conceptual Questions

Exercise:

Problem:

Give additional examples of resonance phenomenon from everyday life.

Exercise:

Problem:

What "damps" the resonance of a forced oscillation? What are the dynamics of "damping" and what physical quantity is involved?

Glossary

natural frequency
: the frequency at which a simple harmonic oscillator oscillates if it is set in motion without a driving force

damping force

a frictional force which converts the mechanical energy of the oscillatory motion into thermal energy

resonance
: the phenomenon of driving a system with a frequency equal to the system's natural frequency

resonate
: a system being driven at its natural frequency

Waves

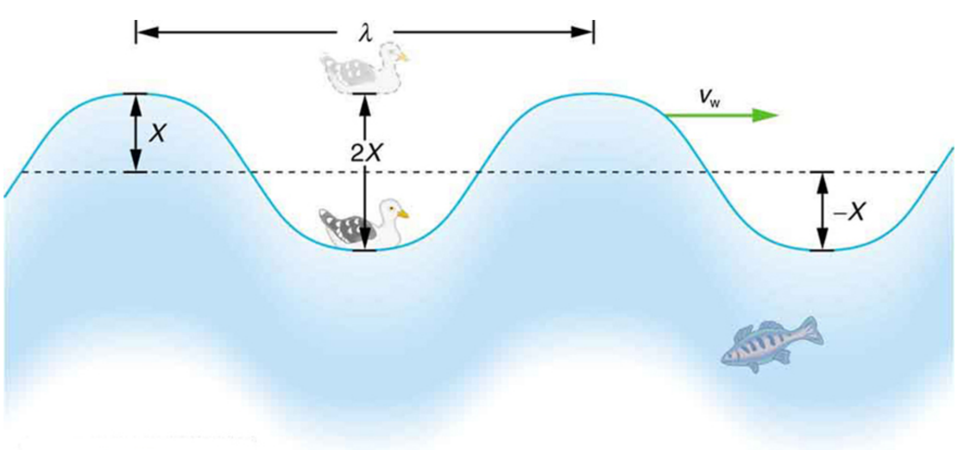
- State the characteristics of a wave.
- Calculate the velocity of wave propagation, given frequency and wavelength

Waves in the ocean behave similarly to all other types of waves. (credit: Steve Jurveston, Flickr)

What do we mean when we say something is a wave? The most intuitive and easiest wave to imagine is the familiar water wave. More precisely, a **wave** is a disturbance that propagates, or moves from the place it was created. For water waves, the disturbance is in the surface of the water, perhaps created by a rock thrown into a pond or by a swimmer splashing the surface repeatedly. For sound waves, the disturbance is a change in air pressure, perhaps created by the oscillating cone inside a speaker. For earthquakes, there are several types of disturbances, including disturbance of Earth’s surface and pressure disturbances under the surface. Even radio waves are most easily understood using an analogy with water waves. Visualizing water waves is useful because there is more to it than just a mental image. Water waves exhibit characteristics common to all waves, such as amplitude, period, frequency and energy. All wave characteristics can be described by a small set of underlying principles.

A wave is a disturbance that propagates, or moves from the place it was created. The simplest waves repeat themselves for several cycles and are associated with simple harmonic motion. Let us start by considering the simplified water wave in [link]. The wave is an up and down disturbance of the water surface. It causes a sea gull to move up and down in simple harmonic motion as the wave crests and troughs (peaks and valleys) pass under the bird. The time for one complete up and down motion is the wave's period T. The wave's frequency is $f = 1/T$, as usual. The wave itself moves to the right in the figure. This movement of the wave is actually the disturbance moving to the right, not the water itself (or the bird would move to the right). We define **wave velocity** v_{w} to be the speed at which the disturbance moves. Wave velocity is sometimes also called the *propagation velocity or propagation speed*, because the disturbance propagates from one location to another.

Note:
Misconception Alert
Many people think that water waves push water from one direction to another. In fact, the particles of water tend to stay in one location, save for moving up and down due to the energy in the wave. The energy moves forward through the water, but the water stays in one place. If you feel yourself pushed in an ocean, what you feel is the energy of the wave, not a rush of water.



An idealized ocean wave passes under a sea gull that bobs up and down in simple harmonic motion. The wave has a wavelength λ, which is the distance between adjacent identical parts of the wave. The up and down disturbance of the surface propagates parallel to the surface at a speed v_{w}.

The water wave in the figure also has a length associated with it, called its **wavelength** λ, the distance between adjacent identical parts of a wave. (λ is the distance parallel to the direction of propagation.) The speed of propagation v_{w} is the distance the wave travels in a given time, which is one wavelength in the time of one period. In equation form, that is

Equation:

$$v_{\text{w}} = \frac{\lambda}{T}$$

or

Equation:

$$v_{\text{w}} = f\lambda.$$

This fundamental relationship holds for all types of waves. For water waves, v_{w} is the speed of a surface wave; for sound, v_{w} is the speed of sound; and for visible light, v_{w} is the speed of light, for example.

One note of caution with [link]: despite the appearance, wave speed v_w is a *not* a function of frequency f and wavelength λ. When f changes (or λ changes), v_w does not change (you will see this explicitly later with sound and light waves). Wave speed is a *property of the medium* the wave travels in. Unless the medium itself changes, the wave speed remains constant. This is true of sound and light waves, as well as almost all other types of waves you will see in this textbook (water waves, waves on a string, etc.). So, what happens when frequency or wavelength does change, then? The

wavelength of frequency changes to compensate. For example, if the frequency doubles for a given wave, instead of the wave speed doubling, the wavelength will decrease to half, so that the product $f\lambda$ remains constant.

Note:
Take-Home Experiment: Waves in a Bowl
Fill a large bowl or basin with water and wait for the water to settle so there are no ripples. Gently drop a cork into the middle of the bowl. Estimate the wavelength and period of oscillation of the water wave that propagates away from the cork. Remove the cork from the bowl and wait for the water to settle again. Gently drop the cork at a height that is different from the first drop. Does the wavelength depend upon how high above the water the cork is dropped?

Example:
Calculate the Velocity of Wave Propagation: Gull in the Ocean
Calculate the wave velocity of the ocean wave in [link] if the distance between wave crests is 10.0 m and the time for a sea gull to bob up and down is 5.00 s.
Strategy
We are asked to find v_{w}. The given information tells us that $\lambda = 10.0\text{ m}$ and $T = 5.00\text{ s}$. Therefore, we can use $v_{\text{w}} = \frac{\lambda}{T}$ to find the wave velocity.
Solution

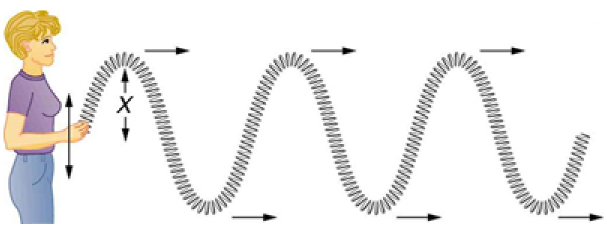
1. Enter the known values into $v_{\text{w}} = \frac{\lambda}{T}$:
   Equation:
   $$v_{\text{w}} = \frac{10.0\text{ m}}{5.00\text{ s}}.$$
2. Solve for v_{w} to find v_{w}= 2.00 m/s.

Discussion

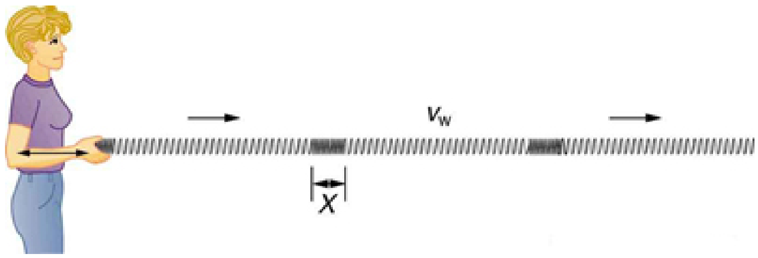
This slow speed seems reasonable for an ocean wave. Note that the wave moves to the right in the figure at this speed, not the varying speed at which the sea gull moves up and down.

Transverse and Longitudinal Waves

A simple wave consists of a periodic disturbance that propagates from one place to another. The wave in [link] propagates in the horizontal direction while the surface is disturbed in the vertical direction. Such a wave is called a **transverse wave** or shear wave; in such a wave, the disturbance is perpendicular to the direction of propagation. In contrast, in a **longitudinal wave** or compressional wave, the disturbance is parallel to the direction of propagation. [link] shows an example of a longitudinal wave. The size of the disturbance is its amplitude X and is completely independent of the speed of propagation v_w.



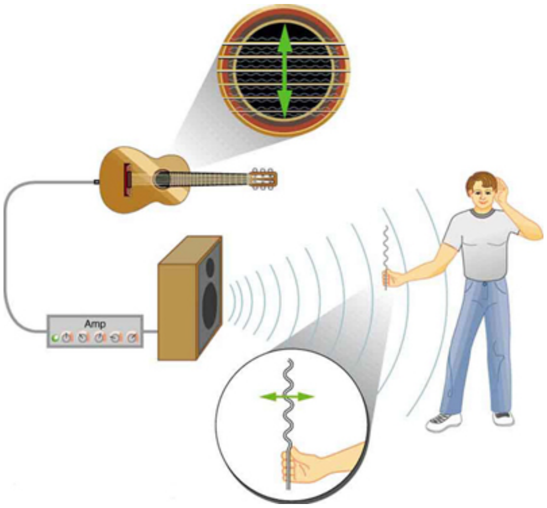
In this example of a transverse wave, the wave propagates horizontally, and the disturbance in the cord is in the vertical direction.



In this example of a longitudinal wave, the wave propagates horizontally, and the disturbance in the cord is also in the horizontal direction.

Waves may be transverse, longitudinal, or *a combination of the two*. (Water waves are actually a combination of transverse and longitudinal. The simplified water wave illustrated in [link] shows no longitudinal motion of the bird.) The waves on the strings of musical instruments are transverse—so are electromagnetic waves, such as visible light.

Sound waves in air and water are longitudinal. Their disturbances are periodic variations in pressure that are transmitted in fluids. Fluids do not have appreciable shear strength, and thus the sound waves in them must be longitudinal or compressional. Sound in solids can be both longitudinal and transverse.



The wave on a guitar string is transverse. The sound wave rattles a sheet of paper in a direction that shows the sound wave is longitudinal.

Earthquake waves under Earth's surface also have both longitudinal and transverse components (called compressional or P-waves and shear or S-waves, respectively). These components have important individual characteristics—they propagate at different speeds, for example. Earthquakes also have surface waves that are similar to surface waves on water.

Exercise:

Check Your Understanding

Problem:

Why is it important to differentiate between longitudinal and transverse waves?

Solution:

In the different types of waves, energy can propagate in a different direction relative to the motion of the wave. This is important to

understand how different types of waves affect the materials around them.

Section Summary

- A wave is a disturbance that moves from the point of creation with a wave velocity v_{w}.
- A wave has a wavelength λ, which is the distance between adjacent identical parts of the wave.
- Wave velocity and wavelength are related to the wave's frequency and period by $v_{\text{w}} = \frac{\lambda}{T}$ or $v_{\text{w}} = f\lambda$. However, wave velocity is a property of the medium and remains constant as wavelength or frequency changes.
- A transverse wave has a disturbance perpendicular to its direction of propagation, whereas a longitudinal wave has a disturbance parallel to its direction of propagation.

Conceptual Questions

Exercise:

Problem:

Give one example of a transverse wave and another of a longitudinal wave, being careful to note the relative directions of the disturbance and wave propagation in each.

Exercise:

Problem:

What is the difference between propagation speed and the frequency of a wave? Does one or both affect wavelength? If so, how?

Exercise:

Problem:

Given a wave of frequency f traveling with wave speed v_w, what happens to the wave speed if the frequency doubles? What happens to the wavelength?

Glossary

longitudinal wave
: a wave in which the disturbance is parallel to the direction of propagation

transverse wave
: a wave in which the disturbance is perpendicular to the direction of propagation

wave velocity
: the speed at which the disturbance moves; also called wave speed, propagation velocity, or propagation speed

wavelength
: the distance between adjacent identical parts of a wave

Wave Interference: Standing Waves and Beats

- Distinguish between constructive interference and destructive interference.
- Explain standing waves in terms of interference, nodes, and antinodes.
- Explain the beat phenomenon in terms of interference.

These waves result from the superposition of several waves from different sources, producing a complex pattern. (credit: waterborough, Wikimedia Commons)

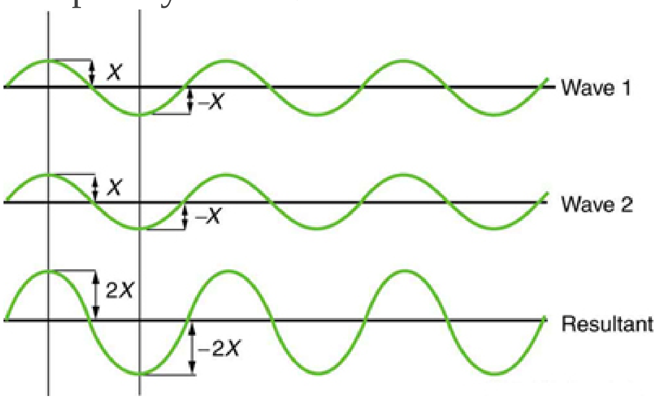
Most waves do not look very simple. They look more like the waves in [link] than like the simple water wave considered in "Waves". (Simple waves may be created by a simple harmonic oscillation, and thus have a sinusoidal shape). Complex waves are more interesting, even beautiful, but they look formidable. Most waves appear complex because they result from several simple waves adding together. Luckily, the rules for adding waves are quite simple.

When two or more waves arrive at the same point, they superimpose themselves on one another. More specifically, the disturbances of waves are superimposed when they come together—a phenomenon called **superposition**. Each disturbance corresponds to a force, and forces add. If

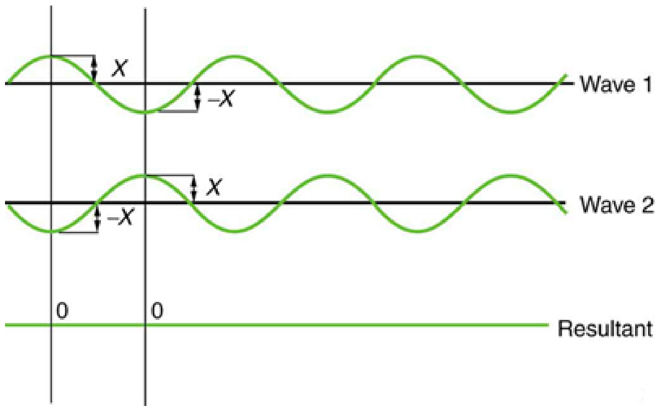
the disturbances are along the same line, then the resulting wave is a simple addition of the disturbances of the individual waves—that is, their amplitudes add. [link] and [link] illustrate superposition in two special cases, both of which produce simple results.

[link] shows two identical waves that arrive at the same point exactly in phase. The crests of the two waves are precisely aligned, as are the troughs. This superposition produces pure **constructive interference**. Because the disturbances add, pure constructive interference produces a wave that has twice the amplitude of the individual waves, but has the same wavelength.

[link] shows two identical waves that arrive exactly out of phase—that is, precisely aligned crest to trough—producing pure **destructive interference**. Because the disturbances are in the opposite direction for this superposition, the resulting amplitude is zero for pure destructive interference—the waves completely cancel.



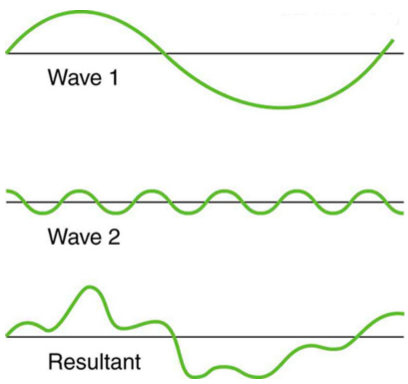
Pure constructive interference of two identical waves produces one with twice the amplitude, but the same wavelength.



Pure destructive interference of two identical waves produces zero amplitude, or complete cancellation.

While pure constructive and pure destructive interference do occur, they require precisely aligned identical waves. The superposition of most waves produces a combination of constructive and destructive interference and can vary from place to place and time to time. Sound from a stereo, for example, can be loud in one spot and quiet in another. Varying loudness means the sound waves add partially constructively and partially destructively at different locations. A stereo has at least two speakers creating sound waves, and waves can reflect from walls. All these waves superimpose. An example of sounds that vary over time from constructive to destructive is found in the combined whine of airplane jets heard by a stationary passenger. The combined sound can fluctuate up and down in volume as the sound from the two engines varies in time from constructive to destructive. These examples are of waves that are similar.

An example of the superposition of two dissimilar waves is shown in [link]. Here again, the disturbances add and subtract, producing a more complicated looking wave.

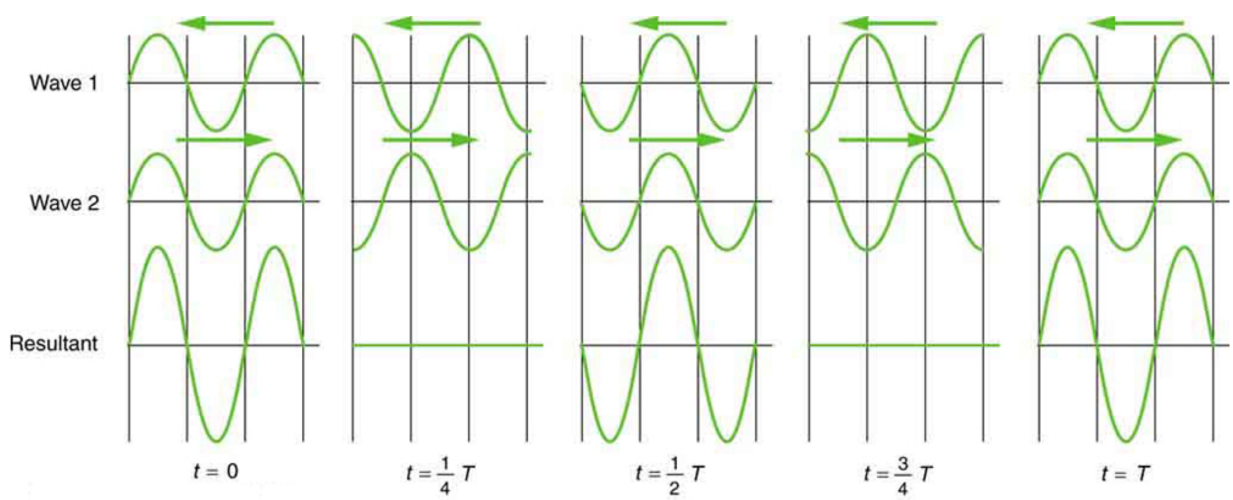


Superposition of non-identical waves exhibits both constructive and destructive interference.

Standing Waves

Sometimes waves do not seem to move; rather, they just vibrate in place. Unmoving waves can be seen on the surface of a glass of milk in a refrigerator, for example. Vibrations from the refrigerator motor create waves on the milk that oscillate up and down but do not seem to move across the surface. These waves are formed by the superposition of two or more moving waves, such as illustrated in [link] for two identical waves moving in opposite directions. The waves move through each other with their disturbances adding as they go by. If the two waves have the same amplitude and wavelength, then they alternate between constructive and destructive interference. The resultant looks like a wave standing in place and, thus, is called a **standing wave**. Waves on the glass of milk are one example of standing waves. There are other standing waves, such as on guitar strings and in organ pipes. With the glass of milk, the two waves that produce standing waves may come from reflections from the side of the glass.

A closer look at earthquakes provides evidence for conditions appropriate for resonance, standing waves, and constructive and destructive interference. A building may be vibrated for several seconds with a driving frequency matching that of the natural frequency of vibration of the building—producing a resonance resulting in one building collapsing while neighboring buildings do not. Often buildings of a certain height are devastated while other taller buildings remain intact. The building height matches the condition for setting up a standing wave for that particular height. As the earthquake waves travel along the surface of Earth and reflect off denser rocks, constructive interference occurs at certain points. Often areas closer to the epicenter are not damaged while areas farther away are damaged.

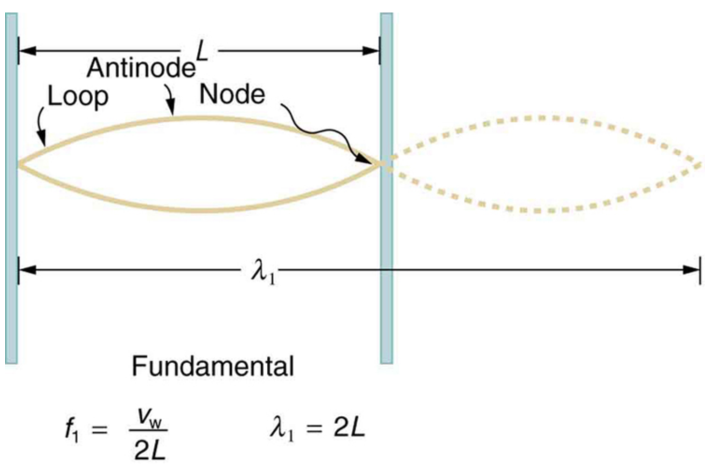


Standing wave created by the superposition of two identical waves moving in opposite directions. The oscillations are at fixed locations in space and result from alternately constructive and destructive interference.

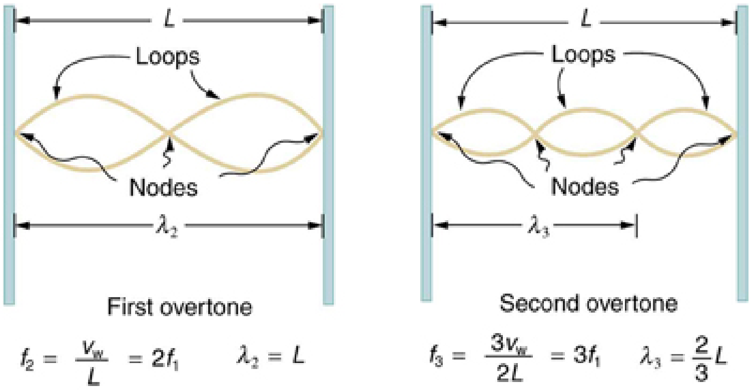
Standing waves are also found on the strings of musical instruments and are due to reflections of waves from the ends of the string. [link] and [link] show three standing waves that can be created on a string that is fixed at both ends. **Nodes** are the points where the string does not move; more

generally, nodes are where the wave disturbance is zero in a standing wave. The fixed ends of strings must be nodes, too, because the string cannot move there. The word **antinode** is used to denote the location of maximum amplitude in standing waves. Standing waves on strings have a frequency that is related to the propagation speed v_w of the disturbance on the string. The wavelength λ is determined by the distance between the points where the string is fixed in place.

The lowest frequency, called the **fundamental frequency**, is thus for the longest wavelength, which is seen to be $\lambda_1 = 2L$. Therefore, the fundamental frequency is $f_1 = v_w/\lambda_1 = v_w/2L$. In this case, the **overtones** or harmonics are multiples of the fundamental frequency. As seen in [link], the first harmonic can easily be calculated since $\lambda_2 = L$. Thus, $f_2 = v_w/\lambda_2 = v_w/2L = 2f_1$. Similarly, $f_3 = 3f_1$, and so on. All of these frequencies can be changed by adjusting the tension in the string. The greater the tension, the greater v_w is and the higher the frequencies. This observation is familiar to anyone who has ever observed a string instrument being tuned. We will see in later chapters that standing waves are crucial to many resonance phenomena, such as in sounding boxes on string instruments.



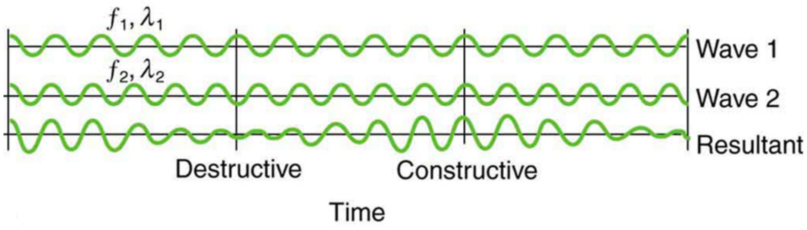
The figure shows a string oscillating at its fundamental frequency.



First and second harmonic frequencies are shown.

Beats

Striking two adjacent keys on a piano produces a warbling combination usually considered to be unpleasant. The superposition of two waves of similar but not identical frequencies is the culprit. Another example is often noticeable in jet aircraft, particularly the two-engine variety, while taxiing. The combined sound of the engines goes up and down in loudness. This varying loudness happens because the sound waves have similar but not identical frequencies. The discordant warbling of the piano and the fluctuating loudness of the jet engine noise are both due to alternately constructive and destructive interference as the two waves go in and out of phase. [link] illustrates this graphically.



Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes. The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude.

The wave resulting from the superposition of two similar-frequency waves has a frequency that is the average of the two. This wave fluctuates in amplitude, or *beats*, with a frequency called the **beat frequency**. We can determine the beat frequency by adding two waves together mathematically. The result,

Equation:

$$f_B = \mid f_1 - f_2 \mid$$

is the beat frequency. The resultant wave has the average frequency of the two superimposed waves, but it also fluctuates in overall amplitude at the beat frequency f_B. This result is valid for all types of waves. However, if it is a sound wave, providing the two frequencies are similar, then what we hear is an average frequency that gets louder and softer (or warbles) at the beat frequency.

Note:

Making Career Connections

Piano tuners use beats routinely in their work. When comparing a note with a tuning fork, they listen for beats and adjust the string until the beats go away (to zero frequency). For example, if the tuning fork has a 256 Hz frequency and two beats per second are heard, then the other frequency is either 254 or 258 Hz. Most keys hit multiple strings, and these strings are actually adjusted until they have nearly the same frequency and give a slow beat for richness. Twelve-string guitars and mandolins are also tuned using beats.

While beats may sometimes be annoying in audible sounds, we will find that beats have many applications. Observing beats is a very useful way to compare similar frequencies. There are applications of beats as apparently disparate as in ultrasonic imaging and radar speed traps.

Exercise:
Check Your Understanding

Problem:

Imagine you are holding one end of a jump rope, and your friend holds the other. If your friend holds her end still, you can move your end up and down, creating a transverse wave. If your friend then begins to move her end up and down, generating a wave in the opposite direction, what resultant wave forms would you expect to see in the jump rope?

Solution:

The rope would alternate between having waves with amplitudes two times the original amplitude and reaching equilibrium with no amplitude at all. The wavelengths will result in both constructive and destructive interference

Exercise:
Check Your Understanding

Problem: Define nodes and antinodes.

Solution:

Nodes are areas of wave interference where there is no motion. Antinodes are areas of wave interference where the motion is at its maximum point.

Exercise:
Check Your Understanding

Problem:

You hook up a stereo system. When you test the system, you notice that in one corner of the room, the sounds seem dull. In another area, the sounds seem excessively loud. Describe how the sound moving about the room could result in these effects.

Solution:

With multiple speakers putting out sounds into the room, and these sounds bouncing off walls, there is bound to be some wave interference. In the dull areas, the interference is probably mostly destructive. In the louder areas, the interference is probably mostly constructive.

Section Summary

- Superposition is the combination of two waves at the same location.
- Constructive interference occurs when two waves are superimposed in phase (both disturbances in the same direction).
- Destructive interference occurs when two waves are superimposed exactly out of phase (two disturbances in opposite directions).
- A standing wave is one in which two waves superimpose to produce a wave that forms a standing pattern of nodes and antinodes, locations of no displacement and of maximum displacement.
- Standing waves form at specific resonance frequencies, with the lowest frequency being the fundamental frequency, and higher frequencies being called overtones or harmonics.

- Beats occur when waves of similar frequencies f_1 and f_2 are superimposed. The resulting amplitude oscillates with a beat frequency given by
  Equation:

$$f_{\mathrm{B}} = \mid f_1 - f_2 \mid .$$

Conceptual Questions

Exercise:

Problem:

How does destructive interference occur? What do you see in a destructive interference?

Exercise:

Problem:

Explain how noise-cancelling headphones work in terms of wave interference phenomena.

Exercise:

Problem: Explain how nodes and antinodes form on standing waves.

Exercise:

Problem:

In the beat phenomenon, the loudness of the sound combined from two sound waves "wobbles" (it increases and decreases over time). Explain how the loudness of the combined sound becomes louder then quieter, even though individual sound waves remain at the same loudness, or amplitude.

Glossary

antinode
: the location of maximum amplitude in standing waves

beat frequency
: the frequency of the amplitude fluctuations of a wave

constructive interference
: when two waves arrive at the same point exactly in phase; that is, the crests of the two waves are precisely aligned, as are the troughs

destructive interference
: when two identical waves arrive at the same point exactly out of phase; that is, precisely aligned crest to trough

fundamental frequency
: the lowest frequency of a periodic waveform

nodes
: the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave

overtones
: multiples of the fundamental frequency of a sound

superposition
: the phenomenon that occurs when two or more waves arrive at the same point

Sound

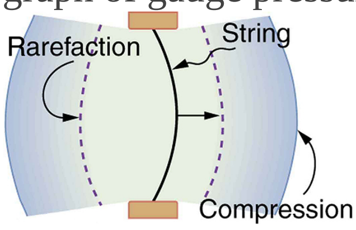
- Define sound and hearing.
- Describe sound as a longitudinal wave.
- Distinguish between audible sound, ultrasound, and infrasound.

This glass has been shattered by a high-intensity sound wave of the same frequency as the resonant frequency of the glass. While the sound is not visible, the effects of the sound prove its existence. (credit: ||read||, Flickr)

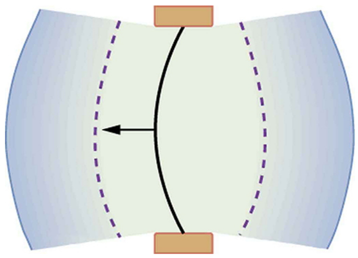
Sound can be used as a familiar illustration of waves. Because hearing is one of our most important senses, it is interesting to see how the physical properties of sound correspond to our perceptions of it. **Hearing** is the perception of sound, just as vision is the perception of visible light. But sound has important applications beyond hearing. Ultrasound, for example, is not heard but can be employed to form medical images and is also used in treatment.

The physical phenomenon of **sound** is defined to be a disturbance of matter that is transmitted from its source outward. Sound is a wave. On the atomic scale, it is a disturbance of atoms that is far more ordered than their thermal motions. In many instances, sound is a periodic wave, and the atoms undergo simple harmonic motion. In this text, we shall explore such periodic sound waves.

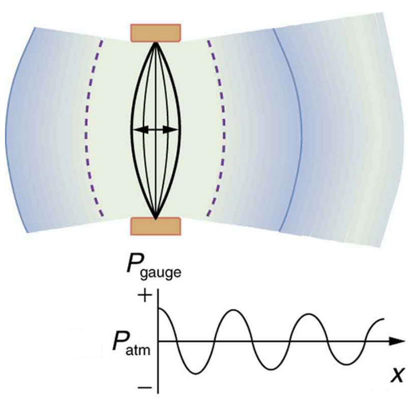
A vibrating string produces a sound wave as illustrated in [link], [link], and [link]. As the string oscillates back and forth, it transfers energy to the air, mostly as thermal energy created by turbulence. But a small part of the string's energy goes into compressing and expanding the surrounding air, creating slightly higher and lower local pressures. These compressions (high pressure regions) and rarefactions (low pressure regions) move out as longitudinal pressure waves having the same frequency as the string—they are the disturbance that is a sound wave. (Sound waves in air and most fluids are longitudinal, because fluids have almost no shear strength. In solids, sound waves can be both transverse and longitudinal.) [link] shows a graph of gauge pressure versus distance from the vibrating string.



A vibrating string moving to the right compresses the air in front of it and expands the air behind it.



As the string moves to the left, it creates another compression and rarefaction as the ones on the right move away from the string.

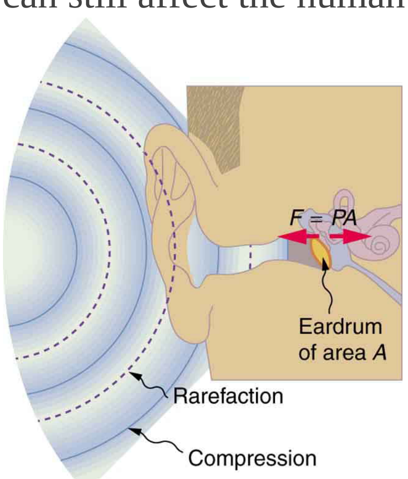


After many vibrations, there are a series of compressions and rarefactions moving out from the string as a sound wave. The graph shows gauge pressure versus

distance from the source. Pressures vary only slightly from atmospheric for ordinary sounds.

The amplitude of a sound wave decreases with distance from its source, because the energy of the wave is spread over a larger and larger area. But it is also absorbed by objects, such as the eardrum in [link], and converted to thermal energy by the viscosity of air. In addition, during each compression a little heat transfers to the air and during each rarefaction even less heat transfers from the air, so that the heat transfer reduces the organized disturbance into random thermal motions. Whether the heat transfer from compression to rarefaction is significant depends on how far apart they are—that is, it depends on wavelength. Wavelength, frequency, amplitude, and speed of propagation are important for sound, as they are for all waves.

When the frequency of the sound wave is in **audible range** of 20 to 20,000 Hz, the vibration of the eardrums at these frequencies can be sensed, which is what we hear. Sound waves of frequency above 20 kHz are called **ultrasound** waves. Some construction equipment produces **infrasound** waves of frequencies less than 20 Hz, which, although they can't be heard, can still affect the human body at large enough amplitudes.



Sound wave compressions and rarefactions travel up the ear canal and force the eardrum to vibrate. There is a net force on the eardrum, since the sound wave pressures differ from the atmospheric pressure found behind the eardrum. A complicated mechanism converts the vibrations to nerve impulses, which are perceived by the person.

Section Summary

- Sound is a disturbance of matter that is transmitted from its source outward.
- Sound is one type of wave.
- Hearing is the perception of sound.

Glossary

audible range
: frequencies between 20 Hz and 20,000 Hz

infrasound
: sound waves below audible frequencies (less than 20 Hz)

hearing
: the perception of sound

sound
: a disturbance of matter that is transmitted from its source outward

ultrasound
: sound waves above audible frequencies (greater than 20,000 Hz)

Speed of Sound, Frequency, and Wavelength

- Define pitch.
- Describe the relationship between the speed of sound, its frequency, and its wavelength.
- Describe the effects on the speed of sound as it travels through various media.
- Describe the effects of temperature on the speed of sound.

When a firework explodes, the light energy is perceived before the sound energy. Sound travels more slowly than light does. (credit: Dominic Alves, Flickr)

Sound, like all waves, travels at a certain speed and has the properties of frequency and wavelength. You can observe direct evidence of the speed of sound while watching a fireworks display. The flash of an explosion is seen well before its sound is heard, implying both that sound travels at a finite speed and that it is much slower than light. You can also directly sense the frequency of a sound. Perception of frequency is called **pitch**. The wavelength of sound is not directly sensed, but indirect evidence is found in the correlation of the size of musical instruments with their pitch. Small

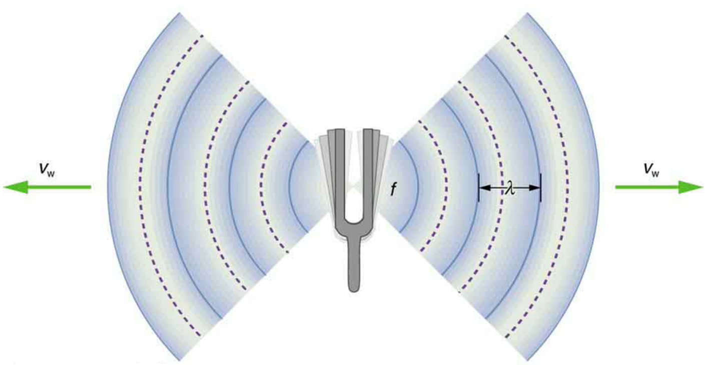
instruments, such as a piccolo, typically make high-pitch sounds, while large instruments, such as a tuba, typically make low-pitch sounds. High pitch means small wavelength, and the size of a musical instrument is directly related to the wavelengths of sound it produces. So a small instrument creates short-wavelength sounds. Similar arguments hold that a large instrument creates long-wavelength sounds.

The relationship of the speed of sound, its frequency, and wavelength is the same as for all waves:

Equation:

$$v_{\text{w}} = f\lambda,$$

where v_{w} is the speed of sound, f is its frequency, and λ is its wavelength. The wavelength of a sound is the distance between adjacent identical parts of a wave—for example, between adjacent compressions as illustrated in [link]. The frequency is the same as that of the source and is the number of waves that pass a point per unit time.



A sound wave emanates from a source vibrating at a frequency f, propagates at v_{w}, and has a wavelength λ.

[link] makes it apparent that the speed of sound varies greatly in different media. The speed of sound in a medium is determined by a combination of the medium’s rigidity (or compressibility in gases) and its density. The

more rigid (or less compressible) the medium, the faster the speed of sound. This observation is analogous to the fact that the frequency of a simple harmonic motion is directly proportional to the stiffness of the oscillating object. The greater the density of a medium, the slower the speed of sound. This observation is analogous to the fact that the frequency of a simple harmonic motion is inversely proportional to the mass of the oscillating object. The speed of sound in air is low, because air is compressible. Because liquids and solids are relatively rigid and very difficult to compress, the speed of sound in such media is generally greater than in gases.

Medium	v_w**(m/s)**
Gases at $0^\circ C$	
Air	331
Carbon dioxide	259
Oxygen	316
Helium	965
Hydrogen	1290
Liquids at $20^\circ C$	
Ethanol	1160
Mercury	1450
Water, fresh	1480

Medium	**v_w(m/s)**
Sea water	1540
Human tissue	1540
Solids (longitudinal or bulk)	
Vulcanized rubber	54
Polyethylene	920
Marble	3810
Glass, Pyrex	5640
Lead	1960
Aluminum	5120
Steel	5960

Speed of Sound in Various Media

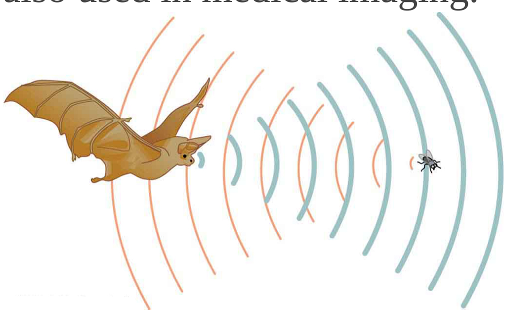
Earthquakes, essentially sound waves in Earth’s crust, are an interesting example of how the speed of sound depends on the rigidity of the medium. Earthquakes have both longitudinal and transverse components, and these travel at different speeds. The bulk modulus of granite is greater than its shear modulus. For that reason, the speed of longitudinal or pressure waves (P-waves) in earthquakes in granite is significantly higher than the speed of transverse or shear waves (S-waves). Both components of earthquakes travel slower in less rigid material, such as sediments. P-waves have speeds of 4 to 7 km/s, and S-waves correspondingly range in speed from 2 to 5 km/s, both being faster in more rigid material. The P-wave gets progressively farther ahead of the S-wave as they travel through Earth’s crust. The time between the P- and S-waves is routinely used to determine the distance to their source, the epicenter of the earthquake.

The speed of sound is affected by temperature in a given medium. For air at sea level, the speed of sound is given by

Equation:

$$v_{\mathrm{w}} = (331\ \mathrm{m/s})\sqrt{\frac{T}{273\ \mathrm{K}}},$$

where the temperature (denoted as T) is in units of kelvin. While not negligible, this is not a strong dependence. At $0^{\circ}\mathrm{C}$, the speed of sound is 331 m/s, whereas at $20.0^{\circ}\mathrm{C}$ it is 343 m/s, less than a 4% increase. [link] shows a use of the speed of sound by a bat to sense distances. Echoes are also used in medical imaging.



A bat uses sound echoes to find its way about and to catch prey. The time for the echo to return is directly proportional to the distance.

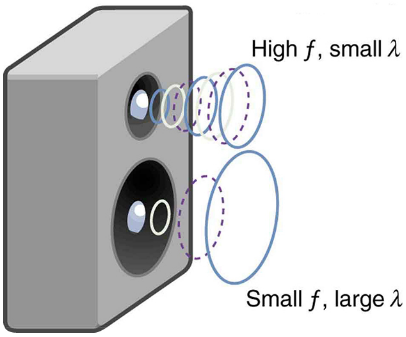
One of the more important properties of sound is that its speed is nearly independent of frequency. This independence is certainly true in open air for sounds in the audible range of 20 to 20,000 Hz. If this independence were not true, you would certainly notice it for music played by a marching band in a football stadium, for example. Suppose that high-frequency sounds traveled faster—then the farther you were from the band, the more the sound from the low-pitch instruments would lag that from the high-pitch ones. But the music from all instruments arrives in cadence independent of

distance, and so all frequencies must travel at nearly the same speed. Recall that

Equation:

$$v_{\text{w}} = f\lambda.$$

In a given medium under fixed conditions, v_{w} is constant, so that there is a relationship between f and λ; the higher the frequency, the smaller the wavelength. See [link] and consider the following example.



Because they travel at the same speed in a given medium, low-frequency sounds must have a greater wavelength than high-frequency sounds. Here, the lower-frequency sounds are emitted by the large speaker, called a woofer, while the higher-frequency sounds are emitted by the small speaker, called a tweeter.

Example:
Calculating Wavelengths: What Are the Wavelengths of Audible Sounds?

Calculate the wavelengths of sounds at the extremes of the audible range, 20 and 20,000 Hz, in 30.0°C air. (Assume that the frequency values are accurate to two significant figures.)

Strategy

To find wavelength from frequency, we can use $v_{\text{w}} = f\lambda$.

Solution

1. Identify knowns. The value for v_{w}, is given by
   Equation:
   $$v_{\text{w}} = (331\ \text{m/s})\sqrt{\frac{T}{273\ \text{K}}}.$$
2. Convert the temperature into kelvin and then enter the temperature into the equation
   Equation:
   $$v_{\text{w}} = (331\ \text{m/s})\sqrt{\frac{303\ \text{K}}{273\ \text{K}}} = 348.7\ \text{m/s}.$$
3. Solve the relationship between speed and wavelength for λ:
   Equation:
   $$\lambda = \frac{v_{\text{w}}}{f}.$$
4. Enter the speed and the minimum frequency to give the maximum wavelength:
   Equation:

$$\lambda_{\text{max}} = \frac{348.7 \text{ m/s}}{20 \text{ Hz}} = 17 \text{ m.}$$

5. Enter the speed and the maximum frequency to give the minimum wavelength:
   Equation:

$$\lambda_{\text{min}} = \frac{348.7 \text{ m/s}}{20{,}000 \text{ Hz}} = 0.017 \text{ m} = 1.7 \text{ cm.}$$

Discussion
Because the product of f multiplied by λ equals a constant, the smaller f is, the larger λ must be, and vice versa.

The speed of sound can change when sound travels from one medium to another. However, the frequency usually remains the same because it is like a driven oscillation and has the frequency of the original source. If v_{w} changes and f remains the same, then the wavelength λ must change. That is, because $v_{\text{w}} = f\lambda$, the higher the speed of a sound, the greater its wavelength for a given frequency.

Note:
Making Connections: Take-Home Investigation—Voice as a Sound Wave
Suspend a sheet of paper so that the top edge of the paper is fixed and the bottom edge is free to move. You could tape the top edge of the paper to the edge of a table. Gently blow near the edge of the bottom of the sheet and note how the sheet moves. Speak softly and then louder such that the sounds hit the edge of the bottom of the paper, and note how the sheet moves. Explain the effects.

Exercise:

Check Your Understanding

Problem:

Imagine you observe two fireworks explode. You hear the explosion of one as soon as you see it. However, you see the other firework for several milliseconds before you hear the explosion. Explain why this is so.

Solution:

Sound and light both travel at definite speeds. The speed of sound is slower than the speed of light. The first firework is probably very close by, so the speed difference is not noticeable. The second firework is farther away, so the light arrives at your eyes noticeably sooner than the sound wave arrives at your ears.

Exercise:
Check Your Understanding

Problem:

You observe two musical instruments that you cannot identify. One plays high-pitch sounds and the other plays low-pitch sounds. How could you determine which is which without hearing either of them play?

Solution:

Compare their sizes. High-pitch instruments are generally smaller than low-pitch instruments because they generate a smaller wavelength.

Section Summary

The relationship of the speed of sound v_{w}, its frequency f, and its wavelength λ is given by
Equation:

$$v_{\text{w}} = f\lambda,$$

which is the same relationship given for all waves.

In air, the speed of sound is related to air temperature T by
Equation:

$$v_{\text{w}} = (331\ \text{m/s})\sqrt{\frac{T}{273\ \text{K}}}.$$

v_{w} is the same for all frequencies and wavelengths.

Conceptual Questions

Exercise:

Problem:

How do sound vibrations of atoms differ from thermal motion?

Exercise:

Problem:

When sound passes from one medium to another where its propagation speed is different, does its frequency or wavelength change? Explain your answer briefly.

Exercise:

Problem:

For a high-fidelity speaker containing a woofer and a tweeter (see [link] for example), why is the woofer larger than the tweeter?

Glossary

pitch
: the perception of the frequency of a sound

Doppler Effect and Sonic Booms

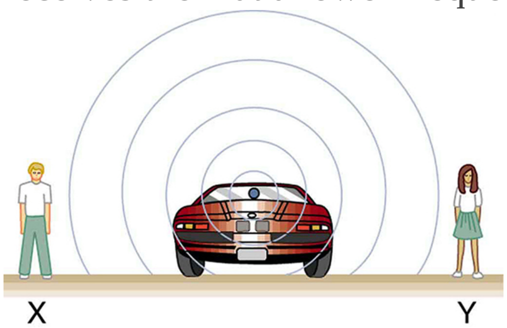
- Define Doppler effect, Doppler shift, and sonic boom.
- Describe the sounds produced by objects moving faster than the speed of sound.

The characteristic sound of a motorcycle buzzing by is an example of the Doppler effect. The high-pitch scream shifts dramatically to a lower-pitch roar as the motorcycle passes by a stationary observer. The closer the motorcycle brushes by, the more abrupt the shift. The faster the motorcycle moves, the greater the shift. We also hear this characteristic shift in frequency for passing race cars, airplanes, and trains. It is so familiar that it is used to imply motion and children often mimic it in play.

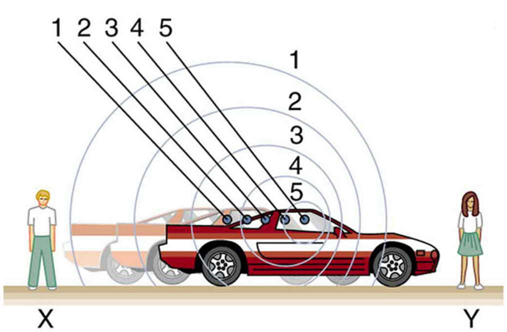
The Doppler effect is an alteration in the observed frequency of a sound due to motion of either the source or the observer. Although less familiar, this effect is easily noticed for a stationary source and moving observer. For example, if you ride a train past a stationary warning bell, you will hear the bell's frequency shift from high to low as you pass by. This change in frequency due to *relative* motion of source and observer is called a **Doppler shift**. The Doppler effect is named for the Austrian physicist and mathematician Christian Johann Doppler (1803–1853), who did experiments with both moving sources and moving observers. Doppler, for example, had musicians play on a moving open train car and also play standing next to the train tracks as a train passed by. Their music was observed both on and off the train, and changes in frequency were measured.

What causes the Doppler shift? [link], [link], and [link] compare sound waves emitted by stationary and moving sources in a stationary air mass. Each disturbance spreads out spherically from the point where the sound was emitted. If the source is stationary, then all of the spheres representing the air compressions in the sound wave centered on the same point, and the stationary observers on either side see the same wavelength and frequency as emitted by the source, as in [link]. If the source is moving, as in [link], then the situation is different. Each compression of the air moves out in a sphere from the point where it was emitted, but the point of emission moves. This moving emission point causes the air compressions to be closer

together on one side and farther apart on the other. Thus, the wavelength is shorter in the direction the source is moving (on the right in [link]), and longer in the opposite direction (on the left in [link]). Finally, if the observers move, as in [link], the frequency at which they receive the compressions changes. The observer moving toward the source receives them at a higher frequency, and the person moving away from the source receives them at a lower frequency.

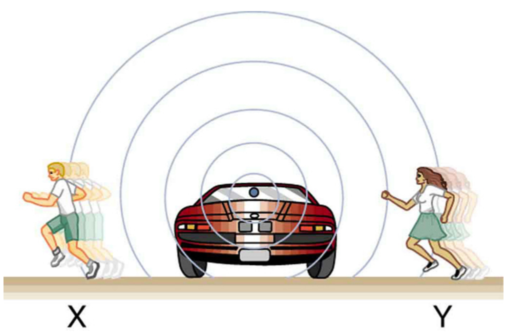


Sounds emitted by a source spread out in spherical waves. Because the source, observers, and air are stationary, the wavelength and frequency are the same in all directions and to all observers.



Sounds emitted by a source moving to the right spread out from the

points at which they were emitted. The wavelength is reduced and, consequently, the frequency is increased in the direction of motion, so that the observer on the right hears a higher-pitch sound. The opposite is true for the observer on the left, where the wavelength is increased and the frequency is reduced.



The same effect is produced when the observers move relative to the source. Motion toward the source increases frequency as the observer on the right passes through more wave crests than she would if stationary. Motion away from the source decreases frequency as the observer

on the left passes through fewer wave crests than he would if stationary.

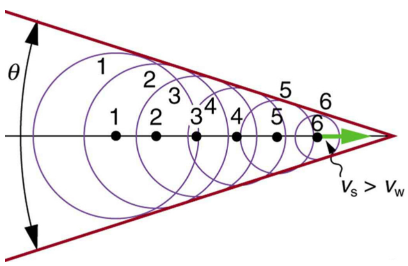
We know that wavelength and frequency are related by $v_{\text{w}} = f\lambda$, where v_{w} is the fixed speed of sound. The sound moves in a medium and has the same speed v_{w} in that medium whether the source is moving or not. Thus f multiplied by λ is a constant. Because the observer on the right in [link] receives a shorter wavelength, the frequency she receives must be higher. Similarly, the observer on the left receives a longer wavelength, and hence he hears a lower frequency. The same thing happens in [link]. A higher frequency is received by the observer moving toward the source, and a lower frequency is received by an observer moving away from the source. In general, then, relative motion of source and observer toward one another increases the received frequency. Relative motion apart decreases frequency. The greater the relative speed is, the greater the effect.

Note:
The Doppler Effect
The Doppler effect occurs not only for sound but for any wave when there is relative motion between the observer and the source. There are Doppler shifts in the frequency of sound, light, and water waves, for example. Doppler shifts can be used to determine velocity, such as when ultrasound is reflected from blood in a medical diagnostic. The recession of galaxies is determined by the shift in the frequencies of light received from them and has implied much about the origins of the universe. Modern physics has been profoundly affected by observations of Doppler shifts.

Sonic Booms to Bow Wakes

What happens to the sound produced by a moving source, such as a jet airplane, that approaches or even exceeds the speed of sound? The answer to this question applies not only to sound but to all other waves as well.

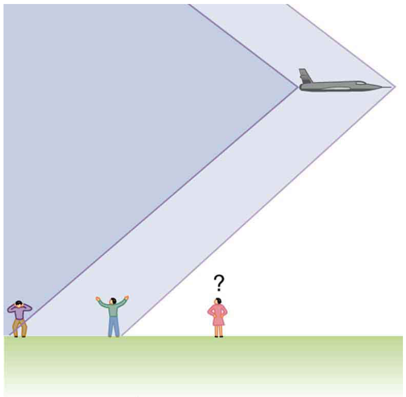
Suppose a jet airplane is coming nearly straight at you, emitting a sound of frequency f_s. The greater the plane's speed v_s, the greater the Doppler shift and the greater the value observed for f_{obs}. Now, as v_s approaches the speed of sound, f_{obs} approaches infinity. At the speed of sound, this result means that in front of the source, each successive wave is superimposed on the previous one because the source moves forward at the speed of sound. The observer gets them all at the same instant, and so the frequency is infinite. (Before airplanes exceeded the speed of sound, some people argued it would be impossible because such constructive superposition would produce pressures great enough to destroy the airplane.) If the source exceeds the speed of sound, no sound is received by the observer until the source has passed, so that the sounds from the approaching source are mixed with those from it when receding. This mixing appears messy, but something interesting happens—a sonic boom is created. (See [link].)



Sound waves from a source that moves faster than the speed of sound spread spherically from the point where they are emitted, but the source moves ahead of each.

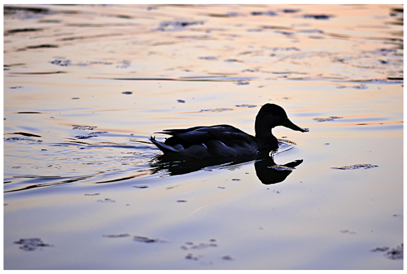
Constructive interference along the lines shown (actually a cone in three dimensions) creates a shock wave called a sonic boom. The faster the speed of the source, the smaller the angle θ.

There is constructive interference along the lines shown (a cone in three dimensions) from similar sound waves arriving there simultaneously. This superposition forms a disturbance called a **sonic boom**, a constructive interference of sound created by an object moving faster than sound. Inside the cone, the interference is mostly destructive, and so the sound intensity there is much less than on the shock wave. An aircraft creates two sonic booms, one from its nose and one from its tail. (See [link].) During television coverage of space shuttle landings, two distinct booms could often be heard. These were separated by exactly the time it would take the shuttle to pass by a point. Observers on the ground often do not see the aircraft creating the sonic boom, because it has passed by before the shock wave reaches them, as seen in [link]. If the aircraft flies close by at low altitude, pressures in the sonic boom can be destructive and break windows as well as rattle nerves. Because of how destructive sonic booms can be, supersonic flights are banned over populated areas of the United States.

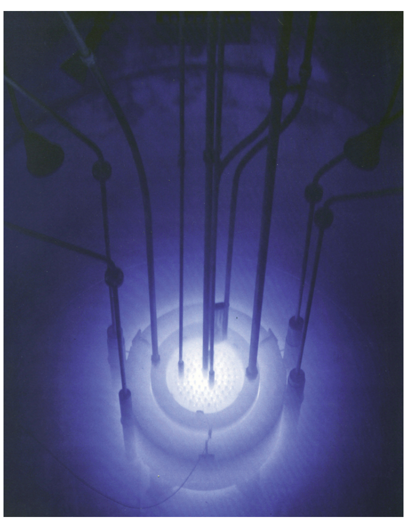


Two sonic booms, created by the nose and tail of an aircraft, are observed on the ground after the plane has passed by.

Sonic booms are one example of a broader phenomenon called bow wakes. A **bow wake**, such as the one in [link], is created when the wave source moves faster than the wave propagation speed. Water waves spread out in circles from the point where created, and the bow wake is the familiar V-shaped wake trailing the source. A more exotic bow wake is created when a subatomic particle travels through a medium faster than the speed of light travels in that medium (the speed of light in vacuum is $c = 2.998 \times 10^8 \text{m/s}$, and this is the maximum speed anything can move, but in water, for example, speed of light slows to $0.75c$, and other particles can move faster). If the particle creates light in its passage, that light spreads on a cone with an angle indicative of the speed of the particle, as illustrated in [link]. Such a bow wake is called Cherenkov radiation and is commonly observed in particle physics.



Bow wake created by a duck. Constructive interference produces the rather structured wake, while there is relatively little wave action inside the wake, where interference is mostly destructive. (credit: Horia Varlan, Flickr)



The blue glow in this research reactor pool is Cherenkov radiation caused by subatomic particles traveling faster than the speed of light in water. (credit: U.S. Nuclear Regulatory Commission)

Doppler shifts and sonic booms are interesting sound phenomena that occur in all types of waves. They can be of considerable use. For example, the Doppler shift in ultrasound can be used to measure blood velocity, while police use the Doppler shift in radar (a microwave) to measure car velocities. In meteorology, the Doppler shift is used to track the motion of storm clouds; such “Doppler Radar” can give velocity and direction and rain or snow potential of imposing weather fronts. In astronomy, we can examine the light emitted from distant galaxies and determine their speed relative to ours. As galaxies move away from us, their light is shifted to a lower frequency, and so to a longer wavelength—the so-called red shift. Such information from galaxies far, far away has allowed us to estimate the age of the universe (from the Big Bang) as about 14 billion years.

Exercise:

Check Your Understanding

Problem:

Why did scientist Christian Doppler observe musicians both on a moving train and also from a stationary point not on the train?

Solution:

Doppler needed to compare the perception of sound when the observer is stationary and the sound source moves, as well as when the sound

source and the observer are both in motion.

Exercise:
Check Your Understanding

Problem:

Describe a situation in your life when you might rely on the Doppler shift to help you either while driving a car or walking near traffic.

Solution:

If I am driving and I hear Doppler shift in an ambulance siren, I would be able to tell when it was getting closer and also if it has passed by. This would help me to know whether I needed to pull over and let the ambulance through.

Section Summary

- The Doppler shift is change in observed frequency of a wave (for example, a sound wave) due to relative motion of the source and the observer, with approaching source and/or observer increasing the observed frequency and receding source and/or observer decreasing the observed frequency.
- A sonic boom is constructive interference of sound created by an object moving faster than sound.
- A shock wave is a type of bow wake created when any wave source moves faster than the wave propagation speed.

Conceptual Questions

Exercise:

Problem:

Explain Doppler shift. How does the measured frequency increase or decrease?

Exercise:

Problem:

Due to efficiency considerations related to its bow wake, the supersonic transport aircraft must maintain a cruising speed that is a constant ratio to the speed of sound (a constant Mach number). If the aircraft flies from warm air into colder air, should it increase or decrease its speed? Explain your answer.

Exercise:

Problem:

When you hear a sonic boom, you often cannot see the plane that made it. Why is that?

Glossary

Doppler shift
: the change in wave frequency due to relative motion of source and observer

sonic boom
: a constructive interference of sound created by an object moving faster than sound

bow wake
: V-shaped disturbance created when the wave source moves faster than the wave propagation speed

Introduction to Rotational Motion and Angular Momentum class="introduction"

The mention of a tornado conjures up images of raw destructive power. Tornadoes blow houses away as if they were made of paper and have been known to pierce tree trunks with pieces of straw. They descend from clouds in funnel-like shapes that spin violently, particularly at the bottom where they are most narrow, producing winds as high as 500 km/h. (credit: Daphne Zaras, U.S. National Oceanic and Atmospheric Administration)

Why do tornadoes spin at all? And why do tornados spin so rapidly? The answer is that air masses that produce tornadoes are themselves rotating, and when the radii of the air masses decrease, their rate of rotation increases. An ice skater increases her spin in an exactly analogous manner as seen in [link]. The skater starts her rotation with outstretched limbs and increases her spin by pulling them in toward her body. The same physics describes the exhilarating spin of a skater and the wrenching force of a tornado.

Clearly, force, energy, and power are associated with rotational motion. These and other aspects of rotational motion are covered in this chapter. We shall see that all important aspects of rotational motion either have already been defined for linear motion or have exact analogs in linear motion. First, we look at angular acceleration—the rotational analog of linear acceleration.

This figure skater increases her rate of spin by pulling her arms and her extended leg closer to her axis of rotation. (credit: Luu, Wikimedia Commons)

https://www.youtube.com/embed/FmnkQ2ytlO8

Angular Acceleration

- Describe uniform circular motion.
- Explain non-uniform circular motion.
- Explain what angular acceleration is.

Previously, we looked at uniform circular motion, where an object moves in a circular path at a constant speed (but changing directions, so it undergoes centripetal acceleration). We can also describe this in terms of a rotation at a constant rate. For example, an object completing a circle every second can be said to rotate at a constant rate of 360 degrees per second about the center of the circle, since there are 360 degrees in one complete circle.

For the formulas and equations you will see in this chapter, we will introduce a mathematically convenient unit of angle called **radians**. An angular measure in radian is defined by the arc length covered by the angle divided by the radius of the circle that the arc length is part of. Do you see how, if the angular size remains the same, as we consider larger circles, with matching longer arc length, the ratio of arc length to the radius of the circle remains constant? Consider this example. The circumference S of a circle of radius r is given by $S = 2\pi r$. So the angular size of a full circle is this circumference (a special kind of arc length) divided by the radius, or 2π (so you could say 2π radians is equal to 360 degrees).

So, imagine an object completing a uniform circular motion of radius r at speed v, and let's say it takes time Δt to complete one full circle. Then using what we learned in kinematics, we know this holds:
Equation:

$$v = \frac{\Delta s}{\Delta t} = \frac{2\pi r}{\Delta t},$$

where Δs is the arc length, or the circumference, in this case. We can also describe this in terms of **angular velocity**, or the rate of change of angle,
Equation:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi}{\Delta t},$$

where change of angle $\Delta\theta$ in this case is the full circle, or 2π radians. We use ω (lower-case omega) to denote angular velocity.

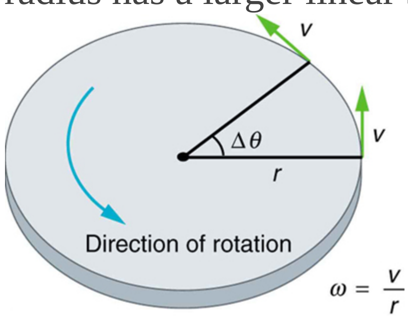
Carefully comparing [link] with [link], we see that following relationships hold:
Equation:

$$v = r\omega$$

and
Equation:

$$\omega = \frac{v}{r}.$$

We are going to use this relationship to relate linear motion with rotation (or angular motion) whenever necessary. A summary with figure is in [link]. In [link], also note that, given the same angular velocity ω, an object at a larger radius has a larger linear speed (does this make intuitive sense?).



This figure shows uniform circular motion and some of its defined quantities.

Angular velocity is not constant when a skater pulls in her arms (speeding up for some reason, which we'll look at why later), when a child starts up a merry-go-round from rest, or when a computer's hard disk slows to a halt when switched off. In all these cases, there is an **angular acceleration**, in which ω changes. The faster the change occurs, the greater the angular acceleration. Angular acceleration α is defined as the rate of change of angular velocity. In equation form, angular acceleration is expressed as follows:

Equation:

$$\alpha = \frac{\Delta\omega}{\Delta t},$$

where $\Delta\omega$ is the **change in angular velocity** and Δt is the change in time. The units of angular acceleration are $(\mathrm{rad/s})/\mathrm{s}$, or $\mathrm{rad/s}^2$. If ω increases, then α is positive. If ω decreases, then α is negative. Hopefully all this sounds rather familiar from a similar discussion in linear kinematics.

Example:

Calculating the Angular Acceleration and Deceleration of a Bike Wheel

Suppose a teenager puts her bicycle on its back and starts the rear wheel spinning from rest to a final angular velocity of 250 rpm in 5.00 s. (a) Calculate the angular acceleration in $\mathrm{rad/s}^2$. (b) If she now slams on the brakes, causing an angular acceleration of $-87.3\ \mathrm{rad/s}^2$, how long does it take the wheel to stop?

Strategy for (a)

The angular acceleration can be found directly from its definition in $\alpha = \frac{\Delta\omega}{\Delta t}$ because the final angular velocity and time are given. We see that $\Delta\omega$ is 250 rpm and Δt is 5.00 s.

Solution for (a)

Entering known information into the definition of angular acceleration, we get

Equation:

$$\begin{aligned}\alpha &= \frac{\Delta\omega}{\Delta t}\\ &= \frac{250\text{ rpm}}{5.00\text{ s}}.\end{aligned}$$

Because $\Delta\omega$ is in revolutions per minute (rpm) and we want the standard units of rad/s^2 for angular acceleration, we need to convert $\Delta\omega$ from rpm to rad/s:

Equation:

$$\begin{aligned}\Delta\omega &= 250\,\frac{\text{rev}}{\text{min}}\cdot\frac{2\pi\text{ rad}}{\text{rev}}\cdot\frac{1\text{ min}}{60\text{ sec}}\\ &= 26.2\,\frac{\text{rad}}{\text{s}}.\end{aligned}$$

Entering this quantity into the expression for α, we get

Equation:

$$\begin{aligned}\alpha &= \frac{\Delta\omega}{\Delta t}\\ &= \frac{26.2\text{ rad/s}}{5.00\text{ s}}\\ &= 5.24\text{ rad/s}^2.\end{aligned}$$

Strategy for (b)

In this part, we know the angular acceleration and the initial angular velocity. We can find the stoppage time by using the definition of angular acceleration and solving for Δt, yielding

Equation:

$$\Delta t = \frac{\Delta\omega}{\alpha}.$$

Solution for (b)

Here the angular velocity decreases from 26.2 rad/s (250 rpm) to zero, so that $\Delta\omega$ is -26.2 rad/s, and α is given to be -87.3 rad/s^2. Thus,

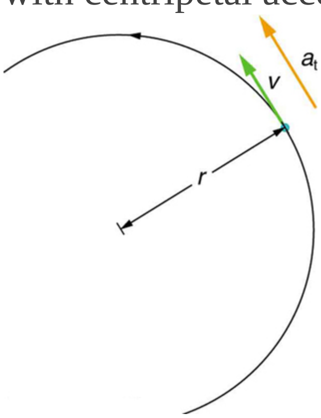
Equation:

$$\begin{aligned}\Delta t &= \frac{-26.2\ \text{rad/s}}{-87.3\ \text{rad/s}^2} \\ &= 0.300\ \text{s.}\end{aligned}$$

Discussion

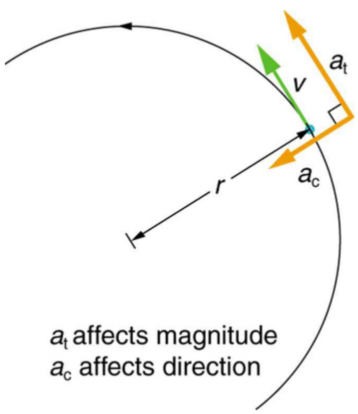
Note that the angular acceleration as the girl spins the wheel is small and positive; it takes 5 s to produce an appreciable angular velocity. When she hits the brake, the angular acceleration is large and negative. The angular velocity quickly goes to zero. In both cases, the relationships are analogous to what happens with linear motion. For example, there is a large deceleration when you crash into a brick wall—the velocity change is large in a short time interval.

If the bicycle in the preceding example had been on its wheels instead of upside-down, it would first have accelerated along the ground and then come to a stop. This connection between circular motion and linear motion needs to be explored. For example, it would be useful to know how linear and angular acceleration are related. In circular motion, linear acceleration is *tangent* to the circle at the point of interest, as seen in [link]. Thus, linear acceleration is called **tangential acceleration** a_{t}. This is not to be confused with centripetal acceleration we covered earlier (see: [link]).



In circular motion, linear acceleration a, occurs as the magnitude of

the velocity changes: a is tangent to the motion. In the context of circular motion, linear acceleration is also called tangential acceleration a_{t}.



Centripetal acceleration a_{c} occurs as the direction of velocity changes; it is perpendicular to the circular motion. Centripetal and tangential acceleration are thus

perpendicular to each other.

This tangential acceleration a_t is related to angular acceleration α in a similar way as how linear velocity is related to angular velocity. So, taking the cues from [link] and [link], tangential acceleration and angular acceleration are related by,

Equation:

$$a_\mathrm{t} = r\alpha,$$

and

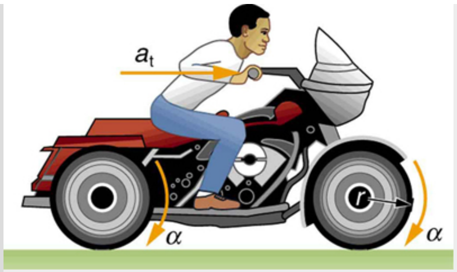
Equation:

$$\alpha = \frac{a_\mathrm{t}}{r}.$$

These equations mean that linear acceleration and angular acceleration are directly proportional. The greater the angular acceleration is, the larger the linear (tangential) acceleration is, and vice versa. For example, the greater the angular acceleration of a car's drive wheels, the greater the acceleration of the car. The radius also matters. For example, the smaller a wheel, the smaller its linear acceleration for a given angular acceleration α.

Example:
Calculating the Angular Acceleration of a Motorcycle Wheel

A powerful motorcycle can accelerate from 0 to 30.0 m/s (about 108 km/h) in 4.20 s. What is the angular acceleration of its 0.320-m-radius wheels? (See [link].)



The linear acceleration of a motorcycle is accompanied by an angular acceleration of its wheels.

Strategy

We are given information about the linear velocities of the motorcycle. Thus, we can find its linear acceleration a_{t}. Then, the expression $\alpha = \frac{a_{\mathrm{t}}}{r}$ can be used to find the angular acceleration.

Solution

The linear acceleration is

Equation:

$$\begin{aligned} a_{\mathrm{t}} &= \frac{\Delta v}{\Delta t} \\ &= \frac{30.0 \text{ m/s}}{4.20 \text{ s}} \\ &= 7.14 \text{ m/s}^2. \end{aligned}$$

We also know the radius of the wheels. Entering the values for a_{t} and r into $\alpha = \frac{a_{\mathrm{t}}}{r}$, we get

Equation:

$$\begin{aligned} \alpha &= \frac{a_{\mathrm{t}}}{r} \\ &= \frac{7.14 \text{ m/s}^2}{0.320 \text{ m}} \\ &= 22.3 \text{ rad/s}^2. \end{aligned}$$

Discussion

Units of radians are dimensionless and appear in any relationship between angular and linear quantities.

So far, we have defined three rotational quantities— θ, ω, and α. These quantities are analogous to the translational quantities x, v, and a. [link] displays rotational quantities, the analogous translational quantities, and the relationships between them.

Rotational	Translational	Relationship
θ	x	$\theta = \frac{x}{r}$
ω	v	$\omega = \frac{v}{r}$
α	a	$\alpha = \frac{a_t}{r}$

Rotational and Translational Quantities

Note:
Making Connections: Take-Home Experiment

Sit down with your feet on the ground on a chair that rotates. Lift one of your legs such that it is unbent (straightened out). Using the other leg, begin to rotate yourself by pushing on the ground. Stop using your leg to push the ground but allow the chair to rotate. From the origin where you began, sketch the angle, angular velocity, and angular acceleration of your leg as a function of time in the form of three separate graphs. Estimate the magnitudes of these quantities.

Exercise:
Check Your Understanding

Problem:

Angular acceleration is a vector, having both magnitude and direction. How do we denote its magnitude and direction? Illustrate with an example.

Solution:

The magnitude of angular acceleration is α and its most common units are $\mathrm{rad/s^2}$. The direction of angular acceleration along a fixed axis is denoted by a + or a – sign, just as the direction of linear acceleration in one dimension is denoted by a + or a – sign. For example, consider a gymnast doing a forward flip. Her angular momentum would be parallel to the mat and to her left. The magnitude of her angular acceleration would be proportional to her angular velocity (spin rate) and her moment of inertia about her spin axis.

Section Summary

- Uniform circular motion is the motion with a constant angular velocity $\omega = \frac{\Delta\theta}{\Delta t}$.
- In non-uniform circular motion, the velocity changes with time and the rate of change of angular velocity (i.e. angular acceleration) is $\alpha = \frac{\Delta\omega}{\Delta t}$.

- Linear or tangential acceleration refers to changes in the magnitude of velocity but not its direction, given as $a_{\mathrm{t}} = \frac{\Delta v}{\Delta t}$.
- Linear or tangential motion variables x, v, and a_t are proportional to the rotational quantities θ, ω, and α. They are also proportional to the radius r of the rotational motion.

Conceptual Questions

Exercise:

Problem:

Analogies exist between rotational and translational physical quantities. Identify the rotational term analogous to each of the following: acceleration, force, mass, work, translational kinetic energy, linear momentum, impulse.

Exercise:

Problem:

Explain why centripetal acceleration changes the direction of velocity in circular motion but not its magnitude.

Exercise:

Problem:

In circular motion, a tangential acceleration can change the magnitude of the velocity but not its direction. Explain your answer.

Exercise:

Problem:

Suppose a piece of food is on the edge of a rotating microwave oven plate. Does it experience nonzero tangential acceleration, centripetal acceleration, or both when: (a) The plate starts to spin? (b) The plate rotates at constant angular velocity? (c) The plate slows to a halt?

Glossary

angular acceleration
: the rate of change of angular velocity with time

angular velocity
: the rate of change of angular position with time

change in angular velocity
: the difference between final and initial values of angular velocity

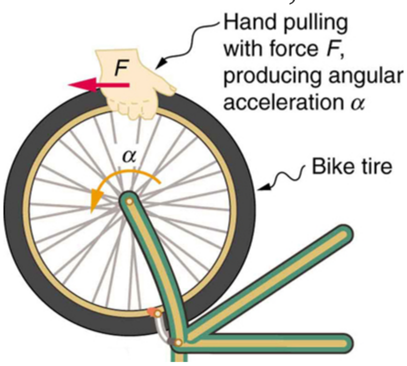
radian
: a unit of angular measure defined by the arc length covered by the angle divided by the radius of the circle that the arc length is part of; one full circle is 2π radians.

tangential acceleration
: the acceleration in a direction tangent to the circle at the point of interest in circular motion

Dynamics of Rotational Motion: Rotational Inertia

- Understand the relationship between torque, rotational inertia and angular acceleration.
- Study the turning effect of torque.
- Study the analogy between force and torque, mass and moment of inertia, and linear acceleration and angular acceleration.

If you have ever spun a bike wheel or pushed a merry-go-round, you know that force is needed to change angular velocity as seen in [link]. In fact, your intuition is reliable in predicting many of the factors that are involved. For example, we know that a door opens slowly if we push too close to its hinges. Furthermore, we know that the more massive the door, the more slowly it opens. The first example implies that the farther the force is applied from the pivot, the greater the angular acceleration; another implication is that angular acceleration is inversely proportional to mass. These relationships should seem very similar to the familiar relationships among force, mass, and acceleration embodied in Newton’s second law of motion. There are, in fact, precise rotational analogs to both force and mass.



Force is required to spin the bike wheel. The greater the force, the greater the angular acceleration produced. The more massive the wheel, the smaller the angular

acceleration. If you push on a spoke closer to the axle, the angular acceleration will be smaller.

To develop the precise relationship among force, mass, radius, and angular acceleration, consider what happens if we exert a force F on a point mass m that is at a distance r from a pivot point, as shown in [link]. Because the force is perpendicular to r, an acceleration $a = \frac{F}{m}$ is obtained in the direction of F. We can rearrange this equation such that $F = \text{ma}$ and then look for ways to relate this expression to expressions for rotational quantities. We note that $a = r\alpha$, and we substitute this expression into $F = \text{ma}$, yielding

Equation:

$$F = \text{mr}\alpha.$$

The turning effectiveness of force is called **torque**. Intuitively, torque increases with a larger **lever arm**, or the perpendicular distance between the center of rotation and the point at which force is being applied (think of a see-saw; to balance the weight of a heavier child on one side with a lighter child on the other side, the lighter child needs to sit farther from the pivot point). So, torque is given by $\tau = rF$. So, if we multiply both sides of the equation above by r, we get torque on the left-hand side. That is,

Equation:

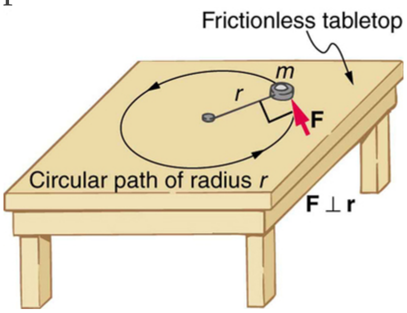
$$\text{rF} = \text{mr}^2\alpha$$

or

Equation:

$$\tau = \text{mr}^2\text{α}.$$

This last equation is the rotational analog of Newton's second law ($F = \mathrm{ma}$), where torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass (or inertia). The quantity mr^2 is called the **rotational inertia** or **moment of inertia** of a point mass m a distance r from the center of rotation.



An object is supported by a horizontal frictionless table and is attached to a pivot point by a cord that supplies centripetal force. A force F is applied to the object perpendicular to the radius r, causing it to accelerate about the pivot point. The force is kept perpendicular to r.

Note:
Making Connections: Rotational Motion Dynamics

Dynamics for rotational motion is completely analogous to linear or translational dynamics. Dynamics is concerned with force and mass and their effects on motion. For rotational motion, we will find direct analogs to force and mass that behave just as we would expect from our earlier experiences.

Rotational Inertia and Moment of Inertia

Before we can consider the rotation of anything other than a point mass like the one in [link], we must extend the idea of rotational inertia to all types of objects. To expand our concept of rotational inertia, we define the **moment of inertia** I of an object to be the sum of mr^2 for all the point masses of which it is composed. That is, $I = \sum \mathrm{mr}^2$. Here I is analogous to m in translational motion. Because of the distance r, the moment of inertia for any object depends on the chosen axis. Actually, calculating I is beyond the scope of this text except for one simple case—that of a hoop, which has all its mass at the same distance from its axis. A hoop's moment of inertia around its axis is therefore MR^2, where M is its total mass and R its radius. (We use M and R for an entire object to distinguish them from m and r for point masses.) [link] shows a variety of formulas of rotational inertia for many different shapes (deriving these formulas require calculus). For our purposes, it is good enough for you to see that rotational inertia is proportional to mass, and that it's proportional to size of object squared. Note that I has units of mass multiplied by distance squared ($\mathrm{kg} \cdot \mathrm{m}^2$), as we might expect from its definition.

The general relationship among torque, moment of inertia, and angular acceleration is
Equation:

$$\text{net } \tau = I\alpha$$

or
Equation:

$$\alpha = \frac{\text{net } \tau}{I},$$

where net τ is the total torque from all forces relative to a chosen axis. For simplicity, we will only consider torques exerted by forces in the plane of the rotation. Such torques are either positive or negative and add like ordinary numbers. The relationship in $\tau = I\alpha$, $\alpha = \frac{\text{net } \tau}{I}$ is the rotational analog to Newton's second law and is very generally applicable. This equation is actually valid for *any* torque, applied to *any* object, relative to *any* axis.

As we might expect, the larger the torque is, the larger the angular acceleration is. For example, the harder a child pushes on a merry-go-round, the faster it accelerates. Furthermore, the more massive a merry-go-round, the slower it accelerates for the same torque. The basic relationship between moment of inertia and angular acceleration is that the larger the moment of inertia, the smaller is the angular acceleration. But there is an additional twist. The moment of inertia depends not only on the mass of an object, but also on its *distribution* of mass relative to the axis around which it rotates. For example, it will be much easier to accelerate a merry-go-round full of children if they stand close to its axis than if they all stand at the outer edge. The mass is the same in both cases, but the moment of inertia is much larger when the children are at the edge.

Note:

Take-Home Experiment

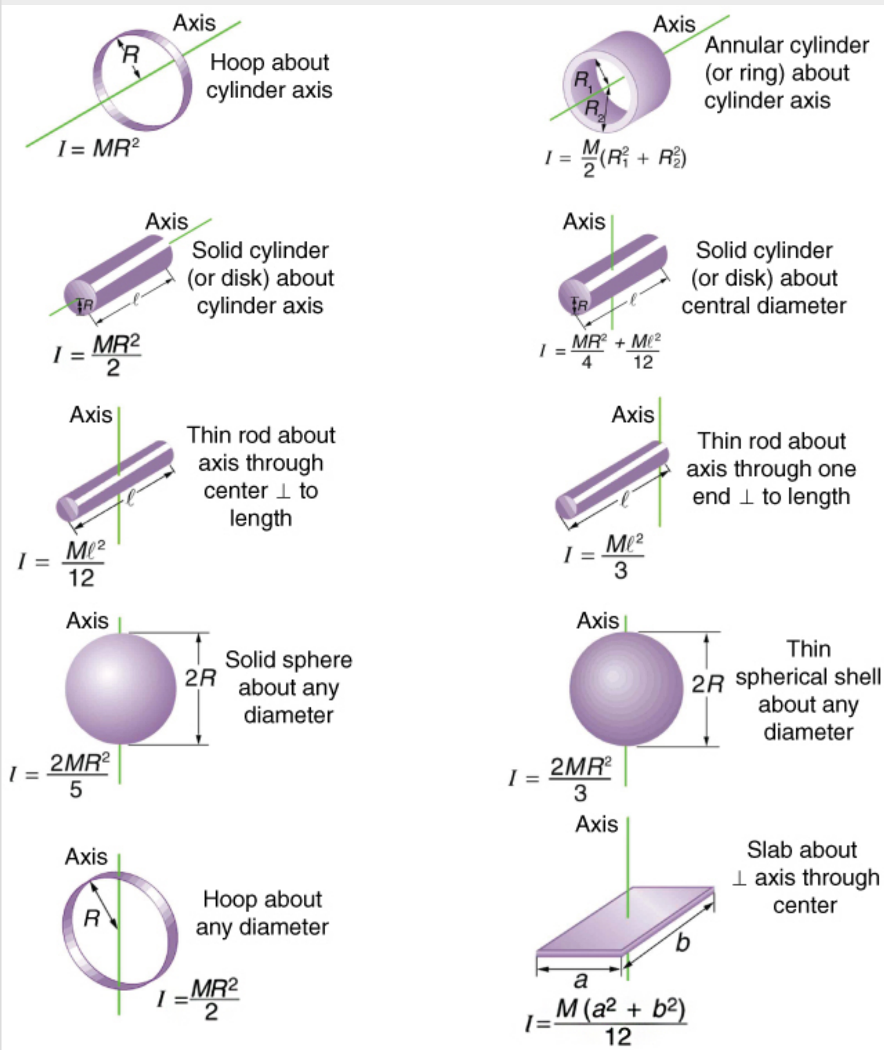
Cut out a circle that has about a 10 cm radius from stiff cardboard. Near the edge of the circle, write numbers 1 to 12 like hours on a clock face. Position the circle so that it can rotate freely about a horizontal axis through its center, like a wheel. (You could loosely nail the circle to a wall.) Hold the circle stationary and with the number 12 positioned at the top, attach a lump of blue putty (sticky material used for fixing posters to walls) at the number 3. How large does the lump need to be to just rotate the circle? Describe how you can change the moment of inertia of the

circle. How does this change affect the amount of blue putty needed at the number 3 to just rotate the circle? Change the circle’s moment of inertia and then try rotating the circle by using different amounts of blue putty. Repeat this process several times.

Note:

Making Connections

In statics, the net torque is zero, and there is no angular acceleration. In rotational motion, net torque is the cause of angular acceleration, exactly as in Newton’s second law of motion for rotation.



Some rotational inertias.

Exercise:
Check Your Understanding

Problem:

Torque is the analog of force and moment of inertia is the analog of mass. Force and mass are physical quantities that depend on only one factor. For example, mass is related solely to the numbers of atoms of various types in an object. Are torque and moment of inertia similarly simple?

Solution:

No. Torque depends on three factors: force magnitude, force direction, and point of application. Moment of inertia depends on both mass and its distribution relative to the axis of rotation. So, while the analogies are precise, these rotational quantities depend on more factors.

Section Summary

- The farther the force is applied from the pivot, the greater is the angular acceleration; angular acceleration is inversely proportional to mass.
- The torque applied on an object is given by lever arm times the applied force: $\tau = rF$.
- Using the torque, we can discover the rotational version of Newton's Second Law. It says $\tau = I\alpha$, where I is rotational inertia, defined for a point mass moving in a circle of radius r as $I = mr^2$.
- The formula for rotational inertia of an extended body can be complicated. But they are all proportional to the mass of the object and proportional to the square of the size of the object.

Conceptual Questions

Exercise:

Problem:

The moment of inertia of a long rod spun around an axis through one end perpendicular to its length is $ML^2/3$. Why is this moment of inertia greater than it would be if you spun a point mass M at the location of the center of mass of the rod (at $L/2$)? (That would be $ML^2/4$.)

Exercise:

Problem:

Why is the moment of inertia of a hoop that has a mass M and a radius R greater than the moment of inertia of a disk that has the same mass and radius? Why is the moment of inertia of a spherical shell that has a mass M and a radius R greater than that of a solid sphere that has the same mass and radius?

Exercise:

Problem:

Give an example in which a small force exerts a large torque. Give another example in which a large force exerts a small torque.

Exercise:

Problem:

While reducing the mass of a racing bike, the greatest benefit is realized from reducing the mass of the tires and wheel rims. Why does this allow a racer to achieve greater accelerations than would an identical reduction in the mass of the bicycle’s frame?

The image shows a side view of a racing bicycle. Can you see evidence in the design of the wheels on this racing bicycle that their moment of inertia has been purposely reduced? (credit: Jesús Rodriguez)

Exercise:

Problem:

A ball slides up a frictionless ramp. It is then rolled without slipping and with the same initial velocity up another frictionless ramp (with the same slope angle). In which case does it reach a greater height, and why?

Glossary

torque
: the turning effectiveness of a force

lever arm
: the perpendicular distance between the center of rotation and the point at which force is being applied

rotational inertia

resistance to change of angular velocity; also called **moment of inertia**; for a point mass, $I = \mathrm{mr}^2$

Rotational Kinetic Energy

- Guess the equation for rotational kinetic energy by analogy.
- Calculate rotational kinetic energy.

In this module, we will learn about work and energy associated with rotational motion. [link] shows a worker using an electric grindstone propelled by a motor. Sparks are flying, and noise and vibration are created as layers of steel are pared from the pole. The stone continues to turn even after the motor is turned off, but it is eventually brought to a stop by friction. Clearly, the motor had to work to get the stone spinning. This work went into heat, light, sound, vibration, and considerable **rotational kinetic energy**.

The motor works in spinning the grindstone, giving it rotational kinetic energy. That energy is then converted to heat, light, sound, and vibration. (credit: U.S. Navy photo by Mass Communication Specialist Seaman Zachary David Bell)

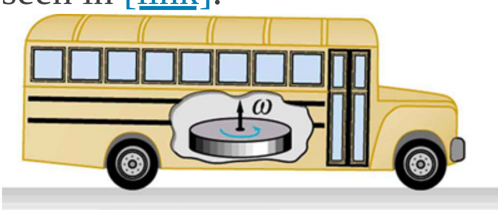
Instead of deriving the formula for rotational kinetic energy by calculating the work done by torque on a rotating object, we can simply guess the

correct formula by analogy. We know the kinetic energy in linear, or translational motion, $\mathrm{KE} = \frac{1}{2}mv^2$. We can find the rotational version of kinetic energy by replacing mass m with rotational version of mass, rotational inertia I, and by replacing speed v with rotational speed ω. Rotational speed is measured in units of radians per second. With these replacements, we get the formula for rotational kinetic energy,

Equation:

$$\mathrm{KE_{rot}} = \frac{1}{2}I\omega^2.$$

Rotational kinetic energy has important effects. Flywheels, for example, can be used to store large amounts of rotational kinetic energy in a vehicle, as seen in [link].



Experimental vehicles, such as this bus, have been constructed in which rotational kinetic energy is stored in a large flywheel. When the bus goes down a hill, its transmission converts its gravitational potential energy into $\mathrm{KE_{rot}}$. It can also convert translational kinetic energy, when the bus stops, into $\mathrm{KE_{rot}}$. The flywheel's energy can then be used to accelerate, to go up another hill, or to keep

the bus from going
against friction.

Helicopter pilots are quite familiar with rotational kinetic energy. They know, for example, that a point of no return will be reached if they allow their blades to slow below a critical angular velocity during flight. The blades lose lift, and it is impossible to immediately get the blades spinning fast enough to regain it. Rotational kinetic energy must be supplied to the blades to get them to rotate faster, and enough energy cannot be supplied in time to avoid a crash. Because of weight limitations, helicopter engines are too small to supply both the energy needed for lift and to replenish the rotational kinetic energy of the blades once they have slowed down. The rotational kinetic energy is put into them before takeoff and must not be allowed to drop below this crucial level. One possible way to avoid a crash is to use the gravitational potential energy of the helicopter to replenish the rotational kinetic energy of the blades by losing altitude and aligning the blades so that the helicopter is spun up in the descent. Of course, if the helicopter's altitude is too low, then there is insufficient time for the blade to regain lift before reaching the ground.

Example:
Calculating Helicopter Energies
A typical small rescue helicopter, similar to the one in [link], has four blades, each is 4.00 m long and has a mass of 50.0 kg. The blades can be approximated as thin rods that rotate about one end of an axis perpendicular to their length. The helicopter has a total loaded mass of 1000 kg. (a) Calculate the rotational kinetic energy in the blades when they rotate at 300 rpm. (b) Calculate the translational kinetic energy of the helicopter when it flies at 20.0 m/s, and compare it with the rotational energy in the blades. (c) To what height could the helicopter be raised if all of the rotational kinetic energy could be used to lift it?
Strategy

Rotational and translational kinetic energies can be calculated from their definitions. The last part of the problem relates to the idea that energy can change form, in this case from rotational kinetic energy to gravitational potential energy.

Solution for (a)

The rotational kinetic energy is

Equation:

$$\text{KE}_{\text{rot}} = \frac{1}{2} I\omega^2.$$

We must convert the angular velocity to radians per second and calculate the moment of inertia before we can find KE_{rot}. The angular velocity ω is

Equation:

$$\omega = \frac{300 \text{ rev}}{1.00 \text{ min}} \cdot \frac{2\pi \text{ rad}}{1 \text{ rev}} \cdot \frac{1.00 \text{ min}}{60.0 \text{ s}} = 31.4 \frac{\text{rad}}{\text{s}}.$$

The moment of inertia of one blade will be that of a thin rod rotated about its end, $I_{\text{rod}} = \frac{1}{3}M\ell^2$. The total I is four times this moment of inertia, because there are four blades. Thus,

Equation:

$$I = 4\frac{M\ell^2}{3} = 4 \times \frac{(50.0 \text{ kg})(4.00 \text{ m})^2}{3} = 1067 \text{ kg} \cdot \text{m}^2.$$

Entering ω and I into the expression for rotational kinetic energy gives

Equation:

$$\begin{aligned} \text{KE}_{\text{rot}} &= 0.5(1067 \text{ kg} \cdot \text{m}^2)(31.4 \text{ rad/s})^2 \\ &= 5.26 \times 10^5 \text{ J} \end{aligned}$$

Solution for (b)

Entering the given values of mass and velocity into formula for translational kinetic energy, we obtain

Equation:

$$\text{KE}_{\text{trans}} = \frac{1}{2}mv^2 = (0.5)(1000\ \text{kg})(20.0\ \text{m/s})^2 = 2.00 \times 10^5\ \text{J}.$$

To compare kinetic energies, we take the ratio of translational kinetic energy to rotational kinetic energy. This ratio is

Equation:

$$\frac{2.00 \times 10^5\ \text{J}}{5.26 \times 10^5\ \text{J}} = 0.380.$$

Solution for (c)

At the maximum height, all rotational kinetic energy will have been converted to gravitational energy. To find this height, we equate those two energies:

Equation:

$$\text{KE}_{\text{rot}} = \text{PE}_{\text{grav}}$$

or

Equation:

$$\frac{1}{2}I\omega^2 = \text{mgh}.$$

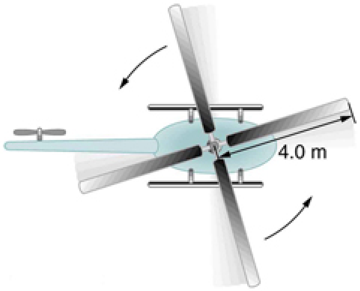
We now solve for h and substitute known values into the resulting equation

Equation:

$$h = \frac{\frac{1}{2}I\omega^2}{\text{mg}} = \frac{5.26 \times 10^5\ \text{J}}{(1000\ \text{kg})\left(9.80\ \text{m/s}^2\right)} = 53.7\ \text{m}.$$

Discussion

The ratio of translational energy to rotational kinetic energy is only 0.380. This ratio tells us that most of the kinetic energy of the helicopter is in its spinning blades—something you probably would not suspect. The 53.7 m height to which the helicopter could be raised with the rotational kinetic energy is also impressive, again emphasizing the amount of rotational kinetic energy in the blades.



(a)

(b)

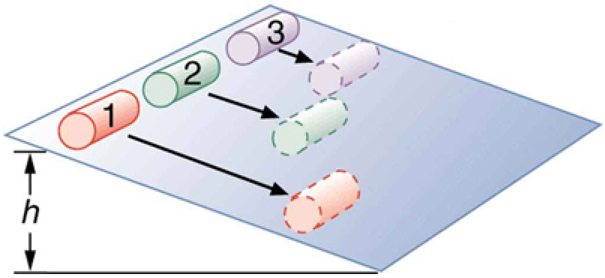
The first image shows how helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades. The second image shows a helicopter from the Auckland

Westpac Rescue Helicopter Service. Over 50,000 lives have been saved since its operations beginning in 1973. Here, a water rescue operation is shown. (credit: 111 Emergency, Flickr)

How Thick Is the Soup? Or Why Don't All Objects Roll Downhill at the Same Rate?

One of the quality controls in a tomato soup factory consists of rolling filled cans down a ramp. If they roll too fast, the soup is too thin. Why should cans of identical size and mass roll down an incline at different rates? And why should the thickest soup roll the slowest?

The easiest way to answer these questions is to consider energy. Suppose each can starts down the ramp from rest. Each can starting from rest means each starts with the same gravitational potential energy $\mathrm{PE_{grav}}$, which is converted entirely to KE, provided each rolls without slipping. KE, however, can take the form of $\mathrm{KE_{trans}}$ or $\mathrm{KE_{rot}}$, and total KE is the sum of the two. If a can rolls down a ramp, it puts part of its energy into rotation, leaving less for translation. Thus, the can goes slower than it would if it slid down. Furthermore, the thin soup does not rotate, whereas the thick soup does, because it sticks to the can. The thick soup thus puts more of the can's original gravitational potential energy into rotation than the thin soup, and the can rolls more slowly, as seen in [link].



Three cans of soup with identical masses race down an incline. The first can has a low friction coating and does not roll but just slides down the incline. It wins because it converts its entire PE into translational KE. The second and third cans both roll down the incline without slipping. The second can contains thin soup and comes in second because part of its initial PE goes into rotating the can (but not the thin soup). The third can contains thick soup. It comes in third because the soup rotates along with the can, taking even more of the initial PE for rotational KE, leaving less for translational KE.

Assuming no losses due to friction, there is only one force doing work—gravity. Therefore the total work done is the change in kinetic energy. As the cans start moving, the potential energy is changing into kinetic energy. Conservation of energy gives

Equation:

$$\mathrm{PE_i} = \mathrm{KE_f}.$$

More specifically,
Equation:

$$\mathrm{PE_{grav}} = \mathrm{KE_{trans}} + \mathrm{KE_{rot}}$$

or
Equation:

$$\mathrm{mgh} = \frac{1}{2}\mathrm{mv}^2 + \frac{1}{2}I\omega^2.$$

So, the initial mgh is divided between translational kinetic energy and rotational kinetic energy; and the greater I is, the less energy goes into translation. If the can slides down without friction, then $\omega = 0$ and all the energy goes into translation; thus, the can goes faster.

Note:
Take-Home Experiment
Locate several cans each containing a different type of food (for example, canned soup with different kinds of soup). First, predict which can will win the race down an inclined plane and explain why. See if your prediction is correct. A good contrast can be obtained between a thick soup concentrate (for example, condensed cream of mushroom) and a thin soup (for example, chicken noodle soup). If you feel adventurous, try other foods, such as canned corn or peas; try predicting how these would compare to, for example, condensed cream of mushroom can in the race down an inclined plane.

Example:
Calculating the Speed of a Cylinder Rolling Down an Incline

Calculate the final speed of a solid cylinder that rolls down a 2.00-m-high incline. The cylinder starts from rest, has a mass of 0.750 kg, and has a radius of 4.00 cm.

Strategy

We can solve for the final velocity using conservation of energy, but we must first express rotational quantities in terms of translational quantities to end up with v as the only unknown.

Solution

Conservation of energy for this situation is written as described above:

Equation:

$$\mathrm{mgh} = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2.$$

Before we can solve for v , we must look up an expression for I for a solid disk: $I_{\text{disk}} = \frac{1}{2}M\ell^2$. Because v and ω are related (note here that the cylinder is rolling without slipping), we must also substitute the relationship $\omega = v/R$ into the expression. These substitutions yield

Equation:

$$\mathrm{mgh} = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mR^2\right)\left(\frac{v^2}{R^2}\right).$$

Interestingly, the cylinder's radius R and mass m cancel, yielding

Equation:

$$\mathrm{gh} = \frac{1}{2}v^2 + \frac{1}{4}v^2 = \frac{3}{4}v^2.$$

Solving algebraically, the equation for the final velocity v gives

Equation:

$$v = \left(\frac{4\mathrm{gh}}{3}\right)^{1/2}.$$

Substituting known values into the resulting expression yields

Equation:

$$v = \left(\frac{4\left(9.80\ \text{m/s}^2\right)(2.00\ \text{m})}{3}\right)^{1/2} = 5.11\ \text{m/s}.$$

Discussion

Because m and R cancel, the result $v = \left(\frac{4}{3}\text{gh}\right)^{1/2}$ is valid for any solid cylinder, implying that all solid cylinders will roll down an incline at the same rate independent of their masses and sizes. (Rolling cylinders down inclines is what Galileo actually did to show that objects fall at the same rate independent of mass.) Note that if the cylinder slid without friction down the incline without rolling, then the entire gravitational potential energy would go into translational kinetic energy. Thus, $\frac{1}{2}\text{mv}^2 = \text{mgh}$ and $v = (2\text{gh})^{1/2}$, which is 22% greater than $(4\text{gh}/3)^{1/2}$. That is, the cylinder would go faster at the bottom.

Exercise:
Check Your Understanding

Problem:

Analogy of Rotational and Translational Kinetic Energy
Is rotational kinetic energy completely analogous to translational kinetic energy? What, if any, are their differences? Give an example of each type of kinetic energy.

Solution:

Yes, rotational and translational kinetic energy are exact analogs. They both are the energy of motion involved with the coordinated (non-random) movement of mass relative to some reference frame. The only difference between rotational and translational kinetic energy is that translational is straight line motion while rotational is not. An example of both kinetic and translational kinetic energy is found in a bike tire while being ridden down a bike path. The rotational motion of the tire means it has rotational kinetic energy while the movement of the bike

along the path means the tire also has translational kinetic energy. If you were to lift the front wheel of the bike and spin it while the bike is stationary, then the wheel would have only rotational kinetic energy relative to the Earth.

Section Summary

- The rotational kinetic energy $\mathrm{KE}_{\mathrm{rot}}$ for an object with a moment of inertia I and an angular velocity ω is given by
  Equation:

$$\mathrm{KE}_{\mathrm{rot}} = \frac{1}{2}I\omega^2.$$

- Helicopters store large amounts of rotational kinetic energy in their blades. This energy must be put into the blades before takeoff and maintained until the end of the flight. The engines do not have enough power to simultaneously provide lift and put significant rotational energy into the blades.
- Work and energy in rotational motion are completely analogous to work and energy in translational motion.

Conceptual Questions

Exercise:

Problem:

Describe the energy transformations involved when a yo-yo is thrown downward and then climbs back up its string to be caught in the user’s hand.

Exercise:

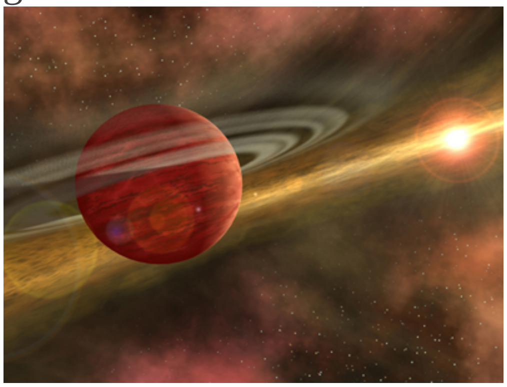
Problem:

What energy transformations are involved when a dragster engine is revved, its clutch let out rapidly, its tires spun, and it starts to accelerate forward? Describe the source and transformation of energy at each step.

Exercise:

Problem:

The Earth has more rotational kinetic energy now than did the cloud of gas and dust from which it formed. Where did this energy come from?



An immense cloud of rotating gas and dust contracted under the influence of gravity to form the Earth and in the process rotational kinetic energy increased. (credit: NASA)

Glossary

rotational kinetic energy

the kinetic energy due to the rotation of an object. This is part of its total kinetic energy

Angular Momentum and Its Conservation

- Understand the analogy between angular momentum and linear momentum.
- Observe the relationship between torque and angular momentum.
- Apply the law of conservation of angular momentum.

Why does Earth keep on spinning? What started it spinning to begin with? And how does an ice skater manage to spin faster and faster simply by pulling her arms in? Why does she not have to exert a torque to spin faster? Questions like these have answers based in angular momentum, the rotational analog to linear momentum.

By now the pattern is clear—every rotational phenomenon has a direct translational analog. It seems quite reasonable, then, to define **angular momentum** L as

Equation:

$$L = I\omega.$$

This equation is an analog to the definition of linear momentum as $p = \mathrm{mv}$. Units for linear momentum are $\mathrm{kg} \cdot \mathrm{m/s}$ while units for angular momentum are $\mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}$. As we would expect, an object that has a large moment of inertia I, such as Earth, has a very large angular momentum. An object that has a large angular velocity ω, such as a centrifuge, also has a rather large angular momentum.

Example:

Calculating Angular Momentum of the Earth

Strategy

No information is given in the statement of the problem; so we must look up pertinent data before we can calculate $L = I\omega$. First, according to [link], the formula for the moment of inertia of a sphere is

Equation:

$$I = \frac{2MR^2}{5}$$

so that

Equation:

$$L = I\omega = \frac{2MR^2\omega}{5}.$$

Earth's mass M is 5.979×10^{24} kg and its radius R is 6.376×10^6 m. The Earth's angular velocity ω is, of course, exactly one revolution per day, but we must covert ω to radians per second to do the calculation in SI units.

Solution

Substituting known information into the expression for L and converting ω to radians per second gives

Equation:

$$\begin{aligned} L &= 0.4\left(5.979 \times 10^{24}\ \text{kg}\right)\left(6.376 \times 10^6\ \text{m}\right)^2\left(\tfrac{1\ \text{rev}}{\text{d}}\right) \\ &= 9.72 \times 10^{37}\ \text{kg} \cdot \text{m}^2 \cdot \text{rev/d}. \end{aligned}$$

Substituting 2π rad for 1 rev and 8.64×10^4 s for 1 day gives

Equation:

$$\begin{aligned} L &= \left(9.72 \times 10^{37}\ \text{kg} \cdot \text{m}^2\right)\left(\frac{2\pi\ \text{rad/rev}}{8.64 \times 10^4\ \text{s/d}}\right)(1\ \text{rev/d}) \\ &= 7.07 \times 10^{33}\ \text{kg} \cdot \text{m}^2\text{/s}. \end{aligned}$$

Discussion

This number is large, demonstrating that Earth, as expected, has a tremendous angular momentum. The answer is approximate, because we have assumed a constant density for Earth in order to estimate its moment of inertia.

When you push a merry-go-round, spin a bike wheel, or open a door, you exert a torque. If the torque you exert is greater than opposing torques, then the rotation accelerates, and angular momentum increases. The greater the net torque, the more rapid the increase in L. The relationship between torque and angular momentum is

Equation:

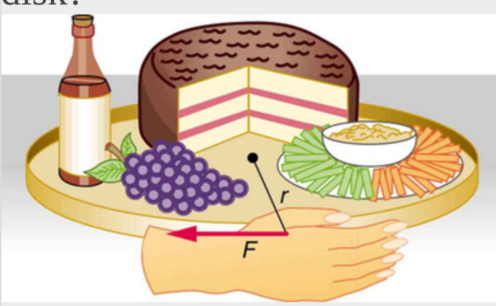
$$\text{net } \tau = \frac{\Delta L}{\Delta t}.$$

This expression is exactly analogous to the relationship between force and linear momentum, $F = \Delta p / \Delta t$. The equation $\text{net } \tau = \frac{\Delta L}{\Delta t}$ is very fundamental and broadly applicable. It is, in fact, the rotational form of Newton's second law.

Example:

Calculating the Torque Putting Angular Momentum Into a Lazy Susan

[link] shows a Lazy Susan food tray being rotated by a person in quest of sustenance. Suppose the person exerts a 2.50 N force perpendicular to the lazy Susan's 0.260-m radius for 0.150 s. (a) What is the final angular momentum of the lazy Susan if it starts from rest, assuming friction is negligible? (b) What is the final angular velocity of the lazy Susan, given that its mass is 4.00 kg and assuming its moment of inertia is that of a disk?



A partygoer exerts a torque on a lazy Susan to

make it rotate. The equation net $\tau = \frac{\Delta L}{\Delta t}$ gives the relationship between torque and the angular momentum produced.

Strategy

We can find the angular momentum by solving net $\tau = \frac{\Delta L}{\Delta t}$ for ΔL, and using the given information to calculate the torque. The final angular momentum equals the change in angular momentum, because the lazy Susan starts from rest. That is, $\Delta L = L$. To find the final velocity, we must calculate ω from the definition of L in $L = I\omega$.

Solution for (a)

Solving net $\tau = \frac{\Delta L}{\Delta t}$ for ΔL gives

Equation:

$$\Delta L = (\text{net } \tau)\Delta \text{t}.$$

Because the force is perpendicular to r, we see that net $\tau = \text{rF}$, so that

Equation:

$$\begin{aligned} L &= \text{rF}\Delta t = (0.260 \text{ m})(2.50 \text{ N})(0.150 \text{ s}) \\ &= 9.75 \times 10^{-2} \text{ kg} \cdot \text{m}^2/\text{s}. \end{aligned}$$

Solution for (b)

The final angular velocity can be calculated from the definition of angular momentum,

Equation:

$$L = I\omega.$$

Solving for ω and substituting the formula for the moment of inertia of a disk into the resulting equation gives

Equation:

$$\omega = \frac{L}{I} = \frac{L}{\frac{1}{2}MR^2}.$$

And substituting known values into the preceding equation yields

Equation:

$$\omega = \frac{9.75 \times 10^{-2}\ \text{kg} \cdot \text{m}^2/\text{s}}{(0.500)(4.00\ \text{kg})(0.260\ \text{m})} = 0.721\ \text{rad/s}.$$

Discussion

Note that the imparted angular momentum does not depend on any property of the object but only on torque and time. The final angular velocity is equivalent to one revolution in 8.71 s (determination of the time period is left as an exercise for the reader), which is about right for a lazy Susan.

Note:

Making Connections: Conservation Laws

Angular momentum, like energy and linear momentum, is conserved. This universally applicable law is another sign of underlying unity in physical laws. Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero.

Conservation of Angular Momentum

We can now understand why Earth keeps on spinning. As we saw in the previous example, $\Delta L = (\text{net } \tau)\Delta t$. This equation means that, to change angular momentum, a torque must act over some period of time. Because Earth has a large angular momentum, a large torque acting over a long time is needed to change its rate of spin. So what external torques are there? Tidal friction exerts torque that is slowing Earth's rotation, but tens of millions of years must pass before the change is very significant. Recent research indicates the length of the day was 18 h some 900 million years

ago. Only the tides exert significant retarding torques on Earth, and so it will continue to spin, although ever more slowly, for many billions of years.

What we have here is, in fact, another conservation law. If the net torque is *zero*, then angular momentum is constant or *conserved*. We can see this rigorously by considering $\text{net } \tau = \frac{\Delta L}{\Delta t}$ for the situation in which the net torque is zero. In that case,

Equation:

$$\text{net}\tau = 0$$

implying that

Equation:

$$\frac{\Delta L}{\Delta t} = 0.$$

If the change in angular momentum ΔL is zero, then the angular momentum is constant; thus,

Equation:

$$L = \text{constant} \ (\text{net } \tau = 0)$$

or

Equation:

$$L = L\prime(\text{net}\tau = 0).$$

These expressions are the **law of conservation of angular momentum**. Conservation laws are as scarce as they are important.

An example of conservation of angular momentum is seen in [link], in which an ice skater is executing a spin. The net torque on her is very close to zero, because there is relatively little friction between her skates and the ice and because the friction is exerted very close to the pivot point. (Both F

and r are small, and so τ is negligibly small.) Consequently, she can spin for quite some time. She can do something else, too. She can increase her rate of spin by pulling her arms and legs in. Why does pulling her arms and legs in increase her rate of spin? The answer is that her angular momentum is constant, so that

Equation:

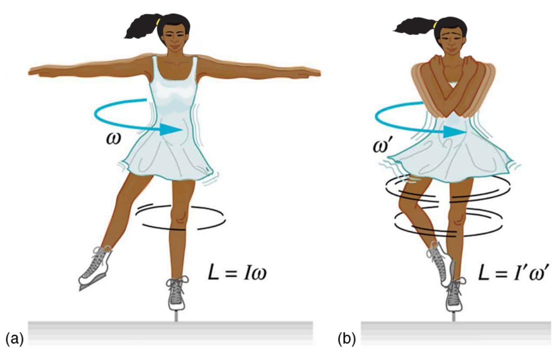
$$L = L\prime.$$

Expressing this equation in terms of the moment of inertia,

Equation:

$$I\omega = I\prime\omega\prime,$$

where the primed quantities refer to conditions after she has pulled in her arms and reduced her moment of inertia. Because $I\prime$ is smaller, the angular velocity $\omega\prime$ must increase to keep the angular momentum constant. The change can be dramatic, as the following example shows.



(a) An ice skater is spinning on the tip of her skate with her arms extended. Her angular momentum is conserved because the net torque on her is negligibly small. In the next image, her

rate of spin increases greatly when she pulls in her arms, decreasing her moment of inertia. The work she does to pull in her arms results in an increase in rotational kinetic energy.

Example:
Calculating the Angular Momentum of a Spinning Skater
Suppose an ice skater, such as the one in [link], is spinning at 0.800 rev/ s with her arms extended. She has a moment of inertia of $2.34\ \text{kg}\cdot\text{m}^2$ with her arms extended and of $0.363\ \text{kg}\cdot\text{m}^2$with her arms close to her body. (These moments of inertia are based on reasonable assumptions about a 60.0-kg skater.) (a) What is her angular velocity in revolutions per second after she pulls in her arms? (b) What is her rotational kinetic energy before and after she does this?

Strategy
In the first part of the problem, we are looking for the skater's angular velocity $\omega\prime$ after she has pulled in her arms. To find this quantity, we use the conservation of angular momentum and note that the moments of inertia and initial angular velocity are given. To find the initial and final kinetic energies, we use the definition of rotational kinetic energy given by

Equation:

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2.$$

Solution for (a)
Because torque is negligible (as discussed above), the conservation of angular momentum given in $I\omega = I\prime\omega\prime$ is applicable. Thus,

Equation:

$$L = L\prime$$

or

Equation:

$$I\omega = I\prime\omega\prime$$

Solving for $\omega\prime$ and substituting known values into the resulting equation gives

Equation:

$$\begin{aligned} \omega\prime &= \frac{I}{I\prime}\omega = \left(\frac{2.34\ \text{kg}\cdot\text{m}^2}{0.363\ \text{kg}\cdot\text{m}^2}\right)(0.800\ \text{rev/s}) \\ &= 5.16\ \text{rev/s}. \end{aligned}$$

Solution for (b)

Rotational kinetic energy is given by

Equation:

$$\text{KE}_{\text{rot}} = \frac{1}{2}I\omega^2.$$

The initial value is found by substituting known values into the equation and converting the angular velocity to rad/s:

Equation:

$$\begin{aligned} \text{KE}_{\text{rot}} &= (0.5)\left(2.34\ \text{kg}\cdot\text{m}^2\right)\left((0.800\ \text{rev/s})(2\pi\ \text{rad/rev})\right)^2 \\ &= 29.6\ \text{J}. \end{aligned}$$

The final rotational kinetic energy is

Equation:

$$\text{KE}_{\text{rot}}\prime = \frac{1}{2}I\prime\omega\prime^2.$$

Substituting known values into this equation gives

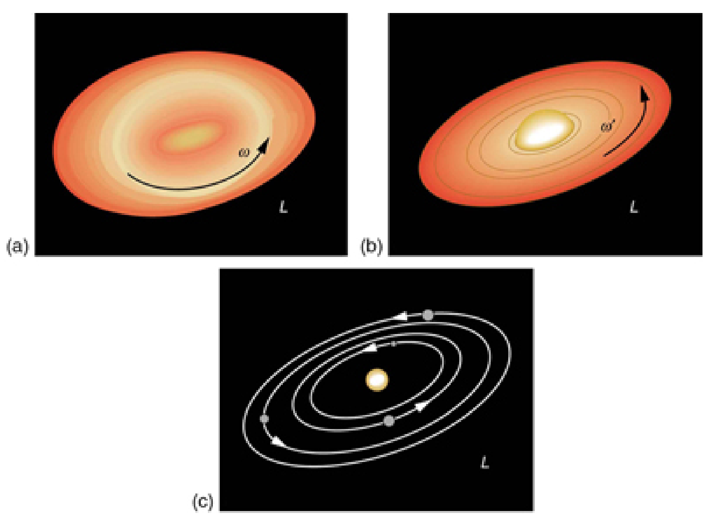
Equation:

$$KE_{\text{rot}}' = (0.5)\left(0.363\ \text{kg}\cdot\text{m}^2\right)\left[(5.16\ \text{rev/s})(2\pi\ \text{rad/rev})\right]^2$$
$$= 191\ \text{J}.$$

Discussion

In both parts, there is an impressive increase. First, the final angular velocity is large, although most world-class skaters can achieve spin rates about this great. Second, the final kinetic energy is much greater than the initial kinetic energy. The increase in rotational kinetic energy comes from work done by the skater in pulling in her arms. This work is internal work that depletes some of the skater's food energy.

There are several other examples of objects that increase their rate of spin because something reduced their moment of inertia. Tornadoes are one example. Storm systems that create tornadoes are slowly rotating. When the radius of rotation narrows, even in a local region, angular velocity increases, sometimes to the furious level of a tornado. Earth is another example. Our planet was born from a huge cloud of gas and dust, the rotation of which came from turbulence in an even larger cloud. Gravitational forces caused the cloud to contract, and the rotation rate increased as a result. (See [link].)



The Solar System coalesced from a cloud of gas and dust that was originally rotating. The orbital motions and spins of the planets are in the same direction as the original spin and conserve the angular momentum of the parent cloud.

In case of human motion, one would not expect angular momentum to be conserved when a body interacts with the environment as its foot pushes off the ground. Astronauts floating in space aboard the International Space Station have no angular momentum relative to the inside of the ship if they are motionless. Their bodies will continue to have this zero value no matter how they twist about as long as they do not give themselves a push off the side of the vessel.

Exercise:

Check Your Undestanding

Problem:

Is angular momentum completely analogous to linear momentum? What, if any, are their differences?

Solution:

Yes, angular and linear momentums are completely analogous. While they are exact analogs they have different units and are not directly inter-convertible like forms of energy are.

Section Summary

- Every rotational phenomenon has a direct translational analog , likewise angular momentum L can be defined as $L = I\omega$.
- This equation is an analog to the definition of linear momentum as $p = \mathrm{mv}$. The relationship between torque and angular momentum is

net $\tau = \frac{\Delta L}{\Delta t}$.

- Angular momentum, like energy and linear momentum, is conserved. This universally applicable law is another sign of underlying unity in physical laws. Angular momentum is conserved when net external torque is zero, just as linear momentum is conserved when the net external force is zero.

Conceptual Questions

Exercise:

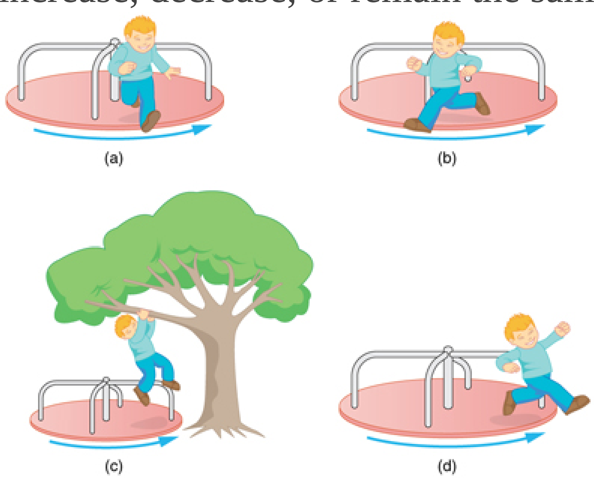
Problem:

When you start the engine of your car with the transmission in neutral, you notice that the car rocks in the opposite sense of the engine’s rotation. Explain in terms of conservation of angular momentum. Is the angular momentum of the car conserved for long (for more than a few seconds)?

Exercise:

Problem:

Suppose a child walks from the outer edge of a rotating merry-go round to the inside. Does the angular velocity of the merry-go-round increase, decrease, or remain the same? Explain your answer.



A child may jump off a merry-

go-round in a variety of directions.

Exercise:

Problem:

Suppose a child gets off a rotating merry-go-round. Does the angular velocity of the merry-go-round increase, decrease, or remain the same if: (a) He jumps off radially? (b) He jumps backward to land motionless? (c) He jumps straight up and hangs onto an overhead tree branch? (d) He jumps off forward, tangential to the edge? Explain your answers. (Refer to [link]).

Exercise:

Problem:

Helicopters have a small propeller on their tail to keep them from rotating in the opposite direction of their main lifting blades. Explain in terms of Newton’s third law why the helicopter body rotates in the opposite direction to the blades.

Exercise:

Problem:

Whenever a helicopter has two sets of lifting blades, they rotate in opposite directions (and there will be no tail propeller). Explain why it is best to have the blades rotate in opposite directions.

Exercise:

Problem:

Describe how work is done by a skater pulling in her arms during a spin. In particular, identify the force she exerts on each arm to pull it in and the distance each moves, noting that a component of the force is in the direction moved. Why is angular momentum not increased by this action?

Exercise:

Problem:

When there is a global heating trend on Earth, the atmosphere expands and the length of the day increases very slightly. Explain why the length of a day increases.

Exercise:

Problem:

Nearly all conventional piston engines have flywheels on them to smooth out engine vibrations caused by the thrust of individual piston firings. Why does the flywheel have this effect?

Exercise:

Problem:

Jet turbines spin rapidly. They are designed to fly apart if something makes them seize suddenly, rather than transfer angular momentum to the plane's wing, possibly tearing it off. Explain how flying apart conserves angular momentum without transferring it to the wing.

Exercise:

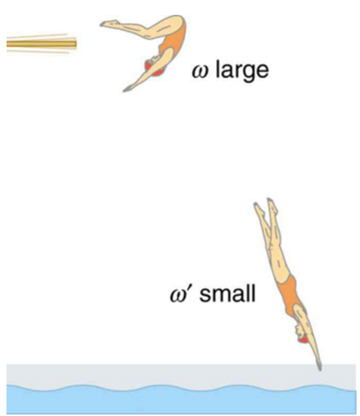
Problem:

An astronaut tightens a bolt on a satellite in orbit. He rotates in a direction opposite to that of the bolt, and the satellite rotates in the same direction as the bolt. Explain why. If a handhold is available on the satellite, can this counter-rotation be prevented? Explain your answer.

Exercise:

Problem:

Competitive divers pull their limbs in and curl up their bodies when they do flips. Just before entering the water, they fully extend their limbs to enter straight down. Explain the effect of both actions on their angular velocities. Also explain the effect on their angular momenta.



The diver spins rapidly when curled up and slows when she extends her limbs before entering the water.

Exercise:

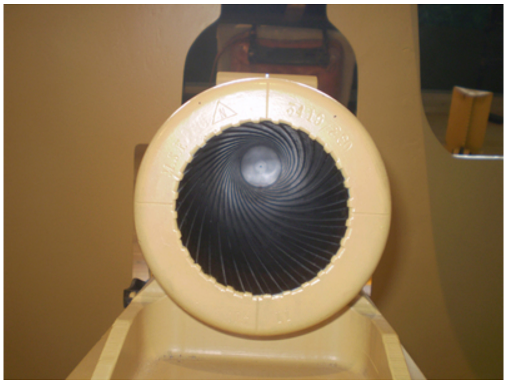
Problem:

Draw a free body diagram to show how a diver gains angular momentum when leaving the diving board.

Exercise:

Problem:

In terms of angular momentum, what is the advantage of giving a football or a rifle bullet a spin when throwing or releasing it?



The image shows a view down the barrel of a cannon, emphasizing its rifling. Rifling in the barrel of a canon causes the projectile to spin just as is the case for rifles (hence the name for the grooves in the barrel). (credit: Elsie esq., Flickr)

Glossary

angular momentum
: the product of moment of inertia and angular velocity

law of conservation of angular momentum
: angular momentum is conserved, i.e., the initial angular momentum is equal to the final angular momentum when no external torque is applied to the system

Gyroscopic Effects: Vector Aspects of Angular Momentum

- Describe the right-hand rule to find the direction of angular velocity, momentum, and torque.
- Explain the gyroscopic effect.
- Study how Earth acts like a gigantic gyroscope.

Angular momentum is a vector and, therefore, *has direction as well as magnitude*. Torque affects both the direction and the magnitude of angular momentum. What is the direction of the angular momentum of a rotating object like the disk in [link]? The figure shows the **right-hand rule** used to find the direction of both angular momentum and angular velocity. Both $\mathbf{L}$ and $\boldsymbol{\omega}$ are vectors—each has direction and magnitude. Both can be represented by arrows. The right-hand rule defines both to be perpendicular to the plane of rotation in the direction shown. Because angular momentum is related to angular velocity by $\mathbf{L} = I\boldsymbol{\omega}$, the direction of $\mathbf{L}$ is the same as the direction of $\boldsymbol{\omega}$. Notice in the figure that both point along the axis of rotation.

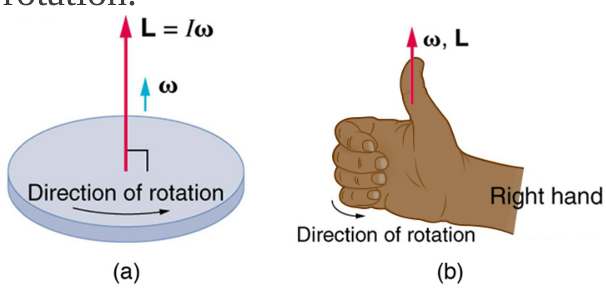


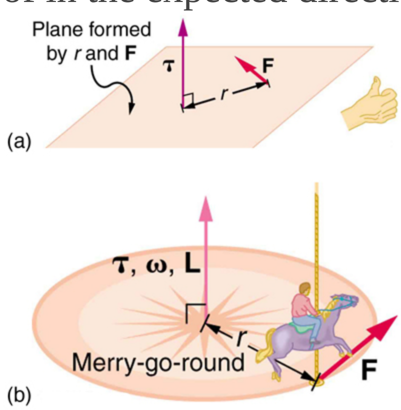
Figure (a) shows a disk is rotating counterclockwise when viewed from above. Figure (b) shows the right-hand rule. The direction of angular velocity $\boldsymbol{\omega}$ size and angular momentum $\mathbf{L}$ are defined to be the direction in which the thumb of your right hand points when you curl your fingers in the direction of the disk’s rotation as shown.

Now, recall that torque changes angular momentum as expressed by
Equation:

$$\text{net } \boldsymbol{\tau} = \frac{\Delta \mathbf{L}}{\Delta t}.$$

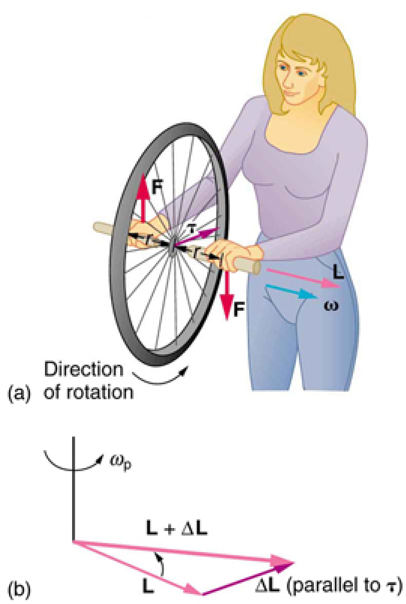
This equation means that the direction of $\Delta \mathbf{L}$ is the same as the direction of the torque $\boldsymbol{\tau}$ that creates it. This result is illustrated in [link], which shows the direction of torque and the angular momentum it creates.

Let us now consider a bicycle wheel with a couple of handles attached to it, as shown in [link]. (This device is popular in demonstrations among physicists, because it does unexpected things.) With the wheel rotating as shown, its angular momentum is to the woman's left. Suppose the person holding the wheel tries to rotate it as in the figure. Her natural expectation is that the wheel will rotate in the direction she pushes it—but what happens is quite different. The forces exerted create a torque that is horizontal toward the person, as shown in [link](a). This torque creates a change in angular momentum $\mathbf{L}$ in the same direction, perpendicular to the original angular momentum $\mathbf{L}$, thus changing the direction of $\mathbf{L}$ but not the magnitude of $\mathbf{L}$. [link] shows how $\Delta \mathbf{L}$ and $\mathbf{L}$ add, giving a new angular momentum with direction that is inclined more toward the person than before. The axis of the wheel has thus moved *perpendicular to the forces exerted on it*, instead of in the expected direction.



In figure (a), the torque is

perpendicular to the plane formed by r and $\mathbf{F}$ and is the direction your right thumb would point to if you curled your fingers in the direction of $\mathbf{F}$. Figure (b) shows that the direction of the torque is the same as that of the angular momentum it produces.

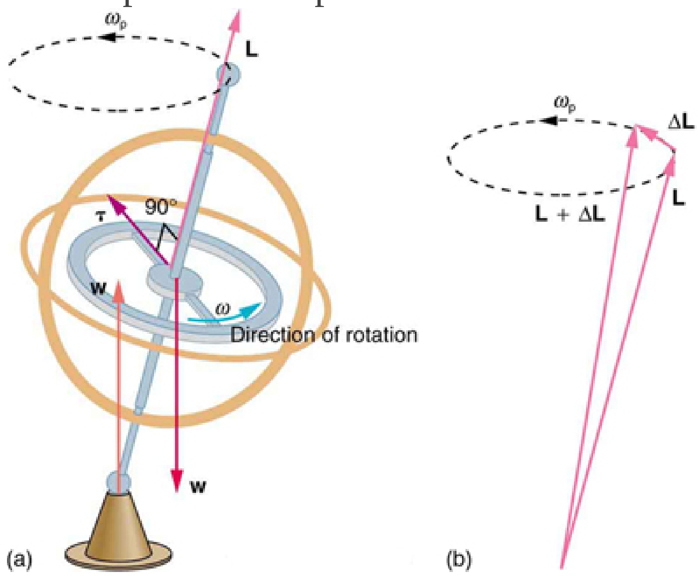


In figure (a), a person holding the spinning bike wheel lifts it with her right hand and pushes down with her left hand in an attempt

to rotate the wheel. This action creates a torque directly toward her. This torque causes a change in angular momentum $\Delta\mathbf{L}$ in exactly the same direction. Figure (b) shows a vector diagram depicting how $\Delta\mathbf{L}$ and $\mathbf{L}$ add, producing a new angular momentum pointing more toward the person. The wheel moves toward the person, perpendicular to the forces she exerts on it.

This same logic explains the behavior of gyroscopes. [link] shows the two forces acting on a spinning gyroscope. The torque produced is perpendicular to the angular momentum, thus the direction of the angular momentum is changed but not its magnitude, as a result of the change in angular momentum produced by the torque. This type of motion is called a **precession**, and we say that the gyroscope *precesses* around a vertical axis, since the torque is always horizontal and perpendicular to $\mathbf{L}$. If the gyroscope is *not* spinning, it acquires initial angular momentum in the direction of the torque ($\mathbf{L} = \Delta\mathbf{L}$) and the change of angular momentum due to torque continues to add in the same direction, causing the gyroscope to rotate around a horizontal axis, falling over just as we would expect.

Earth itself acts like a gigantic gyroscope. Its angular momentum is along its axis and points at Polaris, the North Star. But Earth is slowly precessing (once in about 26,000 years) due to the torque of the Sun and the Moon on its nonspherical shape.



As seen in figure (a), the forces on a spinning gyroscope are its weight and the supporting force from the stand. These forces create a horizontal torque on the gyroscope, which create a change in angular momentum $\Delta\mathbf{L}$ that is also horizontal. In figure (b), $\Delta\mathbf{L}$ and $\mathbf{L}$ add to produce a new angular momentum with the same magnitude, but different direction, so that the gyroscope precesses in the direction shown instead of falling over.

Exercise:
Check Your Understanding

Problem:

Rotational kinetic energy is associated with angular momentum? Does that mean that rotational kinetic energy is a vector?

Solution:

No, energy is always a scalar whether motion is involved or not. No form of energy has a direction in space and you can see that rotational kinetic energy does not depend on the direction of motion just as linear kinetic energy is independent of the direction of motion.

Section Summary

- Torque is perpendicular to the plane formed by r and $\mathbf{F}$ and is the direction your right thumb would point if you curled the fingers of your right hand in the direction of $\mathbf{F}$. The direction of the torque is thus the same as that of the change in angular momentum it produces.
- The gyroscope precesses around a vertical axis, since the torque is always horizontal and perpendicular to $\mathbf{L}$. If the gyroscope is not spinning, it acquires angular momentum in the direction of the torque ($\mathbf{L} = \Delta\mathbf{L}$), and it rotates about a horizontal axis, falling over just as we would expect.
- Earth itself acts like a gigantic gyroscope. Its angular momentum is along its axis and points at Polaris, the North Star.

Conceptual Questions

Exercise:

Problem:

While driving his motorcycle at highway speed, a physics student notices that pulling back lightly on the right handlebar tips the cycle to the left and produces a left turn. Explain why this happens.

Exercise:

Problem:

Gyroscopes used in guidance systems to indicate directions in space must have an angular momentum that does not change in direction. Yet they are often subjected to large forces and accelerations. How can the direction of their angular momentum be constant when they are accelerated?

Exercise:

Problem:

Consider a gyroscope or a spinning bicycle wheel as in [link] or [link]. Under the torque due to gravitational force, they precess around a vertical axis. If this gyroscope or spinning bicycle wheel started out by spinning faster (larger angular momentum), will it precess faster or slower?

Glossary

precession
: a type of motion that an object possessing angular momentum undergoes, as a torque perpendicular to the direction of angular momentum is applied, causing the direction of angular momentum to change; the circular wobbling motion traced by a slightly tilted circular top is an example

right-hand rule
: direction of angular velocity ω and angular momentum L in which the thumb of your right hand points when you curl your fingers in the direction of the disk's rotation

Introduction to Fluids
class="introduction"

The fluid essential to all life has a beauty of its own. It also helps support the weight of this swimmer. (credit: Terren, Wikimedia Commons)

Much of what we value in life is fluid: a breath of fresh winter air; the hot blue flame in our gas cooker; the water we drink, swim in, and bathe in; the

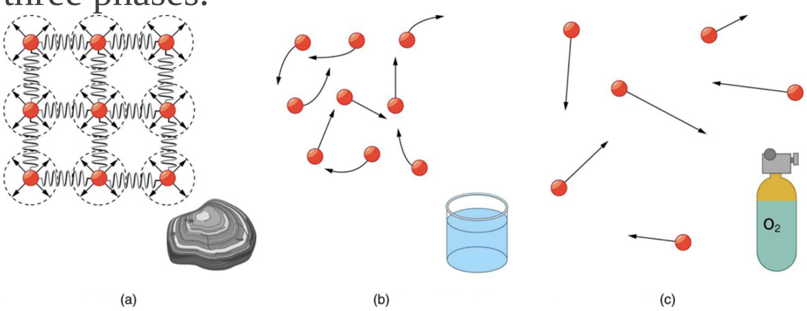
blood in our veins. What exactly is a fluid? Can we understand fluids with the laws already presented, or will new laws emerge from their study? The physical characteristics of fluids and some of the laws that govern their behavior are the topics of this chapter.

Also by their very definition, fluids flow. Examples come easily—a column of smoke rises from a camp fire, water streams from a fire hose, blood courses through your veins. Why does rising smoke curl and twist? How does a nozzle increase the speed of water emerging from a hose? How does the body regulate blood flow? The physics of fluids in motion—fluid dynamics—allows us to answer these and many other questions.

What Is a Fluid?

- State the common phases of matter.
- Explain the physical characteristics of solids, liquids, and gases.
- Describe the arrangement of atoms in solids, liquids, and gases.

Matter most commonly exists as a solid, liquid, or gas; these states are known as the three common *phases of matter*. Solids have a definite shape and a specific volume, liquids have a definite volume but their shape changes depending on the container in which they are held, and gases have neither a definite shape nor a specific volume as their molecules move to fill the container in which they are held. (See [link].) Liquids and gases are considered to be fluids because they yield to shearing forces, whereas solids resist them. Note that the extent to which fluids yield to shearing forces (and hence flow easily and quickly) depends on a quantity called the viscosity. We can understand the phases of matter and what constitutes a fluid by considering the forces between atoms that make up matter in the three phases.



(a) Atoms in a solid always have the same neighbors, held near home by forces represented here by springs. These atoms are essentially in contact with one another. A rock is an example of a solid. This rock retains its shape because of the forces holding its atoms together. (b) Atoms in a liquid are also in close contact but can slide over one another. Forces between them strongly resist attempts to push them closer together and also hold them in close contact. Water is an example of a liquid. Water can

flow, but it also remains in an open container because of the forces between its atoms. (c) Atoms in a gas are separated by distances that are considerably larger than the size of the atoms themselves, and they move about freely. A gas must be held in a closed container to prevent it from moving out freely.

Atoms in *solids* are in close contact, with forces between them that allow the atoms to vibrate but not to change positions with neighboring atoms. (These forces can be thought of as springs that can be stretched or compressed, but not easily broken.) Thus a solid *resists* all types of stress. A solid cannot be easily deformed because the atoms that make up the solid are not able to move about freely. Solids also resist compression, because their atoms form part of a lattice structure in which the atoms are a relatively fixed distance apart. Under compression, the atoms would be forced into one another. Most of the examples we have studied so far have involved solid objects which deform very little when stressed.

Note:
Connections: Submicroscopic Explanation of Solids and Liquids
Atomic and molecular characteristics explain and underlie the macroscopic characteristics of solids and fluids. This present section is devoted entirely to the submicroscopic explanation of solids and liquids.

In contrast, *liquids* deform easily when stressed and do not spring back to their original shape once the force is removed because the atoms are free to slide about and change neighbors—that is, they *flow* (so they are a type of fluid), with the molecules held together by their mutual attraction. When a liquid is placed in a container with no lid on, it remains in the container (providing the container has no holes below the surface of the liquid!).

Because the atoms are closely packed, liquids, like solids, resist compression.

Atoms in *gases* are separated by distances that are large compared with the size of the atoms. The forces between gas atoms are therefore very weak, except when the atoms collide with one another. Gases thus not only flow (and are therefore considered to be fluids) but they are relatively easy to compress because there is much space and little force between atoms. When placed in an open container gases, unlike liquids, will escape. The major distinction is that gases are easily compressed, whereas liquids are not. We shall generally refer to both gases and liquids simply as **fluids**, and make a distinction between them only when they behave differently.

Section Summary

- A fluid is a state of matter that yields to sideways or shearing forces. Liquids and gases are both fluids. Fluid statics is the physics of stationary fluids.

Conceptual Questions

Exercise:

Problem:

What physical characteristic distinguishes a fluid from a solid?

Exercise:

Problem:

Which of the following substances are fluids at room temperature: air, mercury, water, glass?

Exercise:

Problem: Why are gases easier to compress than liquids and solids?

Exercise:

Problem: How do gases differ from liquids?

Glossary

fluids
: liquids and gases; a fluid is a state of matter that yields to shearing forces

Density

- Define density.
- Calculate the mass of an object from its density and volume.
- Compare and contrast the densities of various substances.

Which weighs more, a ton of feathers or a ton of bricks? This old riddle plays with the distinction between mass and density. A ton is a ton, of course; but bricks have much greater density than feathers, and so we are tempted to think of them as heavier. (See [link].)

Density, as you will see, is an important characteristic of substances. It is crucial, for example, in determining whether an object sinks or floats in a fluid. Density is the mass per unit volume of a substance or object. In equation form, density is defined as
Equation:

$$\rho = \frac{m}{V},$$

where the Greek letter ρ (rho) is the symbol for density, m is the mass, and V is the volume occupied by the substance.

Note:
Density
Density is mass per unit volume.
Equation:

$$\rho = \frac{m}{V},$$

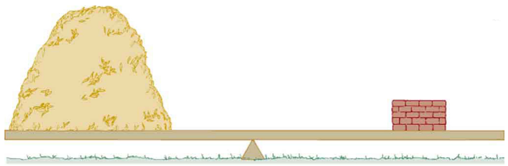
where ρ is the symbol for density, m is the mass, and V is the volume occupied by the substance.

In the riddle regarding the feathers and bricks, the masses are the same, but the volume occupied by the feathers is much greater, since their density is much lower. The SI unit of density is kg/m^3, representative values are given in [link]. The metric system was originally devised so that water would have a density of $1\ \text{g/cm}^3$, equivalent to $10^3\ \text{kg/m}^3$. Thus the basic mass unit, the kilogram, was first devised to be the mass of 1000 mL of water, which has a volume of 1000 cm^3.

Substance	$\rho(10^3\ \text{kg/m}^3\ \text{or g/mL})$	**Substance**	$\rho(10^3\ \text{kg/m}^3\ \text{or g/mL})$	**Substance**	$\rho($
Solids		**Liquids**		**Gases**	
Aluminum	2.7	Water (4ºC)	1.000	Air	

Substance	$\rho(10^3\ \mathrm{kg/m^3}\ \text{or}\ \mathrm{g/mL})$	**Substance**	$\rho(10^3\ \mathrm{kg/m^3}\ \text{or}\ \mathrm{g/mL})$	**Substance**	$\rho($
Brass	8.44	Blood	1.05	Carbon dioxide	
Copper (average)	8.8	Sea water	1.025	Carbon monoxide	
Gold	19.32	Mercury	13.6	Hydrogen	
Iron or steel	7.8	Ethyl alcohol	0.79	Helium	
Lead	11.3	Petrol	0.68	Methane	
Polystyrene	0.10	Glycerin	1.26	Nitrogen	
Tungsten	19.30	Olive oil	0.92	Nitrous oxide	
Uranium	18.70			Oxygen	
Concrete	2.30–3.0			Steam $(100^{o}\ \mathrm{C})$	
Cork	0.24				
Glass, common (average)	2.6				
Granite	2.7				
Earth’s crust	3.3				
Wood	0.3–0.9				
Ice (0°C)	0.917				
Bone	1.7–2.0				

Densities of Various Substances



A ton of feathers and a ton of bricks have the same mass, but the feathers make a much bigger pile because they have a much lower density.

As you can see by examining [link], the density of an object may help identify its composition. The density of gold, for example, is about 2.5 times the density of iron, which is about 2.5 times the density of aluminum. Density also reveals something about the phase of the matter and its substructure. Notice that the densities of liquids and solids are roughly comparable, consistent with the fact that their atoms are in close contact. The densities of gases are much less than those of liquids and solids, because the atoms in gases are separated by large amounts of empty space.

Note:
Take-Home Experiment Sugar and Salt
A pile of sugar and a pile of salt look pretty similar, but which weighs more? If the volumes of both piles are the same, any difference in mass is due to their different densities (including the air space between crystals). Which do you think has the greater density? What values did you find? What method did you use to determine these values?

Example:
Calculating the Mass of a Reservoir From Its Volume
A reservoir has a surface area of $50.0\ \text{km}^2$ and an average depth of 40.0 m. What mass of water is held behind the dam? (See [link] for a view of a large reservoir—the Three Gorges Dam site on the Yangtze River in central China.)
Strategy
We can calculate the volume V of the reservoir from its dimensions, and find the density of water ρ in [link]. Then the mass m can be found from the definition of density
Equation:

$$\rho = \frac{m}{V}.$$

Solution
Solving equation $\rho = m/V$ for m gives $m = \rho V$.
The volume V of the reservoir is its surface area A times its average depth h:
Equation:

$$\begin{aligned} V &= \text{Ah} = \left(50.0\ \text{km}^2\right)\left(40.0\ \text{m}\right) \\ &= \left[\left(50.0\ \text{km}^2\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right)^2\right]\left(40.0\ \text{m}\right) = 2.00 \times 10^9\ \text{m}^3 \end{aligned}$$

The density of water ρ from [link] is $1.000 \times 10^3\ \text{kg/m}^3$. Substituting V and ρ into the expression for mass gives
Equation:

$$
\begin{aligned}
m &= \left(1.00\times10^{3}\ \mathrm{kg/m^{3}}\right)\left(2.00\times10^{9}\ \mathrm{m^{3}}\right)\\
&= 2.00\times10^{12}\ \mathrm{kg}.
\end{aligned}
$$

Discussion

A large reservoir contains a very large mass of water. In this example, the weight of the water in the reservoir is $\mathrm{mg} = 1.96\times10^{13}\ \mathrm{N}$, where g is the acceleration due to the Earth's gravity (about $9.80\ \mathrm{m/s^{2}}$). It is reasonable to ask whether the dam must supply a force equal to this tremendous weight. The answer is no. As we shall see in the following sections, the force the dam must supply can be much smaller than the weight of the water it holds back.

Three Gorges Dam in central China. When completed in 2008, this became the world's largest hydroelectric plant, generating power equivalent to that generated by 22 average-sized nuclear power plants. The concrete dam is 181 m high and 2.3 km across. The reservoir made by this dam is 660 km long. Over 1 million people were displaced by the creation of the reservoir. (credit: Le Grand Portage)

Section Summary

- Density is the mass per unit volume of a substance or object. In equation form, density is defined as
  Equation:

$$\rho = \frac{m}{V}.$$

- The SI unit of density is $\mathrm{kg/m^{3}}$.

Conceptual Questions

Exercise:

Problem: Approximately how does the density of air vary with altitude?

Exercise:

Problem:

Give an example in which density is used to identify the substance composing an object. Would information in addition to average density be needed to identify the substances in an object composed of more than one material?

Exercise:

Problem:

[link] shows a glass of ice water filled to the brim. Will the water overflow when the ice melts? Explain your answer.

Glossary

density
: the mass per unit volume of a substance or object

Pressure

- Define pressure.
- Explain the relationship between pressure and force.
- Calculate force given pressure and area.

You have no doubt heard the word **pressure** being used in relation to blood (high or low blood pressure) and in relation to the weather (high- and low-pressure weather systems). These are only two of many examples of pressures in fluids. Pressure P is defined as

Equation:

$$P = \frac{F}{A}$$

where F is a force applied to an area A that is perpendicular to the force.

Note:

Pressure

Pressure is defined as the force divided by the area perpendicular to the force over which the force is applied, or

Equation:

$$P = \frac{F}{A}.$$

A given force can have a significantly different effect depending on the area over which the force is exerted, as shown in [link]. The SI unit for pressure is the *pascal*, where

Equation:

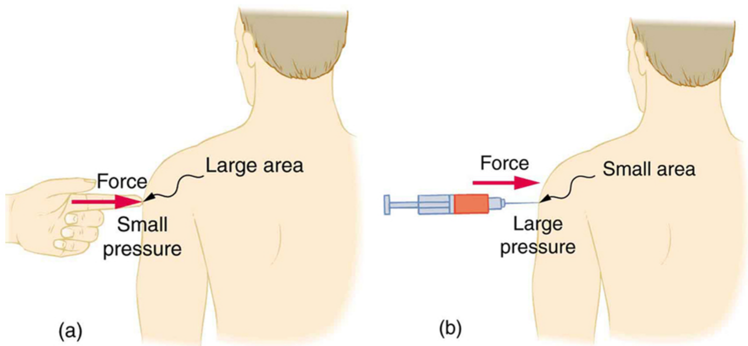
$$1 \text{ Pa} = 1 \text{ N/m}^2.$$

In addition to the pascal, there are many other units for pressure that are in common use. In meteorology, atmospheric pressure is often described in units of millibar (mb), where

Equation:

$$100\text{ mb} = 1 \times 10^5\text{ Pa}\ .$$

Pounds per square inch $\left(\text{lb/in}^2\text{ or psi}\right)$ is still sometimes used as a measure of tire pressure, and millimeters of mercury (mm Hg) is still often used in the measurement of blood pressure. Pressure is defined for all states of matter but is particularly important when discussing fluids.



(a) While the person being poked with the finger might be irritated, the force has little lasting effect. (b) In contrast, the same force applied to an area the size of the sharp end of a needle is great enough to break the skin.

Example:

Calculating Force Exerted by the Air: What Force Does a Pressure Exert?

An astronaut is working outside the International Space Station where the atmospheric pressure is essentially zero. The pressure gauge on her air tank

reads 6.90×10^6 Pa. What force does the air inside the tank exert on the flat end of the cylindrical tank, a disk 0.150 m in diameter?

Strategy

We can find the force exerted from the definition of pressure given in $P = \frac{F}{A}$, provided we can find the area A acted upon.

Solution

By rearranging the definition of pressure to solve for force, we see that

Equation:

$$F = \text{PA}.$$

Here, the pressure P is given, as is the area of the end of the cylinder A, given by $A = \pi r^2$. Thus,

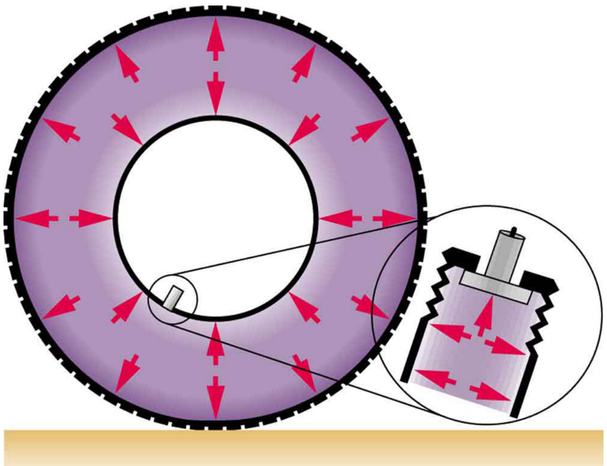
Equation:

$$\begin{array}{lll} F & = & \left(6.90 \times 10^6 \text{ N/m}^2\right)(3.14)(0.0750 \text{ m})^2 \\ & = & 1.22 \times 10^5 \text{ N}. \end{array}$$

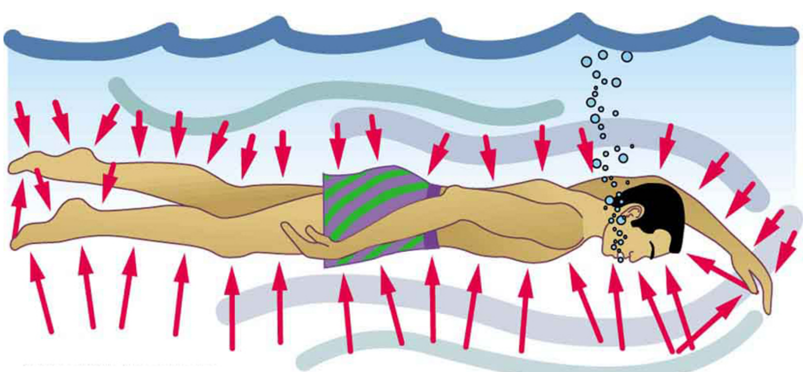
Discussion

Wow! No wonder the tank must be strong. Since we found $F = \text{PA}$, we see that the force exerted by a pressure is directly proportional to the area acted upon as well as the pressure itself.

The force exerted on the end of the tank is perpendicular to its inside surface. This direction is because the force is exerted by a static or stationary fluid. We have already seen that fluids cannot *withstand* shearing (sideways) forces; they cannot *exert* shearing forces, either. Fluid pressure has no direction, being a scalar quantity. The forces due to pressure have well-defined directions: they are always exerted perpendicular to any surface. (See the tire in [link], for example.) Finally, note that pressure is exerted on all surfaces. Swimmers, as well as the tire, feel pressure on all sides. (See [link].)



Pressure inside this tire exerts forces perpendicular to all surfaces it contacts. The arrows give representative directions and magnitudes of the forces exerted at various points. Note that static fluids do not exert shearing forces.



Pressure is exerted on all sides of this swimmer, since the water would flow into the space he occupies if he were not there. The arrows represent the directions and magnitudes of the forces exerted at various points on the swimmer. Note that the forces are larger underneath, due to greater depth,

giving a net upward or buoyant force that is balanced by the weight of the swimmer.

Section Summary

- Pressure is the force per unit perpendicular area over which the force is applied. In equation form, pressure is defined as
  Equation:
  $$P = \frac{F}{A}.$$
- The SI unit of pressure is pascal and $1\ \text{Pa} = 1\ \text{N/m}^2$.

Conceptual Questions

Exercise:

Problem:

How is pressure related to the sharpness of a knife and its ability to cut?

Exercise:

Problem:

Why does a dull hypodermic needle hurt more than a sharp one?

Exercise:

Problem:

The outward force on one end of an air tank was calculated in [link]. How is this force balanced? (The tank does not accelerate, so the force must be balanced.)

Exercise:

Problem:

Why is force exerted by static fluids always perpendicular to a surface?

Exercise:

Problem:

In a remote location near the North Pole, an iceberg floats in a lake. Next to the lake (assume it is not frozen) sits a comparably sized glacier sitting on land. If both chunks of ice should melt due to rising global temperatures (and the melted ice all goes into the lake), which ice chunk would give the greatest increase in the level of the lake water, if any?

Exercise:

Problem:

How do jogging on soft ground and wearing padded shoes reduce the pressures to which the feet and legs are subjected?

Exercise:

Problem:

Toe dancing (as in ballet) is much harder on toes than normal dancing or walking. Explain in terms of pressure.

Exercise:

Problem:

How do you convert pressure units like millimeters of mercury, centimeters of water, and inches of mercury into units like newtons per meter squared without resorting to a table of pressure conversion factors?

Glossary

pressure

the force per unit area perpendicular to the force, over which the force acts

Pressure Due to the Weight of Fluid

- Define pressure in terms of weight.
- Explain the variation of pressure with depth in a fluid.
- Calculate density given pressure and altitude.

If your ears have ever popped on a plane flight or ached during a deep dive in a swimming pool, you have experienced the effect of depth on pressure in a fluid. At the Earth’s surface, the air pressure exerted on you is a result of the weight of air above you. This pressure is reduced as you climb up in altitude and the weight of air above you decreases. Under water, the pressure exerted on you increases with increasing depth. In this case, the pressure being exerted upon you is a result of both the weight of water above you *and* that of the atmosphere above you. You may notice an air pressure change on an elevator ride that transports you many stories, but you need only dive a meter or so below the surface of a pool to feel a pressure increase. The difference is that water is much denser than air, about 775 times as dense.

Consider the container in [link]. Its bottom supports the weight of the fluid in it. Let us calculate the pressure exerted on the bottom by the weight of the fluid. That **pressure** is the weight of the fluid mg divided by the area A supporting it (the area of the bottom of the container):
Equation:

$$P=\frac{\text{mg}}{A}.$$

We can find the mass of the fluid from its volume and density:
Equation:

$$m=\rho V.$$

The volume of the fluid V is related to the dimensions of the container. It is
Equation:

$$V=\text{Ah},$$

where A is the cross-sectional area and h is the depth. Combining the last two equations gives

Equation:

$$m = \rho \text{Ah}.$$

If we enter this into the expression for pressure, we obtain

Equation:

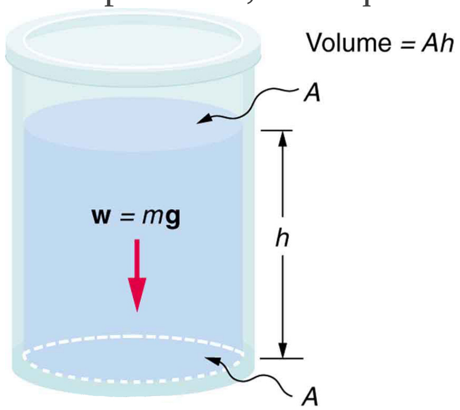
$$P = \frac{(\rho \text{Ah})g}{A}.$$

The area cancels, and rearranging the variables yields

Equation:

$$P = \rho g h.$$

This value is the **pressure due to the weight of a fluid**. The equation has general validity beyond the special conditions under which it is derived here. Even if the container were not there, the surrounding fluid would still exert this pressure, keeping the fluid static. Thus the equation $P = \rho g h$ represents the pressure due to the weight of any fluid of *average density* ρ at any depth h below its surface. For liquids, which are nearly incompressible, this equation holds to great depths.



The bottom of this container supports the entire weight of the fluid in it. The vertical sides cannot exert an upward force on the fluid (since it cannot withstand a shearing force), and so the bottom must support it all.

Example:

Calculating the Average Pressure and Force Exerted: What Force Must a Dam Withstand?

In [link], we calculated the mass of water in a large reservoir. We will now consider the pressure and force acting on the dam retaining water. (See [link].) The dam is 500 m wide, and the water is 80.0 m deep at the dam. (a) What is the average pressure on the dam due to the water? (b) Calculate the force exerted against the dam and compare it with the weight of water in the dam (previously found to be 1.96×10^{13} N).

Strategy for (a)

The average pressure P due to the weight of the water is the pressure at the average depth h of 40.0 m, since pressure increases linearly with depth.

Solution for (a)

The average pressure due to the weight of a fluid is

Equation:

$$P = \rho g h.$$

Entering the density of water from [link] and taking h to be the average depth of 40.0 m, we obtain

Equation:

$$\begin{aligned} P &= \left(10^3 \ \frac{\text{kg}}{\text{m}^3}\right)\left(9.80 \ \frac{\text{m}}{\text{s}^2}\right)(40.0 \ \text{m}) \\ &= 3.92 \times 10^5 \ \frac{\text{N}}{\text{m}^2} = 392 \ \text{kPa}. \end{aligned}$$

Strategy for (b)

The force exerted on the dam by the water is the average pressure times the area of contact:

Equation:

$$F = PA.$$

Solution for (b)

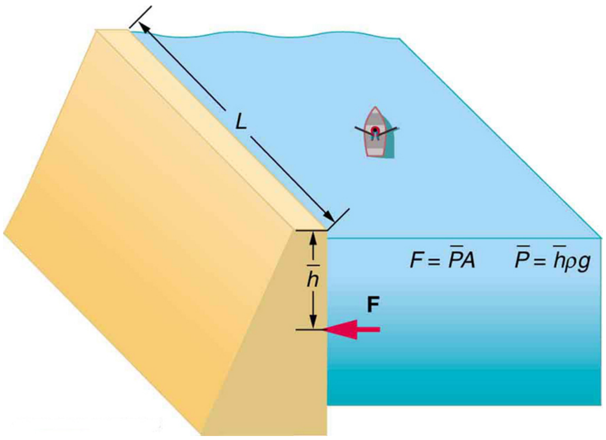
We have already found the value for P. The area of the dam is $A = 80.0 \ \text{m} \times 500 \ \text{m} = 4.00 \times 10^4 \ \text{m}^2$, so that

Equation:

$$\begin{aligned} F &= (3.92 \times 10^5 \ \text{N/m}^2)(4.00 \times 10^4 \ \text{m}^2) \\ &= 1.57 \times 10^{10} \ \text{N}. \end{aligned}$$

Discussion

Although this force seems large, it is small compared with the 1.96×10^{13} N weight of the water in the reservoir—in fact, it is only 0.0800% of the weight. Note that the pressure found in part (a) is completely independent of the width and length of the lake—it depends only on its average depth at the dam. Thus the force depends only on the water's average depth and the dimensions of the dam, *not* on the horizontal extent of the reservoir. In the diagram, the thickness of the dam increases with depth to balance the increasing force due to the increasing pressure.



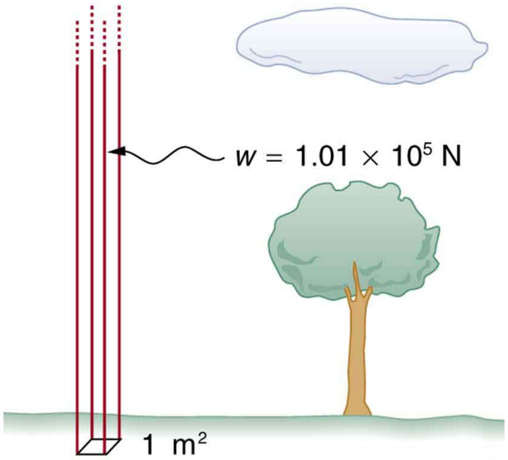
The dam must withstand the force exerted against it by the water it retains. This force is small compared with the weight of the water behind the dam.

Atmospheric pressure is another example of pressure due to the weight of a fluid, in this case due to the weight of *air* above a given height. The atmospheric pressure at the Earth's surface varies a little due to the large-scale flow of the atmosphere induced by the Earth's rotation (this creates weather "highs" and "lows"). However, the average pressure at sea level is given by the *standard atmospheric pressure* P_{atm}, measured to be

Equation:

$$1\text{ atmosphere (atm)} = P_{\text{atm}} = 1.01 \times 10^5\text{ N/m}^2 = 101\text{ kPa}.$$

This relationship means that, on average, at sea level, a column of air above 1.00 m^2 of the Earth's surface has a weight of $1.01 \times 10^5\text{ N}$, equivalent to 1 atm. (See [link].)



Atmospheric pressure at sea level averages $1.01 \times 10^5 \text{ Pa}$ (equivalent to 1 atm), since the column of air over this 1 m^2, extending to the top of the atmosphere, weighs $1.01 \times 10^5 \text{ N}$.

Example:
Calculating Depth Below the Surface of Water: What Depth of Water Creates the Same Pressure as the Entire Atmosphere?
Calculate the depth below the surface of water at which the pressure due to the weight of the water equals 1.00 atm.
Strategy
We begin by solving the equation $P = \rho g h$ for depth h:
Equation:

$$h = \frac{P}{\rho g}.$$

Then we take P to be 1.00 atm and ρ to be the density of the water that creates the pressure.

Solution

Entering the known values into the expression for h gives

Equation:

$$h = \frac{1.01 \times 10^5 \text{ N/m}^2}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 10.3 \text{ m.}$$

Discussion

Just 10.3 m of water creates the same pressure as 120 km of air (height of uppermost layers of atmosphere). Since water is nearly incompressible, we can neglect any change in its density over this depth.

What do you suppose is the *total* pressure at a depth of 10.3 m in a swimming pool? Does the atmospheric pressure on the water's surface affect the pressure below? The answer is yes. This seems only logical, since both the water's weight and the atmosphere's weight must be supported. So the *total* pressure at a depth of 10.3 m is 2 atm—half from the water above and half from the air above.

Section Summary

- Pressure is the weight of the fluid mg divided by the area A supporting it (the area of the bottom of the container):
  Equation:

  $$P = \frac{\text{mg}}{A}.$$

- Pressure due to the weight of a liquid is given by
  Equation:

  $$P = \rho g h,$$

where P is the pressure, ρ is the density of the liquid, g is the acceleration due to gravity, and h is the height of the liquid.

Conceptual Questions

Exercise:

Problem:

Atmospheric pressure exerts a large force (equal to the weight of the atmosphere above your body—about 10 tons) on the top of your body when you are lying on the beach sunbathing. Why are you able to get up?

Exercise:

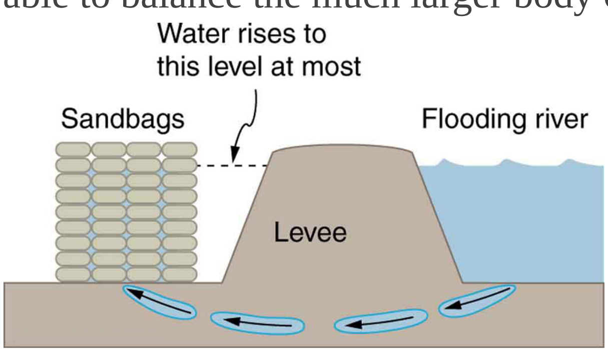
Problem:

What are two reasons why mercury rather than water is used in barometers?

Exercise:

Problem:

[link] shows how sandbags placed around a leak outside a river levee can effectively stop the flow of water under the levee. Explain how the small amount of water inside the column formed by the sandbags is able to balance the much larger body of water behind the levee.



Because the river level is very high, it has started to leak under the levee. Sandbags are placed

around the leak, and the water held by them rises until it is the same level as the river, at which point the water there stops rising.

Exercise:

Problem:

Is there a net force on a dam due to atmospheric pressure? Explain your answer.

Exercise:

Problem:

Does atmospheric pressure add to the gas pressure in a rigid tank? In a toy balloon? When, in general, does atmospheric pressure *not* affect the total pressure in a fluid?

Exercise:

Problem:

You can break a strong wine bottle by pounding a cork into it with your fist, but the cork must press directly against the liquid filling the bottle—there can be no air between the cork and liquid. Explain why the bottle breaks, and why it will not if there is air between the cork and liquid.

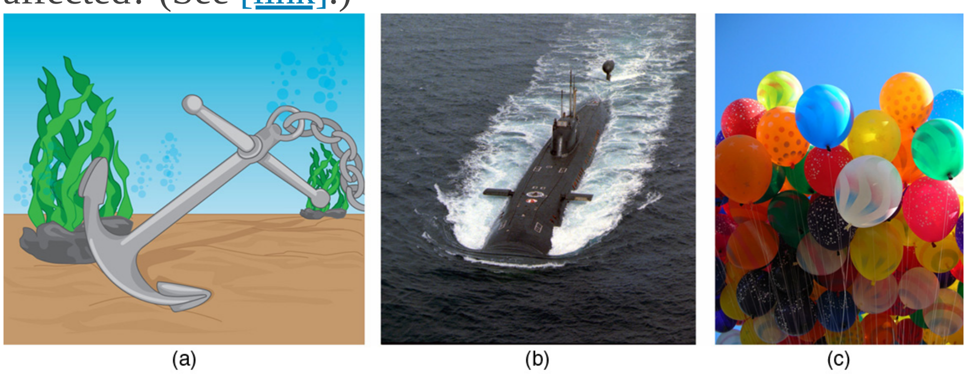
Glossary

pressure due to the weight of fluid
: pressure at a depth below a fluid surface due to its weight; given by $P = \rho g h$

Archimedes’ Principle

- Define buoyant force.
- State Archimedes’ principle.
- Understand why objects float or sink.
- Understand the relationship between density and Archimedes’ principle.

When you rise from lounging in a warm bath, your arms feel strangely heavy. This is because you no longer have the buoyant support of the water. Where does this buoyant force come from? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected? (See [link].)

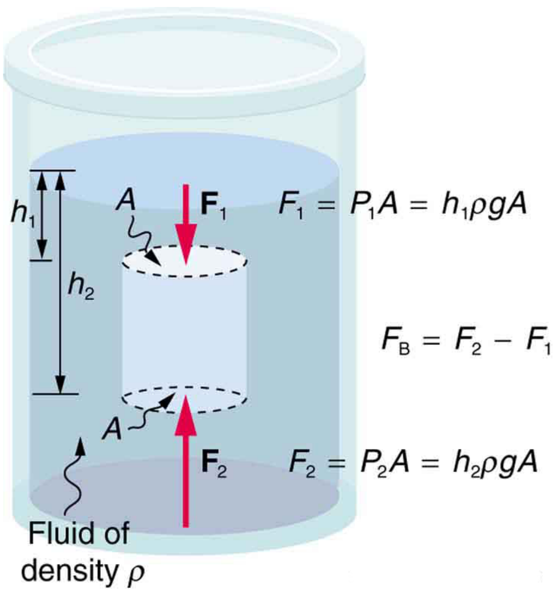


(a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (credit: Allied Navy) (c) Helium-filled balloons tug upward on their strings, demonstrating air’s buoyant effect. (credit: Crystl)

Answers to all these questions, and many others, are based on the fact that pressure increases with depth in a fluid. This means that the upward force on the bottom of an object in a fluid is greater than the downward force on the top of the object. There is a net upward, or **buoyant force** on any object in any fluid. (See [link].) If the buoyant force is greater than the object’s

weight, the object will rise to the surface and float. If the buoyant force is less than the object’s weight, the object will sink. If the buoyant force equals the object’s weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.

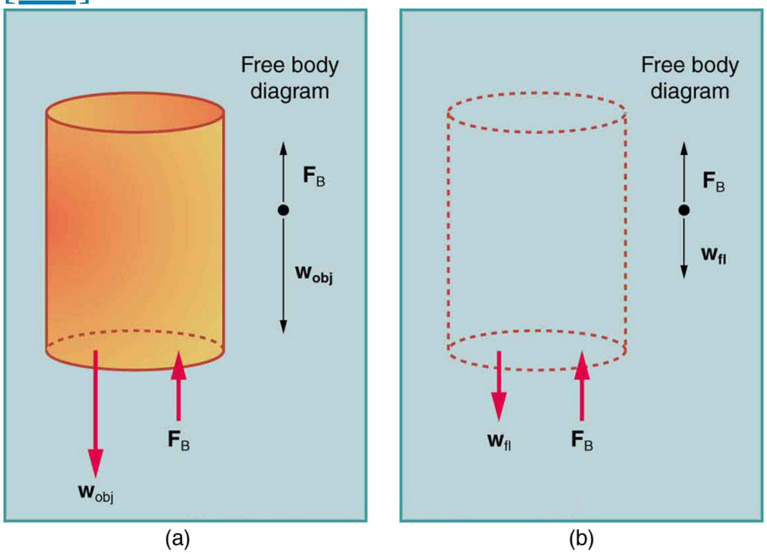
> **Note:**
> Buoyant Force
> The buoyant force is the net upward force on any object in any fluid.



Pressure due to the weight of a fluid increases with depth since $P = \rho gh$. This pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder. Their difference is the buoyant

force $\mathbf{F}_{\mathrm{B}}$. (Horizontal forces cancel.)

Just how great is this buoyant force? To answer this question, think about what happens when a submerged object is removed from a fluid, as in [link].



(a) An object submerged in a fluid experiences a buoyant force F_{B}. If F_{B} is greater than the weight of the object, the object will rise. If F_{B} is less than the weight of the object, the object will sink. (b) If the object is removed, it is replaced by fluid having weight w_{fl}. Since this weight is supported by surrounding fluid, the buoyant force must equal the weight of the fluid displaced. That is, $F_{\mathrm{B}} = w_{\mathrm{fl}}$,a statement of Archimedes' principle.

The space it occupied is filled by fluid having a weight w_{fl}. This weight is supported by the surrounding fluid, and so the buoyant force must equal w_{fl}, the weight of the fluid displaced by the object. It is a tribute to the genius

of the Greek mathematician and inventor Archimedes (ca. 287–212 B.C.) that he stated this principle long before concepts of force were well established. Stated in words, **Archimedes' principle** is as follows: The buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

Equation:

$$F_{\text{B}} = w_{\text{fl}},$$

where F_{B} is the buoyant force and w_{fl} is the weight of the fluid displaced by the object. Archimedes' principle is valid in general, for any object in any fluid, whether partially or totally submerged.

Note:

Archimedes' Principle

According to this principle the buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

Equation:

$$F_{\text{B}} = w_{\text{fl}},$$

where F_{B} is the buoyant force and w_{fl} is the weight of the fluid displaced by the object.

Humm … High-tech body swimsuits were introduced in 2008 in preparation for the Beijing Olympics. One concern (and international rule) was that these suits should not provide any buoyancy advantage. How do you think that this rule could be verified?

Note:

Making Connections: Take-Home Investigation

The density of aluminum foil is 2.7 times the density of water. Take a piece of foil, roll it up into a ball and drop it into water. Does it sink? Why or why not? Can you make it sink?

Floating and Sinking

Drop a lump of clay in water. It will sink. Then mold the lump of clay into the shape of a boat, and it will float. Because of its shape, the boat displaces more water than the lump and experiences a greater buoyant force. The same is true of steel ships.

Example:
Calculating buoyant force: dependency on shape
(a) Calculate the buoyant force on 10,000 metric tons $(1.00 \times 10^7 \text{ kg})$ of solid steel completely submerged in water, and compare this with the steel's weight. (b) What is the maximum buoyant force that water could exert on this same steel if it were shaped into a boat that could displace $1.00 \times 10^5 \text{ m}^3$ of water?
Strategy for (a)
To find the buoyant force, we must find the weight of water displaced. We can do this by using the densities of water and steel given earlier (see "Density"). We note that, since the steel is completely submerged, its volume and the water's volume are the same. Once we know the volume of water, we can find its mass and weight.
Solution for (a)
First, we use the definition of density $\rho = \frac{m}{V}$ to find the steel's volume, and then we substitute values for mass and density. This gives
Equation:

$$V_{\text{st}} = \frac{m_{\text{st}}}{\rho_{\text{st}}} = \frac{1.00 \times 10^7 \text{ kg}}{7.8 \times 10^3 \text{ kg/m}^3} = 1.28 \times 10^3 \text{ m}^3.$$

Because the steel is completely submerged, this is also the volume of water displaced, V_{w}. We can now find the mass of water displaced from the relationship between its volume and density, both of which are known. This gives

Equation:

$$\begin{aligned} m_{\mathrm{w}} &= \rho_{\mathrm{w}} V_{\mathrm{w}} = (1.000 \times 10^3\ \mathrm{kg/m^3})(1.28 \times 10^3\ \mathrm{m^3}) \\ &= 1.28 \times 10^6\ \mathrm{kg.} \end{aligned}$$

By Archimedes' principle, the weight of water displaced is $m_{\mathrm{w}} g$, so the buoyant force is

Equation:

$$\begin{aligned} F_{\mathrm{B}} &= w_{\mathrm{w}} = m_{\mathrm{w}} g = \left(1.28 \times 10^6\ \mathrm{kg}\right)\left(9.80\ \mathrm{m/s^2}\right) \\ &= 1.3 \times 10^7\ \mathrm{N.} \end{aligned}$$

The steel's weight is $m_{\mathrm{w}} g = 9.80 \times 10^7\ \mathrm{N}$, which is much greater than the buoyant force, so the steel will remain submerged. Note that the buoyant force is rounded to two digits because the density of steel is given to only two digits.

Strategy for (b)

Here we are given the maximum volume of water the steel boat can displace. The buoyant force is the weight of this volume of water.

Solution for (b)

The mass of water displaced is found from its relationship to density and volume, both of which are known. That is,

Equation:

$$\begin{aligned} m_{\mathrm{w}} &= \rho_{\mathrm{w}} V_{\mathrm{w}} = \left(1.000 \times 10^3\ \mathrm{kg/m^3}\right)\left(1.00 \times 10^5\ \mathrm{m^3}\right) \\ &= 1.00 \times 10^8\ \mathrm{kg.} \end{aligned}$$

The maximum buoyant force is the weight of this much water, or

Equation:

$$\begin{aligned} F_{\mathrm{B}} &= w_{\mathrm{w}} = m_{\mathrm{w}} g = \left(1.00 \times 10^{8}\ \mathrm{kg}\right)\left(9.80\ \mathrm{m/s^{2}}\right) \\ &= 9.80 \times 10^{8}\ \mathrm{N}. \end{aligned}$$

Discussion
The maximum buoyant force is ten times the weight of the steel, meaning the ship can carry a load nine times its own weight without sinking.

Note:
Making Connections: Take-Home Investigation
A piece of household aluminum foil is 0.016 mm thick. Use a piece of foil that measures 10 cm by 15 cm. (a) What is the mass of this amount of foil? (b) If the foil is folded to give it four sides, and paper clips or washers are added to this “boat,” what shape of the boat would allow it to hold the most “cargo” when placed in water? Test your prediction.

Density and Archimedes’ Principle

Density plays a crucial role in Archimedes’ principle. The average density of an object is what ultimately determines whether it floats. If its average density is less than that of the surrounding fluid, it will float. This is because the fluid, having a higher density, contains more mass and hence more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink.

The extent to which a floating object is submerged depends on how the object’s density is related to that of the fluid. In [link], for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship loaded. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or
Equation:

$$\text{fraction submerged} = \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{V_{\text{fl}}}{V_{\text{obj}}}.$$

The volume submerged equals the volume of fluid displaced, which we call V_{fl}. Now we can obtain the relationship between the densities by substituting $\rho = \frac{m}{V}$ into the expression. This gives

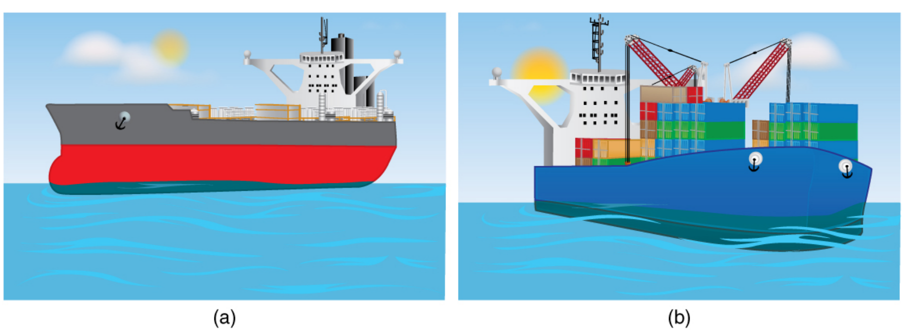
Equation:

$$\frac{V_{\text{fl}}}{V_{\text{obj}}} = \frac{m_{\text{fl}}/\rho_{\text{fl}}}{m_{\text{obj}}/\rho_{\text{obj}}},$$

where ρ_{obj} is the average density of the object and ρ_{fl} is the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, and so they cancel from the equation, leaving

Equation:

$$\text{fraction submerged} = \frac{\rho_{\text{obj}}}{\rho_{\text{fl}}}.$$



An unloaded ship (a) floats higher in the water than a loaded ship (b).

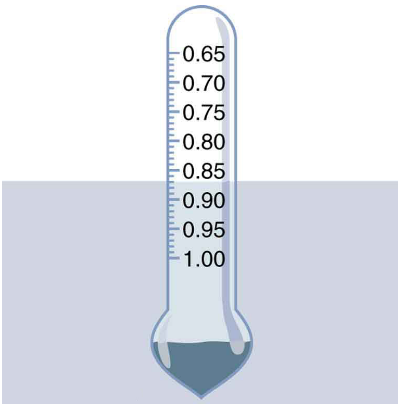
We use this last relationship to measure densities. This is done by measuring the fraction of a floating object that is submerged—for example, with a hydrometer. It is useful to define the ratio of the density of an object to a fluid (usually water) as **specific gravity**:

Equation:

$$\text{specific gravity} = \frac{\rho}{\rho_{\text{w}}},$$

where ρ is the average density of the object or substance and ρ_{w} is the density of water at 4.00°C. Specific gravity is dimensionless, independent of whatever units are used for ρ. If an object floats, its specific gravity is less than one. If it sinks, its specific gravity is greater than one. Moreover, the fraction of a floating object that is submerged equals its specific gravity. If an object's specific gravity is exactly 1, then it will remain suspended in the fluid, neither sinking nor floating. Scuba divers try to obtain this state so that they can hover in the water. We measure the specific gravity of fluids, such as battery acid, radiator fluid, and urine, as an indicator of their condition. One device for measuring specific gravity is shown in [link].

Note:
Specific Gravity
Specific gravity is the ratio of the density of an object to a fluid (usually water).



This hydrometer is floating in a fluid of specific gravity 0.87. The glass hydrometer is filled with air and weighted with lead at the bottom. It floats highest in the densest fluids and has been calibrated and labeled so that specific gravity can be read from it directly.

Example:
Calculating Average Density: Floating Woman
Suppose a 60.0-kg woman floats in freshwater with 97.0% of her volume submerged when her lungs are full of air. What is her average density?
Strategy
We can find the woman’s density by solving the equation
Equation:

$$\text{fraction submerged} = \frac{\rho_{\text{obj}}}{\rho_{\text{fl}}}$$

for the density of the object. This yields

Equation:

$$\rho_{\text{obj}} = \rho_{\text{person}} = (\text{fraction submerged}) \cdot \rho_{\text{fl}}.$$

We know both the fraction submerged and the density of water, and so we can calculate the woman's density.

Solution

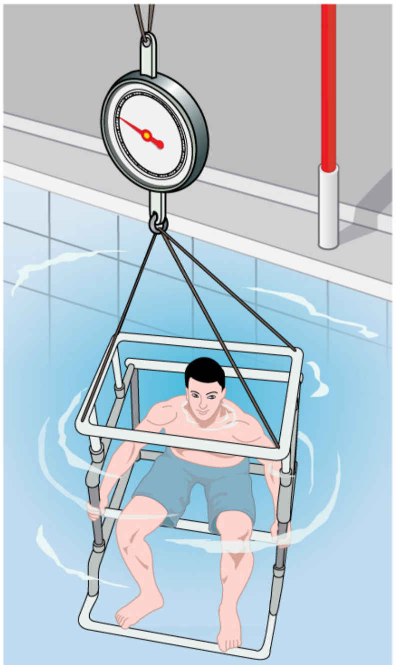
Entering the known values into the expression for her density, we obtain

Equation:

$$\rho_{\text{person}} = 0.970 \cdot \left(10^3 \frac{\text{kg}}{\text{m}^3}\right) = 970 \frac{\text{kg}}{\text{m}^3}.$$

Discussion

Her density is less than the fluid density. We expect this because she floats. Body density is one indicator of a person's percent body fat, of interest in medical diagnostics and athletic training. (See [link].)



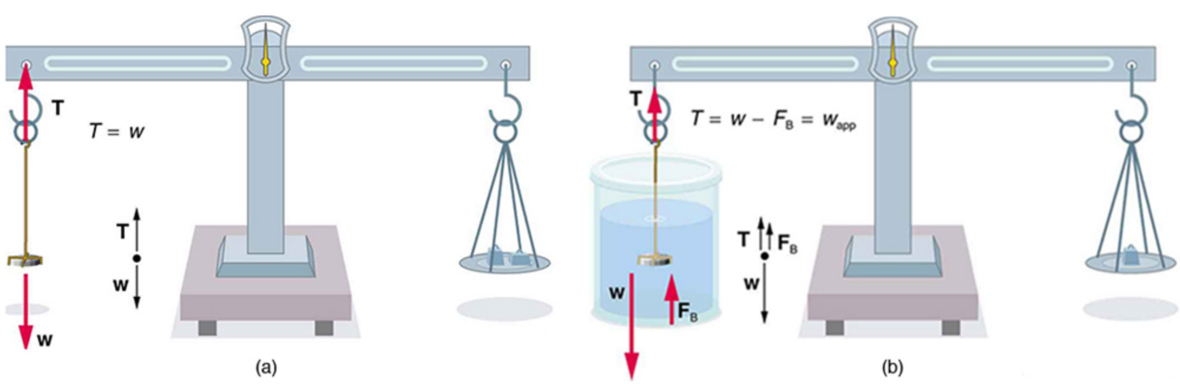
Subject in a "fat tank," where he is weighed while completely submerged as part of a body density determination. The subject must completely empty his lungs and hold a metal weight in order to sink. Corrections are made for the residual air in his lungs (measured separately) and the metal weight. His corrected submerged weight, his weight in air,

and pinch tests of strategic fatty areas are used to calculate his percent body fat.

There are many obvious examples of lower-density objects or substances floating in higher-density fluids—oil on water, a hot-air balloon, a bit of cork in wine, an iceberg, and hot wax in a “lava lamp,” to name a few. Less obvious examples include lava rising in a volcano and mountain ranges floating on the higher-density crust and mantle beneath them. Even seemingly solid Earth has fluid characteristics.

More Density Measurements

One of the most common techniques for determining density is shown in [link].



(a) A coin is weighed in air. (b) The apparent weight of the coin is determined while it is completely submerged in a fluid of known density. These two measurements are used to calculate the density of the coin.

An object, here a coin, is weighed in air and then weighed again while submerged in a liquid. The density of the coin, an indication of its authenticity, can be calculated if the fluid density is known. This same technique can also be used to determine the density of the fluid if the density of the coin is known. All of these calculations are based on Archimedes' principle.

Archimedes' principle states that the buoyant force on the object equals the weight of the fluid displaced. This, in turn, means that the object *appears* to weigh less when submerged; we call this measurement the object's *apparent weight*. The object suffers an *apparent weight loss* equal to the weight of the fluid displaced. Alternatively, on balances that measure mass, the object suffers an *apparent mass loss* equal to the mass of fluid displaced. That is

Equation:

$$\text{apparent weight loss} = \text{weight of fluid displaced}$$

or

Equation:

$$\text{apparent mass loss} = \text{mass of fluid displaced.}$$

The next example illustrates the use of this technique.

Example:

Calculating Density: Is the Coin Authentic?

The mass of an ancient Greek coin is determined in air to be 8.630 g. When the coin is submerged in water as shown in [link], its apparent mass is 7.800 g. Calculate its density, given that water has a density of $1.000\ \text{g/cm}^3$ and that effects caused by the wire suspending the coin are negligible.

Strategy

To calculate the coin's density, we need its mass (which is given) and its volume. The volume of the coin equals the volume of water displaced. The volume of water displaced V_w can be found by solving the equation for density $\rho = \frac{m}{V}$ for V.

Solution

The volume of water is $V_\mathrm{w} = \frac{m_\mathrm{w}}{\rho_\mathrm{w}}$ where m_w is the mass of water displaced. As noted, the mass of the water displaced equals the apparent mass loss, which is $m_\mathrm{w} = 8.630\ \mathrm{g} - 7.800\ \mathrm{g} = 0.830\ \mathrm{g}$. Thus the volume of water is $V_\mathrm{w} = \frac{0.830\ \mathrm{g}}{1.000\ \mathrm{g/cm^3}} = 0.830\ \mathrm{cm^3}$. This is also the volume of the coin, since it is completely submerged. We can now find the density of the coin using the definition of density:

Equation:

$$\rho_\mathrm{c} = \frac{m_\mathrm{c}}{V_\mathrm{c}} = \frac{8.630\ \mathrm{g}}{0.830\ \mathrm{cm^3}} = 10.4\ \mathrm{g/cm^3}.$$

Discussion

You see that this density is very close to that of pure silver, appropriate for this type of ancient coin. Most modern counterfeits are not pure silver.

This brings us back to Archimedes' principle and how it came into being. As the story goes, the king of Syracuse gave Archimedes the task of determining whether the royal crown maker was supplying a crown of pure gold. The purity of gold is difficult to determine by color (it can be diluted with other metals and still look as yellow as pure gold), and other analytical techniques had not yet been conceived. Even ancient peoples, however, realized that the density of gold was greater than that of any other then-known substance. Archimedes purportedly agonized over his task and had his inspiration one day while at the public baths, pondering the support the water gave his body. He came up with his now-famous principle, saw how to apply it to determine density, and ran naked down the streets of Syracuse crying "Eureka!" (Greek for "I have found it"). Similar behavior can be observed in contemporary physicists from time to time!

Section Summary

- Buoyant force is the net upward force on any object in any fluid. If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.
- Archimedes' principle states that the buoyant force on an object equals the weight of the fluid it displaces.
- Specific gravity is the ratio of the density of an object to a fluid (usually water).

Conceptual Questions

Exercise:

Problem:

More force is required to pull the plug in a full bathtub than when it is empty. Does this contradict Archimedes' principle? Explain your answer.

Exercise:

Problem:

Do fluids exert buoyant forces in a "weightless" environment, such as in the space shuttle? Explain your answer.

Exercise:

Problem:

Will the same ship float higher in salt water than in freshwater? Explain your answer.

Exercise:

Problem:

Marbles dropped into a partially filled bathtub sink to the bottom. Part of their weight is supported by buoyant force, yet the downward force on the bottom of the tub increases by exactly the weight of the marbles. Explain why.

Glossary

Archimedes' principle
: the buoyant force on an object equals the weight of the fluid it displaces

buoyant force
: the net upward force on any object in any fluid

specific gravity
: the ratio of the density of an object to a fluid (usually water)

Flow Rate and Its Relation to Velocity

- Calculate flow rate.
- Define units of volume.
- Describe incompressible fluids.
- Explain the consequences of the equation of continuity.

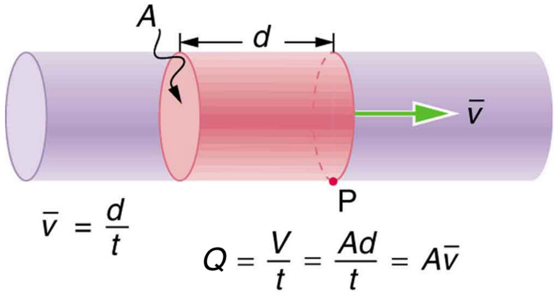
Flow rate Q is defined to be the volume of fluid passing by some location through an area during a period of time, as seen in [link]. In symbols, this can be written as

Equation:

$$Q=\frac{V}{t},$$

where V is the volume and t is the elapsed time.

The SI unit for flow rate is m^3/s, but a number of other units for Q are in common use. For example, the heart of a resting adult pumps blood at a rate of 5.00 liters per minute (L/min). Note that a **liter** (L) is 1/1000 of a cubic meter or 1000 cubic centimeters ($10^{-3}\ \text{m}^3$ or $10^3\ \text{cm}^3$). In this text we shall use whatever metric units are most convenient for a given situation.



Flow rate is the volume of fluid per unit time flowing past a point through the area A. Here the shaded cylinder of fluid flows past point P in a uniform pipe in time t. The volume of the cylinder is Ad

and the average velocity is $v = d/t$ so that the flow rate is $Q = \text{Ad}/t = Av$.

Example:
Calculating Volume from Flow Rate: The Heart Pumps a Lot of Blood in a Lifetime

How many cubic meters of blood does the heart pump in a 75-year lifetime, assuming the average flow rate is 5.00 L/min?

Strategy

Time and flow rate Q are given, and so the volume V can be calculated from the definition of flow rate.

Solution

Solving $Q = V/t$ for volume gives

Equation:

$$V = \text{Qt}.$$

Substituting known values yields

Equation:

$$\begin{array}{lll} V & = & \left(\frac{5.00\text{ L}}{1\text{ min}}\right)(75\text{ y})\left(\frac{1\text{ m}^3}{10^3\text{ L}}\right)\left(5.26\times 10^5\,\frac{\text{min}}{\text{y}}\right) \\ & = & 2.0\times 10^5\text{ m}^3. \end{array}$$

Discussion

This amount is about 200,000 tons of blood. For comparison, this value is equivalent to about 200 times the volume of water contained in a 6-lane 50-m lap pool.

Flow rate and velocity are related, but quite different, physical quantities. To make the distinction clear, think about the flow rate of a river. The

greater the velocity of the water, the greater the flow rate of the river. But flow rate also depends on the size of the river. A rapid mountain stream carries far less water than the Amazon River in Brazil, for example. The precise relationship between flow rate Q and velocity v is

Equation:

$$Q = Av,$$

where A is the cross-sectional area and v is the average velocity. This equation seems logical enough. The relationship tells us that flow rate is directly proportional to both the magnitude of the average velocity (hereafter referred to as the speed) and the size of a river, pipe, or other conduit. The larger the conduit, the greater its cross-sectional area. [link] illustrates how this relationship is obtained. The shaded cylinder has a volume

Equation:

$$V = \mathrm{Ad},$$

which flows past the point P in a time t. Dividing both sides of this relationship by t gives

Equation:

$$\frac{V}{t} = \frac{\mathrm{Ad}}{t}.$$

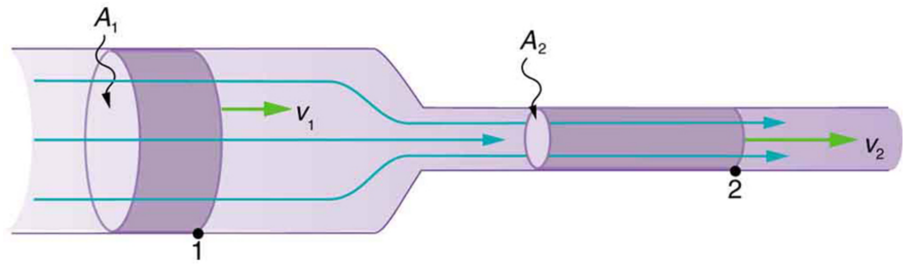
We note that $Q = V/t$ and the average speed is $v = d/t$. Thus the equation becomes $Q = Av$.

[link] shows an incompressible fluid flowing along a pipe of decreasing radius. Because the fluid is incompressible, the same amount of fluid must flow past any point in the tube in a given time to ensure continuity of flow. In this case, because the cross-sectional area of the pipe decreases, the velocity must necessarily increase. This logic can be extended to say that the flow rate must be the same at all points along the pipe. In particular, for points 1 and 2,

Equation:

$$\left.\begin{aligned} Q_1 &= Q_2 \\ A_1 v_1 &= A_2 v_2 \end{aligned}\right\}.$$

This is called the equation of continuity and is valid for any incompressible fluid. The consequences of the equation of continuity can be observed when water flows from a hose into a narrow spray nozzle: it emerges with a large speed—that is the purpose of the nozzle. Conversely, when a river empties into one end of a reservoir, the water slows considerably, perhaps picking up speed again when it leaves the other end of the reservoir. In other words, speed increases when cross-sectional area decreases, and speed decreases when cross-sectional area increases.



When a tube narrows, the same volume occupies a greater length. For the same volume to pass points 1 and 2 in a given time, the speed must be greater at point 2. The process is exactly reversible. If the fluid flows in the opposite direction, its speed will decrease when the tube widens. (Note that the relative volumes of the two cylinders and the corresponding velocity vector arrows are not drawn to scale.)

Since liquids are essentially incompressible, the equation of continuity is valid for all liquids. However, gases are compressible, and so the equation must be applied with caution to gases if they are subjected to compression or expansion.

Example:

Calculating Fluid Speed: Speed Increases When a Tube Narrows

A nozzle with a radius of 0.250 cm is attached to a garden hose with a radius of 0.900 cm. The flow rate through hose and nozzle is 0.500 L/s. Calculate the speed of the water (a) in the hose and (b) in the nozzle.

Strategy

We can use the relationship between flow rate and speed to find both velocities. We will use the subscript 1 for the hose and 2 for the nozzle.

Solution for (a)

First, we solve $Q = Av$ for v_1 and note that the cross-sectional area is $A = \pi r^2$, yielding

Equation:

$$v_1 = \frac{Q}{A_1} = \frac{Q}{\pi r_1^2}.$$

Substituting known values and making appropriate unit conversions yields

Equation:

$$v_1 = \frac{(0.500\ \mathrm{L/s})(10^{-3}\ \mathrm{m^3/L})}{\pi(9.00 \times 10^{-3}\ \mathrm{m})^2} = 1.96\ \mathrm{m/s}.$$

Solution for (b)

We could repeat this calculation to find the speed in the nozzle v_2, but we will use the equation of continuity to give a somewhat different insight. Using the equation which states

Equation:

$$A_1 v_1 = A_2 v_2,$$

solving for v_2 and substituting πr^2 for the cross-sectional area yields

Equation:

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi r_1^2}{\pi r_2^2} v_1 = \frac{r_{1^2}}{r_{2^2}} v_1.$$

Substituting known values,

Equation:

$$v_2 = \frac{(0.900\ \text{cm})^2}{(0.250\ \text{cm})^2} 1.96\ \text{m/s} = 25.5\ \text{m/s}.$$

Discussion

A speed of 1.96 m/s is about right for water emerging from a nozzleless hose. The nozzle produces a considerably faster stream merely by constricting the flow to a narrower tube.

The solution to the last part of the example shows that speed is inversely proportional to the *square* of the radius of the tube, making for large effects when radius varies. We can blow out a candle at quite a distance, for example, by pursing our lips, whereas blowing on a candle with our mouth wide open is quite ineffective.

In many situations, including in the cardiovascular system, branching of the flow occurs. The blood is pumped from the heart into arteries that subdivide into smaller arteries (arterioles) which branch into very fine vessels called capillaries. In this situation, continuity of flow is maintained but it is the *sum* of the flow rates in each of the branches in any portion along the tube that is maintained. The equation of continuity in a more general form becomes

Equation:

$$n_1 A_1 v_1 = n_2 A_2 v_2,$$

where n_1 and n_2 are the number of branches in each of the sections along the tube.

Example:

Calculating Flow Speed and Vessel Diameter: Branching in the Cardiovascular System

The aorta is the principal blood vessel through which blood leaves the heart in order to circulate around the body. (a) Calculate the average speed of the blood in the aorta if the flow rate is 5.0 L/min. The aorta has a radius of 10 mm. (b) Blood also flows through smaller blood vessels known as capillaries. When the rate of blood flow in the aorta is 5.0 L/min, the speed of blood in the capillaries is about 0.33 mm/s. Given that the average diameter of a capillary is 8.0 μm, calculate the number of capillaries in the blood circulatory system.

Strategy

We can use $Q = Av$ to calculate the speed of flow in the aorta and then use the general form of the equation of continuity to calculate the number of capillaries as all of the other variables are known.

Solution for (a)

The flow rate is given by $Q = Av$ or $v = \frac{Q}{\pi r^2}$ for a cylindrical vessel. Substituting the known values (converted to units of meters and seconds) gives

Equation:

$$v = \frac{(5.0\ \text{L/min})\left(10^{-3}\ \text{m}^3/\text{L}\right)(1\ \text{min}/60\ \text{s})}{\pi(0.010\ \text{m})^2} = 0.27\ \text{m/s}.$$

Solution for (b)

Using $n_1 A_1 v_1 = n_2 A_2 v_1$, assigning the subscript 1 to the aorta and 2 to the capillaries, and solving for n_2 (the number of capillaries) gives $n_2 = \frac{n_1 A_1 v_1}{A_2 v_2}$. Converting all quantities to units of meters and seconds and substituting into the equation above gives

Equation:

$$n_2 = \frac{(1)(\pi)\left(10 \times 10^{-3}\ \text{m}\right)^2 (0.27\ \text{m/s})}{(\pi)\left(4.0 \times 10^{-6}\ \text{m}\right)^2 \left(0.33 \times 10^{-3}\ \text{m/s}\right)} = 5.0 \times 10^9\ \text{capillaries}.$$

Discussion

Note that the speed of flow in the capillaries is considerably reduced relative to the speed in the aorta due to the significant increase in the total cross-sectional area at the capillaries. This low speed is to allow sufficient

time for effective exchange to occur although it is equally important for the flow not to become stationary in order to avoid the possibility of clotting. Does this large number of capillaries in the body seem reasonable? In active muscle, one finds about 200 capillaries per mm^3, or about 200×10^6 per 1 kg of muscle. For 20 kg of muscle, this amounts to about 4×10^9 capillaries.

Section Summary

- Flow rate Q is defined to be the volume V flowing past a point in time t, or $Q = \frac{V}{t}$ where V is volume and t is time.
- The SI unit of volume is m^3.
- Another common unit is the liter (L), which is $10^{-3}\ \text{m}^3$.
- Flow rate and velocity are related by $Q = Av$ where A is the cross-sectional area of the flow and v is its average velocity.
- For incompressible fluids, flow rate at various points is constant. That is,

  Equation:

$$\begin{aligned} Q_1 &= Q_2 \\ A_1 v_1 &= A_2 v_2 \\ n_1 A_1 v_1 &= n_2 A_2 v_2 \end{aligned} \quad .$$

Conceptual Questions

Exercise:

Problem:

What is the difference between flow rate and fluid velocity? How are they related?

Exercise:

Problem:

Many figures in the text show streamlines. Explain why fluid velocity is greatest where streamlines are closest together. (Hint: Consider the relationship between fluid velocity and the cross-sectional area through which it flows.)

Exercise:

Problem:

Identify some substances that are incompressible and some that are not.

Glossary

flow rate
: abbreviated Q, it is the volume V that flows past a particular point during a time t, or $Q = V/t$

liter
: a unit of volume, equal to 10^{-3} m^3

Bernoulli's Equation

- Explain the terms in Bernoulli's equation.
- Explain how Bernoulli's equation is related to conservation of energy.
- Explain how to derive Bernoulli's principle from Bernoulli's equation.
- Calculate with Bernoulli's principle.
- List some applications of Bernoulli's principle.

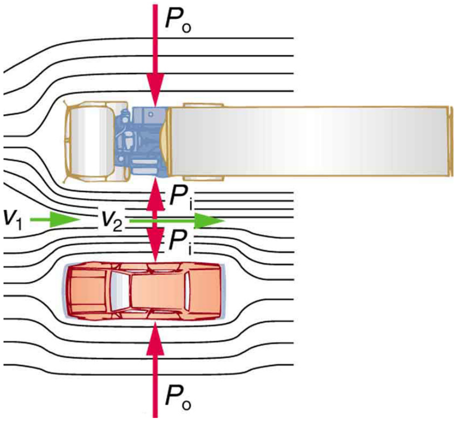
When a fluid flows into a narrower channel, its speed increases. That means its kinetic energy also increases. Where does that change in kinetic energy come from? The increased kinetic energy comes from the net work done on the fluid to push it into the channel and the work done on the fluid by the gravitational force, if the fluid changes vertical position. Recall the work-energy theorem,

Equation:

$$W_{\text{net}} = \frac{1}{2}\text{mv}^2 - \frac{1}{2}\text{mv}_0^2.$$

There is a pressure difference when the channel narrows. This pressure difference results in a net force on the fluid: recall that pressure times area equals force. The net work done increases the fluid's kinetic energy. As a result, the *pressure will drop in a rapidly-moving fluid*, whether or not the fluid is confined to a tube.

There are a number of common examples of pressure dropping in rapidly-moving fluids. Shower curtains have a disagreeable habit of bulging into the shower stall when the shower is on. The high-velocity stream of water and air creates a region of lower pressure inside the shower, and standard atmospheric pressure on the other side. The pressure difference results in a net force inward pushing the curtain in. You may also have noticed that when passing a truck on the highway, your car tends to veer toward it. The reason is the same—the high velocity of the air between the car and the truck creates a region of lower pressure, and the vehicles are pushed together by greater pressure on the outside. (See [link].) This effect was observed as far back as the mid-1800s, when it was found that trains passing in opposite directions tipped precariously toward one another.



An overhead view of a car passing a truck on a highway. Air passing between the vehicles flows in a narrower channel and must increase its speed (v_2 is greater than v_1), causing the pressure between them to drop (P_i is less than P_o). Greater pressure on the outside pushes the car and truck together.

Note:

Making Connections: Take-Home Investigation with a Sheet of Paper

Hold the short edge of a sheet of paper parallel to your mouth with one hand on each side of your mouth. The page should slant downward over your hands. Blow over the top of the page. Describe what happens and explain the reason for this behavior.

Bernoulli’s Equation

The relationship between pressure and velocity in fluids is described quantitatively by **Bernoulli’s equation**, named after its discoverer, the Swiss scientist Daniel Bernoulli (1700–1782). Bernoulli’s equation states that for an incompressible, frictionless fluid, the following sum is constant:
Equation:

$$P+\frac{1}{2}\rho v^2+\rho \mathrm{gh}=\text{constant},$$

where P is the absolute pressure, ρ is the fluid density, v is the velocity of the fluid, h is the height above some reference point, and g is the acceleration due to gravity. If we follow a small volume of fluid along its path, various quantities in the sum may change, but the total remains constant. Let the subscripts 1 and 2 refer to any two points along the path that the bit of fluid follows; Bernoulli’s equation becomes
Equation:

$$P_1+\frac{1}{2}\rho v_1^2+\rho\, gh_1=P_2+\frac{1}{2}\rho v_2^2+\rho\, gh_2\,.$$

Bernoulli’s equation is a form of the conservation of energy principle. Note that the second and third terms are the kinetic and potential energy with m replaced by ρ. In fact, each term in the equation has units of energy per unit volume. We can prove this for the second term by substituting $\rho=m/V$ into it and gathering terms:
Equation:

$$\frac{1}{2}\rho v^2=\frac{\frac{1}{2}\mathrm{mv}^2}{V}=\frac{\mathrm{KE}}{V}.$$

So $\frac{1}{2}\rho v^2$ is the kinetic energy per unit volume. Making the same substitution into the third term in the equation, we find
Equation:

$$\rho\, gh = \frac{mgh}{V} = \frac{\mathrm{PE_g}}{V},$$

so ρgh is the gravitational potential energy per unit volume. Note that pressure P has units of energy per unit volume, too. Since $P = F/A$, its units are $\mathrm{N/m^2}$. If we multiply these by m/m, we obtain $\mathrm{N \cdot m/m^3 = J/m^3}$, or energy per unit volume. Bernoulli's equation is, in fact, just a convenient statement of conservation of energy for an incompressible fluid in the absence of friction.

Note:
Making Connections: Conservation of Energy
Conservation of energy applied to fluid flow produces Bernoulli's equation. The net work done by the fluid's pressure results in changes in the fluid's KE and $\mathrm{PE_g}$ per unit volume. If other forms of energy are involved in fluid flow, Bernoulli's equation can be modified to take these forms into account. Such forms of energy include thermal energy dissipated because of fluid viscosity.

The general form of Bernoulli's equation has three terms in it, and it is broadly applicable. To understand it better, we will look at a number of specific situations that simplify and illustrate its use and meaning.

Bernoulli's Equation for Static Fluids

Let us first consider the very simple situation where the fluid is static—that is, $v_1 = v_2 = 0$. Bernoulli's equation in that case is
Equation:

$$P_1 + \rho\, gh_1 = P_2 + \rho\, gh_2\,.$$

We can further simplify the equation by taking $h_2 = 0$ (we can always choose some height to be zero, just as we often have done for other situations involving the gravitational force, and take all other heights to be relative to this). In that case, we get

Equation:

$$P_2 = P_1 + \rho gh_1 .$$

This equation tells us that, in static fluids, pressure increases with depth. As we go from point 1 to point 2 in the fluid, the depth increases by h_1, and consequently, P_2 is greater than P_1 by an amount ρgh_1. In the very simplest case, P_1 is zero at the top of the fluid, and we get the familiar relationship $P = \rho gh$. (Recall that $P = \rho gh$ and $\Delta \mathrm{PE_g} = \mathrm{mgh}$.) Bernoulli's equation includes the fact that the pressure due to the weight of a fluid is ρgh. Although we introduce Bernoulli's equation for fluid flow, it includes much of what we studied for static fluids in the preceding chapter.

Bernoulli's Principle—Bernoulli's Equation at Constant Depth

Another important situation is one in which the fluid moves but its depth is constant—that is, $h_1 = h_2$. Under that condition, Bernoulli's equation becomes

Equation:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2.$$

Situations in which fluid flows at a constant depth are so important that this equation is often called **Bernoulli's principle**. It is Bernoulli's equation for fluids at constant depth. (Note again that this applies to a small volume of fluid as we follow it along its path.) As we have just discussed, pressure drops as speed increases in a moving fluid. We can see this from Bernoulli's principle. For example, if v_2 is greater than v_1 in the equation, then P_2 must be less than P_1 for the equality to hold.

Example:
Calculating Pressure: Pressure Drops as a Fluid Speeds Up
In [link], we found that the speed of water in a hose increased from 1.96 m/s to 25.5 m/s going from the hose to the nozzle. Calculate the pressure in the hose, given that the absolute pressure in the nozzle is $1.01 \times 10^5 \text{ N/m}^2$ (atmospheric, as it must be) and assuming level, frictionless flow.

Strategy
Level flow means constant depth, so Bernoulli's principle applies. We use the subscript 1 for values in the hose and 2 for those in the nozzle. We are thus asked to find P_1.

Solution
Solving Bernoulli's principle for P_1 yields

Equation:

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho(v_2^2 - v_1^2).$$

Substituting known values,

Equation:

$$\begin{aligned} P_1 &= 1.01 \times 10^5 \text{ N/m}^2 \\ &\quad + \tfrac{1}{2}(10^3 \text{ kg/m}^3)\left[(25.5 \text{ m/s})^2 - (1.96 \text{ m/s})^2\right] \\ &= 4.24 \times 10^5 \text{ N/m}^2. \end{aligned}$$

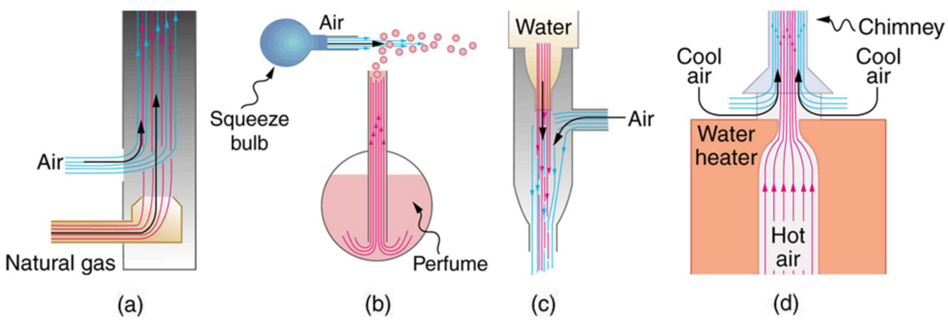
Discussion
This absolute pressure in the hose is greater than in the nozzle, as expected since v is greater in the nozzle. The pressure P_2 in the nozzle must be atmospheric since it emerges into the atmosphere without other changes in conditions.

Applications of Bernoulli's Principle

There are a number of devices and situations in which fluid flows at a constant height and, thus, can be analyzed with Bernoulli's principle.

Entrainment

People have long put the Bernoulli principle to work by using reduced pressure in high-velocity fluids to move things about. With a higher pressure on the outside, the high-velocity fluid forces other fluids into the stream. This process is called *entrainment*. Entrainment devices have been in use since ancient times, particularly as pumps to raise water small heights, as in draining swamps, fields, or other low-lying areas. Some other devices that use the concept of entrainment are shown in [link].



Examples of entrainment devices that use increased fluid speed to create low pressures, which then entrain one fluid into another. (a) A Bunsen burner uses an adjustable gas nozzle, entraining air for proper combustion. (b) An atomizer uses a squeeze bulb to create a jet of air that entrains drops of perfume. Paint sprayers and carburetors use very similar techniques to move their respective liquids. (c) A common aspirator uses a high-speed stream of water to create a region of lower pressure. Aspirators may be used as suction pumps in dental and surgical situations or for draining a flooded basement or producing a reduced pressure in a vessel. (d) The chimney of a water heater is designed to entrain air into the pipe leading through the ceiling.

Wings and Sails

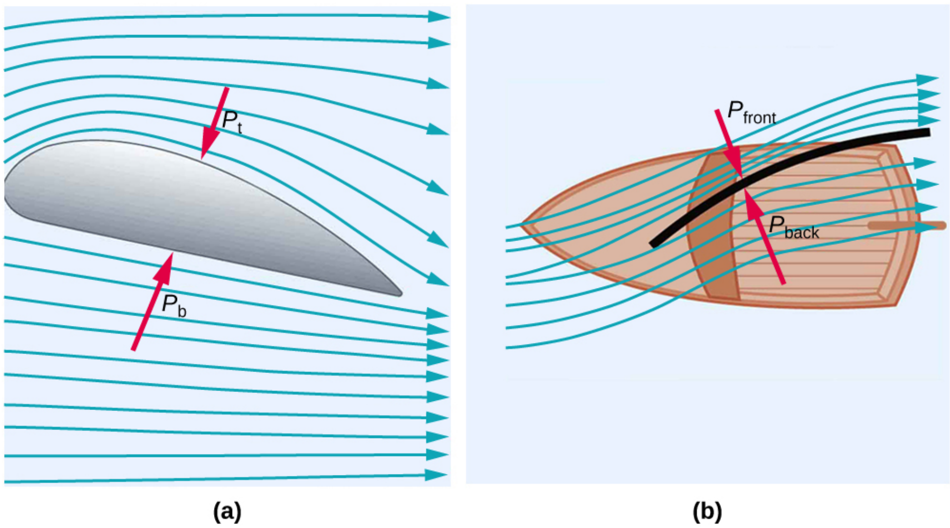
The airplane wing is a beautiful example of Bernoulli’s principle in action. [link](a) shows the characteristic shape of a wing. The wing is tilted upward at a small angle and the upper surface is longer, causing air to flow faster over it. The pressure on top of the wing is therefore reduced, creating a net upward force or lift. (Wings can also gain lift by pushing air downward, utilizing the conservation of momentum principle. The deflected air molecules result in an upward force on the wing — Newton’s third law.) Sails also have the characteristic shape of a wing. (See [link](b).) The pressure on the front side of the sail, P_{front}, is lower than the pressure on the back of the sail, P_{back}. This results in a forward force and even allows you to sail into the wind.

Note:
Making Connections: Take-Home Investigation with Two Strips of Paper
For a good illustration of Bernoulli’s principle, make two strips of paper, each about 15 cm long and 4 cm wide. Hold the small end of one strip up to your lips and let it drape over your finger. Blow across the paper. What happens? Now hold two strips of paper up to your lips, separated by your fingers. Blow between the strips. What happens?

Velocity measurement

[link] shows two devices that measure fluid velocity based on Bernoulli’s principle. The manometer in [link](a) is connected to two tubes that are small enough not to appreciably disturb the flow. The tube facing the oncoming fluid creates a dead spot having zero velocity ($v_1 = 0$) in front of it, while fluid passing the other tube has velocity v_2. This means that Bernoulli’s principle as stated in $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$ becomes
Equation:

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2.$$



(a) The Bernoulli principle helps explain lift generated by a wing. (b) Sails use the same technique to generate part of their thrust.

Thus pressure P_2 over the second opening is reduced by $\frac{1}{2}\rho v_2^2$, and so the fluid in the manometer rises by h on the side connected to the second opening, where

Equation:

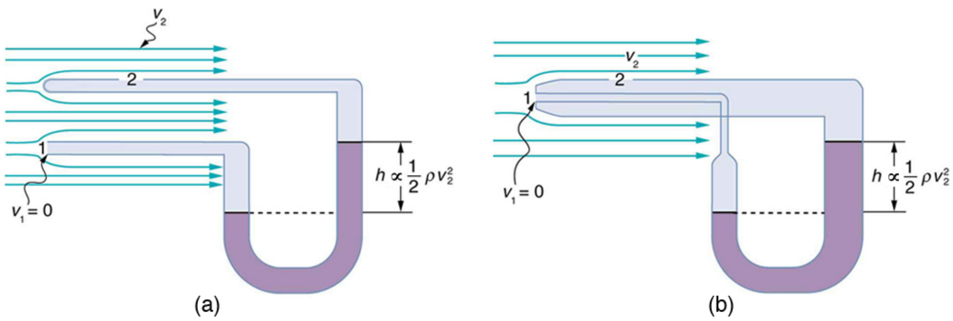
$$h \propto \frac{1}{2}\rho v_2^2.$$

(Recall that the symbol $\propto$ means "proportional to.") Solving for v_2, we see that

Equation:

$$v_2 \propto \sqrt{h}.$$

[link](b) shows a version of this device that is in common use for measuring various fluid velocities; such devices are frequently used as air speed indicators in aircraft.



Measurement of fluid speed based on Bernoulli's principle. (a) A manometer is connected to two tubes that are close together and small enough not to disturb the flow. Tube 1 is open at the end facing the flow. A dead spot having zero speed is created there. Tube 2 has an opening on the side, and so the fluid has a speed v across the opening; thus, pressure there drops. The difference in pressure at the manometer is $\frac{1}{2}\rho v_2^2$, and so h is proportional to $\frac{1}{2}\rho v_2^2$. (b) This type of velocity measuring device is a Prandtl tube, also known as a pitot tube.

Summary

- Bernoulli's equation states that the sum on each side of the following equation is constant, or the same at any two points in an incompressible frictionless fluid:
  Equation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho\, gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho \mathrm{gh}_2.$$

- Bernoulli's principle is Bernoulli's equation applied to situations in which depth is constant. The terms involving depth (or height h) subtract out, yielding
  Equation:
  $$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2.$$
- Bernoulli's principle has many applications, including entrainment, wings and sails, and velocity measurement.

Conceptual Questions

Exercise:

Problem:

You can squirt water a considerably greater distance by placing your thumb over the end of a garden hose and then releasing, than by leaving it completely uncovered. Explain how this works.

Exercise:

Problem:

Water is shot nearly vertically upward in a decorative fountain and the stream is observed to broaden as it rises. Conversely, a stream of water falling straight down from a faucet narrows. Explain why, and discuss whether surface tension enhances or reduces the effect in each case.

Exercise:

Problem:

Look back to [link]. Answer the following two questions. Why is P_{o} less than atmospheric? Why is P_{o} greater than P_{i}?

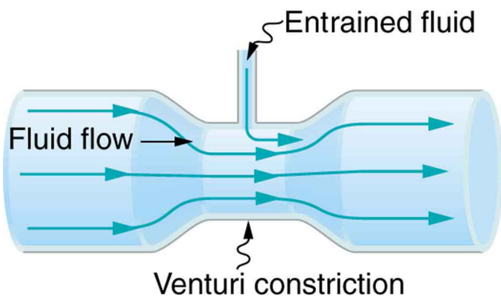
Exercise:

Problem: Give an example of entrainment not mentioned in the text.

Exercise:

Problem:

Many entrainment devices have a constriction, called a Venturi, such as shown in [link]. How does this bolster entrainment?



A tube with a narrow segment designed to enhance entrainment is called a Venturi. These are very commonly used in carburetors and aspirators.

Exercise:

Problem:

Some chimney pipes have a T-shape, with a crosspiece on top that helps draw up gases whenever there is even a slight breeze. Explain how this works in terms of Bernoulli's principle.

Exercise:

Problem:

Is there a limit to the height to which an entrainment device can raise a fluid? Explain your answer.

Exercise:

Problem:

Why is it preferable for airplanes to take off into the wind rather than with the wind?

Exercise:

Problem:

Roofs are sometimes pushed off vertically during a tropical cyclone, and buildings sometimes explode outward when hit by a tornado. Use Bernoulli's principle to explain these phenomena.

Exercise:

Problem: Why does a sailboat need a keel?

Exercise:

Problem:

It is dangerous to stand close to railroad tracks when a rapidly moving commuter train passes. Explain why atmospheric pressure would push you toward the moving train.

Exercise:

Problem:

Water pressure inside a hose nozzle can be less than atmospheric pressure due to the Bernoulli effect. Explain in terms of energy how the water can emerge from the nozzle against the opposing atmospheric pressure.

Exercise:

Problem:

A perfume bottle or atomizer sprays a fluid that is in the bottle. ([link].) How does the fluid rise up in the vertical tube in the bottle?

Atomizer: perfume bottle with tube to carry perfume up through the bottle. (credit: Antonia Foy, Flickr)

Exercise:

Problem:

If you lower the window on a car while moving, an empty plastic bag can sometimes fly out the window. Why does this happen?

Glossary

Bernoulli's equation
: the equation resulting from applying conservation of energy to an incompressible frictionless fluid: $P + 1/2pv^2 + pgh = \text{constant}$, through the fluid

Bernoulli’s principle

Bernoulli’s equation applied at constant depth: $P_1 + 1/2pv_1^2 = P_2 + 1/2pv_2^2$

Introduction to Thermal Physics
class="introduction"

The welder’s gloves and helmet protect him from the electric arc that transfers enough thermal energy to melt the rod, spray sparks, and burn the retina of an unprotected eye. The thermal energy can be felt on exposed skin a few meters away, and its light can be seen for kilometers. (credit: Kevin S. O’Brien/U.S. Navy)

Heat is something familiar to each of us. We feel the warmth of the summer Sun, the chill of a clear summer night, the heat of coffee after a winter stroll, and the cooling effect of our sweat. Heat transfer is maintained by temperature differences. Manifestations of **heat transfer**—the movement of heat energy from one place or material to another—are apparent throughout the universe. Heat from beneath Earth’s surface is brought to the surface in flows of incandescent lava. The Sun warms Earth’s surface and is the source of much of the energy we find on it. Rising levels of atmospheric carbon dioxide threaten to trap more of the Sun’s energy, perhaps fundamentally altering the ecosphere. In space, supernovas explode, briefly radiating more heat than an entire galaxy does.

What is heat? How do we define it? How is it related to temperature? What are heat’s effects? How is it related to other forms of energy and to work? We will find that, in spite of the richness of the phenomena, there is a small set of underlying physical principles that unite the subjects and tie them to other fields.

Most notably, heat transfer can be used to do work. It can also be converted to any other form of energy. A car engine, for example, burns fuel for heat transfer into a gas. Work is done by the gas as it exerts a force through a distance, converting its energy into a variety of other forms—into the car’s

kinetic or gravitational potential energy; into electrical energy to run the spark plugs, radio, and lights; and back into stored energy in the car’s battery. But most of the heat transfer produced from burning fuel in the engine does not do work on the gas-the engine is quite inefficient.

Basic physical laws govern how heat transfer for doing work takes place and place insurmountable limits onto its efficiency. This chapter will explore these laws as well as some applications and concepts associated with them. These topics are part of thermodynamics—the study of heat transfer and its relationship to doing work.

In a typical thermometer like this one, the alcohol, with a red dye, expands more rapidly than the glass containing it. When the thermometer’s temperature increases, the liquid from the bulb is forced into the narrow tube, producing a large change in the length of the column for a small change in temperature. (credit: Chemical Engineer, Wikimedia Commons)

Temperature

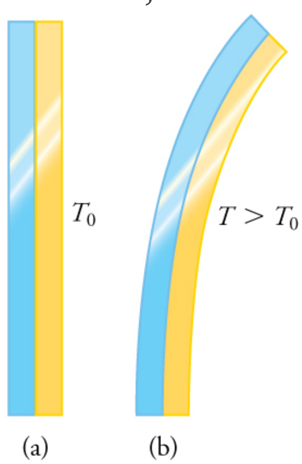
- Define temperature.
- Convert temperatures between the Celsius, Fahrenheit, and Kelvin scales.
- Define thermal equilibrium.
- State the zeroth law of thermodynamics.

The concept of temperature has evolved from the common concepts of hot and cold. Human perception of what feels hot or cold is a relative one. For example, if you place one hand in hot water and the other in cold water, and then place both hands in tepid water, the tepid water will feel cool to the hand that was in hot water, and warm to the one that was in cold water. The scientific definition of temperature is less ambiguous than your senses of hot and cold. **Temperature** is operationally defined to be what we measure with a thermometer. (Many physical quantities are defined solely in terms of how they are measured. We shall see later how temperature is related to the kinetic energies of atoms and molecules, a more physical explanation.) Two accurate thermometers, one placed in hot water and the other in cold water, will show the hot water to have a higher temperature. If they are then placed in the tepid water, both will give identical readings (within measurement uncertainties). In this section, we discuss temperature, its measurement by thermometers, and its relationship to thermal equilibrium. Again, temperature is the quantity measured by a thermometer.

Note:
Misconception Alert: Human Perception vs. Reality
On a cold winter morning, the wood on a porch feels warmer than the metal of your bike. The wood and bicycle are in thermal equilibrium with the outside air, and are thus the same temperature. They *feel* different because of the difference in the way that they conduct heat away from your skin. The metal conducts heat away from your body faster than the wood does. This is just one example demonstrating that the human sense of hot and cold is not determined by temperature alone.
Another factor that affects our perception of temperature is humidity. Most people feel much hotter on hot, humid days than on hot, dry days. This is

because on humid days, sweat does not evaporate from the skin as efficiently as it does on dry days. It is the evaporation of sweat (or water from a sprinkler or pool) that cools us off.

Any physical property that depends on temperature, and whose response to temperature is reproducible, can be used as the basis of a thermometer. Because many physical properties depend on temperature, the variety of thermometers is remarkable. For example, volume increases with temperature for most substances. This property is the basis for the common alcohol thermometer, the old mercury thermometer, and the bimetallic strip ([link]). Other properties used to measure temperature include electrical resistance and color, as shown in [link], and the emission of infrared radiation, as shown in [link].



The curvature of a bimetallic strip depends on temperature. (a) The strip is straight at the starting temperature, where its two

components have the same length. (b) At a higher temperature, this strip bends to the right, because the metal on the left has expanded more than the metal on the right.

Each of the six squares on this plastic (liquid crystal) thermometer contains a film of a different heat-sensitive liquid crystal material. Below 95ºF, all six squares are black. When the plastic thermometer is exposed to temperature that increases to 95ºF, the first liquid crystal square changes color. When the temperature increases above 96.8ºF the second liquid crystal square also

changes color, and so forth. (credit: Arkrishna, Wikimedia Commons)

Fireman Jason Ormand uses a pyrometer to check the temperature of an aircraft carrier’s ventilation system. Infrared radiation (whose emission varies with temperature) from the vent is measured and a temperature readout is quickly produced.

Infrared measurements are also frequently used as a measure of body temperature. These modern thermometers, placed in the ear canal, are more accurate than alcohol thermometers placed under the tongue or in the armpit. (credit: Lamel J. Hinton/U.S. Navy)

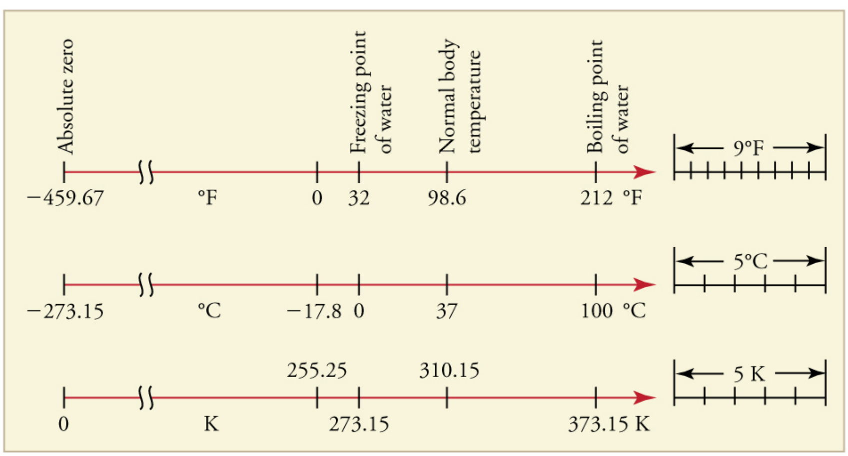
Temperature Scales

Thermometers are used to measure temperature according to well-defined scales of measurement, which use pre-defined reference points to help compare quantities. The three most common temperature scales are the Fahrenheit, Celsius, and Kelvin scales. A temperature scale can be created by identifying two easily reproducible temperatures. The freezing and boiling temperatures of water at standard atmospheric pressure are commonly used.

The **Celsius** scale (which replaced the slightly different *centigrade* scale) has the freezing point of water at 0°C and the boiling point at 100°C. Its unit is the **degree Celsius**$(^{\circ}\text{C})$. On the **Fahrenheit** scale (still the most frequently used in the United States), the freezing point of water is at 32°F

and the boiling point is at $212°F$. The unit of temperature on this scale is the **degree Fahrenheit** $(°F)$. Note that a temperature difference of one degree Celsius is greater than a temperature difference of one degree Fahrenheit. Only 100 Celsius degrees span the same range as 180 Fahrenheit degrees, thus one degree on the Celsius scale is 1.8 times larger than one degree on the Fahrenheit scale $180/100 = 9/5$.

The **Kelvin** scale is the temperature scale that is commonly used in science. It is an *absolute temperature* scale defined to have 0 K at the lowest possible temperature, called **absolute zero**. The official temperature unit on this scale is the *kelvin*, which is abbreviated K, and is not accompanied by a degree sign. The freezing and boiling points of water are 273.15 K and 373.15 K, respectively. Thus, the magnitude of temperature differences is the same in units of kelvins and degrees Celsius. Unlike other temperature scales, the Kelvin scale is an absolute scale. It is used extensively in scientific work because a number of physical quantities, such as the volume of an ideal gas, are directly related to absolute temperature. The kelvin is the SI unit used in scientific work.

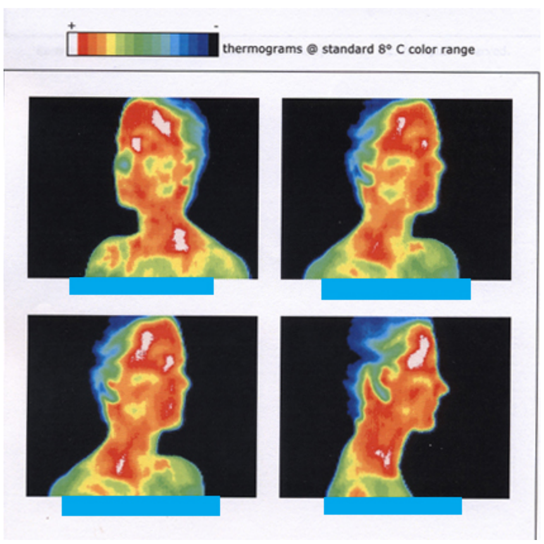


Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown.

The relationships between the three common temperature scales is shown in [link].

Temperature Ranges in the Universe

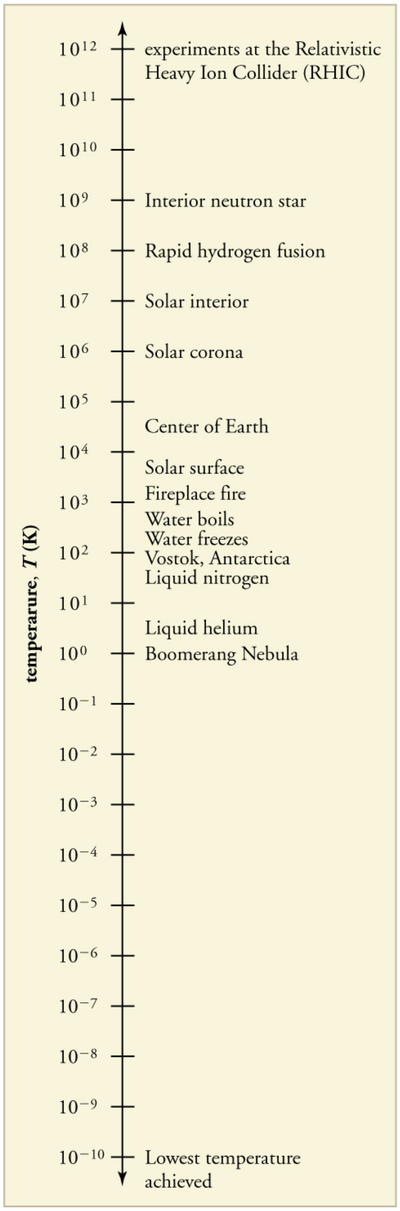
[link] shows the wide range of temperatures found in the universe. Human beings have been known to survive with body temperatures within a small range, from $24^{\circ}C$ to $44^{\circ}C$ $(75^{\circ}F$ to $111^{\circ}F)$. The average normal body temperature is usually given as $37.0^{\circ}C$ $(98.6^{\circ}F)$, and variations in this temperature can indicate a medical condition: a fever, an infection, a tumor, or circulatory problems (see [link]).



This image of radiation from a person's body (an infrared thermograph) shows the location of temperature abnormalities in the upper body. Dark blue corresponds to cold areas and red to white corresponds to hot areas.

An elevated temperature might be an indication of malignant tissue (a cancerous tumor in the breast, for example), while a depressed temperature might be due to a decline in blood flow from a clot. In this case, the abnormalities are caused by a condition called hyperhidrosis. (credit: Porcelina81, Wikimedia Commons)

The lowest temperatures ever recorded have been measured during laboratory experiments: $4.5 \times 10^{-10}\ \text{K}$ at the Massachusetts Institute of Technology (USA), and $1.0 \times 10^{-10}\ \text{K}$ at Helsinki University of Technology (Finland). In comparison, the coldest recorded place on Earth's surface is Vostok, Antarctica at 183 K (-89°C), and the coldest place (outside the lab) known in the universe is the Boomerang Nebula, with a temperature of 1 K.



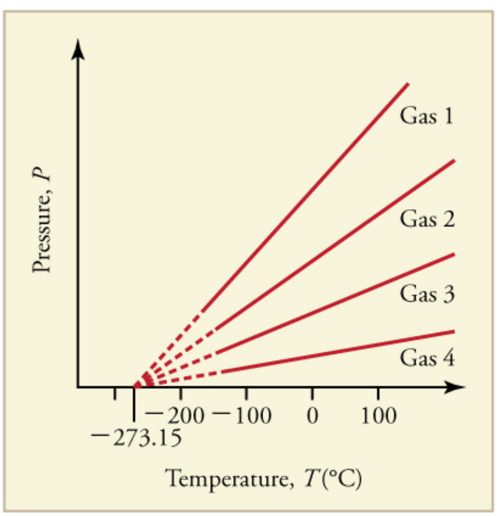
Each increment on this logarithmic scale indicates an increase by a factor of ten, and thus illustrates the tremendous range of temperatures in nature. Note that zero on a

logarithmic scale would occur off the bottom of the page at infinity.

Note:

Making Connections: Absolute Zero

What is absolute zero? Absolute zero is the temperature at which all molecular motion has ceased. The concept of absolute zero arises from the behavior of gases. [link] shows how the pressure of gases at a constant volume decreases as temperature decreases. Various scientists have noted that the pressures of gases extrapolate to zero at the same temperature, $-273.15^{\circ}C$. This extrapolation implies that there is a lowest temperature. This temperature is called *absolute zero*. Today we know that most gases first liquefy and then freeze, and it is not actually possible to reach absolute zero. The numerical value of absolute zero temperature is $-273.15^{\circ}C$ or 0 K.



Graph of pressure versus

temperature for various gases kept at a constant volume. Note that all of the graphs extrapolate to zero pressure at the same temperature.

Thermal Equilibrium and the Zeroth Law of Thermodynamics

Thermometers actually take their *own* temperature, not the temperature of the object they are measuring. This raises the question of how we can be certain that a thermometer measures the temperature of the object with which it is in contact. It is based on the fact that any two systems placed in *thermal contact* (meaning heat transfer can occur between them) will reach the same temperature. That is, heat will flow from the hotter object to the cooler one until they have exactly the same temperature. The objects are then in **thermal equilibrium**, and no further changes will occur. The systems interact and change because their temperatures differ, and the changes stop once their temperatures are the same. Thus, if enough time is allowed for this transfer of heat to run its course, the temperature a thermometer registers *does* represent the system with which it is in thermal equilibrium. Thermal equilibrium is established when two bodies are in contact with each other and can freely exchange energy.

Furthermore, experimentation has shown that if two systems, A and B, are in thermal equilibrium with each another, and B is in thermal equilibrium with a third system C, then A is also in thermal equilibrium with C. This conclusion may seem obvious, because all three have the same temperature, but it is basic to thermodynamics. It is called the **zeroth law of thermodynamics**.

Note:

The Zeroth Law of Thermodynamics
If two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

This law was postulated in the 1930s, after the first and second laws of thermodynamics had been developed and named. It is called the *zeroth law* because it comes logically before the first and second laws. An example of this law in action is seen in babies in incubators: babies in incubators normally have very few clothes on, so to an observer they look as if they may not be warm enough. However, the temperature of the air, the cot, and the baby is the same, because they are in thermal equilibrium, which is accomplished by maintaining air temperature to keep the baby comfortable.
Exercise:
Check Your Understanding

Problem: Does the temperature of a body depend on its size?

Solution:

No, the system can be divided into smaller parts each of which is at the same temperature. We say that the temperature is an *intensive* quantity. Intensive quantities are independent of size.

Section Summary

- Temperature is the quantity measured by a thermometer.
- Temperature is related to the average kinetic energy of atoms and molecules in a system.
- Absolute zero is the temperature at which there is no molecular motion.
- There are three main temperature scales: Celsius, Fahrenheit, and Kelvin.

- Temperatures on one scale can be converted to temperatures on another scale using the following equations:
  Equation:

  $$T_{°F} = \frac{9}{5} T_{°C} + 32$$

  Equation:

  $$T_{°C} = \frac{5}{9} \left(T_{°F} - 32\right)$$

  Equation:

  $$T_{K} = T_{°C} + 273.15$$

  Equation:

  $$T_{°C} = T_{K} - 273.15$$

- Systems are in thermal equilibrium when they have the same temperature.
- Thermal equilibrium occurs when two bodies are in contact with each other and can freely exchange energy.
- The zeroth law of thermodynamics states that when two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

Conceptual Questions

Exercise:

Problem:

What does it mean to say that two systems are in thermal equilibrium?

Exercise:

Problem:

Give an example of a physical property that varies with temperature and describe how it is used to measure temperature.

Exercise:

Problem:

When a cold alcohol thermometer is placed in a hot liquid, the column of alcohol goes *down* slightly before going up. Explain why.

Exercise:

Problem:

If you add boiling water to a cup at room temperature, what would you expect the final equilibrium temperature of the unit to be? You will need to include the surroundings as part of the system. Consider the zeroth law of thermodynamics.

Problems & Exercises

Exercise:

Problem:

What is the Fahrenheit temperature of a person with a $39.0^{\circ}C$ fever?

Solution:

$102^{\circ}F$

Exercise:

Problem:

Frost damage to most plants occurs at temperatures of $28.0^{\circ}F$ or lower. What is this temperature on the Kelvin scale?

Exercise:

Problem:

To conserve energy, room temperatures are kept at $68.0^{\circ}F$ in the winter and $78.0^{\circ}F$ in the summer. What are these temperatures on the Celsius scale?

Solution:

$20.0^{\circ}C$ and $25.6^{\circ}C$

Exercise:

Problem:

A tungsten light bulb filament may operate at 2900 K. What is its Fahrenheit temperature? What is this on the Celsius scale?

Exercise:

Problem:

The surface temperature of the Sun is about 5750 K. What is this temperature on the Fahrenheit scale?

Solution:

$9890^{\circ}F$

Exercise:

Problem:

One of the hottest temperatures ever recorded on the surface of Earth was $134^{\circ}F$ in Death Valley, CA. What is this temperature in Celsius degrees? What is this temperature in Kelvin?

Exercise:

Problem:

(a) Suppose a cold front blows into your locale and drops the temperature by 40.0 Fahrenheit degrees. How many degrees Celsius does the temperature decrease when there is a 40.0°F decrease in temperature? (b) Show that any change in temperature in Fahrenheit degrees is nine-fifths the change in Celsius degrees.

Solution:

(a) 22.2°C

(b)
$$\begin{array}{lll} \Delta T(^{\circ}\text{F}) & = & T_2(^{\circ}\text{F}) - T_1(^{\circ}\text{F}) \\ & = & \frac{9}{5}T_2(^{\circ}\text{C}) + 32.0^{\circ} - \left(\frac{9}{5}T_1(^{\circ}\text{C}) + 32.0^{\circ}\right) \\ & = & \frac{9}{5}\left(T_2(^{\circ}\text{C}) - T_1(^{\circ}\text{C})\right) = \frac{9}{5}\Delta T(^{\circ}\text{C}) \end{array}$$

Exercise:

Problem:

(a) At what temperature do the Fahrenheit and Celsius scales have the same numerical value? (b) At what temperature do the Fahrenheit and Kelvin scales have the same numerical value?

Glossary

temperature
: the quantity measured by a thermometer

Celsius scale
: temperature scale in which the freezing point of water is 0°C and the boiling point of water is 100°C

degree Celsius
: unit on the Celsius temperature scale

Fahrenheit scale

temperature scale in which the freezing point of water is $32^{\circ}F$ and the boiling point of water is $212^{\circ}F$

degree Fahrenheit
unit on the Fahrenheit temperature scale

Kelvin scale
temperature scale in which 0 K is the lowest possible temperature, representing absolute zero

absolute zero
the lowest possible temperature; the temperature at which all molecular motion ceases

thermal equilibrium
the condition in which heat no longer flows between two objects that are in contact; the two objects have the same temperature

zeroth law of thermodynamics
law that states that if two objects are in thermal equilibrium, and a third object is in thermal equilibrium with one of those objects, it is also in thermal equilibrium with the other object

The Ideal Gas Law

- State and explain the ideal gas law using Boltzmann's constant
- Use the ideal gas law to calculate pressure change, temperature change, volume change, or the number of molecules in a given volume.

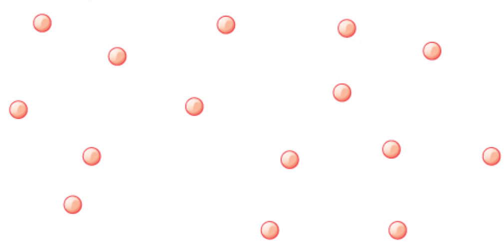
The air inside this hot air balloon flying over Putrajaya, Malaysia, is hotter than the ambient air. As a result, the balloon experiences a buoyant force pushing it upward. (credit: Kevin Poh, Flickr)

In this section, we continue to explore the thermal behavior of gases. In particular, we examine the characteristics of atoms and molecules that

compose gases. (Most gases, for example nitrogen, N_2, and oxygen, O_2, are composed of two or more atoms. We will primarily use the term “molecule” in discussing a gas because the term can also be applied to monatomic gases, such as helium.)

Gases are easily compressed. Gases expand and contract very rapidly with temperature changes. In addition, you will note that most gases expand at the *same* rate, or have the same β. This raises the question as to why gases should all act in nearly the same way, when liquids and solids have widely varying expansion rates.

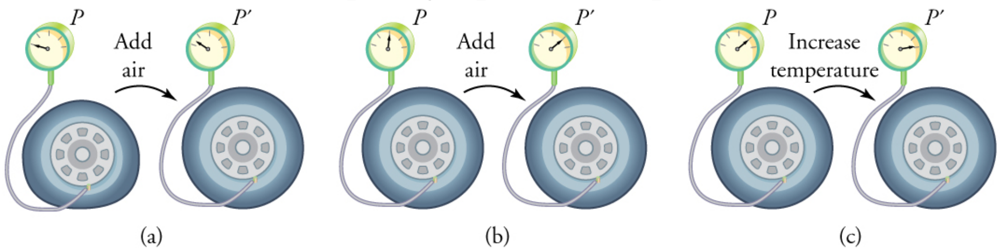
The answer lies in the large separation of atoms and molecules in gases, compared to their sizes, as illustrated in [link]. Because atoms and molecules have large separations, forces between them can be ignored, except when they collide with each other during collisions. The motion of atoms and molecules (at temperatures well above the boiling temperature) is fast, such that the gas occupies all of the accessible volume and the expansion of gases is rapid. In contrast, in liquids and solids, atoms and molecules are closer together and are quite sensitive to the forces between them.



Atoms and molecules in a gas are typically widely separated, as shown. Because the forces between them are quite weak at these distances, the properties of a gas depend more on the number of atoms per unit volume and on

temperature than on the type of atom.

To get some idea of how pressure, temperature, and volume of a gas are related to one another, consider what happens when you pump air into an initially deflated tire. The tire's volume first increases in direct proportion to the amount of air injected, without much increase in the tire pressure. Once the tire has expanded to nearly its full size, the walls limit volume expansion. If we continue to pump air into it, the pressure increases. The pressure will further increase when the car is driven and the tires move. Most manufacturers specify optimal tire pressure for cold tires. (See [link].)



(a) When air is pumped into a deflated tire, its volume first increases without much increase in pressure. (b) When the tire is filled to a certain point, the tire walls resist further expansion and the pressure increases with more air. (c) Once the tire is inflated, its pressure increases with temperature.

At room temperatures, collisions between atoms and molecules can be ignored. In this case, the gas is called an ideal gas, in which case the relationship between the pressure, volume, and temperature is given by the equation of state called the ideal gas law.

Note:
Ideal Gas Law
The **ideal gas law** states that

Equation:

$$\mathrm{PV} = \mathrm{NkT},$$

where P is the absolute pressure of a gas, V is the volume it occupies, N is the number of atoms and molecules in the gas, and T is its absolute temperature. The constant k is called the **Boltzmann constant** in honor of Austrian physicist Ludwig Boltzmann (1844–1906) and has the value

Equation:

$$k = 1.38 \times 10^{-23}\ \mathrm{J/K}.$$

The ideal gas law can be derived from basic principles, but was originally deduced from experimental measurements of Charles' law (that volume occupied by a gas is proportional to temperature at a fixed pressure) and from Boyle's law (that for a fixed temperature, the product PV is a constant). In the ideal gas model, the volume occupied by its atoms and molecules is a negligible fraction of V. The ideal gas law describes the behavior of real gases under most conditions. (Note, for example, that N is the total number of atoms and molecules, independent of the type of gas.)

Let us see how the ideal gas law is consistent with the behavior of filling the tire when it is pumped slowly and the temperature is constant. At first, the pressure P is essentially equal to atmospheric pressure, and the volume V increases in direct proportion to the number of atoms and molecules N put into the tire. Once the volume of the tire is constant, the equation $\mathrm{PV} = \mathrm{NkT}$ predicts that the pressure should increase in proportion to *the number N of atoms and molecules*.

Example:
Calculating Pressure Changes Due to Temperature Changes: Tire Pressure

Suppose your bicycle tire is fully inflated, with an absolute pressure of 7.00×10^5 Pa (a gauge pressure of just under $90.0\ \mathrm{lb/in}^2$) at a temperature of 18.0ºC. What is the pressure after its temperature has risen to 35.0ºC? Assume that there are no appreciable leaks or changes in volume.

Strategy

The pressure in the tire is changing only because of changes in temperature. First we need to identify what we know and what we want to know, and then identify an equation to solve for the unknown.

We know the initial pressure $P_0 = 7.00 \times 10^5\ \mathrm{Pa}$, the initial temperature $T_0 = 18.0ºC$, and the final temperature $T_\mathrm{f} = 35.0ºC$. We must find the final pressure P_f. How can we use the equation $PV = NkT$? At first, it may seem that not enough information is given, because the volume V and number of atoms N are not specified. What we can do is use the equation twice: $P_0V_0 = NkT_0$ and $P_\mathrm{f}V_\mathrm{f} = NkT_\mathrm{f}$. If we divide $P_\mathrm{f}V_\mathrm{f}$ by P_0V_0 we can come up with an equation that allows us to solve for P_f.

Equation:

$$\frac{P_\mathrm{f}V_\mathrm{f}}{P_0V_0} = \frac{N_\mathrm{f}kT_\mathrm{f}}{N_0kT_0}$$

Since the volume is constant, V_f and V_0 are the same and they cancel out. The same is true for N_f and N_0, and k, which is a constant. Therefore,

Equation:

$$\frac{P_\mathrm{f}}{P_0} = \frac{T_\mathrm{f}}{T_0}.$$

We can then rearrange this to solve for P_f:

Equation:

$$P_\mathrm{f} = P_0\frac{T_\mathrm{f}}{T_0},$$

where the temperature must be in units of kelvins, because T_0 and T_f are absolute temperatures.

Solution

1. Convert temperatures from Celsius to Kelvin.
Equation:

$$T_0 = (18.0 + 273)\text{K} = 291\text{ K}$$
$$T_\text{f} = (35.0 + 273)\text{K} = 308\text{ K}$$

2. Substitute the known values into the equation.
Equation:

$$P_\text{f} = P_0 \frac{T_\text{f}}{T_0} = 7.00 \times 10^5\text{ Pa}\left(\frac{308\text{ K}}{291\text{ K}}\right) = 7.41 \times 10^5\text{ Pa}$$

Discussion
The final temperature is about 6% greater than the original temperature, so the final pressure is about 6% greater as well. Note that *absolute* pressure and *absolute* temperature must be used in the ideal gas law.

Note:
Making Connections: Take-Home Experiment—Refrigerating a Balloon
Inflate a balloon at room temperature. Leave the inflated balloon in the refrigerator overnight. What happens to the balloon, and why?

Example:
Calculating the Number of Molecules in a Cubic Meter of Gas
How many molecules are in a typical object, such as gas in a tire or water in a drink? We can use the ideal gas law to give us an idea of how large N typically is.
Calculate the number of molecules in a cubic meter of gas at standard temperature and pressure (STP), which is defined to be 0^oC and atmospheric pressure.
Strategy
Because pressure, volume, and temperature are all specified, we can use the ideal gas law $\text{PV} = \text{NkT}$, to find N.

Solution
1. Identify the knowns.
Equation:

$$\begin{aligned} T &= 0^{\mathrm{o}}\mathrm{C} = 273\ \mathrm{K} \\ P &= 1.01 \times 10^{5}\ \mathrm{Pa} \\ V &= 1.00\ \mathrm{m}^{3} \\ k &= 1.38 \times 10^{-23}\ \mathrm{J/K} \end{aligned}$$

2. Identify the unknown: number of molecules, N.
3. Rearrange the ideal gas law to solve for N.
Equation:

$$\mathrm{PV} = \mathrm{NkT}$$
$$N = \frac{\mathrm{PV}}{\mathrm{kT}}$$

4. Substitute the known values into the equation and solve for N.
Equation:

$$N = \frac{\mathrm{PV}}{\mathrm{kT}} = \frac{\left(1.01 \times 10^{5}\ \mathrm{Pa}\right)\left(1.00\ \mathrm{m}^{3}\right)}{\left(1.38 \times 10^{-23}\ \mathrm{J/K}\right)(273\ \mathrm{K})} = 2.68 \times 10^{25}\ \mathrm{molecules}$$

Discussion
The calculated number, 2.68×10^{25}, is certainly very large. You might say that the volume of a cubic meter is also large ($1\ \mathrm{m}^{3} = 1000\ \mathrm{L}$), but even in a small volume of $1\ \mathrm{cm}^{3}$, which is about size of a thimble ($1\ \mathrm{cm}^{3} = 10^{-6}\ \mathrm{m}^{3}$), a gas at STP has 2.68×10^{19} molecules in it (still a very large number). Once again, note that N is the same for all types or mixtures of gases.

Section Summary

- The ideal gas law relates the pressure and volume of a gas to the number of gas molecules and the temperature of the gas.

- The ideal gas law can be written in terms of the number of molecules of gas:
  Equation:
  $$\mathrm{PV} = \mathrm{NkT},$$
  where P is pressure, V is volume, T is temperature, N is number of molecules, and k is the Boltzmann constant
  Equation:
  $$k = 1.38 \times 10^{-23}\ \mathrm{J/K}.$$
- The ideal gas law is generally valid at temperatures well above the boiling temperature.

Conceptual Questions

Exercise:

Problem:

Under what circumstances would you expect a gas to behave significantly differently than predicted by the ideal gas law?

Exercise:

Problem:

A constant-volume gas thermometer contains a fixed amount of gas. What property of the gas is measured to indicate its temperature?

Problems & Exercises

Exercise:

Problem:

The gauge pressure in your car tires is $2.50 \times 10^5 \ \mathrm{N/m^2}$ at a temperature of $35.0^\circ\mathrm{C}$ when you drive it onto a ferry boat to Alaska. What is their gauge pressure later, when their temperature has dropped to $-40.0^\circ\mathrm{C}$?

Solution:

1.62 atm

Exercise:

Problem:

Convert an absolute pressure of $7.00 \times 10^5 \ \mathrm{N/m^2}$ to gauge pressure in $\mathrm{lb/in^2}$. (This value was stated to be just less than $90.0 \ \mathrm{lb/in^2}$ in [link]. Is it?)

Exercise:

Problem:

Suppose a gas-filled incandescent light bulb is manufactured so that the gas inside the bulb is at atmospheric pressure when the bulb has a temperature of $20.0^\circ\mathrm{C}$. (a) Find the gauge pressure inside such a bulb when it is hot, assuming its average temperature is $60.0^\circ\mathrm{C}$ (an approximation) and neglecting any change in volume due to thermal expansion or gas leaks. (b) The actual final pressure for the light bulb will be less than calculated in part (a) because the glass bulb will expand. What will the actual final pressure be, taking this into account? Is this a negligible difference?

Solution:

(a) 0.136 atm

(b) 0.135 atm. The difference between this value and the value from part (a) is negligible.

Exercise:

Problem:

Large helium-filled balloons are used to lift scientific equipment to high altitudes. (a) What is the pressure inside such a balloon if it starts out at sea level with a temperature of 10.0°C and rises to an altitude where its volume is twenty times the original volume and its temperature is -50.0°C? (b) What is the gauge pressure? (Assume atmospheric pressure is constant.)

Exercise:

Problem:

In the text, it was shown that $N/V = 2.68 \times 10^{25}\ \text{m}^{-3}$ for gas at STP. (a) Show that this quantity is equivalent to $N/V = 2.68 \times 10^{19}\ \text{cm}^{-3}$, as stated. (b) About how many atoms are there in one μm^3 (a cubic micrometer) at STP? (c) What does your answer to part (b) imply about the separation of atoms and molecules?

Exercise:

Problem:

An airplane passenger has $100\ \text{cm}^3$ of air in his stomach just before the plane takes off from a sea-level airport. What volume will the air have at cruising altitude if cabin pressure drops to $7.50 \times 10^4\ \text{N/m}^2$?

Exercise:

Problem:

An expensive vacuum system can achieve a pressure as low as $1.00 \times 10^{-7}\ \text{N/m}^2$ at 20°C. How many atoms are there in a cubic centimeter at this pressure and temperature?

Exercise:

Problem:

The number density of gas atoms at a certain location in the space above our planet is about $1.00 \times 10^{11}\ \text{m}^{-3}$, and the pressure is $2.75 \times 10^{-10}\ \text{N/m}^2$ in this space. What is the temperature there?

Solution:

-73.9°C

Exercise:

Problem:

A bicycle tire has a pressure of $7.00 \times 10^5\ \text{N/m}^2$ at a temperature of 18.0°C and contains 2.00 L of gas. What will its pressure be if you let out an amount of air that has a volume of $100\ \text{cm}^3$ at atmospheric pressure? Assume tire temperature and volume remain constant.

Exercise:

Problem:

A high-pressure gas cylinder contains 50.0 L of toxic gas at a pressure of $1.40 \times 10^7\ \text{N/m}^2$ and a temperature of 25.0°C. Its valve leaks after the cylinder is dropped. The cylinder is cooled to dry ice temperature (-78.5°C) to reduce the leak rate and pressure so that it can be safely repaired. (a) What is the final pressure in the tank, assuming a negligible amount of gas leaks while being cooled and that there is no phase change? (b) What is the final pressure if one-tenth of the gas escapes? (c) To what temperature must the tank be cooled to reduce the pressure to 1.00 atm (assuming the gas does not change phase and that there is no leakage during cooling)? (d) Does cooling the tank appear to be a practical solution?

Solution:

(a) $9.14 \times 10^6\ \text{N/m}^2$

(b) $8.23 \times 10^6\ \text{N/m}^2$

(c) 2.16 K

(d) No. The final temperature needed is much too low to be easily achieved for a large object.

Exercise:

Problem:

(a) What is the gauge pressure in a 25.0°C car tire containing 3.60 mol of gas in a 30.0 L volume? (b) What will its gauge pressure be if you add 1.00 L of gas originally at atmospheric pressure and 25.0°C? Assume the temperature returns to 25.0°C and the volume remains constant.

Glossary

ideal gas law
: the physical law that relates the pressure and volume of a gas to the number of gas molecules or number of moles of gas and the temperature of the gas

Boltzmann constant
: k , a physical constant that relates energy to temperature; $k = 1.38 \times 10^{-23}\ \text{J/K}$

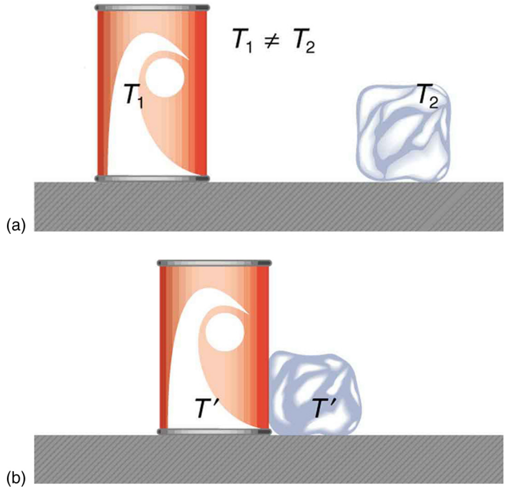
Heat

- Define heat as transfer of energy.

We have defined work as force times distance and learned that work done on an object changes its kinetic energy. We have also seen that temperature is proportional to the (average) kinetic energy of atoms and molecules. We say that a thermal system has a certain internal energy: its internal energy is higher if the temperature is higher. If two objects at different temperatures are brought in contact with each other, energy is transferred from the hotter to the colder object until equilibrium is reached and the bodies reach thermal equilibrium (i.e., they are at the same temperature). No work is done by either object, because no force acts through a distance. The transfer of energy is caused by the temperature difference, and ceases once the temperatures are equal. These observations lead to the following definition of **heat**: Heat is the spontaneous transfer of energy due to a temperature difference.

Heat is often confused with temperature. For example, we may say the heat was unbearable, when we actually mean that the temperature was high. Heat is a form of energy, whereas temperature is not. The misconception arises because we are sensitive to the flow of heat, rather than the temperature.

Owing to the fact that heat is a form of energy, it has the SI unit of *joule* (J). The *calorie* (cal) is a common unit of energy, defined as the energy needed to change the temperature of 1.00 g of water by 1.00°C —specifically, between 14.5°C and 15.5°C, since there is a slight temperature dependence. Perhaps the most common unit of heat is the **kilocalorie** (kcal), which is the energy needed to change the temperature of 1.00 kg of water by 1.00°C. Since mass is most often specified in kilograms, kilocalorie is commonly used. Food calories (given the notation Cal, and sometimes called “big calorie”) are actually kilocalories ($1\text{ kilocalorie} = 1000\text{ calories}$), a fact not easily determined from package labeling.



In figure (a) the soft drink and the ice have different temperatures, T_1 and T_2, and are not in thermal equilibrium. In figure (b), when the soft drink and ice are allowed to interact, energy is transferred until they reach the same temperature $T\prime$, achieving equilibrium. Heat transfer occurs due to the difference in temperatures. In fact, since the soft drink and ice are both in contact with the surrounding air and bench, the equilibrium temperature will be the same for both.

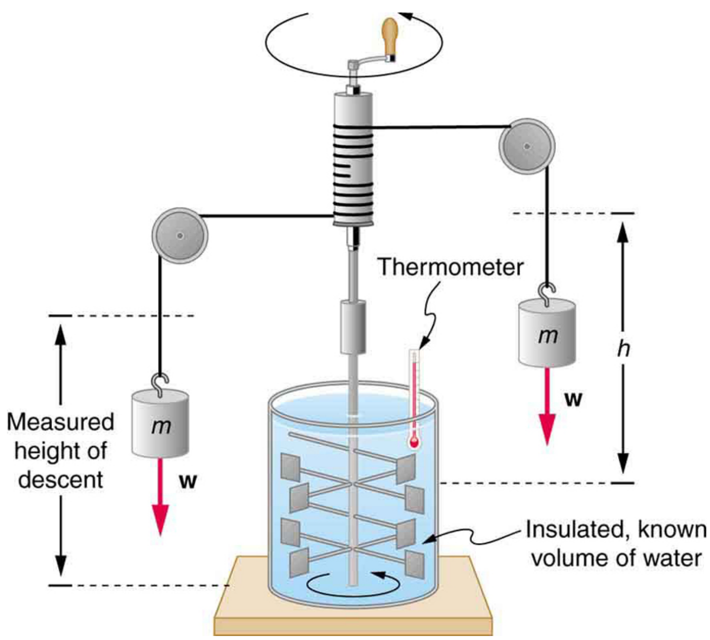
Mechanical Equivalent of Heat

It is also possible to change the temperature of a substance by doing work. Work can transfer energy into or out of a system. This realization helped establish the fact that heat is a form of energy. James Prescott Joule (1818–1889) performed many experiments to establish the **mechanical equivalent of heat**—*the work needed to produce the same effects as heat transfer*. In terms of the units used for these two terms, the best modern value for this equivalence is

Equation:

$$1.000\ \text{kcal} = 4186\ \text{J}.$$

We consider this equation as the conversion between two different units of energy.



Schematic depiction of Joule's experiment that established the equivalence of heat and work.

The figure above shows one of Joule’s most famous experimental setups for demonstrating the mechanical equivalent of heat. It demonstrated that work and heat can produce the same effects, and helped establish the principle of conservation of energy. Gravitational potential energy (PE) (work done by the gravitational force) is converted into kinetic energy (KE), and then randomized by viscosity and turbulence into increased average kinetic energy of atoms and molecules in the system, producing a temperature increase. His contributions to the field of thermodynamics were so significant that the SI unit of energy was named after him.

Heat added or removed from a system changes its internal energy and thus its temperature. Such a temperature increase is observed while cooking. However, adding heat does not necessarily increase the temperature. An example is melting of ice; that is, when a substance changes from one phase to another. Work done on the system or by the system can also change the internal energy of the system. Joule demonstrated that the temperature of a system can be increased by stirring. If an ice cube is rubbed against a rough surface, work is done by the frictional force. A system has a well-defined internal energy, but we cannot say that it has a certain “heat content” or “work content”. We use the phrase “heat transfer” to emphasize its nature.

Exercise:

Check Your Understanding

Problem:

Two samples (A and B) of the same substance are kept in a lab. Someone adds 10 kilojoules (kJ) of heat to one sample, while 10 kJ of work is done on the other sample. How can you tell to which sample the heat was added?

Solution:

Heat and work both change the internal energy of the substance. However, the properties of the sample only depend on the internal energy so that it is impossible to tell whether heat was added to sample A or B.

Summary

- Heat and work are the two distinct methods of energy transfer.
- Heat is energy transferred solely due to a temperature difference.
- Any energy unit can be used for heat transfer, and the most common are kilocalorie (kcal) and joule (J).
- Kilocalorie is defined to be the energy needed to change the temperature of 1.00 kg of water between 14.5^{o}C and 15.5^{o}C.
- The mechanical equivalent of this heat transfer is $1.00\text{ kcal}\ =\ 4186\text{ J}$.

Conceptual Questions

Exercise:

Problem: How is heat transfer related to temperature?

Exercise:

Problem:

Describe a situation in which heat transfer occurs. What are the resulting forms of energy?

Exercise:

Problem:

When heat transfers into a system, is the energy stored as heat? Explain briefly.

Glossary

heat
: the spontaneous transfer of energy due to a temperature difference

kilocalorie
: $1\text{ kilocalorie} = 1000\text{ calories}$

mechanical equivalent of heat

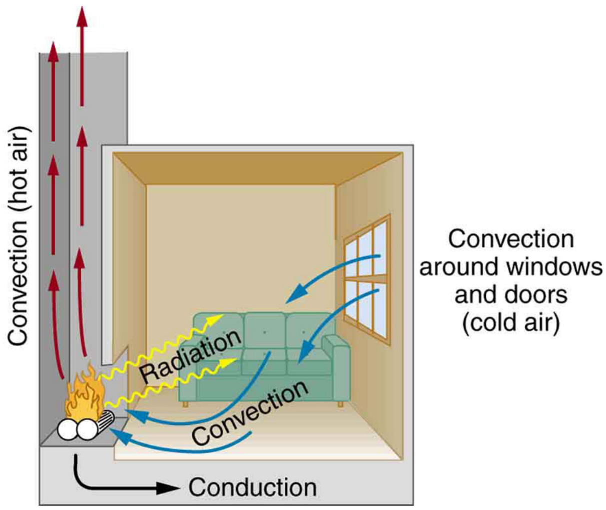
the work needed to produce the same effects as heat transfer

Heat Transfer Methods

- Discuss the three different methods of heat transfer.

Equally as interesting as the effects of heat transfer on a system are the methods by which this occurs. Whenever there is a temperature difference, heat transfer occurs. Heat transfer may occur rapidly, such as through a cooking pan, or slowly, such as through the walls of a picnic ice chest. We can control rates of heat transfer by choosing materials (such as thick wool clothing for the winter), controlling air movement (such as the use of weather stripping around doors), or by choice of color (such as a white roof to reflect summer sunlight). So many processes involve heat transfer, so that it is hard to imagine a situation where no heat transfer occurs. Yet every process involving heat transfer takes place by only three methods:

1. **Conduction** is heat transfer through stationary matter by physical contact. (The matter is stationary on a macroscopic scale—we know there is thermal motion of the atoms and molecules at any temperature above absolute zero.) Heat transferred between the electric burner of a stove and the bottom of a pan is transferred by conduction.
2. **Convection** is the heat transfer by the macroscopic movement of a fluid. This type of transfer takes place in a forced-air furnace and in weather systems, for example.
3. Heat transfer by **radiation** occurs when microwaves, infrared radiation, visible light, or another form of electromagnetic radiation is emitted or absorbed. An obvious example is the warming of the Earth by the Sun. A less obvious example is thermal radiation from the human body.



In a fireplace, heat transfer occurs by all three methods: conduction, convection, and radiation. Radiation is responsible for most of the heat transferred into the room. Heat transfer also occurs through conduction into the room, but at a much slower rate. Heat transfer by convection also occurs through cold air entering the room around windows and hot air leaving the room by rising up the chimney.

[link] illustrates these three methods of heat transfer occurring in one system. While we skip detailed discussion of methods of heat transfer, the one thing all three methods-conduction, convection, and radiation-share is they are driven by temperature difference.

Exercise:

Check Your Understanding

Problem:

Name an example from daily life (different from the text) for each mechanism of heat transfer.

Solution:

Conduction: Heat transfers into your hands as you hold a hot cup of coffee.

Convection: Heat transfers as the barista "steams" cold milk to make hot *cocoa*.

Radiation: Reheating a cold cup of coffee in a microwave oven.

Summary

- Heat is transferred by three different methods: conduction, convection, and radiation.

Conceptual Questions

Exercise:

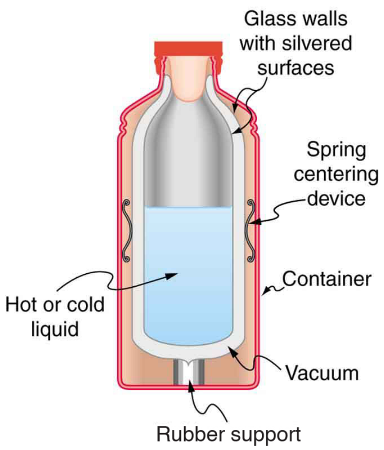
Problem:

What are the main methods of heat transfer from the hot core of Earth to its surface? From Earth's surface to outer space?

When our bodies get too warm, they respond by sweating and increasing blood circulation to the surface to transfer thermal energy away from the core. What effect will this have on a person in a 40.0°C hot tub?

[link] shows a cut-away drawing of a thermos bottle (also known as a Dewar flask), which is a device designed specifically to slow down all forms of heat transfer. Explain the functions of the various parts, such as the

vacuum, the silvering of the walls, the thin-walled long glass neck, the rubber support, the air layer, and the stopper.



The construction of a thermos bottle is designed to inhibit all methods of heat transfer.

Glossary

conduction
: heat transfer through stationary matter by physical contact

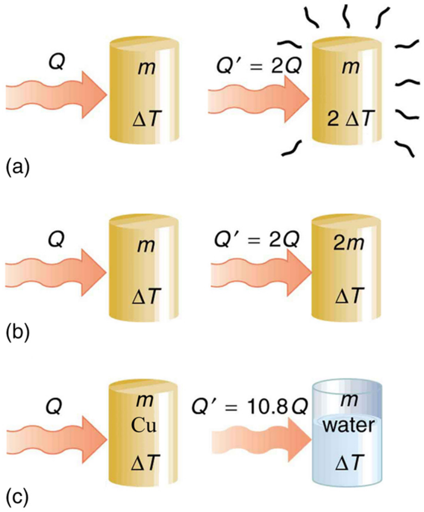
convection
: heat transfer by the macroscopic movement of fluid

radiation
: heat transfer which occurs when microwaves, infrared radiation, visible light, or other electromagnetic radiation is emitted or absorbed

Temperature Change and Heat Capacity

- Observe heat transfer and change in temperature and mass.
- Calculate final temperature after heat transfer between two objects.

One of the major effects of heat transfer is temperature change: heating increases the temperature while cooling decreases it. We assume that there is no phase change and that no work is done on or by the system. Experiments show that the transferred heat depends on three factors—the change in temperature, the mass of the system, and the substance and phase of the substance.



The heat Q transferred to cause a temperature change depends on the magnitude of the temperature change, the mass of the system, and the substance and phase involved. (a) The amount of heat transferred is directly proportional to the temperature change. To double the temperature change of a mass m, you need to add twice the heat. (b) The amount of heat transferred is also directly proportional to the mass. To cause an equivalent temperature change in a

doubled mass, you need to add twice the heat. (c) The amount of heat transferred depends on the substance and its phase. If it takes an amount Q of heat to cause a temperature change ΔT in a given mass of copper, it will take 10.8 times that amount of heat to cause the equivalent temperature change in the same mass of water assuming no phase change in either substance.

The dependence on temperature change and mass are easily understood. Owing to the fact that the (average) kinetic energy of an atom or molecule is proportional to the absolute temperature, the internal energy of a system is proportional to the absolute temperature and the number of atoms or molecules. Owing to the fact that the transferred heat is equal to the change in the internal energy, the heat is proportional to the mass of the substance and the temperature change. The transferred heat also depends on the substance so that, for example, the heat necessary to raise the temperature is less for alcohol than for water. For the same substance, the transferred heat also depends on the phase (gas, liquid, or solid).

Note:
Heat Transfer and Temperature Change
The quantitative relationship between heat transfer and temperature change contains all three factors:
Equation:

$$Q = \mathrm{mc}\Delta T,$$

where Q is the symbol for heat transfer, m is the mass of the substance, and ΔT is the change in temperature. The symbol c stands for **specific heat** and depends on the material and phase. The specific heat is the amount of heat necessary to change the

temperature of 1.00 kg of mass by 1.00°C. The specific heat c is a property of the substance; its SI unit is $\text{J}/(\text{kg}\cdot\text{K})$ or $\text{J}/(\text{kg}\cdot{}^{\circ}\text{C})$. Recall that the temperature change (ΔT) is the same in units of kelvin and degrees Celsius. If heat transfer is measured in kilocalories, then *the unit of specific heat* is $\text{kcal}/(\text{kg}\cdot{}^{\circ}\text{C})$.

Values of specific heat must generally be looked up in tables, because there is no simple way to calculate them. In general, the specific heat also depends on the temperature. [link] lists representative values of specific heat for various substances. Except for gases, the temperature and volume dependence of the specific heat of most substances is weak. We see from this table that the specific heat of water is five times that of glass and ten times that of iron, which means that it takes five times as much heat to raise the temperature of water the same amount as for glass and ten times as much heat to raise the temperature of water as for iron. In fact, water has one of the largest specific heats of any material, which is important for sustaining life on Earth.

Example:
Calculating the Required Heat: Heating Water in an Aluminum Pan

A 0.500 kg aluminum pan on a stove is used to heat 0.250 liters of water from 20.0°C to 80.0°C. (a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

Strategy

The pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and the pan is increased by the same amount. We use the equation for the heat transfer for the given temperature change and mass of water and aluminum. The specific heat values for water and aluminum are given in [link].

Solution

Because water is in thermal contact with the aluminum, the pan and the water are at the same temperature.

1. Calculate the temperature difference:
   Equation:
   $$\Delta T = T_{\text{f}} - T_{\text{i}} = 60.0^{\circ}\text{C}.$$
2. Calculate the mass of water. Because the density of water is $1000\ \text{kg}/\text{m}^3$, one liter of water has a mass of 1 kg, and the mass of 0.250 liters of water is $m_{\text{w}} = 0.250\ \text{kg}$.
3. Calculate the heat transferred to the water. Use the specific heat of water in [link]:
   Equation:

$$Q_{\mathrm{w}} = m_{\mathrm{w}} c_{\mathrm{w}} \Delta T = (0.250\ \mathrm{kg})(4186\ \mathrm{J/kg^{o}C})(60.0^{\mathrm{o}}\mathrm{C}) = 62.8\ \mathrm{kJ}.$$

4. Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in [link]:
   Equation:

   $$Q_{\mathrm{Al}} = m_{\mathrm{Al}} c_{\mathrm{Al}} \Delta T = (0.500\ \mathrm{kg})(900\ \mathrm{J/kg^{o}C})(60.0^{\mathrm{o}}\mathrm{C}) = 27.0 \times 10^{4}\mathrm{J} = 27.0\ \mathrm{kJ}.$$

5. Compare the percentage of heat going into the pan versus that going into the water. First, find the total transferred heat:
   Equation:

   $$Q_{\mathrm{Total}} = Q_{\mathrm{W}} + Q_{\mathrm{Al}} = 62.8\ \mathrm{kJ} + 27.0\ \ \mathrm{kJ} = 89.8\ \mathrm{kJ}.$$

Thus, the amount of heat going into heating the pan is
Equation:

$$\frac{27.0\ \mathrm{kJ}}{89.8\ \mathrm{kJ}} \times 100\% = 30.1\%,$$

and the amount going into heating the water is
Equation:

$$\frac{62.8\ \mathrm{kJ}}{89.8\ \mathrm{kJ}} \times 100\% = 69.9\%.$$

Discussion
In this example, the heat transferred to the container is a significant fraction of the total transferred heat. Although the mass of the pan is twice that of the water, the specific heat of water is over four times greater than that of aluminum. Therefore, it takes a bit more than twice the heat to achieve the given temperature change for the water as compared to the aluminum pan.

The smoking brakes on this truck are a visible evidence of the mechanical equivalent of heat.

Example:
Calculating the Temperature Increase from the Work Done on a Substance: Truck Brakes Overheat on Downhill Runs

Truck brakes used to control speed on a downhill run do work, converting gravitational potential energy into increased internal energy (higher temperature) of the brake material. This conversion prevents the gravitational potential energy from being converted into kinetic energy of the truck. The problem is that the mass of the truck is large compared with that of the brake material absorbing the energy, and the temperature increase may occur too fast for sufficient heat to transfer from the brakes to the environment.

Calculate the temperature increase of 100 kg of brake material with an average specific heat of 800 $\text{J/kg} \cdot {}^{\text{o}}\text{C}$ if the material retains 10% of the energy from a 10,000-kg truck descending 75.0 m (in vertical displacement) at a constant speed.

Strategy

If the brakes are not applied, gravitational potential energy is converted into kinetic energy. When brakes are applied, gravitational potential energy is converted into internal energy of the brake material. We first calculate the gravitational potential energy (Mgh) that the entire truck loses in its descent and then find the temperature increase produced in the brake material alone.

Solution

1. Calculate the change in gravitational potential energy as the truck goes downhill
   Equation:

$$\text{Mgh} = (10{,}000\text{ kg})\left(9.80\text{ m/s}^2\right)(75.0\text{ m}) = 7.35 \times 10^6\text{ J}.$$

2. Calculate the temperature from the heat transferred using $Q = (0.10)Mgh$ (0.10 for 10% of energy being retained by brake material) and
   Equation:

$$\Delta T = \frac{Q}{\text{mc}},$$

   where m is the mass of the brake material. Insert the values $m = 100$ kg and $c = 800\ \text{J/kg·ºC}$ to find
   Equation:

$$\Delta T = \frac{\left(7.35 \times 10^5\text{ J}\right)}{(100\text{ kg})(800\text{ J/kgºC})} = 9.2\text{ ºC}.$$

Discussion
The same idea underlies the recent hybrid technology of cars, where mechanical energy (gravitational potential energy) is converted by the brakes into electrical energy (battery).

Substances	Specific heat (*c*)	
Solids	J/kg·ºC	kcal/kg·ºC[footnote] These values are identical in units of cal/g ·ºC.
Aluminum	900	0.215
Asbestos	800	0.19
Concrete, granite (average)	840	0.20
Copper	387	0.0924

Substances	**Specific heat (*c*)**	
Glass	840	0.20
Gold	129	0.0308
Human body (average at 37 °C)	3500	0.83
Ice (average, -50°C to 0°C)	2090	0.50
Iron, steel	452	0.108
Lead	128	0.0305
Silver	235	0.0562
Wood	1700	0.4
Liquids		
Benzene	1740	0.415
Ethanol	2450	0.586
Glycerin	2410	0.576
Mercury	139	0.0333
Water (15.0 °C)	4186	1.000
Gases [footnote] c_v at constant volume and at 20.0ºC, except as noted, and at 1.00 atm average pressure. Values in parentheses are c_p at a constant pressure of 1.00 atm.		
Air (dry)	721 (1015)	0.172 (0.242)
Ammonia	1670 (2190)	0.399 (0.523)

Substances	Specific heat (c)	
Carbon dioxide	638 (833)	0.152 (0.199)
Nitrogen	739 (1040)	0.177 (0.248)
Oxygen	651 (913)	0.156 (0.218)
Steam (100°C)	1520 (2020)	0.363 (0.482)

Specific Heats[footnote] of Various Substances
The values for solids and liquids are at constant volume and at $25^{\circ}\mathrm{C}$, except as noted.

Note that [link] is an illustration of the mechanical equivalent of heat. Alternatively, the temperature increase could be produced by a blow torch instead of mechanically.

Example:
Calculating the Final Temperature When Heat Is Transferred Between Two Bodies: Pouring Cold Water in a Hot Pan
Suppose you pour 0.250 kg of $20.0^{\circ}\mathrm{C}$ water (about a cup) into a 0.500-kg aluminum pan off the stove with a temperature of $150^{\circ}\mathrm{C}$. Assume that the pan is placed on an insulated pad and that a negligible amount of water boils off. What is the temperature when the water and pan reach thermal equilibrium a short time later?
Strategy
The pan is placed on an insulated pad so that little heat transfer occurs with the surroundings. Originally the pan and water are not in thermal equilibrium: the pan is at a higher temperature than the water. Heat transfer then restores thermal equilibrium once the water and pan are in contact. Because heat transfer between the pan and water takes place rapidly, the mass of evaporated water is negligible and the magnitude of the heat lost by the pan is equal to the heat gained by the water. The exchange of heat stops once a thermal equilibrium between the pan and the water is achieved. The heat exchange can be written as $\mid Q_{\text{hot}} \mid = Q_{\text{cold}}$.
Solution

1. Use the equation for heat transfer $Q = \mathrm{mc}\Delta T$ to express the heat lost by the aluminum pan in terms of the mass of the pan, the specific heat of aluminum, the initial temperature of the pan, and the final temperature:
   Equation:

$$Q_{\text{hot}} = m_{\text{Al}}c_{\text{Al}}(T_{\text{f}} - 150^{\text{o}}\text{C}).$$

2. Express the heat gained by the water in terms of the mass of the water, the specific heat of water, the initial temperature of the water and the final temperature:
   Equation:

$$Q_{\text{cold}} = m_{\text{W}}c_{\text{W}}(T_{\text{f}} - 20.0^{\text{o}}\text{C}).$$

3. Note that $Q_{\text{hot}} < 0$ and $Q_{\text{cold}} > 0$ and that they must sum to zero because the heat lost by the hot pan must be the same as the heat gained by the cold water:
   Equation:

$$\begin{aligned} Q_{\text{cold}} + Q_{\text{hot}} &= 0, \\ Q_{\text{cold}} &= -\text{Q}_{\text{hot}}, \\ m_{\text{W}}c_{\text{W}}(T_{\text{f}} - 20.0^{\text{o}}\text{C}) &= -\text{m}_{\text{Al}}c_{\text{Al}}(T_{\text{f}} - 150^{\text{o}}\text{C.}) \end{aligned}$$

4. This an equation for the unknown final temperature, T_{f}
5. Bring all terms involving T_{f} on the left hand side and all other terms on the right hand side. Solve for T_{f},
   Equation:

$$T_{\text{f}} = \frac{m_{\text{Al}}c_{\text{Al}}(150^{\text{o}}\text{C}) + m_{\text{W}}c_{\text{W}}(20.0^{\text{o}}\text{C})}{m_{\text{Al}}c_{\text{Al}} + m_{\text{W}}c_{\text{W}}},$$

   and insert the numerical values:
   Equation:

$$\begin{aligned} T_{\text{f}} &= \frac{(0.500\text{ kg})(900\text{ J/kg}^{\text{o}}\text{C})(150^{\text{o}}\text{C}) + (0.250\text{ kg})(4186\text{ J/kg}^{\text{o}}\text{C})(20.0^{\text{o}}\text{C})}{(0.500\text{ kg})(900\text{ J/kg}^{\text{o}}\text{C}) + (0.250\text{ kg})(4186\text{ J/kg}^{\text{o}}\text{C})} \\ &= \frac{88430\text{ J}}{1496.5\text{ J/}^{\text{o}}\text{C}} \\ &= 59.1^{\text{o}}\text{C}. \end{aligned}$$

Discussion

This is a typical *calorimetry* problem—two bodies at different temperatures are brought in contact with each other and exchange heat until a common temperature is reached. Why is the final temperature so much closer to 20.0^{o}C than 150^{o}C? The reason is that water has a greater specific heat than most common substances and thus undergoes a small temperature change for a given heat transfer. A large body of water, such as a lake, requires a large amount of heat to increase its temperature appreciably. This explains why the temperature of a lake stays relatively constant during a day even when the temperature change of the air is large. However, the water temperature does change over longer times (e.g., summer to winter).

Note:
Take-Home Experiment: Temperature Change of Land and Water
What heats faster, land or water?
To study differences in heat capacity:

- Place equal masses of dry sand (or soil) and water at the same temperature into two small jars. (The average density of soil or sand is about 1.6 times that of water, so you can achieve approximately equal masses by using 50% more water by volume.)
- Heat both (using an oven or a heat lamp) for the same amount of time.
- Record the final temperature of the two masses.
- Now bring both jars to the same temperature by heating for a longer period of time.
- Remove the jars from the heat source and measure their temperature every 5 minutes for about 30 minutes.

Which sample cools off the fastest? This activity replicates the phenomena responsible for land breezes and sea breezes.

Exercise:
Check Your Understanding

Problem:

If 25 kJ is necessary to raise the temperature of a block from $25^{\circ}\mathrm{C}$ to $30^{\circ}\mathrm{C}$, how much heat is necessary to heat the block from $45^{\circ}\mathrm{C}$ to $50^{\circ}\mathrm{C}$?

Solution:

The heat transfer depends only on the temperature difference. Since the temperature differences are the same in both cases, the same 25 kJ is necessary in the second case.

Summary

- The transfer of heat Q that leads to a change ΔT in the temperature of a body with mass m is $Q=\mathrm{mc}\Delta T$, where c is the specific heat of the material. This relationship can also be considered as the definition of specific heat.

Conceptual Questions

Exercise:

Problem:

What three factors affect the heat transfer that is necessary to change an object’s temperature?

Exercise:

Problem:

The brakes in a car increase in temperature by ΔT when bringing the car to rest from a speed v. How much greater would ΔT be if the car initially had twice the speed? You may assume the car to stop sufficiently fast so that no heat transfers out of the brakes.

Glossary

specific heat
: the amount of heat necessary to change the temperature of 1.00 kg of a substance by 1.00 ºC

Phase Change and Latent Heat

- Describe phase changes.
- Explain the relationship between phase changes and heat transfer.

So far we have discussed temperature change due to heat transfer. No temperature change occurs from heat transfer if ice melts and becomes liquid water (i.e., during a phase change). For example, consider water dripping from icicles melting on a roof warmed by the Sun. Conversely, water freezes in an ice tray cooled by lower-temperature surroundings.

Heat from the air transfers to the ice causing it to melt. (credit: Mike Brand)

Energy is required to melt a solid because the cohesive bonds between the molecules in the solid must be broken apart such that, in the liquid, the molecules can move around at comparable kinetic energies; thus, there is no rise in temperature. Similarly, energy is needed to vaporize a liquid, because molecules in a liquid interact with each other via attractive forces. There is no temperature change until a phase change is complete. The temperature of a cup of soda initially at 0ºC stays at 0ºC until all the ice has melted. Conversely, energy is released during freezing and condensation, usually in the form of thermal energy. Work is done by cohesive forces when molecules are brought together. The corresponding energy must be given off (dissipated) to allow them to stay together [link].

The energy involved in a phase change depends on two major factors: the number and strength of bonds or force pairs. The number of bonds is proportional to the number of molecules and thus to the mass of the sample. The strength of forces depends on the type of molecules. The heat Q required to change the phase of a sample of mass m is given by

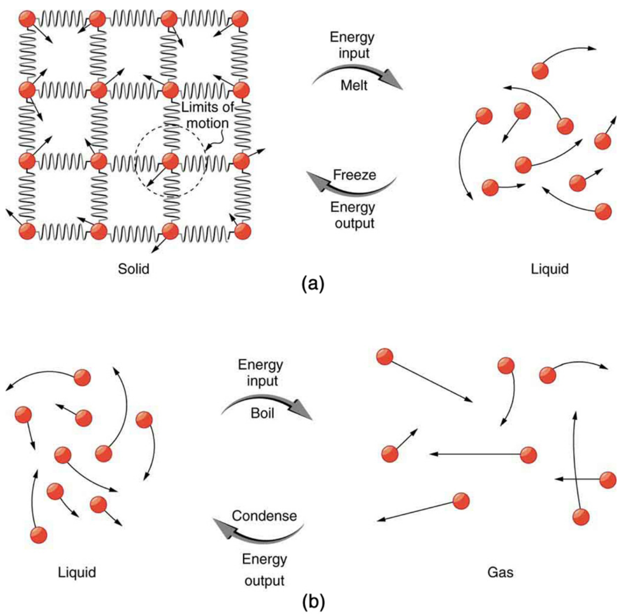
Equation:

$$Q=\mathrm{mL_f}\;(\mathrm{melting/freezing}),$$

Equation:

$$Q=\mathrm{mL_v}\;(\mathrm{vaporization/condensation}),$$

where the latent heat of fusion, L_{f}, and latent heat of vaporization, L_{v}, are material constants that are determined experimentally. See ([link]).



(a) Energy is required to partially overcome the attractive forces between molecules in a solid to form a liquid. That same energy must be removed for freezing to take place. (b) Molecules are separated by large distances when going from liquid to vapor, requiring significant energy to overcome molecular attraction. The same energy must be removed for condensation to take place. There is no temperature change until a phase change is complete.

Latent heat is measured in units of J/kg. Both L_{f} and L_{v} depend on the substance, particularly on the strength of its molecular forces as noted earlier. L_{f} and L_{v} are collectively called **latent heat coefficients**. They are *latent*, or hidden, because in phase changes, energy enters or leaves a system without causing a temperature change in the system; so, in effect, the energy is hidden. [link] lists representative values of L_{f} and L_{v}, together with melting and boiling points.

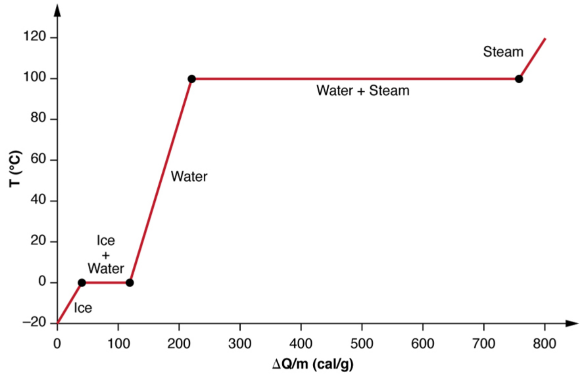
The table shows that significant amounts of energy are involved in phase changes. Let us look, for example, at how much energy is needed to melt a kilogram of ice at $0^{\mathrm{o}}\mathrm{C}$ to produce a kilogram of water at $0^{\circ}\mathrm{C}$. Using the equation for a change in temperature and the value for water from [link], we find that $Q = mL_{\mathrm{f}} = (1.0\ \mathrm{kg})(334\ \mathrm{kJ/kg}) = 334\ \mathrm{kJ}$ is the energy to melt a kilogram of ice. This is a lot of energy as it represents the same amount of energy needed to raise the temperature of 1 kg of liquid water from $0^{\mathrm{o}}\mathrm{C}$ to $79.8^{\mathrm{o}}\mathrm{C}$. Even more energy is required to vaporize water; it would take 2256 kJ to change 1 kg of liquid water at the normal boiling point ($100^{\mathrm{o}}\mathrm{C}$ at atmospheric pressure) to steam (water vapor). This example shows that the energy for a phase change is enormous compared to energy associated with temperature changes without a phase change.

		L_f			L_v	
Substance	**Melting point (ºC)**	**kJ/kg**	**kcal/kg**	**Boiling point (°C)**	**kJ/kg**	**kcal/kg**
Helium	−269.7	5.23	1.25	−268.9	20.9	4.99
Hydrogen	−259.3	58.6	14.0	−252.9	452	108
Nitrogen	−210.0	25.5	6.09	−195.8	201	48.0
Oxygen	−218.8	13.8	3.30	−183.0	213	50.9
Ethanol	−114	104	24.9	78.3	854	204
Ammonia	−75		108	−33.4	1370	327
Mercury	−38.9	11.8	2.82	357	272	65.0
Water	0.00	334	79.8	100.0	2256[footnote] At 37.0°C (body temperature), the heat of vaporization L_{v} for water is 2430 kJ/kg or 580 kcal/kg	539[footnote] At 37.0°C (body temperature), the heat of vaporization L_{v} for water is 2430 kJ/kg or 580 kcal/kg
Sulfur	119	38.1	9.10	444.6	326	77.9
Lead	327	24.5	5.85	1750	871	208
Antimony	631	165	39.4	1440	561	134
Aluminum	660	380	90	2450	11400	2720
Silver	961	88.3	21.1	2193	2336	558
Gold	1063	64.5	15.4	2660	1578	377
Copper	1083	134	32.0	2595	5069	1211
Uranium	1133	84	20	3900	1900	454
Tungsten	3410	184	44	5900	4810	1150

Heats of Fusion and Vaporization [footnote]
Values quoted at the normal melting and boiling temperatures at standard atmospheric pressure (1 atm).

Phase changes can have a tremendous stabilizing effect even on temperatures that are not near the melting and boiling points, because evaporation and condensation (conversion of a gas into a liquid state) occur even at temperatures below the boiling point. Take, for example, the fact that air temperatures in humid climates rarely go above $35.0^{\circ}\mathrm{C}$, which is because most heat transfer goes into evaporating water into the air. Similarly, temperatures in humid weather rarely fall below the dew point because enormous heat is released when water vapor condenses.

We examine the effects of phase change more precisely by considering adding heat into a sample of ice at $-20^{\circ}\mathrm{C}$ ([link]). The temperature of the ice rises linearly, absorbing heat at a constant rate of $0.50\ \mathrm{cal/g}\cdot^{\circ}\mathrm{C}$ until it reaches $0^{\circ}\mathrm{C}$. Once at this temperature, the ice begins to melt until all the ice has melted, absorbing 79.8 cal/g of heat. The temperature remains constant at $0^{\circ}\mathrm{C}$ during this phase change. Once all the ice has melted, the temperature of the liquid water rises, absorbing heat at a new constant rate of $1.00\ \mathrm{cal/g}\cdot^{\circ}\mathrm{C}$. At $100^{\circ}\mathrm{C}$, the water begins to boil and the temperature again remains constant while the water absorbs 539 cal/g of heat during this phase change. When all the liquid has become steam vapor, the temperature rises again, absorbing heat at a rate of $0.482\ \mathrm{cal/g}\cdot^{\circ}\mathrm{C}$.



A graph of temperature versus energy added. The system is constructed so that no vapor evaporates while ice warms to become liquid water, and so that, when vaporization occurs, the vapor remains in of the system. The long stretches of constant temperature values at $0^{\circ}\mathrm{C}$ and $100^{\circ}\mathrm{C}$ reflect the large latent heat of melting and vaporization, respectively.

Water can evaporate at temperatures below the boiling point. More energy is required than at the boiling point, because the kinetic energy of water molecules at temperatures below $100^{\circ}\mathrm{C}$ is less than that at $100^{\circ}\mathrm{C}$, hence less energy is available from random thermal motions. Take, for example, the fact that, at body temperature, perspiration from the skin requires a heat input of 2428 kJ/kg, which is about 10 percent higher than the latent heat of vaporization at $100^{\circ}\mathrm{C}$. This heat comes from the skin, and thus provides an effective cooling mechanism in hot weather. High humidity inhibits evaporation, so that body temperature might rise, leaving unevaporated sweat on your brow.

Example:
Calculate Final Temperature from Phase Change: Cooling Soda with Ice Cubes

Three ice cubes are used to chill a soda at 20ºC with mass $m_{\text{soda}} = 0.25\text{ kg}$. The ice is at 0ºC and each ice cube has a mass of 6.0 g. Assume that the soda is kept in a foam container so that heat loss can be ignored. Assume the soda has the same heat capacity as water. Find the final temperature when all ice has melted.

Strategy

The ice cubes are at the melting temperature of 0ºC. Heat is transferred from the soda to the ice for melting. Melting of ice occurs in two steps: first the phase change occurs and solid (ice) transforms into liquid water at the melting temperature, then the temperature of this water rises. Melting yields water at 0ºC, so more heat is transferred from the soda to this water until the water plus soda system reaches thermal equilibrium,

Equation:

$$Q_{\text{ice}} = -Q_{\text{soda}}.$$

The heat transferred to the ice is $Q_{\text{ice}} = m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{W}}(T_{\text{f}} - 0\text{ºC})$. The heat given off by the soda is $Q_{\text{soda}} = m_{\text{soda}}c_{\text{W}}(T_{\text{f}} - 20\text{ºC})$. Since no heat is lost, $Q_{\text{ice}} = -Q_{\text{soda}}$, so that

Equation:

$$m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{W}}(T_{\text{f}} - 0\text{ºC}) = -m_{\text{soda}}c_{\text{W}}(T_{\text{f}} - 20\text{ºC}).$$

Bring all terms involving T_{f} on the left-hand-side and all other terms on the right-hand-side. Solve for the unknown quantity T_{f}:

Equation:

$$T_{\text{f}} = \frac{m_{\text{soda}}c_{\text{W}}(20\text{ºC}) - m_{\text{ice}}L_{\text{f}}}{(m_{\text{soda}} + m_{\text{ice}})c_{\text{W}}}.$$

Solution

1. Identify the known quantities. The mass of ice is $m_{\text{ice}} = 3\times6.0\text{ g} = 0.018\text{ kg}$ and the mass of soda is $m_{\text{soda}} = 0.25\text{ kg}$.
2. Calculate the terms in the numerator:

   Equation:

   $$m_{\text{soda}}c_{\text{W}}(20\text{ºC}) = (0.25\text{ kg})(4186\text{ J/kg}\cdot^{\text{o}}\text{ C})(20\text{ºC}) = 20{,}930\text{ J}$$

   and

   Equation:

   $$m_{\text{ice}}L_{\text{f}} = (0.018\text{ kg})(334{,}000\text{ J/kg})\text{=}6012\text{ J}.$$

3. Calculate the denominator:

   Equation:

   $$(m_{\text{soda}} + m_{\text{ice}})c_{\text{W}} = (0.25\text{ kg} + 0.018\text{ kg})(4186\text{ K/(kg}\cdot^{\text{o}}\text{ C)=}1122\text{ J/ºC}.$$

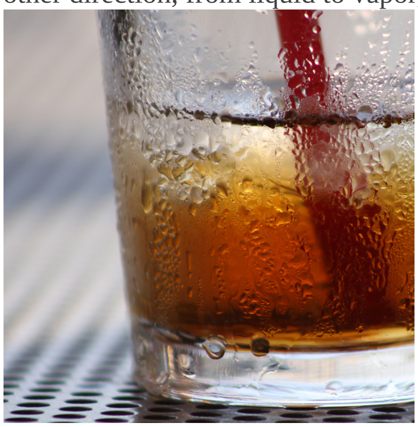
4. Calculate the final temperature:

   Equation:

   $$T_{\text{f}} = \frac{20{,}930\text{ J} - 6012\text{ J}}{1122\text{ J/ºC}} = 13\text{ºC}.$$

Discussion
This example illustrates the enormous energies involved during a phase change. The mass of ice is about 7 percent the mass of water but leads to a noticeable change in the temperature of soda. Although we assumed that the ice was at the freezing temperature, this is incorrect: the typical temperature is $-6^{\circ}\mathrm{C}$. However, this correction gives a final temperature that is essentially identical to the result we found. Can you explain why?

We have seen that vaporization requires heat transfer to a liquid from the surroundings, so that energy is released by the surroundings. Condensation is the reverse process, increasing the temperature of the surroundings. This increase may seem surprising, since we associate condensation with cold objects—the glass in the figure, for example. However, energy must be removed from the condensing molecules to make a vapor condense. The energy is exactly the same as that required to make the phase change in the other direction, from liquid to vapor, and so it can be calculated from $Q = \mathrm{mL_v}$.



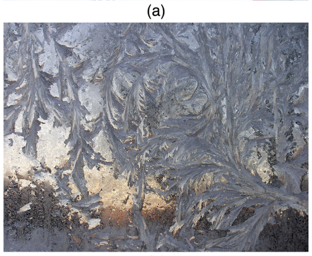
Condensation forms on this glass of iced tea because the temperature of the nearby air is reduced to below the dew point. The air cannot hold as much water as it did at room temperature, and so water condenses. Energy is released when the water condenses, speeding the melting of the ice in the glass. (credit: Jenny Downing)

Note:
Real-World Application
Energy is also released when a liquid freezes. This phenomenon is used by fruit growers in Florida to protect oranges when the temperature is close to the freezing point $(0^{\circ}\mathrm{C})$. Growers spray water on the plants in orchards so that the water freezes and heat is released to the growing oranges on the trees. This prevents the temperature inside the orange from dropping below freezing, which would damage the fruit.

The ice on these trees released large amounts of energy when it froze, helping to prevent the temperature of the trees from dropping below 0ºC. Water is intentionally sprayed on orchards to help prevent hard frosts. (credit: Hermann Hammer)

Sublimation is the transition from solid to vapor phase. You may have noticed that snow can disappear into thin air without a trace of liquid water, or the disappearance of ice cubes in a freezer. The reverse is also true: Frost can form on very cold windows without going through the liquid stage. A popular effect is the making of “smoke” from dry ice, which is solid carbon dioxide. Sublimation occurs because the equilibrium vapor pressure of solids is not zero. Certain air fresheners use the sublimation of a solid to inject a perfume into the room. Moth balls are a slightly toxic example of a phenol (an organic compound) that sublimates, while some solids, such as osmium tetroxide, are so toxic that they must be kept in sealed containers to prevent human exposure to their sublimation-produced vapors.

(a)



(b)

Direct transitions between solid and vapor are common, sometimes useful,

and even beautiful. (a) Dry ice sublimates directly to carbon dioxide gas. The visible vapor is made of water droplets. (credit: Windell Oskay) (b) Frost forms patterns on a very cold window, an example of a solid formed directly from a vapor. (credit: Liz West)

All phase transitions involve heat. In the case of direct solid-vapor transitions, the energy required is given by the equation $Q = \mathrm{mL_s}$, where L_s is the **heat of sublimation**, which is the energy required to change 1.00 kg of a substance from the solid phase to the vapor phase. L_s is analogous to L_f and L_v, and its value depends on the substance. Sublimation requires energy input, so that dry ice is an effective coolant, whereas the reverse process (i.e., frosting) releases energy. The amount of energy required for sublimation is of the same order of magnitude as that for other phase transitions.

The material presented in this section and the preceding section allows us to calculate any number of effects related to temperature and phase change. In each case, it is necessary to identify which temperature and phase changes are taking place and then to apply the appropriate equation. Keep in mind that heat transfer and work can cause both temperature and phase changes.

Exercise:

Check Your Understanding

Problem:

Why does snow remain on mountain slopes even when daytime temperatures are higher than the freezing temperature?

Solution:

Snow is formed from ice crystals and thus is the solid phase of water. Because enormous heat is necessary for phase changes, it takes a certain amount of time for this heat to be accumulated from the air, even if the air is above 0ºC. The warmer the air is, the faster this heat exchange occurs and the faster the snow melts.

Summary

- Most substances can exist either in solid, liquid, and gas forms, which are referred to as “phases.”
- Phase changes occur at fixed temperatures for a given substance at a given pressure, and these temperatures are called boiling and freezing (or melting) points.
- During phase changes, heat absorbed or released is given by:
  Equation:

$$Q = \mathrm{mL},$$

where L is the latent heat coefficient.

Conceptual Questions

Exercise:

Problem:

Heat transfer can cause temperature and phase changes. What else can cause these changes?

Exercise:

Problem:

How does the latent heat of fusion of water help slow the decrease of air temperatures, perhaps preventing temperatures from falling significantly below $0^{\circ}\mathrm{C}$, in the vicinity of large bodies of water?

Exercise:

Problem: What is the temperature of ice right after it is formed by freezing water?

Exercise:

Problem:

If you place $0^{\circ}\mathrm{C}$ ice into $0^{\circ}\mathrm{C}$ water in an insulated container, what will happen? Will some ice melt, will more water freeze, or will neither take place?

Exercise:

Problem:

What effect does condensation on a glass of ice water have on the rate at which the ice melts? Will the condensation speed up the melting process or slow it down?

Exercise:

Problem:

In very humid climates where there are numerous bodies of water, such as in Florida, it is unusual for temperatures to rise above about $35^{\circ}\mathrm{C}(95^{\circ}\mathrm{F})$. In deserts, however, temperatures can rise far above this. Explain how the evaporation of water helps limit high temperatures in humid climates.

Exercise:

Problem:

In winters, it is often warmer in San Francisco than in nearby Sacramento, 150 km inland. In summers, it is nearly always hotter in Sacramento. Explain how the bodies of water surrounding San Francisco moderate its extreme temperatures.

Exercise:

Problem:

Putting a lid on a boiling pot greatly reduces the heat transfer necessary to keep it boiling. Explain why.

Exercise:

Problem:

Freeze-dried foods have been dehydrated in a vacuum. During the process, the food freezes and must be heated to facilitate dehydration. Explain both how the vacuum speeds up dehydration and why the food freezes as a result.

Exercise:

Problem:

When still air cools by radiating at night, it is unusual for temperatures to fall below the dew point. Explain why.

Exercise:

Problem:

In a physics classroom demonstration, an instructor inflates a balloon by mouth and then cools it in liquid nitrogen. When cold, the shrunken balloon has a small amount of light blue liquid in it, as well as some snow-like crystals. As it warms up, the liquid boils, and part of the crystals sublimate, with some crystals lingering for awhile and then producing a liquid. Identify the blue liquid and the two solids in the cold balloon. Justify your identifications using data from [link].

Glossary

heat of sublimation
: the energy required to change a substance from the solid phase to the vapor phase

latent heat coefficient
: a physical constant equal to the amount of heat transferred for every 1 kg of a substance during the change in phase of the substance

sublimation
: the transition from the solid phase to the vapor phase

The First Law of Thermodynamics

- Define the first law of thermodynamics.
- Describe how conservation of energy relates to the first law of thermodynamics.
- Identify instances of the first law of thermodynamics working in everyday situations.

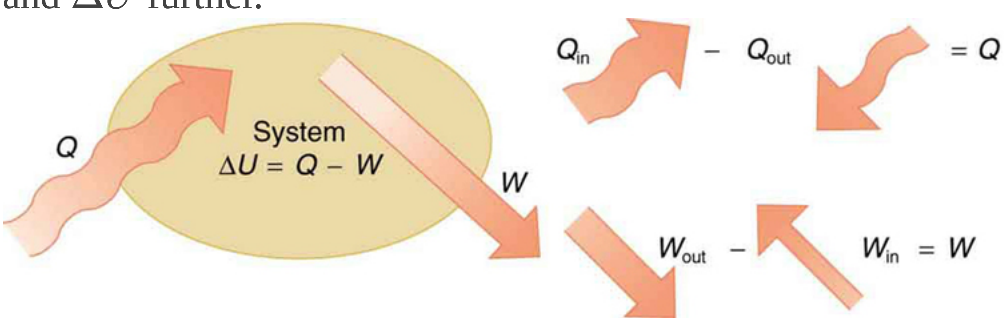
This boiling tea kettle represents energy in motion. The water in the kettle is turning to water vapor because heat is being transferred from the stove to the kettle. As the entire system gets hotter, work is done—from the evaporation of the water to the whistling of the kettle. (credit: Gina Hamilton)

If we are interested in how heat transfer is converted into doing work, then the conservation of energy principle is important. The first law of thermodynamics applies the conservation of energy principle to systems where heat transfer and doing work are the methods of transferring energy into and out of the system. The **first law of thermodynamics** states that the change in internal energy of a system equals the net heat transfer *into* the system minus the net work done *by* the system. In equation form, the first law of thermodynamics is

Equation:

$$\Delta U = Q - W.$$

Here ΔU is the *change in internal energy* U of the system. Q is the *net heat transferred into the system*—that is, Q is the sum of all heat transfer into and out of the system. W is the *net work done by the system*—that is, W is the sum of all work done on or by the system. We use the following sign conventions: if Q is positive, then there is a net heat transfer into the system; if W is positive, then there is net work done by the system. So positive Q adds energy to the system and positive W takes energy from the system. Thus $\Delta U = Q - W$. Note also that if more heat transfer into the system occurs than work done, the difference is stored as internal energy. Heat engines are a good example of this—heat transfer into them takes place so that they can do work. (See [link].) We will now examine Q, W, and ΔU further.



The first law of thermodynamics is the conservation-of-energy principle stated for a system where heat and work are the methods of transferring energy for a system in thermal equilibrium. Q represents the net heat transfer—it is the sum of all heat transfers into and out of the

system. Q is positive for net heat transfer *into* the system. W is the total work done on and by the system. W is positive when more work is done *by* the system than on it. The change in the internal energy of the system, ΔU, is related to heat and work by the first law of thermodynamics, $\Delta U = Q - W$.

Note:
Making Connections: Law of Thermodynamics and Law of Conservation of Energy
The first law of thermodynamics is actually the law of conservation of energy stated in a form most useful in thermodynamics. The first law gives the relationship between heat transfer, work done, and the change in internal energy of a system.

Heat *Q* and Work *W*

Heat transfer (Q) and doing work (W) are the two everyday means of bringing energy into or taking energy out of a system. The processes are quite different. Heat transfer, a less organized process, is driven by temperature differences. Work, a quite organized process, involves a macroscopic force exerted through a distance. Nevertheless, heat and work can produce identical results.For example, both can cause a temperature increase. Heat transfer into a system, such as when the Sun warms the air in a bicycle tire, can increase its temperature, and so can work done on the system, as when the bicyclist pumps air into the tire. Once the temperature increase has occurred, it is impossible to tell whether it was caused by heat transfer or by doing work. This uncertainty is an important point. Heat transfer and work are both energy in transit—neither is stored as such in a system. However, both can change the internal energy U of a system.

Internal energy is a form of energy completely different from either heat or work.

Internal Energy U

We can think about the internal energy of a system in two different but consistent ways. The first is the atomic and molecular view, which examines the system on the atomic and molecular scale. The **internal energy** U of a system is the sum of the kinetic and potential energies of its atoms and molecules. Recall that kinetic plus potential energy is called mechanical energy. Thus internal energy is the sum of atomic and molecular mechanical energy. Because it is impossible to keep track of all individual atoms and molecules, we must deal with averages and distributions. A second way to view the internal energy of a system is in terms of its macroscopic characteristics, which are very similar to atomic and molecular average values.

Macroscopically, we define the change in internal energy ΔU to be that given by the first law of thermodynamics:
Equation:

$$\Delta U = Q - W.$$

Many detailed experiments have verified that $\Delta U = Q - W$, where ΔU is the change in total kinetic and potential energy of all atoms and molecules in a system. It has also been determined experimentally that the internal energy U of a system depends only on the state of the system and *not how it reached that state*. More specifically, U is found to be a function of a few macroscopic quantities (pressure, volume, and temperature, for example), independent of past history such as whether there has been heat transfer or work done. This independence means that if we know the state of a system, we can calculate changes in its internal energy U from a few macroscopic variables.

Note:

Making Connections: Macroscopic and Microscopic

In thermodynamics, we often use the macroscopic picture when making calculations of how a system behaves, while the atomic and molecular picture gives underlying explanations in terms of averages and distributions. We shall see this again in later sections of this chapter. For example, in the topic of entropy, calculations will be made using the atomic and molecular view.

Section Summary

- The first law of thermodynamics is given as $\Delta U = Q - W$, where ΔU is the change in internal energy of a system, Q is the net heat transfer (the sum of all heat transfer into and out of the system), and W is the net work done (the sum of all work done on or by the system).
- Both Q and W are energy in transit; only ΔU represents an independent quantity capable of being stored.
- The internal energy U of a system depends only on the state of the system and not how it reached that state.

Conceptual Questions

Exercise:

Problem:

Describe the photo of the tea kettle at the beginning of this section in terms of heat transfer, work done, and internal energy. How is heat being transferred? What is the work done and what is doing it? How does the kettle maintain its internal energy?

Exercise:

Problem:

The first law of thermodynamics and the conservation of energy are clearly related. How do they differ in the types of energy considered?

Exercise:

Problem:

Heat transfer Q and work done W are always energy in transit, whereas internal energy U is energy stored in a system. Give an example of each type of energy, and state specifically how it is either in transit or resides in a system.

Exercise:

Problem:

How do heat transfer and internal energy differ? In particular, which can be stored as such in a system and which cannot?

Exercise:

Problem:

If you run down some stairs and stop, what happens to your kinetic energy and your initial gravitational potential energy?

Exercise:

Problem:

Give an explanation of how food energy (calories) can be viewed as molecular potential energy (consistent with the atomic and molecular definition of internal energy).

Exercise:

Problem:

Identify the type of energy transferred to your body in each of the following as either internal energy, heat transfer, or doing work: (a) basking in sunlight; (b) eating food; (c) riding an elevator to a higher floor.

Glossary

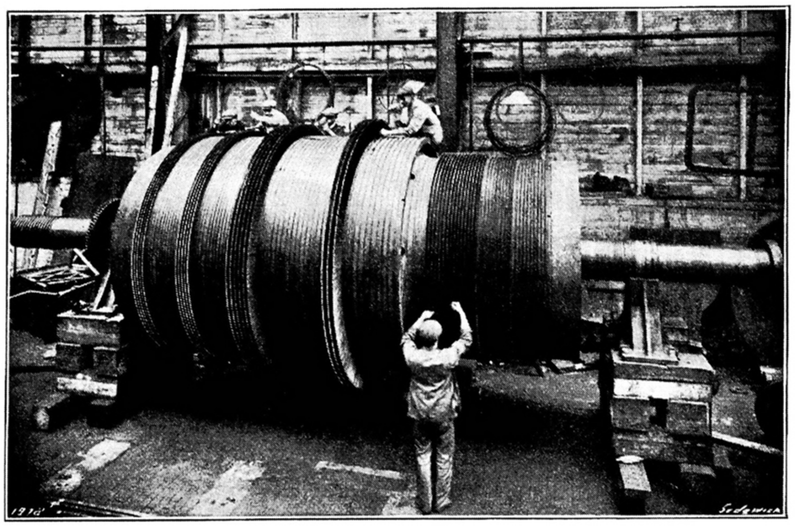
first law of thermodynamics
: states that the change in internal energy of a system equals the net heat transfer *into* the system minus the net work done *by* the system

internal energy
: the sum of the kinetic and potential energies of a system's atoms and molecules

human metabolism
: conversion of food into heat transfer, work, and stored fat

The First Law of Thermodynamics and Heat Engine Processes

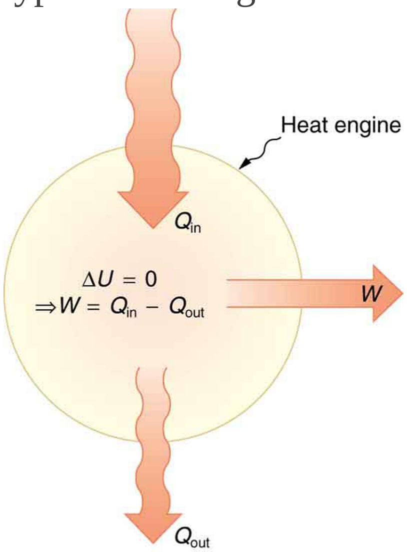
- Describe the processes of a simple heat engine.
- Explain the differences among the simple thermodynamic processes—isobaric, isochoric, isothermal, and adiabatic.
- Explain the relationship between work done by a gas and change in its volume.



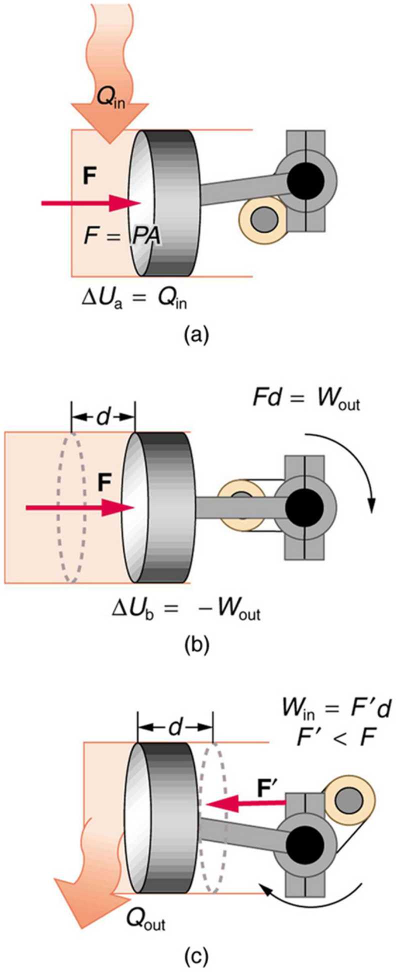
Beginning with the Industrial Revolution, humans have harnessed power through the use of the first law of thermodynamics, before we even understood it completely. This photo, of a steam engine at the Turbinia Works, dates from 1911, a mere 61 years after the first explicit statement of the first law of thermodynamics by Rudolph Clausius. (credit: public domain; author unknown)

One of the most important things we can do with heat transfer is to use it to do work for us. Such a device is called a **heat engine**. Car engines and steam turbines that generate electricity are examples of heat engines. [link]

shows schematically how the first law of thermodynamics applies to the typical heat engine.



Schematic representation of a heat engine, governed, of course, by the first law of thermodynamics (and other laws of thermodynamics we will discuss later).



(a) Heat transfer to the gas in a cylinder increases the internal energy of the gas, creating higher pressure and temperature. (b) The force exerted on the movable cylinder does work as the gas expands. Gas pressure and temperature decrease when it expands, indicating

that the gas's internal energy has been decreased by doing work. (c) Heat transfer to the environment further reduces pressure in the gas so that the piston can be more easily returned to its starting position.

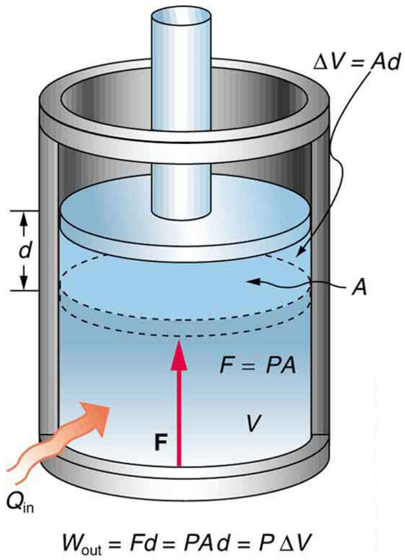
The illustrations above show one of the ways in which heat transfer does work. Fuel combustion produces heat transfer to a gas in a cylinder, increasing the pressure of the gas and thereby the force it exerts on a movable piston. The gas does work on the outside world, as this force moves the piston through some distance. Heat transfer to the gas cylinder results in work being done. To repeat this process, the piston needs to be returned to its starting point. Heat transfer now occurs from the gas to the surroundings so that its pressure decreases, and a force is exerted by the surroundings to push the piston back through some distance. Variations of this process are employed daily in hundreds of millions of heat engines. Here, we consider some of the thermodynamic processes on which heat engines are based.

Work Done by a Gas

A process by which a gas does work on a piston at constant pressure is called an **isobaric process**. Since the pressure is constant, the force exerted is constant and the work done is given as

Equation:

$$P\Delta V.$$



An isobaric expansion of a gas requires heat transfer to keep the pressure constant. Since pressure is constant, the work done is $P\Delta V$.

Recall from mechanics that work done by a force F on an object undergoing displacement d is

Equation:

$$W = \text{Fd}.$$

See the symbols as shown in [link]. Now force is pressure times area ($F = \text{PA}$), and so

Equation:

$$W = \text{PAd}.$$

Because the volume of a cylinder is its cross-sectional area A times its length d, we see that $\mathrm{Ad} = \Delta V$, the change in volume; thus,
Equation:

$$W = P\Delta V \text{ (isobaric process).}$$

Note that if ΔV is positive, then W is positive, meaning that positive work is done by the gas on the outside world.

(Note that the pressure involved in this work that we've called P is the pressure of the gas *inside* the tank. If we call the pressure outside the tank P_{ext}, an expanding gas would be working *against* the external pressure; the work done would therefore be $W = -P_{\text{ext}}\Delta V$ (isobaric process). There are some—especially chemists—who use this definition of work, and not the definition based on internal pressure, as the basis of the First Law of Thermodynamics. This definition reverses the sign conventions for work, and results in a statement of the first law that becomes $\Delta U = Q + W$. In this textbook, we will use the *physics* convention of using work done by the system on the surrounding, not the other way around.)

This is the key lesson from the above derivation: *a gas expanding under pressure does work on its surrounding, and unless additional energy is added through heat transfer, the internal energy of the gas decreases*. We will examine the experimental results that come about as a consequence of this fact later.

Thermodynamic Processes

We introduced the **isobaric process** above in discussing work done by a gas. Isobaric process is an example of a **thermodynamic process**. A thermodynamic process describes a change that happens to a gas, which results in change in its pressure (P), volume (V), and/or temperature (T). An isobaric process is a thermodynamic process that takes place under constant pressure (so the volume and temperature of the gas may change in an isobaric process).

There are three more named thermodynamic processes. These processes are given special names because, like the isobaric process, they occur under some restrictions, which gives them their special properties, as described briefly below. These three additional named thermodynamic processes are: isochoric, isothermal, and adiabatic processes.

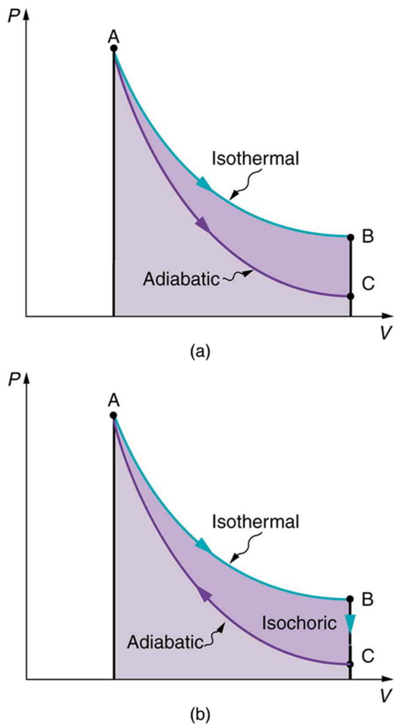
An **isochoric process** is a thermodynamic process in which no change in volume takes place. Because the work done by a gas is proportional to the change in volume, in an isochoric process, no work is done by (or on) the gas. Instead, in an isochoric process, a heat transfer takes place, and the energy from the heat transfer goes into increasing (or decreasing) the internal energy of the gas, increasing (or decreasing) its temperature.

An **isothermal process** is a thermodynamic process in which no change in temperature takes place. A gas expanding isothermally, for example, does work on the surrounding, but its internal energy (as represented by the temperature) does not change, because enough heat flows in to balance out the energy expended in doing work. This is consistent with the first law of thermodynamics ($0 = \varDelta U = Q - W$, because $Q = W$). An isothermal process occurs if a thermodynamic process in a gas occurs slowly enough so that the gas remains in thermal equilibrium with the surrounding at all times.

The adiabatic process is, in some sense, the opposite of an isothermal process. In an **adiabatic process**, no heat transfer takes place (that is, $Q = 0$). This may happen because the gas is well-insulated from the surrounding. It may also happen because the process occurs so quickly that no significant heat transfer can take place. In an adiabatic expansion, for example, the internal energy of the gas decreases, because of the work done by the gas in expansion. This is perhaps the clearest experimental evidence one can observe that it takes work for a gas to expand under pressure.

[link] illustrates these three processes on a plot of pressure and volume (a PV diagram). In an isothermal process, as the gas expands, the pressure decreases. This can be predicted from the ideal gas law ($PV = NkT$). Since the temperature is constant, if volume increases, the pressure must decrease, to keep PV constant. You can also see that in an adiabatic process, the pressure decreases with expanding volume more steeply than

an isothermal process, because in an adiabatic process, the temperature is not constant, but it decreases. So with increasing V, the pressure decrease even more rapidly, so that PV actually decreases (for decreasing temperature).



(a) The upper curve is an isothermal process ($\Delta T = 0$), whereas the lower curve is an adiabatic process ($Q = 0$). Both start from the same point A, but the isothermal process does more work than the adiabatic because heat transfer into the gas

takes place to keep its temperature constant. This keeps the pressure higher all along the isothermal path than along the adiabatic path, producing more work. The adiabatic path thus ends up with a lower pressure and temperature at point C, even though the final volume is the same as for the isothermal process. (b) The cycle ABCA produces a net work output.

Reversible Processes

Both isothermal and adiabatic processes such as shown in [link] are reversible in principle. A **reversible process** is one in which both the system and its environment can return to exactly the states they were in by following the reverse path. The reverse isothermal and adiabatic paths are BA and CA, respectively. Real macroscopic processes are never exactly reversible. In the previous examples, our system is a gas (like that in [link]), and its environment is the piston, cylinder, and the rest of the universe. If there are any energy-dissipating mechanisms, such as friction or turbulence, then heat transfer to the environment occurs for either direction of the piston. So, for example, if the path BA is followed and there is friction, then

the gas will be returned to its original state but the environment will not—it will have been heated in both directions. Reversibility requires the direction of heat transfer to reverse for the reverse path. Since dissipative mechanisms cannot be completely eliminated, real processes cannot be reversible.

There must be reasons that real macroscopic processes cannot be reversible. We can imagine them going in reverse. For example, heat transfer occurs spontaneously from hot to cold and never spontaneously the reverse. Yet it would not violate the first law of thermodynamics for this to happen. In fact, all spontaneous processes, such as bubbles bursting, never go in reverse. There is a second thermodynamic law that forbids them from going in reverse. When we study this law, we will learn something about nature and also find that such a law limits the efficiency of heat engines. We will find that heat engines with the greatest possible theoretical efficiency would have to use reversible processes, and even they cannot convert all heat transfer into doing work. [link] summarizes the simpler thermodynamic processes and their definitions.

Isobaric	Constant pressure $$W = P\Delta V$$
Isochoric	Constant volume $$W = 0$$
Isothermal	Constant temperature $$Q = W$$

Adiabatic	No heat transfer $$Q = 0$$

Summary of Simple Thermodynamic Processes

Section Summary

- One of the important implications of the first law of thermodynamics is that machines can be harnessed to do work that humans previously did by hand or by external energy supplies such as running water or the heat of the Sun. A machine that uses heat transfer to do work is known as a heat engine.
- There are several simple processes, used by heat engines, that flow from the first law of thermodynamics. Among them are the isobaric, isochoric, isothermal and adiabatic processes.
- These processes differ from one another based on how they affect pressure, volume, temperature, and heat transfer.
- If the work done is performed on the outside environment, work (W) will be a positive value. If the work done is done to the heat engine system, work (W) will be a negative value.
- Some thermodynamic processes, including isothermal and adiabatic processes, are reversible in theory; that is, both the thermodynamic system and the environment can be returned to their initial states. However, because of loss of energy owing to the second law of thermodynamics, complete reversibility does not work in practice.

Conceptual Questions

Exercise:

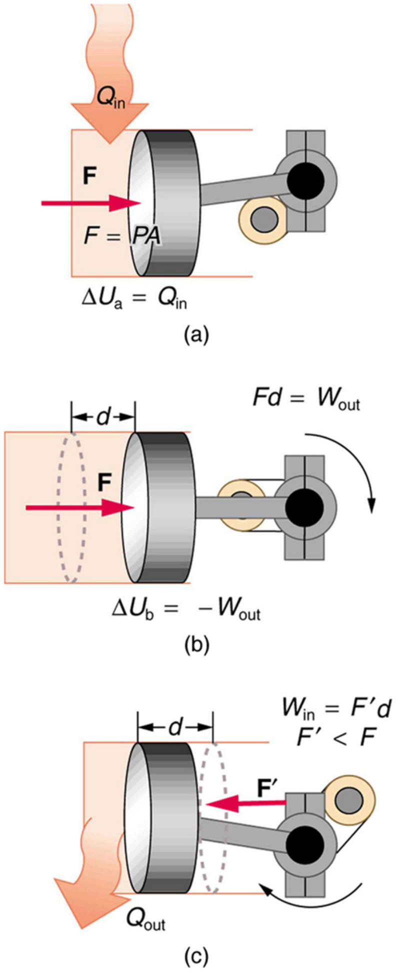
Problem:

A great deal of effort, time, and money has been spent in the quest for the so-called perpetual-motion machine, which is defined as a hypothetical machine that operates or produces useful work indefinitely and/or a hypothetical machine that produces more work or energy than it consumes. Explain, in terms of heat engines and the first law of thermodynamics, why or why not such a machine is likely to be constructed.

Exercise:

Problem:

One method of converting heat transfer into doing work is for heat transfer into a gas to take place, which expands, doing work on a piston, as shown in the figure below. (a) Is the heat transfer converted directly to work in an isobaric process, or does it go through another form first? Explain your answer. (b) What about in an isothermal process? (c) What about in an adiabatic process (where heat transfer occurred prior to the adiabatic process)?



Exercise:

Problem:

Would the previous question make any sense for an isochoric process? Explain your answer.

Exercise:

Problem:

We ordinarily say that $\Delta U = 0$ for an isothermal process. Does this assume no phase change takes place? Explain your answer.

Exercise:

Problem:

The temperature of a rapidly expanding gas decreases. Explain why in terms of the first law of thermodynamics. (Hint: Consider whether the gas does work and whether heat transfer occurs rapidly into the gas through conduction.)

Exercise:

Problem:

A real process may be nearly adiabatic if it occurs over a very short time. How does the short time span help the process to be adiabatic?

Exercise:

Problem:

It is unlikely that a process can be isothermal unless it is a very slow process. Explain why. Is the same true for isobaric and isochoric processes? Explain your answer.

Glossary

heat engine
: a machine that uses heat transfer to do work

isobaric process
: constant-pressure process in which a gas does work

isochoric process
: a constant-volume process

isothermal process
: a constant-temperature process

adiabatic process
: a process in which no heat transfer takes place

reversible process

a process in which both the heat engine system and the external environment theoretically can be returned to their original states

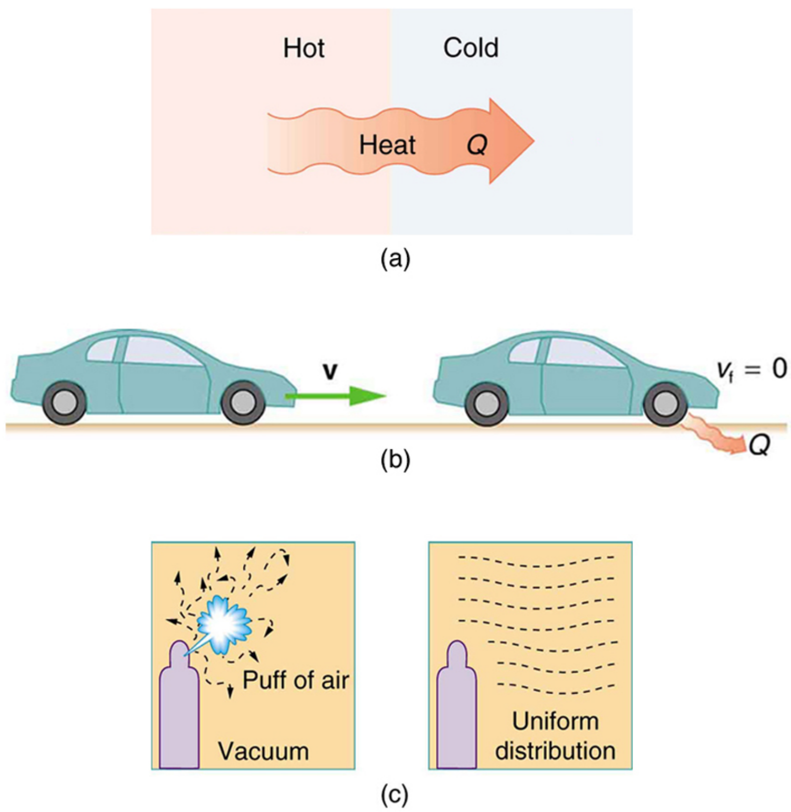
Introduction to the Second Law of Thermodynamics: Heat Engines and Their Efficiency

- State the expressions of the second law of thermodynamics.
- Calculate the efficiency of a coal-fired electricity plant, using second law characteristics.
- Describe and define the Otto cycle.

These ice floes melt during the Arctic summer. Some of them refreeze in the winter, but the second law of thermodynamics predicts that it would be extremely unlikely for the water molecules contained in these particular floes to reform the distinctive alligator-like shape they formed when the picture was taken in the summer of 2009. (credit: Patrick Kelley, U.S. Coast Guard, U.S. Geological Survey)

The second law of thermodynamics deals with the direction taken by spontaneous processes. Many processes occur spontaneously in one direction only—that is, they are irreversible, under a given set of

conditions. Here are some examples of irreversible processes seen in day-to-day life: a glass breaking, heat transferring from a hot object to a cold object, conversion of kinetic energy into thermal energy, and a puff of gas expanding from a corner of a vacuum chamber into the entire chamber. If you saw a video of glass un-breaking, or two objects in thermal contact where one gets hotter and the other gets colder, or a hot stationary object spontaneously cooling off and moving in some direction, or a gas in a chamber "regrouping" into a corner, you would say this is a video run backwards. This is because you intuitively recognize these are examples of **irreversible process**, and these processes in nature only take place in the forward direction, not reverse (See [link]).



Examples of one-way processes in nature. (a) Heat transfer occurs spontaneously from hot to cold and not from cold to hot. (b) The brakes of this car convert its kinetic energy to heat transfer to the environment. The reverse process is impossible. (c) The burst of gas let into this vacuum chamber quickly

expands to uniformly fill every part of the chamber. The random motions of the gas molecules will never return them to the corner.

The fact that certain processes never occur suggests that there is a law forbidding them to occur. The first law of thermodynamics would allow them to occur—none of those processes violate conservation of energy. The law that forbids these processes is called the second law of thermodynamics. We shall see that the second law can be stated in many ways that may seem different, but which in fact are equivalent. Like all natural laws, the second law of thermodynamics gives insights into nature, and its several statements imply that it is broadly applicable, fundamentally affecting many apparently disparate processes.

The already familiar direction of heat transfer from hot to cold is the basis of our first version of the **second law of thermodynamics**.

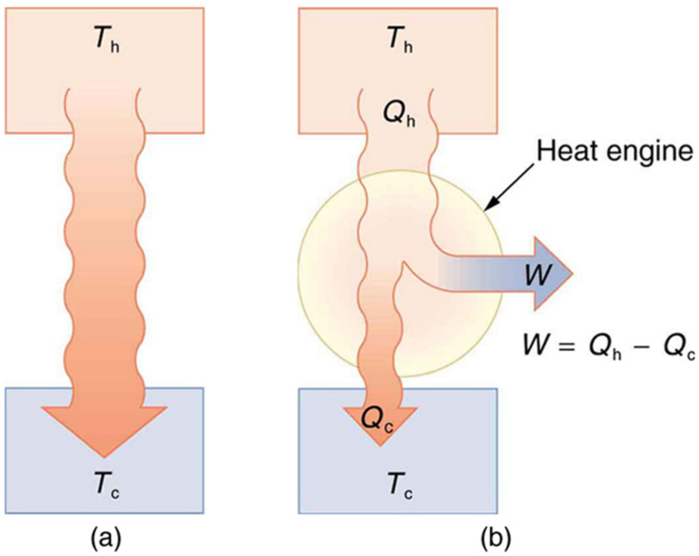
Note:
The Second Law of Thermodynamics (first expression)
Heat transfer occurs spontaneously from higher- to lower-temperature bodies but never spontaneously in the reverse direction.

Another way of stating this: It is impossible for any process to have as its sole result heat transfer from a cooler to a hotter object.

Heat Engines

Now let us consider a device that uses heat transfer to do work. As noted in the previous section, such a device is called a heat engine, and one is shown schematically in [link](b). Gasoline and diesel engines, jet engines, and

steam turbines are all heat engines that do work by using part of the heat transfer from some source. Heat transfer from the hot object (or hot reservoir) is denoted as Q_{h}, while heat transfer into the cold object (or cold reservoir) is Q_{c}, and the work done by the engine is W. The temperatures of the hot and cold reservoirs are T_{h} and T_{c}, respectively.



(a) Heat transfer occurs spontaneously from a hot object to a cold one, consistent with the second law of thermodynamics. (b) A heat engine, represented here by a circle, uses part of the heat transfer to do work. The hot and cold objects are called the hot and cold reservoirs. Q_{h} is the heat transfer out of the hot reservoir, W is the work output, and Q_{c} is the heat transfer into the cold reservoir.

Because the hot reservoir is heated externally, which is energy intensive, it is important that the work is done as efficiently as possible. In fact, we

would like W to equal Q_{h}, and for there to be no heat transfer to the environment ($Q_{\mathrm{c}} = 0$). Unfortunately, this is impossible. The **second law of thermodynamics** also states, with regard to using heat transfer to do work (the second expression of the second law):

Note:
The Second Law of Thermodynamics (second expression)
It is impossible in any system for heat transfer from a reservoir to completely convert to work in a cyclical process in which the system returns to its initial state.

A **cyclical process** brings a system, such as the gas in a cylinder, back to its original state at the end of every cycle. Most heat engines, such as reciprocating piston engines and rotating turbines, use cyclical processes. The second law, just stated in its second form, clearly states that such engines cannot have perfect conversion of heat transfer into work done. Before going into the underlying reasons for the limits on converting heat transfer into work, we need to explore the relationships among W, Q_{h}, and Q_{c}, and to define the efficiency of a cyclical heat engine. As noted, a cyclical process brings the system back to its original condition at the end of every cycle. Such a system's internal energy U is the same at the beginning and end of every cycle—that is, $\Delta U = 0$. The first law of thermodynamics states that

Equation:

$$\Delta U = Q - W,$$

where Q is the *net* heat transfer during the cycle ($Q = Q_{\mathrm{h}} - Q_{\mathrm{c}}$) and W is the net work done by the system. Since $\Delta U = 0$ for a complete cycle, we have

Equation:

$$0 = Q - W,$$

so that

Equation:

$$W = Q.$$

Thus the net work done by the system equals the net heat transfer into the system, or

Equation:

$$W = Q_{\mathrm{h}} - Q_{\mathrm{c}} \text{ (cyclical process)},$$

just as shown schematically in [link](b). The problem is that in all processes, there is some heat transfer Q_{c} to the environment—and usually a very significant amount at that.

In the conversion of energy to work, we are always faced with the problem of getting less out than we put in. We define *conversion efficiency* Eff to be the ratio of useful work output to the energy input (or, in other words, the ratio of what we get to what we spend). In that spirit, we define the efficiency of a heat engine to be its net work output W divided by heat transfer to the engine Q_{h}; that is,

Equation:

$$\mathrm{Eff} = \frac{W}{Q_{\mathrm{h}}}.$$

Since $W = Q_{\mathrm{h}} - Q_{\mathrm{c}}$ in a cyclical process, we can also express this as

Equation:

$$\mathrm{Eff} = \frac{Q_{\mathrm{h}} - Q_{\mathrm{c}}}{Q_{\mathrm{h}}} = 1 - \frac{Q_{\mathrm{c}}}{Q_{\mathrm{h}}} \text{ (cyclical process)},$$

making it clear that an efficiency of 1, or 100%, is possible only if there is no heat transfer to the environment ($Q_{\mathrm{c}} = 0$). Note that all Qs are positive.

The direction of heat transfer is indicated by a plus or minus sign. For example, Q_{c} is out of the system and so is preceded by a minus sign.

Example:
Daily Work Done by a Coal-Fired Power Station, Its Efficiency and Carbon Dioxide Emissions
A coal-fired power station is a huge heat engine. It uses heat transfer from burning coal to do work to turn turbines, which are used to generate electricity. In a single day, a large coal power station has 2.50×10^{14} J of heat transfer from coal and 1.48×10^{14} J of heat transfer into the environment. (a) What is the work done by the power station? (b) What is the efficiency of the power station? (c) In the combustion process, the following chemical reaction occurs: $\text{C} + \text{O}_2 \rightarrow \text{CO}_2$. This implies that every 12 kg of coal puts 12 kg + 16 kg + 16 kg = 44 kg of carbon dioxide into the atmosphere. Assuming that 1 kg of coal can provide 2.5×10^{6} J of heat transfer upon combustion, how much CO_2 is emitted per day by this power plant?
Strategy for (a)
We can use $W = Q_{\text{h}} - Q_{\text{c}}$ to find the work output W, assuming a cyclical process is used in the power station. In this process, water is boiled under pressure to form high-temperature steam, which is used to run steam turbine-generators, and then condensed back to water to start the cycle again.
Solution for (a)
Work output is given by:
Equation:

$$W = Q_{\text{h}} - Q_{\text{c}}.$$

Substituting the given values:
Equation:

$$\begin{aligned} W &= 2.50\times10^{14}\text{ J}-1.48\times10^{14}\text{ J} \\ &= 1.02 \times 10^{14}\text{ J}. \end{aligned}$$

Strategy for (b)

The efficiency can be calculated with $\mathrm{Eff} = \frac{W}{Q_\mathrm{h}}$ since Q_h is given and work W was found in the first part of this example.

Solution for (b)

Efficiency is given by: $\mathrm{Eff} = \frac{W}{Q_\mathrm{h}}$. The work W was just found to be 1.02×10^{14} J, and Q_h is given, so the efficiency is

Equation:

$$\begin{aligned} Eff &= \frac{1.02\times10^{14}\text{ J}}{2.50\times10^{14}\text{ J}} \\ &= 0.408\text{, or }40.8\% \end{aligned}$$

Strategy for (c)

The daily consumption of coal is calculated using the information that each day there is 2.50×10^{14} J of heat transfer from coal. In the combustion process, we have $\mathrm{C} + \mathrm{O_2} \rightarrow \mathrm{CO_2}$. So every 12 kg of coal puts 12 kg + 16 kg + 16 kg = 44 kg of $\mathrm{CO_2}$ into the atmosphere.

Solution for (c)

The daily coal consumption is

Equation:

$$\frac{2.50\times10^{14}\text{ J}}{2.50\times10^{6}\text{ J/kg}} = 1.0\times10^{8}\text{ kg.}$$

Assuming that the coal is pure and that all the coal goes toward producing carbon dioxide, the carbon dioxide produced per day is

Equation:

$$1.0\times10^{8}\text{ kg coal}\times\frac{44\text{ kg CO}_2}{12\text{ kg coal}} = 3.7\times10^{8}\text{ kg CO}_2.$$

This is 370,000 metric tons of $\mathrm{CO_2}$ produced every day.

Discussion

If all the work output is converted to electricity in a period of one day, the average power output is 1180 MW (this is left to you as an end-of-chapter problem). This value is about the size of a large-scale conventional power plant. The efficiency found is acceptably close to the value of 42% given

for coal power stations. It means that fully 59.2% of the energy is heat transfer to the environment, which usually results in warming lakes, rivers, or the ocean near the power station, and is implicated in a warming planet generally. While the laws of thermodynamics limit the efficiency of such plants—including plants fired by nuclear fuel, oil, and natural gas—the heat transfer to the environment could be, and sometimes is, used for heating homes or for industrial processes. The generally low cost of energy has not made it economical to make better use of the waste heat transfer from most heat engines. Coal-fired power plants produce the greatest amount of CO_2 per unit energy output (compared to natural gas or oil), making coal the least efficient fossil fuel.

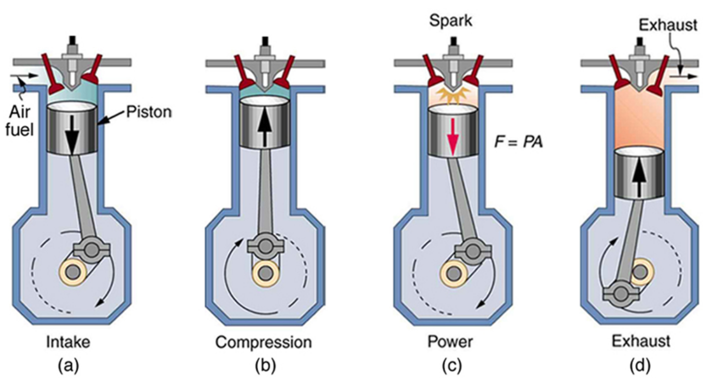
With the information given in [link], we can find characteristics such as the efficiency of a heat engine without any knowledge of how the heat engine operates, but looking further into the mechanism of the engine will give us greater insight. [link] illustrates the operation of the common four-stroke gasoline engine. The four steps shown complete this heat engine's cycle, bringing the gasoline-air mixture back to its original condition.

The **Otto cycle** shown in [link](a) is used in four-stroke internal combustion engines, although in fact the true Otto cycle paths do not correspond exactly to the strokes of the engine.

The adiabatic process AB corresponds to the nearly adiabatic compression stroke of the gasoline engine. In both cases, work is done on the system (the gas mixture in the cylinder), increasing its temperature and pressure. Along path BC of the Otto cycle, heat transfer Q_{h} into the gas occurs at constant volume, causing a further increase in pressure and temperature. This process corresponds to burning fuel in an internal combustion engine, and takes place so rapidly that the volume is nearly constant. Path CD in the Otto cycle is an adiabatic expansion that does work on the outside world, just as the power stroke of an internal combustion engine does in its nearly adiabatic expansion. The work done by the system along path CD is greater than the work done on the system along path AB, because the pressure is greater, and so there is a net work output. Along path DA in the Otto cycle, heat transfer Q_{c} from the gas at constant volume reduces its temperature

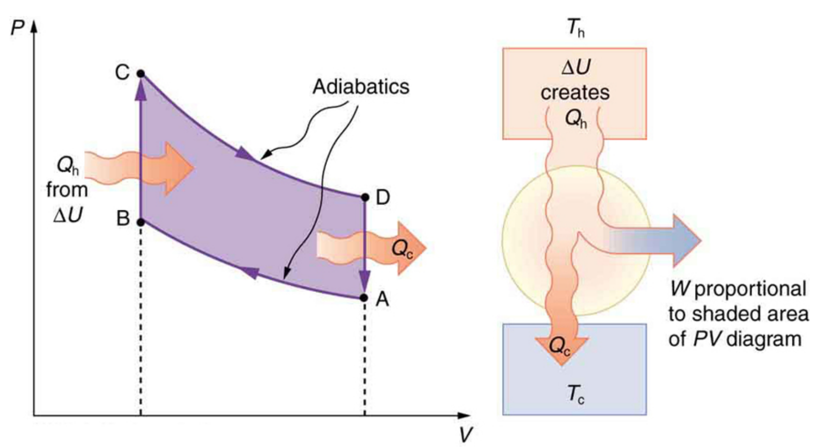
and pressure, returning it to its original state. In an internal combustion engine, this process corresponds to the exhaust of hot gases and the intake of an air-gasoline mixture at a considerably lower temperature. In both cases, heat transfer into the environment occurs along this final path.

The net work done by a cyclical process is the area inside the closed path on a PV diagram, such as that inside path ABCDA in [link]. Note that in every imaginable cyclical process, it is absolutely necessary for heat transfer from the system to occur in order to get a net work output. In the Otto cycle, heat transfer occurs along path DA. If no heat transfer occurs, then the return path is the same, and the net work output is zero. The lower the temperature on the path AB, the less work has to be done to compress the gas. The area inside the closed path is then greater, and so the engine does more work and is thus more efficient. Similarly, the higher the temperature along path CD, the more work output there is. (See [link].) So efficiency is related to the temperatures of the hot and cold reservoirs. In the next section, we shall see what the absolute limit to the efficiency of a heat engine is, and how it is related to temperature.



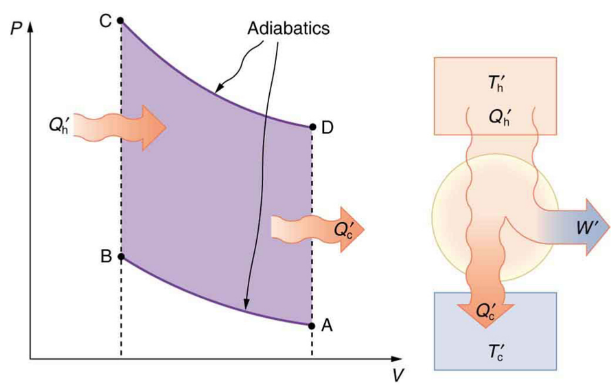
In the four-stroke internal combustion gasoline engine, heat transfer into work takes place in the cyclical process shown here. The piston is connected to a rotating crankshaft, which both takes work out of and does work on the gas in the cylinder. (a)

Air is mixed with fuel during the intake stroke. (b) During the compression stroke, the air-fuel mixture is rapidly compressed in a nearly adiabatic process, as the piston rises with the valves closed. Work is done on the gas. (c) The power stroke has two distinct parts. First, the air-fuel mixture is ignited, converting chemical potential energy into thermal energy almost instantaneously, which leads to a great increase in pressure. Then the piston descends, and the gas does work by exerting a force through a distance in a nearly adiabatic process. (d) The exhaust stroke expels the hot gas to prepare the engine for another cycle, starting again with the intake stroke.



PV diagram for a simplified Otto cycle, analogous to that employed in an internal combustion engine. Point A corresponds to

the start of the compression stroke of an internal combustion engine. Paths AB and CD are adiabatic and correspond to the compression and power strokes of an internal combustion engine, respectively. Paths BC and DA are isochoric and accomplish similar results to the ignition and exhaust-intake portions, respectively, of the internal combustion engine’s cycle. Work is done on the gas along path AB, but more work is done by the gas along path CD, so that there is a net work output.



This Otto cycle produces a greater work output than the one in [link], because the starting temperature of path CD is higher and the starting temperature of path AB is lower. The area inside the loop is greater, corresponding to greater net work output.

Section Summary

- The two expressions of the second law of thermodynamics are: (i) Heat transfer occurs spontaneously from higher- to lower-temperature bodies but never spontaneously in the reverse direction; and (ii) It is impossible in any system for heat transfer from a reservoir to completely convert to work in a cyclical process in which the system returns to its initial state. These expressions describe the direction of irreversible processes that occur in nature.
- Cyclical processes are processes that return to their original state at the end of every cycle.
- In a cyclical process, such as a heat engine, the net work done by the system equals the net heat transfer into the system, or $W = Q_{\text{h}} - Q_{\text{c}}$, where Q_{h} is the heat transfer from the hot object (hot reservoir), and Q_{c} is the heat transfer into the cold object (cold reservoir).
- Efficiency can be expressed as $\text{Eff} = \frac{W}{Q_{\text{h}}}$, the ratio of work output divided by the amount of energy input.
- The four-stroke gasoline engine is often explained in terms of the Otto cycle, which is a repeating sequence of processes that convert heat into work.

Conceptual Questions

Exercise:

Problem:

Imagine you are driving a car up Pike’s Peak in Colorado. To raise a car weighing 1000 kilograms a distance of 100 meters would require about a million joules. You could raise a car 12.5 kilometers with the energy in a gallon of gas. Driving up Pike's Peak (a mere 3000-meter climb) should consume a little less than a quart of gas. But other considerations have to be taken into account. Explain, in terms of efficiency, what factors may keep you from realizing your ideal energy use on this trip.

Exercise:

Problem:

Is a temperature difference necessary to operate a heat engine? State why or why not.

Exercise:

Problem:

Definitions of efficiency vary depending on how energy is being converted. Compare the definitions of efficiency for the human body and heat engines. How does the definition of efficiency in each relate to the type of energy being converted into doing work?

Exercise:

Problem:

Why—other than the fact that the second law of thermodynamics says reversible engines are the most efficient—should heat engines employing reversible processes be more efficient than those employing irreversible processes? Consider that dissipative mechanisms are one cause of irreversibility.

Glossary

irreversible process
: a process which occurs in only one direction in nature; a process that cannot be exactly reversed

second law of thermodynamics
: heat transfer flows from a hotter to a cooler object, never the reverse, and some heat energy in any process is lost to available work in a cyclical process

cyclical process
: a process in which the path returns to its original state at the end of every cycle

Otto cycle

a thermodynamic cycle, consisting of a pair of adiabatic processes and a pair of isochoric processes, that converts heat into work, e.g., the four-stroke engine cycle of intake, compression, ignition, and exhaust

Carnot’s Perfect Heat Engine: The Second Law of Thermodynamics Restated

- Identify a Carnot cycle.
- Calculate maximum theoretical efficiency of a nuclear reactor.
- Explain how dissipative processes affect the ideal Carnot engine.

This novelty toy, known as the drinking bird, is an example of Carnot’s engine. It contains methylene chloride (mixed with a dye) in the abdomen, which boils at a very low temperature—about 100°F. To operate, one gets the bird’s head wet. As the water evaporates, fluid moves up into the head, causing the bird to become top-heavy and dip forward back into the water. This cools down the methylene chloride in the head, and it moves back into the abdomen, causing the bird to become bottom heavy and tip up. Except for a very small input of energy—the original head-wetting—the bird becomes a perpetual motion machine of sorts. (credit: Arabesk.nl, Wikimedia Commons)

We know from the second law of thermodynamics that a heat engine cannot be 100% efficient, since there must always be some heat transfer Q_c to the environment, which is often called waste heat. How efficient, then, can a heat engine be? This question was answered at a theoretical level in 1824 by a young French engineer, Sadi Carnot (1796–1832), in his study of the then-emerging heat engine technology crucial to the Industrial Revolution. He devised a theoretical cycle, now called the **Carnot cycle**, which is the most efficient cyclical process possible. The second law of thermodynamics can be restated in terms of the Carnot cycle, and so what Carnot actually discovered was this fundamental law. Any heat engine employing the Carnot cycle is called a **Carnot engine**.

What is crucial to the Carnot cycle—and, in fact, defines it—is that only reversible processes are used. Irreversible processes involve dissipative factors, such as friction and turbulence. This increases heat transfer Q_c to the environment and reduces the efficiency of the engine. Obviously, then, reversible processes are superior.

Note:
Carnot Engine
Stated in terms of reversible processes, the **second law of thermodynamics** has a third form:
A Carnot engine operating between two given temperatures has the greatest possible efficiency of any heat engine operating between these two temperatures. Furthermore, all engines employing only reversible processes have this same maximum efficiency when operating between the same given temperatures.

[link] shows the PV diagram for a Carnot cycle. The cycle comprises two isothermal and two adiabatic processes. Recall that both isothermal and adiabatic processes are, in principle, reversible.

Carnot also determined the efficiency of a perfect heat engine—that is, a Carnot engine. It is always true that the efficiency of a cyclical heat engine is given by:

Equation:

$$\mathrm{Eff} = \frac{Q_\mathrm{h} - Q_\mathrm{c}}{Q_\mathrm{h}} = 1 - \frac{Q_\mathrm{c}}{Q_\mathrm{h}}.$$

What Carnot found was that for a perfect heat engine, the ratio $Q_\mathrm{c}/Q_\mathrm{h}$ equals the ratio of the absolute temperatures of the heat reservoirs. That is, $Q_\mathrm{c}/Q_\mathrm{h} = T_\mathrm{c}/T_\mathrm{h}$ for a Carnot engine, so that the maximum or **Carnot efficiency** Eff_C is given by

Equation:

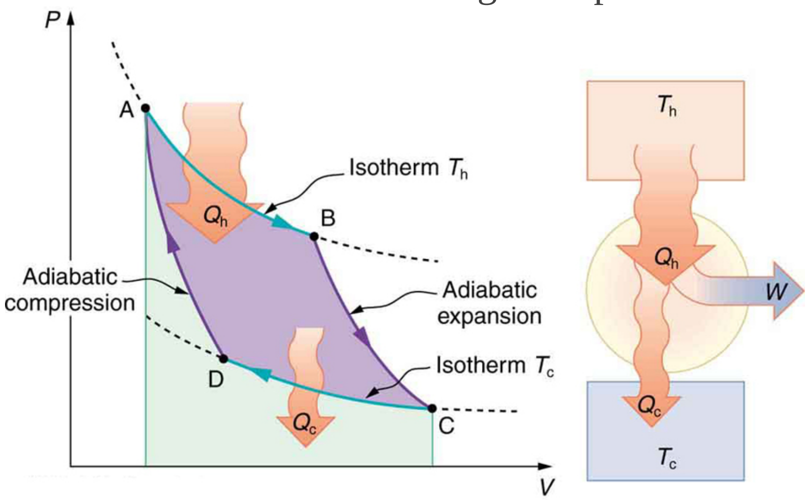
$$Eff_\mathrm{C} = 1 - \frac{T_\mathrm{c}}{T_\mathrm{h}},$$

where T_h and T_c are in kelvins (or any other absolute temperature scale). No real heat engine can do as well as the Carnot efficiency—an actual efficiency of about 0.7 of this maximum is usually the best that can be accomplished. But the ideal Carnot engine, like the drinking bird above, while a fascinating novelty, has zero power. This makes it unrealistic for any applications.

Carnot's interesting result implies that 100% efficiency would be possible only if $T_\mathrm{c} = 0\ \mathrm{K}$ —that is, only if the cold reservoir were at absolute zero, a practical and theoretical impossibility. But the physical implication is this —the only way to have all heat transfer go into doing work is to remove *all* thermal energy, and this requires a cold reservoir at absolute zero.

It is also apparent that the greatest efficiencies are obtained when the ratio $T_\mathrm{c}/T_\mathrm{h}$ is as small as possible. Just as discussed for the Otto cycle in the previous section, this means that efficiency is greatest for the highest possible temperature of the hot reservoir and lowest possible temperature of the cold reservoir. (This setup increases the area inside the closed loop on the PV diagram; also, it seems reasonable that the greater the temperature

difference, the easier it is to divert the heat transfer to work.) The actual reservoir temperatures of a heat engine are usually related to the type of heat source and the temperature of the environment into which heat transfer occurs. Consider the following example.

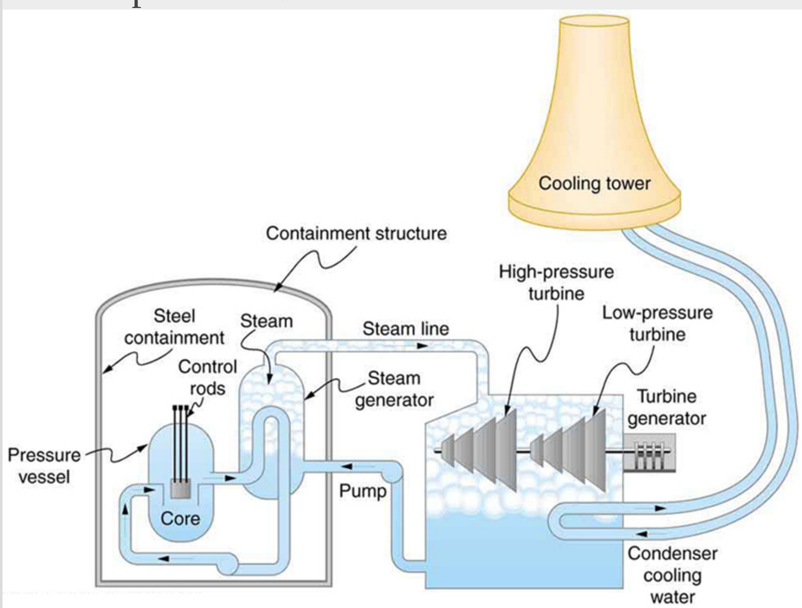


PV diagram for a Carnot cycle, employing only reversible isothermal and adiabatic processes. Heat transfer Q_h occurs into the working substance during the isothermal path AB, which takes place at constant temperature T_h. Heat transfer Q_c occurs out of the working substance during the isothermal path CD, which takes place at constant temperature T_c. The net work output W equals the area inside the path ABCDA. Also shown is a schematic of a Carnot engine operating between hot and cold reservoirs at temperatures T_h and T_c. Any heat engine using reversible processes and operating between these two temperatures will have the same maximum efficiency as the Carnot engine.

Example:

Maximum Theoretical Efficiency for a Nuclear Reactor

A nuclear power reactor has pressurized water at $300^{\circ}C$. (Higher temperatures are theoretically possible but practically not, due to limitations with materials used in the reactor.) Heat transfer from this water is a complex process (see [link]). Steam, produced in the steam generator, is used to drive the turbine-generators. Eventually the steam is condensed to water at $27^{\circ}C$ and then heated again to start the cycle over. Calculate the maximum theoretical efficiency for a heat engine operating between these two temperatures.



Schematic diagram of a pressurized water nuclear reactor and the steam turbines that convert work into electrical energy. Heat exchange is used to generate steam, in part to avoid contamination of the generators with radioactivity. Two turbines are used because this is less expensive than operating a single generator that produces the same amount of electrical energy. The steam is condensed to liquid before being returned to the heat exchanger, to keep exit steam pressure low and aid the flow of steam through the turbines (equivalent to using a

lower-temperature cold reservoir). The considerable energy associated with condensation must be dissipated into the local environment; in this example, a cooling tower is used so there is no direct heat transfer to an aquatic environment. (Note that the water going to the cooling tower does not come into contact with the steam flowing over the turbines.)

Strategy

Since temperatures are given for the hot and cold reservoirs of this heat engine, $Eff_{\mathrm{C}} = 1 - \frac{T_{\mathrm{c}}}{T_{\mathrm{h}}}$ can be used to calculate the Carnot (maximum theoretical) efficiency. Those temperatures must first be converted to kelvins.

Solution

The hot and cold reservoir temperatures are given as $300^{\mathrm{o}}\mathrm{C}$ and $27.0^{\mathrm{o}}\mathrm{C}$, respectively. In kelvins, then, $T_{\mathrm{h}} = 573\ \mathrm{K}$ and $T_{\mathrm{c}} = 300\ \mathrm{K}$, so that the maximum efficiency is

Equation:

$$Eff_{\mathrm{C}} = 1 - \frac{T_{\mathrm{c}}}{T_{\mathrm{h}}}.$$

Thus,

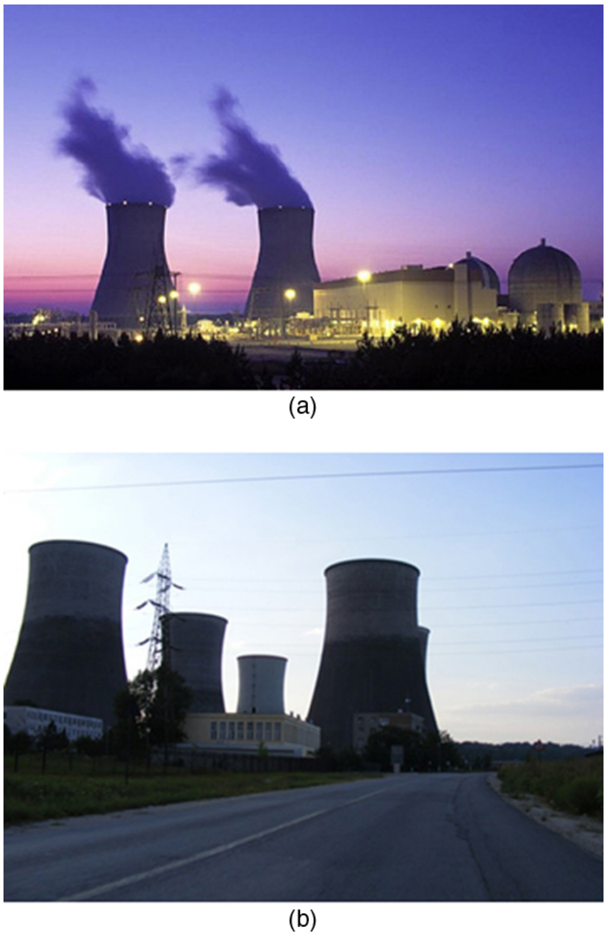
Equation:

$$\begin{array}{lll} Eff_{\mathrm{C}} & = & 1 - \frac{300\ \mathrm{K}}{573\ \mathrm{K}} \\ & = & 0.476, \text{ or } 47.6\%. \end{array}$$

Discussion

A typical nuclear power station’s actual efficiency is about 35%, a little better than 0.7 times the maximum possible value, a tribute to superior engineering. Electrical power stations fired by coal, oil, and natural gas have greater actual efficiencies (about 42%), because their boilers can reach higher temperatures and pressures. The cold reservoir temperature in

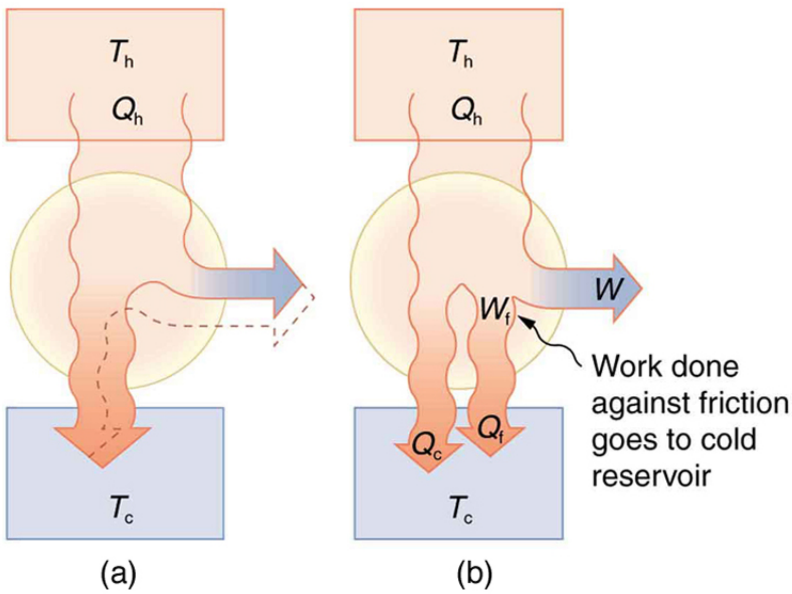
any of these power stations is limited by the local environment. [link] shows (a) the exterior of a nuclear power station and (b) the exterior of a coal-fired power station. Both have cooling towers into which water from the condenser enters the tower near the top and is sprayed downward, cooled by evaporation.



(a) A nuclear power station (credit: BlatantWorld.com) and (b) a coal-fired power station. Both have cooling towers in which water evaporates into the environment, representing Q_{c}. The nuclear reactor, which supplies Q_{h}, is housed inside

the dome-shaped containment buildings. (credit: Robert & Mihaela Vicol, publicphoto.org)

Since all real processes are irreversible, the actual efficiency of a heat engine can never be as great as that of a Carnot engine, as illustrated in [link](a). Even with the best heat engine possible, there are always dissipative processes in peripheral equipment, such as electrical transformers or car transmissions. These further reduce the overall efficiency by converting some of the engine's work output back into heat transfer, as shown in [link](b).



Real heat engines are less efficient than Carnot engines. (a) Real engines use irreversible processes, reducing the heat transfer to work. Solid lines represent the actual process; the dashed lines are what a Carnot engine would do between the same two reservoirs. (b) Friction and other dissipative processes in the output mechanisms of a heat engine convert some

of its work output into heat transfer to the environment.

Section Summary

- The Carnot cycle is a theoretical cycle that is the most efficient cyclical process possible. Any engine using the Carnot cycle, which uses only reversible processes (adiabatic and isothermal), is known as a Carnot engine.
- Any engine that uses the Carnot cycle enjoys the maximum theoretical efficiency.
- While Carnot engines are ideal engines, in reality, no engine achieves Carnot's theoretical maximum efficiency, since dissipative processes, such as friction, play a role. Carnot cycles without heat loss may be possible at absolute zero, but this has never been seen in nature.

Conceptual Questions

Exercise:

Problem:

Think about the drinking bird at the beginning of this section ([link]). Although the bird enjoys the theoretical maximum efficiency possible, if left to its own devices over time, the bird will cease "drinking." What are some of the dissipative processes that might cause the bird's motion to cease?

Exercise:

Problem:

Can improved engineering and materials be employed in heat engines to reduce heat transfer into the environment? Can they eliminate heat transfer into the environment entirely?

Exercise:

Problem:

Does the second law of thermodynamics alter the conservation of energy principle?

Problem Exercises

Exercise:

Problem:

A certain gasoline engine has an efficiency of 30.0%. What would the hot reservoir temperature be for a Carnot engine having that efficiency, if it operates with a cold reservoir temperature of 200°C?

Solution:

403°C

Exercise:

Problem:

A gas-cooled nuclear reactor operates between hot and cold reservoir temperatures of 700°C and 27.0°C. (a) What is the maximum efficiency of a heat engine operating between these temperatures? (b) Find the ratio of this efficiency to the Carnot efficiency of a standard nuclear reactor (found in [link]).

Exercise:

Problem:

(a) What is the hot reservoir temperature of a Carnot engine that has an efficiency of 42.0% and a cold reservoir temperature of 27.0°C? (b) What must the hot reservoir temperature be for a real heat engine that achieves 0.700 of the maximum efficiency, but still has an efficiency of 42.0% (and a cold reservoir at 27.0°C)? (c) Does your answer imply practical limits to the efficiency of car gasoline engines?

Solution:

(a) $244^\circ C$

(b) $477^\circ C$

(c)Yes, since automobiles engines cannot get too hot without overheating, their efficiency is limited.

Exercise:

Problem:

Steam locomotives have an efficiency of 17.0% and operate with a hot steam temperature of $425^\circ C$. (a) What would the cold reservoir temperature be if this were a Carnot engine? (b) What would the maximum efficiency of this steam engine be if its cold reservoir temperature were $150^\circ C$?

Exercise:

Problem:

Practical steam engines utilize $450^\circ C$ steam, which is later exhausted at $270^\circ C$. (a) What is the maximum efficiency that such a heat engine can have? (b) Since $270^\circ C$ steam is still quite hot, a second steam engine is sometimes operated using the exhaust of the first. What is the maximum efficiency of the second engine if its exhaust has a temperature of $150^\circ C$? (c) What is the overall efficiency of the two engines? (d) Show that this is the same efficiency as a single Carnot engine operating between $450^\circ C$ and $150^\circ C$. Explicitly show how you follow the steps in the Problem-Solving Strategies for Thermodynamics.

Solution:

(a) $Eff_1 = 1 - \frac{T_{c,1}}{T_{h,1}} = 1 - \frac{543\text{ K}}{723\text{ K}} = 0.249 \text{ or } 24.9\%$

(b) $Eff_2 = 1 - \frac{423\text{ K}}{543\text{ K}} = 0.221 \text{ or } 22.1\%$

(c) $Eff_1 = 1 - \frac{T_{\text{c},1}}{T_{\text{h},1}} \Rightarrow T_{\text{c},1} = T_{\text{h},1}(1, -, eff_1)$
similarly, $T_{\text{c},2} = T_{\text{h},2}(1 - Eff_2)$
using $T_{\text{h},2} = T_{\text{c},1}$ in above equation gives
$T_{\text{c},2} = T_{\text{h},1}(1 - Eff_1)(1 - Eff_2) \equiv T_{\text{h},1}(1 - Eff_{\text{overall}})$
$\therefore (1 - Eff_{\text{overall}}) = (1 - Eff_1)(1 - Eff_2)$
$Eff_{\text{overall}} = 1 - (1 - 0.249)(1 - 0.221) = 41.5\%$

(d) $Eff_{\text{overall}} = 1 - \frac{423\text{ K}}{723\text{ K}} = 0.415 \text{ or } 41.5\%$

Exercise:

Problem:

A coal-fired electrical power station has an efficiency of 38%. The temperature of the steam leaving the boiler is 550°C. What percentage of the maximum efficiency does this station obtain? (Assume the temperature of the environment is 20°C.)

Exercise:

Problem:

Would you be willing to financially back an inventor who is marketing a device that she claims has 25 kJ of heat transfer at 600 K, has heat transfer to the environment at 300 K, and does 12 kJ of work? Explain your answer.

Solution:

The heat transfer to the cold reservoir is $Q_{\text{c}} = Q_{\text{h}} - W = 25\text{ kJ} - 12\text{ kJ} = 13\text{ kJ}$, so the efficiency is $Eff = 1 - \frac{Q_{\text{c}}}{Q_{\text{h}}} = 1 - \frac{13\text{ kJ}}{25\text{ kJ}} = 0.48$. The Carnot efficiency is $Eff_{\text{C}} = 1 - \frac{T_{\text{c}}}{T_{\text{h}}} = 1 - \frac{300\text{ K}}{600\text{ K}} = 0.50$. The actual efficiency is 96% of the Carnot efficiency, which is much higher than the best-ever achieved of about 70%, so her scheme is likely to be fraudulent.

Exercise:

Problem: **Unreasonable Results**

(a) Suppose you want to design a steam engine that has heat transfer to the environment at $270^{\circ}C$ and has a Carnot efficiency of 0.800. What temperature of hot steam must you use? (b) What is unreasonable about the temperature? (c) Which premise is unreasonable?

Exercise:

Problem: **Unreasonable Results**

Calculate the cold reservoir temperature of a steam engine that uses hot steam at $450^{\circ}C$ and has a Carnot efficiency of 0.700. (b) What is unreasonable about the temperature? (c) Which premise is unreasonable?

Solution:

(a) $-56.3^{\circ}C$

(b) The temperature is too cold for the output of a steam engine (the local environment). It is below the freezing point of water.

(c) The assumed efficiency is too high.

Glossary

Carnot cycle
: a cyclical process that uses only reversible processes, the adiabatic and isothermal processes

Carnot engine
: a heat engine that uses a Carnot cycle

Carnot efficiency
: the maximum theoretical efficiency for a heat engine

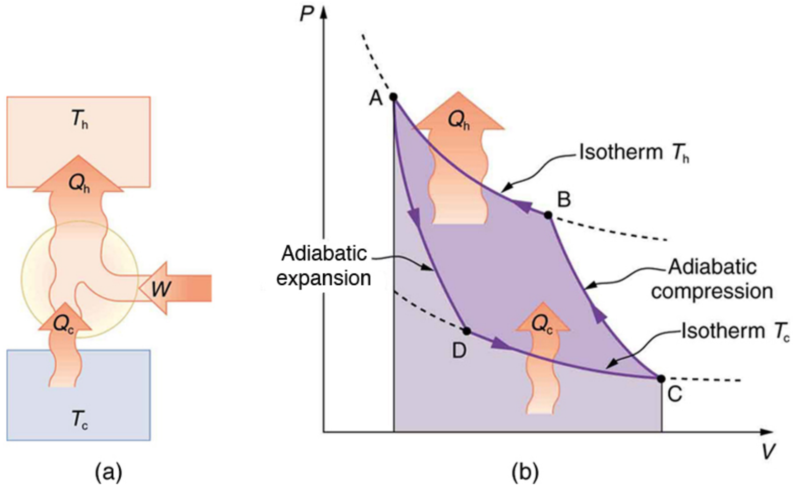
Applications of Thermodynamics: Heat Pumps and Refrigerators

- Describe the use of heat engines in heat pumps and refrigerators.
- Demonstrate how a heat pump works to warm an interior space.
- Explain the differences between heat pumps and refrigerators.
- Calculate a heat pump’s coefficient of performance.

Almost every home contains a refrigerator. Most people don’t realize they are also sharing their homes with a heat pump. (credit: Id1337x, Wikimedia Commons)

Heat pumps, air conditioners, and refrigerators utilize heat transfer from cold to hot. They are heat engines run backward. We say backward, rather than reverse, because except for Carnot engines, all heat engines, though they can be run backward, cannot truly be reversed. Heat transfer occurs from a cold reservoir Q_{c} and into a hot one. This requires work input W, which is also converted to heat transfer. Thus the heat transfer to the hot reservoir is $Q_{\mathrm{h}} = Q_{\mathrm{c}} + W$. (Note that Q_{h}, Q_{c}, and W are positive, with their directions indicated on schematics rather than by sign.) A heat pump’s mission is for heat transfer Q_{h} to occur into a warm environment, such as a home in the winter. The mission of air conditioners and refrigerators is for

heat transfer Q_{c} to occur from a cool environment, such as chilling a room or keeping food at lower temperatures than the environment. (Actually, a heat pump can be used both to heat and cool a space. It is essentially an air conditioner and a heating unit all in one. In this section we will concentrate on its heating mode.)

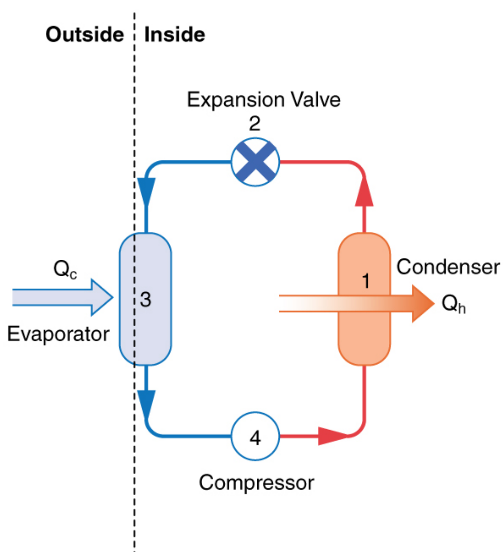


Heat pumps, air conditioners, and refrigerators are heat engines operated backward. The one shown here is based on a Carnot (reversible) engine. (a) Schematic diagram showing heat transfer from a cold reservoir to a warm reservoir with a heat pump. The directions of W, Q_{h}, and Q_{c} are opposite what they would be in a heat engine. (b) PV diagram for a Carnot cycle similar to that in [link] but reversed, following path ADCBA. The area inside the loop is negative, meaning there is a net work input. There is heat transfer Q_{c} into the system from a cold reservoir along path DC, and heat transfer Q_{h} out of the system into a hot reservoir along path BA.

Heat Pumps

The great advantage of using a heat pump to keep your home warm, rather than just burning fuel, is that a heat pump supplies $Q_{\mathrm{h}} = Q_{\mathrm{c}} + W$. Heat transfer is from the outside air, even at a temperature below freezing, to the indoor space. You only pay for W, and you get an additional heat transfer of Q_{c} from the outside at no cost; in many cases, at least twice as much energy is transferred to the heated space as is used to run the heat pump. When you burn fuel to keep warm, you pay for all of it. The disadvantage is that the work input (required by the second law of thermodynamics) is sometimes more expensive than simply burning fuel, especially if the work is done by electrical energy.

The basic components of a heat pump in its heating mode are shown in [link]. A working fluid such as a non-CFC refrigerant is used. In the outdoor coils (the evaporator), heat transfer Q_{c} occurs to the working fluid from the cold outdoor air, turning it into a gas.



A simple heat pump has four basic components: (1) condenser, (2) expansion valve, (3) evaporator, and (4) compressor. In the

heating mode, heat transfer Q_c occurs to the working fluid in the evaporator (3) from the colder outdoor air, turning it into a gas. The electrically driven compressor (4) increases the temperature and pressure of the gas and forces it into the condenser coils (1) inside the heated space. Because the temperature of the gas is higher than the temperature in the room, heat transfer from the gas to the room occurs as the gas condenses to a liquid. The working fluid is then cooled as it flows back through an expansion valve (2) to the outdoor evaporator coils.

The electrically driven compressor (work input W) raises the temperature and pressure of the gas and forces it into the condenser coils that are inside the heated space. Because the temperature of the gas is higher than the temperature inside the room, heat transfer to the room occurs and the gas condenses to a liquid. The liquid then flows back through a pressure-reducing valve to the outdoor evaporator coils, being cooled through expansion. (In a cooling cycle, the evaporator and condenser coils exchange roles and the flow direction of the fluid is reversed.)

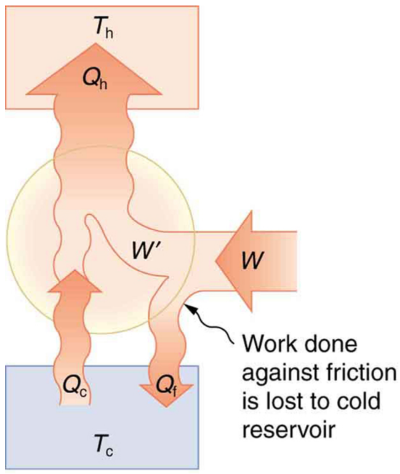
The quality of a heat pump is judged by how much heat transfer Q_{h} occurs into the warm space compared with how much work input W is required. In the spirit of taking the ratio of what you get to what you spend, we define a **heat pump's coefficient of performance** (COP_{hp}) to be

Equation:

$$COP_{\mathrm{hp}} = \frac{Q_{\mathrm{h}}}{W}.$$

Since the efficiency of a heat engine is $Eff = W/Q_{\mathrm{h}}$, we see that $COP_{\mathrm{hp}} = 1/Eff$, an important and interesting fact. First, since the efficiency of any heat engine is less than 1, it means that COP_{hp} is always greater than 1—that is, a heat pump always has more heat transfer Q_{h} than work put into it. Second, it means that heat pumps work best when temperature differences are small. The efficiency of a perfect, or Carnot, engine is $Eff_{\mathrm{C}} = 1 - (T_{\mathrm{c}}/T_{\mathrm{h}})$; thus, the smaller the temperature difference, the smaller the efficiency and the greater the COP_{hp} (because $COP_{\mathrm{hp}} = 1/Eff$). In other words, heat pumps do not work as well in very cold climates as they do in more moderate climates.

Friction and other irreversible processes reduce heat engine efficiency, but they do *not* benefit the operation of a heat pump—instead, they reduce the work input by converting part of it to heat transfer back into the cold reservoir before it gets into the heat pump.



When a real heat engine is run backward, some of the intended work input (W) goes into heat transfer before it gets into the heat engine, thereby reducing its coefficient of performance COP_{hp}. In this figure, W’ represents the portion of W that goes into the heat pump, while the remainder of W is lost in the form of frictional heat (Q_f) to the cold reservoir. If all of W had gone into the heat pump, then Q_{h} would have

been greater. The best heat pump uses adiabatic and isothermal processes, since, in theory, there would be no dissipative processes to reduce the heat transfer to the hot reservoir.

Example:

The Best *COP* $_{\text{hp}}$ of a Heat Pump for Home Use

A heat pump used to warm a home must employ a cycle that produces a working fluid at temperatures greater than typical indoor temperature so that heat transfer to the inside can take place. Similarly, it must produce a working fluid at temperatures that are colder than the outdoor temperature so that heat transfer occurs from outside. Its hot and cold reservoir temperatures therefore cannot be too close, placing a limit on its COP_{hp}. (See [link].) What is the best coefficient of performance possible for such a heat pump, if it has a hot reservoir temperature of 45.0°C and a cold reservoir temperature of -15.0°C?

Strategy

A Carnot engine reversed will give the best possible performance as a heat pump. As noted above, $COP_{\text{hp}} = 1/Eff$, so that we need to first calculate the Carnot efficiency to solve this problem.

Solution

Carnot efficiency in terms of absolute temperature is given by**:**

Equation:

$$Eff_{\text{C}} = 1 - \frac{T_{\text{c}}}{T_{\text{h}}}.$$

The temperatures in kelvins are $T_{\text{h}} = 318 \text{ K}$ and $T_{\text{c}} = 258 \text{ K}$, so that
Equation:

$$Eff_{\text{C}} = 1 - \frac{258 \text{ K}}{318 \text{ K}} = 0.1887.$$

Thus, from the discussion above,
Equation:

$$COP_{\text{hp}} = \frac{1}{\text{Eff}} = \frac{1}{0.1887} = 5.30,$$

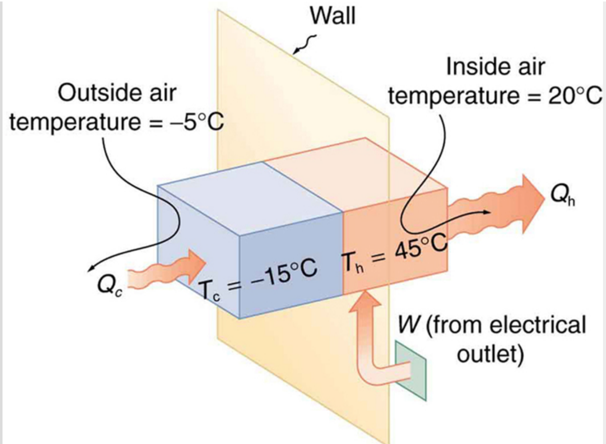
or
Equation:

$$COP_{\text{hp}} = \frac{Q_{\text{h}}}{W} = 5.30,$$

so that
Equation:

$$Q_{\text{h}} = 5.30 \text{ W}.$$

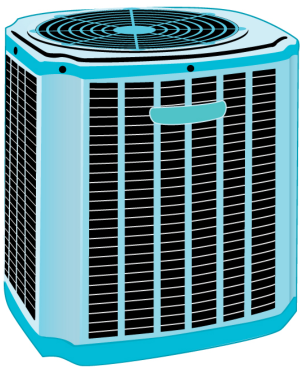
Discussion
This result means that the heat transfer by the heat pump is 5.30 times as much as the work put into it. It would cost 5.30 times as much for the same heat transfer by an electric room heater as it does for that produced by this heat pump. This is not a violation of conservation of energy. Cold ambient air provides 4.3 J per 1 J of work from the electrical outlet.



Heat transfer from the outside to the inside, along with work done to run the pump, takes place in the heat pump of the example above**.** Note that the cold temperature produced by the heat pump is lower than the outside temperature, so that heat transfer into the working fluid occurs. The pump's compressor produces a temperature greater than the indoor temperature in order for heat transfer into the house to occur.

Real heat pumps do not perform quite as well as the ideal one in the previous example; their values of COP_{hp} range from about 2 to 4. This range means that the heat transfer Q_{h} from the heat pumps is 2 to 4 times as great as the work W put into them. Their economical feasibility is still limited, however, since W is usually supplied by electrical energy that costs more per joule than heat transfer by burning fuels like natural gas. Furthermore, the initial cost of a heat pump is greater than that of many

furnaces, so that a heat pump must last longer for its cost to be recovered. Heat pumps are most likely to be economically superior where winter temperatures are mild, electricity is relatively cheap, and other fuels are relatively expensive. Also, since they can cool as well as heat a space, they have advantages where cooling in summer months is also desired. Thus some of the best locations for heat pumps are in warm summer climates with cool winters. [link] shows a heat pump, called a “*reverse cycle*” or “*split-system cooler*” in some countries.



In hot weather, heat transfer occurs from air inside the room to air outside, cooling the room. In cool weather, heat transfer occurs from air outside to air inside, warming the room. This switching is

achieved by reversing the direction of flow of the working fluid.

Air Conditioners and Refrigerators

Air conditioners and refrigerators are designed to cool something down in a warm environment. As with heat pumps, work input is required for heat transfer from cold to hot, and this is expensive. The quality of air conditioners and refrigerators is judged by how much heat transfer Q_{c} occurs from a cold environment compared with how much work input W is required. What is considered the benefit in a heat pump is considered waste heat in a refrigerator. We thus define the **coefficient of performance** (COP_{ref}) of an air conditioner or refrigerator to be

Equation:

$$COP_{\text{ref}} = \frac{Q_{\text{c}}}{W}.$$

Noting again that $Q_{\text{h}} = Q_{\text{c}} + W$, we can see that an air conditioner will have a lower coefficient of performance than a heat pump, because $COP_{\text{hp}} = Q_{\text{h}}/W$ and Q_{h} is greater than Q_{c}. In this module's Problems and Exercises, you will show that

Equation:

$$COP_{\text{ref}} = COP_{\text{hp}} - 1$$

for a heat engine used as either an air conditioner or a heat pump operating between the same two temperatures. Real air conditioners and refrigerators typically do remarkably well, having values of COP_{ref} ranging from 2 to 6.

These numbers are better than the COP_{hp} values for the heat pumps mentioned above, because the temperature differences are smaller, but they are less than those for Carnot engines operating between the same two temperatures.

A type of COP rating system called the "energy efficiency rating" (EER) has been developed. This rating is an example where non-SI units are still used and relevant to consumers. To make it easier for the consumer, Australia, Canada, New Zealand, and the U.S. use an Energy Star Rating out of 5 stars—the more stars, the more energy efficient the appliance. EERs are expressed in mixed units of British thermal units (Btu) per hour of heating or cooling divided by the power input in watts. Room air conditioners are readily available with EERs ranging from 6 to 12. Although not the same as the COPs just described, these EERs are good for comparison purposes—the greater the EER, the cheaper an air conditioner is to operate (but the higher its purchase price is likely to be).

The EER of an air conditioner or refrigerator can be expressed as
Equation:

$$EER = \frac{Q_{\mathrm{c}}/t_1}{W/t_2},$$

where Q_{c} is the amount of heat transfer from a cold environment in British thermal units, t_1 is time in hours, W is the work input in joules, and t_2 is time in seconds.

Note:
Problem-Solving Strategies for Thermodynamics

1. *Examine the situation to determine whether heat, work, or internal energy are involved.* Look for any system where the primary methods of transferring energy are heat and work. Heat engines, heat pumps, refrigerators, and air conditioners are examples of such systems.

2. *Identify the system of interest and draw a labeled diagram of the system showing energy flow.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns).* A written list is useful. Maximum efficiency means a Carnot engine is involved. Efficiency is not the same as the coefficient of performance.
4. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).* Be sure to distinguish heat transfer into a system from heat transfer out of the system, as well as work input from work output. In many situations, it is useful to determine the type of process, such as isothermal or adiabatic.
5. *Solve the appropriate equation for the quantity to be determined (the unknown).*
6. *Substitute the known quantities along with their units into the appropriate equation and obtain numerical solutions complete with units.*
7. *Check the answer to see if it is reasonable: Does it make sense?* For example, efficiency is always less than 1, whereas coefficients of performance are greater than 1.

Section Summary

- An artifact of the second law of thermodynamics is the ability to heat an interior space using a heat pump. Heat pumps compress cold ambient air and, in so doing, heat it to room temperature without violation of conservation principles.
- To calculate the heat pump's coefficient of performance, use the equation $COP_{\text{hp}} = \frac{Q_{\text{h}}}{W}$.
- A refrigerator is a heat pump; it takes warm ambient air and expands it to chill it.

Conceptual Questions

Exercise:

Problem:

Explain why heat pumps do not work as well in very cold climates as they do in milder ones. Is the same true of refrigerators?

Exercise:

Problem:

In some Northern European nations, homes are being built without heating systems of any type. They are very well insulated and are kept warm by the body heat of the residents. However, when the residents are not at home, it is still warm in these houses. What is a possible explanation?

Exercise:

Problem:

Why do refrigerators, air conditioners, and heat pumps operate most cost-effectively for cycles with a small difference between T_{h} and T_{c}? (Note that the temperatures of the cycle employed are crucial to its COP.)

Exercise:

Problem:

Grocery store managers contend that there is *less* total energy consumption in the summer if the store is kept at a *low* temperature. Make arguments to support or refute this claim, taking into account that there are numerous refrigerators and freezers in the store.

Exercise:

Problem:

Can you cool a kitchen by leaving the refrigerator door open?

Problem Exercises

Exercise:

Problem:

What is the coefficient of performance of an ideal heat pump that has heat transfer from a cold temperature of -25.0°C to a hot temperature of 40.0°C?

Solution:

4.82

Exercise:

Problem:

Suppose you have an ideal refrigerator that cools an environment at -20.0°C and has heat transfer to another environment at 50.0°C. What is its coefficient of performance?

Exercise:

Problem:

What is the best coefficient of performance possible for a hypothetical refrigerator that could make liquid nitrogen at -200°C and has heat transfer to the environment at 35.0°C?

Solution:

0.311

Exercise:

Problem:

In a very mild winter climate, a heat pump has heat transfer from an environment at 5.00°C to one at 35.0°C. What is the best possible coefficient of performance for these temperatures? Explicitly show how you follow the steps in the Problem-Solving Strategies for Thermodynamics.

Exercise:

Problem:

(a) What is the best coefficient of performance for a heat pump that has a hot reservoir temperature of 50.0^{o}C and a cold reservoir temperature of -20.0^{o}C? (b) How much heat transfer occurs into the warm environment if $3.60 \times 10^7 \text{ J}$ of work ($10.0\text{kW} \cdot \text{h}$) is put into it? (c) If the cost of this work input is $10.0 \text{ cents/kW} \cdot \text{h}$, how does its cost compare with the direct heat transfer achieved by burning natural gas at a cost of 85.0 cents per therm. (A therm is a common unit of energy for natural gas and equals $1.055 \times 10^8 \text{ J}$.)

Solution:

(a) 4.61

(b) $1.66 \times 10^8 \text{ J}$ or $3.97 \times 10^4 \text{ kcal}$

(c) To transfer $1.66 \times 10^8 \text{ J}$, heat pump costs $1.00, natural gas costs $1.34.

Exercise:

Problem:

(a) What is the best coefficient of performance for a refrigerator that cools an environment at -30.0^{o}C and has heat transfer to another environment at 45.0^{o}C? (b) How much work in joules must be done for a heat transfer of 4186 kJ from the cold environment? (c) What is the cost of doing this if the work costs 10.0 cents per $3.60 \times 10^6 \text{ J}$ (a kilowatt-hour)? (d) How many kJ of heat transfer occurs into the warm environment? (e) Discuss what type of refrigerator might operate between these temperatures.

Exercise:

Problem:

Suppose you want to operate an ideal refrigerator with a cold temperature of -10.0^{o}C, and you would like it to have a coefficient of performance of 7.00. What is the hot reservoir temperature for such a refrigerator?

Solution:

27.6^{o}C

Exercise:

Problem:

An ideal heat pump is being considered for use in heating an environment with a temperature of 22.0^{o}C. What is the cold reservoir temperature if the pump is to have a coefficient of performance of 12.0?

Exercise:

Problem:

A 4-ton air conditioner removes $5.06\times10^{7}\ \text{J}$ (48,000 British thermal units) from a cold environment in 1.00 h. (a) What energy input in joules is necessary to do this if the air conditioner has an energy efficiency rating (EER) of 12.0? (b) What is the cost of doing this if the work costs 10.0 cents per $3.60\times10^{6}\ \text{J}$ (one kilowatt-hour)? (c) Discuss whether this cost seems realistic. Note that the energy efficiency rating (EER) of an air conditioner or refrigerator is defined to be the number of British thermal units of heat transfer from a cold environment per hour divided by the watts of power input.

Solution:

(a) $1.44 \times 10^{7}\ \text{J}$

(b) 40 cents

(c) This cost seems quite realistic; it says that running an air conditioner all day would cost $9.59 (if it ran continuously).

Exercise:

Problem:

Show that the coefficients of performance of refrigerators and heat pumps are related by $COP_{\text{ref}} = COP_{\text{hp}} - 1$.

Start with the definitions of the COP s and the conservation of energy relationship between Q_{h}, Q_{c}, and W.

Glossary

heat pump
: a machine that generates heat transfer from cold to hot

coefficient of performance
: for a heat pump, it is the ratio of heat transfer at the output (the hot reservoir) to the work supplied; for a refrigerator or air conditioner, it is the ratio of heat transfer from the cold reservoir to the work supplied

Entropy and the Second Law of Thermodynamics: Disorder and the Unavailability of Energy

- Define entropy and calculate the increase of entropy in a system with reversible and irreversible processes.
- Calculate the increasing disorder of a system.

The ice in this drink is slowly melting. Eventually the liquid will reach thermal equilibrium, as predicted by the second law of thermodynamics. (credit: Jon Sullivan, PDPhoto.org)

There is yet another way of expressing the second law of thermodynamics. This version relates to a concept called **entropy**. By examining it, we shall see that the directions associated with the second law—heat transfer from

hot to cold, for example—are related to the tendency in nature for systems to become disordered and for less energy to be available for use as work. The entropy of a system can in fact be shown to be a measure of its disorder and of the unavailability of energy to do work.

Note:
Making Connections: Entropy, Energy, and Work
Recall that the simple definition of energy is the ability to do work. Entropy is a measure of how much energy is not available to do work. Although all forms of energy are interconvertible, and all can be used to do work, it is not always possible, even in principle, to convert the entire available energy into work. That unavailable energy is of interest in thermodynamics, because the field of thermodynamics arose from efforts to convert heat to work.

We can see how entropy is defined by recalling our discussion of the Carnot engine. We noted that for a Carnot cycle, and hence for any reversible processes, $Q_{\mathrm{c}}/Q_{\mathrm{h}} = T_{\mathrm{c}}/T_{\mathrm{h}}$. Rearranging terms yields
Equation:

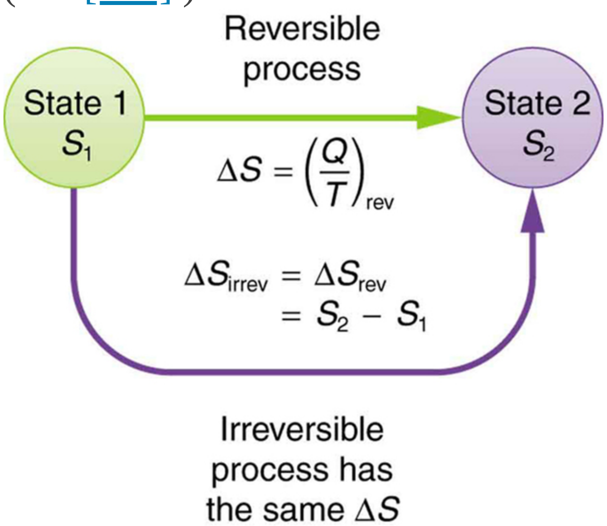
$$\frac{Q_{\mathrm{c}}}{T_{\mathrm{c}}} = \frac{Q_{\mathrm{h}}}{T_{\mathrm{h}}}$$

for any reversible process. Q_{c} and Q_{h} are absolute values of the heat transfer at temperatures T_{c} and T_{h}, respectively. This ratio of Q/T is defined to be the **change in entropy** ΔS for a reversible process,
Equation:

$$\Delta S = \left(\frac{Q}{T}\right)_{\mathrm{rev}},$$

where Q is the heat transfer, which is positive for heat transfer into and negative for heat transfer out of, and T is the absolute temperature at which the reversible process takes place. The SI unit for entropy is joules per kelvin (J/K). If temperature changes during the process, then it is usually a good approximation (for small changes in temperature) to take T to be the average temperature, avoiding the need to use integral calculus to find ΔS.

The definition of ΔS is strictly valid only for reversible processes, such as used in a Carnot engine. However, we can find ΔS precisely even for real, irreversible processes. The reason is that the entropy S of a system, like internal energy U, depends only on the state of the system and not how it reached that condition. Entropy is a property of state. Thus the change in entropy ΔS of a system between state 1 and state 2 is the same no matter how the change occurs. We just need to find or imagine a reversible process that takes us from state 1 to state 2 and calculate ΔS for that process. That will be the change in entropy for any process going from state 1 to state 2. (See [link].)



When a system goes from state 1 to state 2, its entropy changes by the same amount ΔS, whether a hypothetical reversible path is followed or a real irreversible path is taken.

Now let us take a look at the change in entropy of a Carnot engine and its heat reservoirs for one full cycle. The hot reservoir has a loss of entropy $\Delta S_{\mathrm{h}} = -Q_{\mathrm{h}}/T_{\mathrm{h}}$, because heat transfer occurs out of it (remember that when heat transfers out, then Q has a negative sign). The cold reservoir has a gain of entropy $\Delta S_{\mathrm{c}} = Q_{\mathrm{c}}/T_{\mathrm{c}}$, because heat transfer occurs into it. (We assume the reservoirs are sufficiently large that their temperatures are constant.) So the total change in entropy is

Equation:

$$\Delta S_{\mathrm{tot}} = \Delta S_{\mathrm{h}} + \Delta S_{\mathrm{c}}.$$

Thus, since we know that $Q_{\mathrm{h}}/T_{\mathrm{h}} = Q_{\mathrm{c}}/T_{\mathrm{c}}$ for a Carnot engine,

Equation:

$$\Delta S_{\mathrm{tot}} = -\frac{Q_{\mathrm{h}}}{T_{\mathrm{h}}} + \frac{Q_{\mathrm{c}}}{T_{\mathrm{c}}} = 0.$$

This result, which has general validity, means that *the total change in entropy for a system in any reversible process is zero.*

The entropy of various parts of the system may change, but the total change is zero. Furthermore, the system does not affect the entropy of its surroundings, since heat transfer between them does not occur. Thus the reversible process changes neither the total entropy of the system nor the entropy of its surroundings. Sometimes this is stated as follows: *Reversible processes do not affect the total entropy of the universe.* Real processes are not reversible, though, and they do change total entropy. We can, however, use hypothetical reversible processes to determine the value of entropy in real, irreversible processes. The following example illustrates this point.

Example:
Entropy Increases in an Irreversible (Real) Process
Spontaneous heat transfer from hot to cold is an irreversible process. Calculate the total change in entropy if 4000 J of heat transfer occurs from

a hot reservoir at $T_{\text{h}} = 600\text{ K}(327^{\text{o}}\text{ C})$ to a cold reservoir at $T_{\text{c}} = 250\text{ K}(-23^{\text{o}}\text{ C})$, assuming there is no temperature change in either reservoir. (See [link].)

Strategy

How can we calculate the change in entropy for an irreversible process when $\Delta S_{\text{tot}} = \Delta S_{\text{h}} + \Delta S_{\text{c}}$ is valid only for reversible processes? Remember that the total change in entropy of the hot and cold reservoirs will be the same whether a reversible or irreversible process is involved in heat transfer from hot to cold. So we can calculate the change in entropy of the hot reservoir for a hypothetical reversible process in which 4000 J of heat transfer occurs from it; then we do the same for a hypothetical reversible process in which 4000 J of heat transfer occurs to the cold reservoir. This produces the same changes in the hot and cold reservoirs that would occur if the heat transfer were allowed to occur irreversibly between them, and so it also produces the same changes in entropy.

Solution

We now calculate the two changes in entropy using $\Delta S_{\text{tot}} = \Delta S_{\text{h}} + \Delta S_{\text{c}}$. First, for the heat transfer from the hot reservoir,

Equation:

$$\Delta S_{\text{h}} = \frac{-Q_{\text{h}}}{T_{\text{h}}} = \frac{-4000\text{ J}}{600\text{ K}} = -6.67\text{ J/K.}$$

And for the cold reservoir,

Equation:

$$\Delta S_{\text{c}} = \frac{Q_{\text{c}}}{T_{\text{c}}} = \frac{4000\text{ J}}{250\text{ K}} = 16.0\text{ J/K.}$$

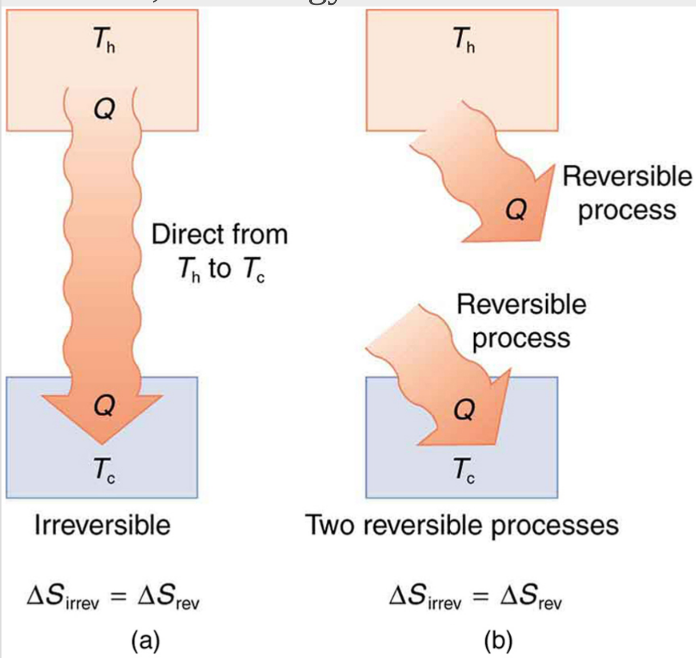
Thus the total is

Equation:

$$\begin{aligned} \Delta S_{\text{tot}} &= \Delta S_{\text{h}} + \Delta S_{\text{c}} \\ &= (-6.67 + 16.0)\text{ J/K} \\ &= 9.33\text{ J/K.} \end{aligned}$$

Discussion

There is an *increase* in entropy for the system of two heat reservoirs undergoing this irreversible heat transfer. We will see that this means there is a loss of ability to do work with this transferred energy. Entropy has increased, and energy has become unavailable to do work.



(a) Heat transfer from a hot object to a cold one is an irreversible process that produces an overall increase in entropy. (b) The same final state and, thus, the same change in entropy is achieved for the objects if reversible heat transfer processes occur between the two objects whose temperatures are the same as the temperatures of the corresponding objects in the irreversible process.

It is reasonable that entropy increases for heat transfer from hot to cold. Since the change in entropy is Q/T, there is a larger change at lower

temperatures. The decrease in entropy of the hot object is therefore less than the increase in entropy of the cold object, producing an overall increase, just as in the previous example. This result is very general:

There is an increase in entropy for any system undergoing an irreversible process.

With respect to entropy, there are only two possibilities: entropy is constant for a reversible process, and it increases for an irreversible process. There is a fourth version of **the second law of thermodynamics stated in terms of entropy**:

The total entropy of a system either increases or remains constant in any process; it never decreases.

For example, heat transfer cannot occur spontaneously from cold to hot, because entropy would decrease.

Entropy is very different from energy. Entropy is *not* conserved but increases in all real processes. Reversible processes (such as in Carnot engines) are the processes in which the most heat transfer to work takes place and are also the ones that keep entropy constant. Thus we are led to make a connection between entropy and the availability of energy to do work.

Order to Disorder

Entropy is related not only to the unavailability of energy to do work—it is also a measure of disorder. This notion was initially postulated by Ludwig Boltzmann in the 1800s. For example, melting a block of ice means taking a highly structured and orderly system of water molecules and converting it into a disorderly liquid in which molecules have no fixed positions. (See [link].) There is a large increase in entropy in the process, as seen in the following example.

Example:

Entropy Associated with Disorder

Find the increase in entropy of 1.00 kg of ice originally at 0º C that is melted to form water at 0º C.

Strategy

As before, the change in entropy can be calculated from the definition of ΔS once we find the energy Q needed to melt the ice.

Solution

The change in entropy is defined as:

Equation:

$$\mathit{\Delta}S=\frac{Q}{T}.$$

Here Q is the heat transfer necessary to melt 1.00 kg of ice and is given by

Equation:

$$Q=mL_{\text{f}},$$

where m is the mass and L_{f} is the latent heat of fusion. $L_{\text{f}}=334\text{ kJ/kg}$ for water, so that

Equation:

$$Q=(1.00\text{ kg})(334\text{ kJ/kg})=3.34\times 10^{5}\text{ J}.$$

Now the change in entropy is positive, since heat transfer occurs into the ice to cause the phase change; thus,

Equation:

$$\Delta S=\frac{Q}{T}=\frac{3.34\times 10^{5}\text{ J}}{T}.$$

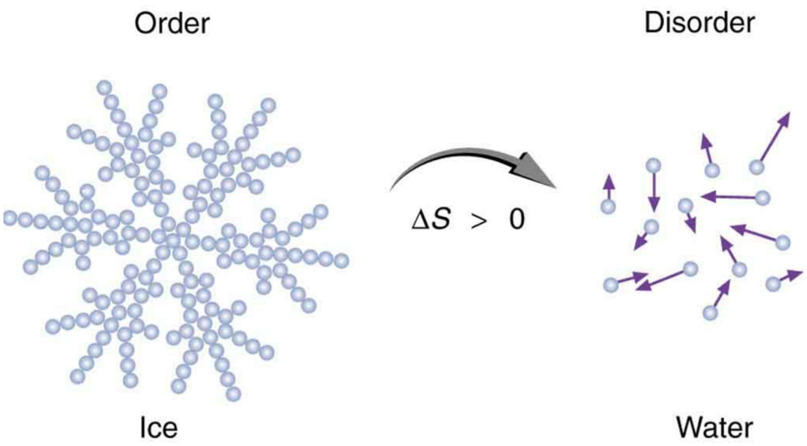
T is the melting temperature of ice. That is, $T=0\text{ºC}=273\text{ K}$. So the change in entropy is

Equation:

$$\begin{array}{lll}\Delta S&=&\frac{3.34\times 10^{5}\text{ J}}{273\text{ K}}\\ &=&1.22\times 10^{3}\text{ J/K}.\end{array}$$

Discussion

This is a significant increase in entropy accompanying an increase in disorder.



When ice melts, it becomes more disordered and less structured. The systematic arrangement of molecules in a crystal structure is replaced by a more random and less orderly movement of molecules without fixed locations or orientations. Its entropy increases because heat transfer occurs into it. Entropy is a measure of disorder.

In another easily imagined example, suppose we mix equal masses of water originally at two different temperatures, say 20.0° C and 40.0° C. The result is water at an intermediate temperature of 30.0° C. Three outcomes have resulted: entropy has increased, some energy has become unavailable to do work, and the system has become less orderly. Let us think about each of these results.

First, entropy has increased for the same reason that it did in the example above. Mixing the two bodies of water has the same effect as heat transfer from the hot one and the same heat transfer into the cold one. The mixing

decreases the entropy of the hot water but increases the entropy of the cold water by a greater amount, producing an overall increase in entropy.

Second, once the two masses of water are mixed, there is only one temperature—you cannot run a heat engine with them. The energy that could have been used to run a heat engine is now unavailable to do work.

Third, the mixture is less orderly, or to use another term, less structured. Rather than having two masses at different temperatures and with different distributions of molecular speeds, we now have a single mass with a uniform temperature.

These three results—entropy, unavailability of energy, and disorder—are not only related but are in fact essentially equivalent.

Section Summary

- Entropy is the loss of energy available to do work.
- Another form of the second law of thermodynamics states that the total entropy of a system either increases or remains constant; it never decreases.
- Change of entropy is zero in a reversible process; it increases in an irreversible process.
- Entropy is also associated with the tendency toward disorder in an *isolated* system.

Conceptual Questions

Exercise:

Problem:

Does a gas become more orderly when it liquefies? Does its entropy change? If so, does the entropy increase or decrease? Explain your answer.

Exercise:

Problem:

Explain how water’s entropy can decrease when it freezes without violating the second law of thermodynamics. Specifically, explain what happens to the entropy of its surroundings.

Exercise:

Problem:

Is a uniform-temperature gas more or less orderly than one with several different temperatures? Which is more structured? In which can heat transfer result in work done without heat transfer from another system?

Exercise:

Problem:

Give an example of a spontaneous process in which a system becomes less ordered and energy becomes less available to do work. What happens to the system’s entropy in this process?

Exercise:

Problem:

What is the change in entropy in an adiabatic process? Does this imply that adiabatic processes are reversible? Can a process be precisely adiabatic for a macroscopic system?

Exercise:

Problem:

Does the entropy of a star increase or decrease as it radiates? Does the entropy of the space into which it radiates (which has a temperature of about 3 K) increase or decrease? What does this do to the entropy of the universe?

Exercise:

Problem:

Explain why a building made of bricks has smaller entropy than the same bricks in a disorganized pile. Do this by considering the number of ways that each could be formed (the number of microstates in each macrostate).

Glossary

entropy
: a measurement of a system's disorder and its inability to do work in a system

change in entropy
: the ratio of heat transfer to temperature Q/T

second law of thermodynamics stated in terms of entropy
: the total entropy of a system either increases or remains constant; it never decreases

Statistical Interpretation of Entropy and the Second Law of Thermodynamics: The Underlying Explanation

- Identify probabilities in entropy.
- Analyze statistical probabilities in entropic systems.

When you toss a coin a large number of times, heads and tails tend to come up in roughly equal numbers. Why doesn’t heads come up 100, 90, or even 80% of the time? (credit: Jon Sullivan, PDPhoto.org)

The various ways of formulating the second law of thermodynamics tell what happens rather than why it happens. Why should heat transfer occur only from hot to cold? Why should energy become ever less available to do work? Why should the universe become increasingly disorderly? The answer is that it is a matter of overwhelming probability. Disorder is simply vastly more likely than order.

When you watch an emerging rain storm begin to wet the ground, you will notice that the drops fall in a disorganized manner both in time and in space. Some fall close together, some far apart, but they never fall in

straight, orderly rows. It is not impossible for rain to fall in an orderly pattern, just highly unlikely, because there are many more disorderly ways than orderly ones. To illustrate this fact, we will examine some random processes, starting with coin tosses.

Coin Tosses

What are the possible outcomes of tossing 5 coins? Each coin can land either heads or tails. On the large scale, we are concerned only with the total heads and tails and not with the order in which heads and tails appear. The following possibilities exist:

Equation:

$$\begin{array}{c}\text{5 heads, 0 tails}\\ \text{4 heads, 1 tail}\\ \text{3 heads, 2 tails}\\ \text{2 heads, 3 tails}\\ \text{1 head, 4 tails}\\ \text{0 head, 5 tails}\end{array}$$

These are what we call macrostates. A **macrostate** is an overall property of a system. It does not specify the details of the system, such as the order in which heads and tails occur or which coins are heads or tails.

Using this nomenclature, a system of 5 coins has the 6 possible macrostates just listed. Some macrostates are more likely to occur than others. For instance, there is only one way to get 5 heads, but there are several ways to get 3 heads and 2 tails, making the latter macrostate more probable. [link] lists of all the ways in which 5 coins can be tossed, taking into account the order in which heads and tails occur. Each sequence is called a **microstate** —a detailed description of every element of a system.

	Individual microstates	Number of microstates
5 heads, 0 tails	HHHHH	1
4 heads, 1 tail	HHHHT, HHHTH, HHTHH, HTHHH, THHHH	5
3 heads, 2 tails	HTHTH, THTHH, HTHHT, THHTH, THHHT HTHTH, THTHH, HTHHT, THHTH, THHHT	10
2 heads, 3 tails	TTTHH, TTHHT, THHTT, HHTTT, TTHTH, THTHT, HTHTT, THTTH, HTTHT, HTTTH	10
1 head, 4 tails	TTTTH, TTTHT, TTHTT, THTTT, HTTTT	5
0 heads, 5 tails	TTTTT	1
		Total: 32

5-Coin Toss

The macrostate of 3 heads and 2 tails can be achieved in 10 ways and is thus 10 times more probable than the one having 5 heads. Not surprisingly, it is equally probable to have the reverse, 2 heads and 3 tails. Similarly, it is equally probable to get 5 tails as it is to get 5 heads. Note that all of these conclusions are based on the crucial assumption that each microstate is equally probable. With coin tosses, this requires that the coins not be

asymmetric in a way that favors one side over the other, as with loaded dice. With any system, the assumption that all microstates are equally probable must be valid, or the analysis will be erroneous.

The two most orderly possibilities are 5 heads or 5 tails. (They are more structured than the others.) They are also the least likely, only 2 out of 32 possibilities. The most disorderly possibilities are 3 heads and 2 tails and its reverse. (They are the least structured.) The most disorderly possibilities are also the most likely, with 20 out of 32 possibilities for the 3 heads and 2 tails and its reverse. If we start with an orderly array like 5 heads and toss the coins, it is very likely that we will get a less orderly array as a result, since 30 out of the 32 possibilities are less orderly. So even if you start with an orderly state, there is a strong tendency to go from order to disorder, from low entropy to high entropy. The reverse can happen, but it is unlikely.

Macrostate		**Number of microstates**
Heads	Tails	(W)
100	0	1
99	1	1.0×10^2
95	5	7.5×10^7
90	10	1.7×10^{13}

Macrostate		Number of microstates
75	25	2.4×10^{23}
60	40	1.4×10^{28}
55	45	6.1×10^{28}
51	49	9.9×10^{28}
50	50	1.0×10^{29}
49	51	9.9×10^{28}
45	55	6.1×10^{28}
40	60	1.4×10^{28}
25	75	2.4×10^{23}

Macrostate		Number of microstates
10	90	1.7×10^{13}
5	95	7.5×10^{7}
1	99	1.0×10^{2}
0	100	1
		Total: 1.27×10^{30}

100-Coin Toss

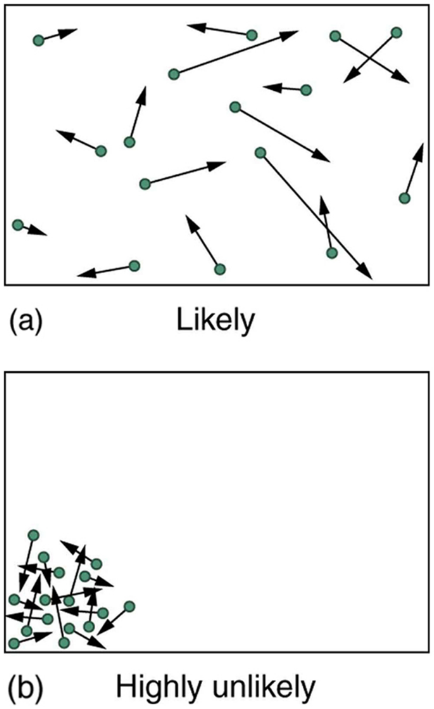
This result becomes dramatic for larger systems. Consider what happens if you have 100 coins instead of just 5. The most orderly arrangements (most structured) are 100 heads or 100 tails. The least orderly (least structured) is that of 50 heads and 50 tails. There is only 1 way (1 microstate) to get the most orderly arrangement of 100 heads. There are 100 ways (100 microstates) to get the next most orderly arrangement of 99 heads and 1 tail (also 100 to get its reverse). And there are 1.0×10^{29} ways to get 50 heads and 50 tails, the least orderly arrangement. [link] is an abbreviated list of the various macrostates and the number of microstates for each macrostate. The total number of microstates—the total number of different ways 100 coins can be tossed—is an impressively large 1.27×10^{30}. Now, if we start with an orderly macrostate like 100 heads and toss the coins, there is a virtual certainty that we will get a less orderly macrostate. If we keep tossing the coins, it is possible, but exceedingly unlikely, that we will ever

get back to the most orderly macrostate. If you tossed the coins once each second, you could expect to get either 100 heads or 100 tails once in 2×10^{22} years! This period is 1 trillion (10^{12}) times longer than the age of the universe, and so the chances are essentially zero. In contrast, there is an 8% chance of getting 50 heads, a 73% chance of getting from 45 to 55 heads, and a 96% chance of getting from 40 to 60 heads. Disorder is highly likely.

Disorder in a Gas

The fantastic growth in the odds favoring disorder that we see in going from 5 to 100 coins continues as the number of entities in the system increases. Let us now imagine applying this approach to perhaps a small sample of gas. Because counting microstates and macrostates involves statistics, this is called **statistical analysis**. The macrostates of a gas correspond to its macroscopic properties, such as volume, temperature, and pressure; and its microstates correspond to the detailed description of the positions and velocities of its atoms. Even a small amount of gas has a huge number of atoms: $1.0\ \text{cm}^3$ of an ideal gas at 1.0 atm and $0^\text{o}\ \text{C}$ has 2.7×10^{19} atoms. So each macrostate has an immense number of microstates. In plain language, this means that there are an immense number of ways in which the atoms in a gas can be arranged, while still having the same pressure, temperature, and so on.

The most likely conditions (or macrostates) for a gas are those we see all the time—a random distribution of atoms in space with a Maxwell-Boltzmann distribution of speeds in random directions, as predicted by kinetic theory. This is the most disorderly and least structured condition we can imagine. In contrast, one type of very orderly and structured macrostate has all of the atoms in one corner of a container with identical velocities. There are very few ways to accomplish this (very few microstates corresponding to it), and so it is exceedingly unlikely ever to occur. (See [link](b).) Indeed, it is so unlikely that we have a law saying that it is impossible, which has never been observed to be violated—the second law of thermodynamics.



(a) The ordinary state of gas in a container is a disorderly, random distribution of atoms or molecules with a Maxwell-Boltzmann distribution of speeds. It is so unlikely that these atoms or molecules would ever end up in one corner of the container that it might as well be impossible. (b) With energy transfer, the gas can be forced into one corner and its entropy greatly reduced. But left alone, it will

spontaneously increase its entropy and return to the normal conditions, because they are immensely more likely.

The disordered condition is one of high entropy, and the ordered one has low entropy. With a transfer of energy from another system, we could force all of the atoms into one corner and have a local decrease in entropy, but at the cost of an overall increase in entropy of the universe. If the atoms start out in one corner, they will quickly disperse and become uniformly distributed and will never return to the orderly original state ([link](b)). Entropy will increase. With such a large sample of atoms, it is possible—but unimaginably unlikely—for entropy to decrease. Disorder is vastly more likely than order.

The arguments that disorder and high entropy are the most probable states are quite convincing. The great Austrian physicist Ludwig Boltzmann (1844–1906)—who, along with Maxwell, made so many contributions to kinetic theory—proved that the entropy of a system in a given state (a macrostate) can be written as

Equation:

$$S = k \ln W,$$

where $k = 1.38 \times 10^{-23}\ \text{J/K}$ is Boltzmann's constant, and $\ln W$ is the natural logarithm of the number of microstates W corresponding to the given macrostate. W is proportional to the probability that the macrostate will occur. Thus entropy is directly related to the probability of a state—the more likely the state, the greater its entropy. Boltzmann proved that this expression for S is equivalent to the definition $\Delta S = Q/T$, which we have used extensively.

Thus the second law of thermodynamics is explained on a very basic level: entropy either remains the same or increases in every process. This phenomenon is due to the extraordinarily small probability of a decrease, based on the extraordinarily larger number of microstates in systems with greater entropy. Entropy *can* decrease, but for any macroscopic system, this outcome is so unlikely that it will never be observed.

Example:

Entropy Increases in a Coin Toss

Suppose you toss 100 coins starting with 60 heads and 40 tails, and you get the most likely result, 50 heads and 50 tails. What is the change in entropy?

Strategy

Noting that the number of microstates is labeled W in [link] for the 100-coin toss, we can use $\Delta S = S_{\text{f}} - S_{\text{i}} = k\ln W_{\text{f}} - k\ln W_{\text{i}}$ to calculate the change in entropy.

Solution

The change in entropy is

Equation:

$$\Delta S = S_{\text{f}} - S_{\text{i}} = k\ln W_{\text{f}} - k\ln W_{\text{i}},$$

where the subscript i stands for the initial 60 heads and 40 tails state, and the subscript f for the final 50 heads and 50 tails state. Substituting the values for W from [link] gives

Equation:

$$\begin{aligned} \Delta S &= (1.38 \times 10^{-23}\text{ J/K})[\ln(1.0 \times 10^{29}) - \ln(1.4 \times 10^{28})] \\ &= 2.7 \times 10^{-23}\text{ J/K} \end{aligned}$$

Discussion

This increase in entropy means we have moved to a less orderly situation. It is not impossible for further tosses to produce the initial state of 60 heads and 40 tails, but it is less likely. There is about a 1 in 90 chance for that decrease in entropy ($-2.7 \times 10^{-23}\text{ J/K}$) to occur. If we calculate the decrease in entropy to move to the most orderly state, we get

$\Delta S = -92 \times 10^{-23}\ \mathrm{J/K}$. There is about a 1 in 10^{30} chance of this change occurring. So while very small decreases in entropy are unlikely, slightly greater decreases are impossibly unlikely. These probabilities imply, again, that for a macroscopic system, a decrease in entropy is impossible. For example, for heat transfer to occur spontaneously from 1.00 kg of $0^{\mathrm{o}}\mathrm{C}$ ice to its $0^{\mathrm{o}}\mathrm{C}$ environment, there would be a decrease in entropy of $1.22 \times 10^{3}\ \mathrm{J/K}$. Given that a ΔS of $10^{-21}\ \mathrm{J/K}$ corresponds to about a 1 in 10^{30} chance, a decrease of this size ($10^{3}\ \mathrm{J/K}$) is an *utter* impossibility. Even for a milligram of melted ice to spontaneously refreeze is impossible.

Note:

Problem-Solving Strategies for Entropy

1. *Examine the situation to determine if entropy is involved.*
2. *Identify the system of interest and draw a labeled diagram of the system showing energy flow.*
3. *Identify exactly what needs to be determined in the problem (identify the unknowns).* A written list is useful.
4. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).* You must carefully identify the heat transfer, if any, and the temperature at which the process takes place. It is also important to identify the initial and final states.
5. *Solve the appropriate equation for the quantity to be determined (the unknown).* Note that the change in entropy can be determined between any states by calculating it for a reversible process.
6. *Substitute the known value along with their units into the appropriate equation, and obtain numerical solutions complete with units.*
7. *To see if it is reasonable: Does it make sense?* For example, total entropy should increase for any real process or be constant for a reversible process. Disordered states should be more probable and have greater entropy than ordered states.

Section Summary

- Disorder is far more likely than order, which can be seen statistically.
- The entropy of a system in a given state (a macrostate) can be written as
  Equation:
  $$S = k \ln W,$$
  where $k = 1.38 \times 10^{-23}\ \text{J/K}$ is Boltzmann's constant, and $\ln W$ is the natural logarithm of the number of microstates W corresponding to the given macrostate.

Conceptual Questions

Exercise:

Problem:

Explain why a building made of bricks has smaller entropy than the same bricks in a disorganized pile. Do this by considering the number of ways that each could be formed (the number of microstates in each macrostate).

Problem Exercises

Exercise:

Problem:

Using [link], verify the contention that if you toss 100 coins each second, you can expect to get 100 heads or 100 tails once in 2×10^{22} years; calculate the time to two-digit accuracy.

Solution:

It should happen twice in every 1.27×10^{30} s or once in every

$$6.35 \times 10^{29}\ \text{s} \quad \begin{array}{c}(6.35 \times 10^{29}\ \text{s})\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\left(\frac{1\ \text{d}}{24\ \text{h}}\right)\left(\frac{1\ \text{y}}{365.25\ \text{d}}\right) \\ = 2.0 \times 10^{22}\ \text{y}\end{array}$$

Exercise:

Problem:

What percent of the time will you get something in the range from 60 heads and 40 tails through 40 heads and 60 tails when tossing 100 coins? The total number of microstates in that range is 1.22×10^{30}. (Consult [link].)

Exercise:

Problem:

(a) If tossing 100 coins, how many ways (microstates) are there to get the three most likely macrostates of 49 heads and 51 tails, 50 heads and 50 tails, and 51 heads and 49 tails? (b) What percent of the total possibilities is this? (Consult [link].)

Solution:

(a) 3.0×10^{29}

(b) 24%

Exercise:

Problem:

(a) What is the change in entropy if you start with 100 coins in the 45 heads and 55 tails macrostate, toss them, and get 51 heads and 49 tails? (b) What if you get 75 heads and 25 tails? (c) How much more likely is 51 heads and 49 tails than 75 heads and 25 tails? (d) Does either outcome violate the second law of thermodynamics?

Exercise:

Problem:

(a) What is the change in entropy if you start with 10 coins in the 5 heads and 5 tails macrostate, toss them, and get 2 heads and 8 tails? (b) How much more likely is 5 heads and 5 tails than 2 heads and 8 tails? (Take the ratio of the number of microstates to find out.) (c) If you were betting on 2 heads and 8 tails would you accept odds of 252 to 45? Explain why or why not.

Solution:

(a) $-2.38\times10^{-23}\ \mathrm{J/K}$

(b) 5.6 times more likely

(c) If you were betting on two heads and 8 tails, the odds of breaking even are 252 to 45, so on average you would break even. So, no, you wouldn't bet on odds of 252 to 45.

Macrostate		**Number of Microstates (*W*)**
Heads	Tails	
10	0	1
9	1	10
8	2	45
7	3	120

Macrostate		Number of Microstates (*W*)
6	4	210
5	5	252
4	6	210
3	7	120
2	8	45
1	9	10
0	10	1
		Total: 1024

10-Coin Toss

Exercise:

Problem:

(a) If you toss 10 coins, what percent of the time will you get the three most likely macrostates (6 heads and 4 tails, 5 heads and 5 tails, 4 heads and 6 tails)? (b) You can realistically toss 10 coins and count the number of heads and tails about twice a minute. At that rate, how long will it take on average to get either 10 heads and 0 tails or 0 heads and 10 tails?

Exercise:

Problem:

(a) Construct a table showing the macrostates and all of the individual microstates for tossing 6 coins. (Use [link] as a guide.) (b) How many macrostates are there? (c) What is the total number of microstates? (d) What percent chance is there of tossing 5 heads and 1 tail? (e) How much more likely are you to toss 3 heads and 3 tails than 5 heads and 1 tail? (Take the ratio of the number of microstates to find out.)

Solution:

(b) 7

(c) 64

(d) 9.38%

(e) 3.33 times more likely (20 to 6)

Exercise:

Problem:

In an air conditioner, 12.65 MJ of heat transfer occurs from a cold environment in 1.00 h. (a) What mass of ice melting would involve the same heat transfer? (b) How many hours of operation would be equivalent to melting 900 kg of ice? (c) If ice costs 20 cents per kg, do you think the air conditioner could be operated more cheaply than by simply using ice? Describe in detail how you evaluate the relative costs.

Glossary

macrostate
: an overall property of a system

microstate
: each sequence within a larger macrostate

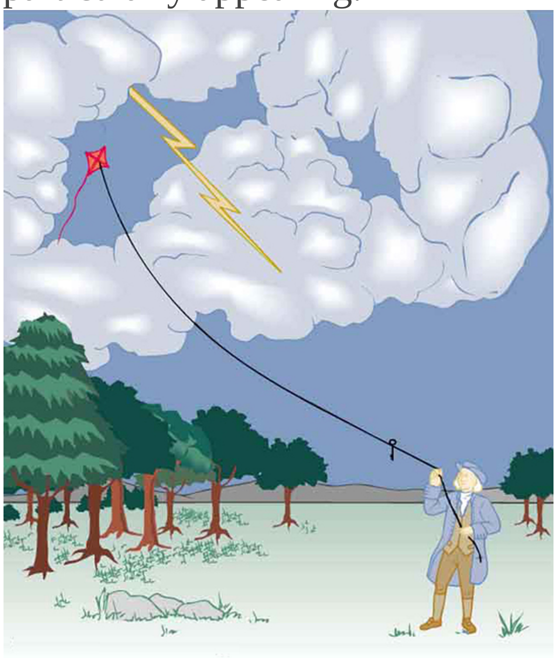
statistical analysis

using statistics to examine data, such as counting microstates and macrostates

Introduction to Electricity class="introduction"

Static electricity from this plastic slide causes the child’s hair to stand on end. The sliding motion stripped electrons away from the child’s body, leaving an excess of positive charges, which repel each other along each strand of hair. (credit: Ken Bosma/Wikimedia Commons)

The image of American politician and scientist Benjamin Franklin (1706–1790) flying a kite in a thunderstorm is familiar to every schoolchild. (See [link].) In this experiment, Franklin demonstrated a connection between lightning and **static electricity**. Sparks were drawn from a key hung on a kite string during an electrical storm. These sparks were like those produced by static electricity, such as the spark that jumps from your finger to a metal doorknob after you walk across a wool carpet. What Franklin demonstrated in his dangerous experiment was a connection between phenomena on two different scales: one the grand power of an electrical storm, the other an effect of more human proportions. Connections like this one reveal the underlying unity of the laws of nature, an aspect we humans find particularly appealing.



When Benjamin Franklin demonstrated that lightning was related to static electricity, he made a connection that is now part of the evidence that all directly experienced forces except the gravitational force are manifestations of the electromagnetic force.

Our understanding of electricity and the range of phenomena that are electrical in nature has vastly expanded since those early days. For example, atomic and molecular interactions, such as the forces of friction, cohesion, and adhesion, are now known to be manifestations of the **electromagnetic force**. Static electricity is just one aspect of the electromagnetic force, which also includes moving electricity and magnetism. Furthermore, All the macroscopic forces that we experience directly, such as the sensations of touch and the tension in a rope, are due to the electromagnetic force, one of the four fundamental forces in nature. The gravitational force, another fundamental force, is actually sensed through the electromagnetic interaction of molecules, such as between those in our feet and those on the top of a bathroom scale. (The other two fundamental forces, the strong nuclear force and the weak nuclear force, cannot be sensed on the human scale.)

In this chapter, we will develop the concepts of electric fields and electric potential (also known as *voltage*), which are used to describe electric force and electrical energy. Consider, for example, great amounts of electrical energy stored in batteries or transmitted cross-country through power lines, or consider electrical signals sent in our nervous systems at molecular levels, with *ions* crossing cell membranes and transferring information, or the role of electricity in our household appliances and devices, including your laptop and smartphones. We will look at the laws that help us describe electric circuits and begin to explore some of the many applications of electricity.

Glossary

static electricity
: a buildup of electric charge on the surface of an object

electromagnetic force
: one of the four fundamental forces of nature; the electromagnetic force consists of static electricity, moving electricity and magnetism

Static Electricity and Charge: Conservation of Charge

- Define electric charge, and describe how the two types of charge interact.
- Describe three common situations that generate static electricity.
- State the law of conservation of charge.

Borneo amber was mined in Sabah, Malaysia, from shale-sandstone-mudstone veins. When a piece of amber is rubbed with a piece of silk, the amber gains more electrons, giving it a net negative charge. At the same time, the silk, having lost electrons, becomes positively charged. (credit: Sebakoamber, Wikimedia Commons)

What makes plastic wrap cling? Static electricity. Not only are applications of static electricity common these days, its existence has been known since ancient times. The first record of its effects dates to ancient Greeks who noted more than 500 years B.C. that polishing amber temporarily enabled it

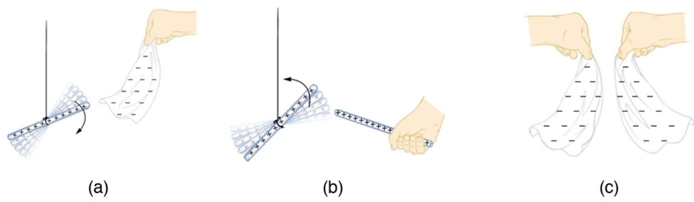
to attract bits of straw (see [link]). The very word *electric* derives from the Greek word for amber (*electron*).

Many of the characteristics of static electricity can be explored by rubbing things together. Rubbing creates the spark you get from walking across a wool carpet, for example. Static cling generated in a clothes dryer and the attraction of straw to recently polished amber also result from rubbing. Similarly, lightning results from air movements under certain weather conditions. You can also rub a balloon on your hair, and the static electricity created can then make the balloon cling to a wall. We also have to be cautious of static electricity, especially in dry climates. When we pump gasoline, we are warned to discharge ourselves (after sliding across the seat) on a metal surface before grabbing the gas nozzle. Attendants in hospital operating rooms must wear booties with aluminum foil on the bottoms to avoid creating sparks which may ignite the oxygen being used.

Some of the most basic characteristics of static electricity include:

- The effects of static electricity are explained by a physical quantity not previously introduced, called electric charge.
- There are only two types of charge, one called positive and the other called negative.
- Like charges repel, whereas unlike charges attract.
- The force between charges decreases with distance.

How do we know there are two types of **electric charge**? When various materials are rubbed together in controlled ways, certain combinations of materials always produce one type of charge on one material and the opposite type on the other. By convention, we call one type of charge “positive”, and the other type “negative.” For example, when glass is rubbed with silk, the glass becomes positively charged and the silk negatively charged. Since the glass and silk have opposite charges, they attract one another like clothes that have rubbed together in a dryer. Two glass rods rubbed with silk in this manner will repel one another, since each rod has positive charge on it. Similarly, two silk cloths so rubbed will repel, since both cloths have negative charge. [link] shows how these simple materials can be used to explore the nature of the force between charges.



A glass rod becomes positively charged when rubbed with silk, while the silk becomes negatively charged. (a) The glass rod is attracted to the silk because their charges are opposite. (b) Two similarly charged glass rods repel. (c) Two similarly charged silk cloths repel.

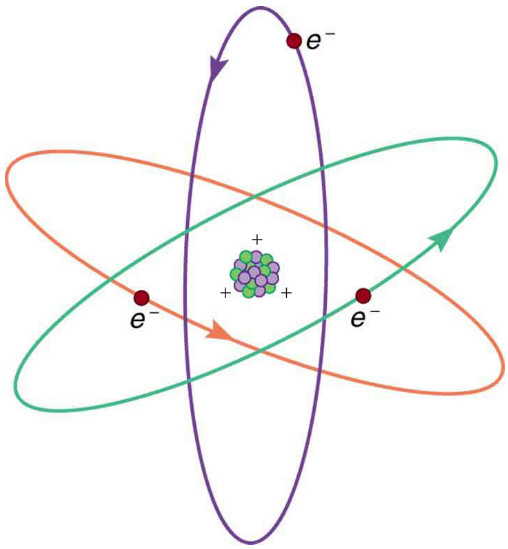
More sophisticated questions arise. Where do these charges come from? Can you create or destroy charge? Is there a smallest unit of charge? Exactly how does the force depend on the amount of charge and the distance between charges? Such questions obviously occurred to Benjamin Franklin and other early researchers, and they interest us even today.

Charge Carried by Electrons and Protons

Franklin wrote in his letters and books that he could see the effects of electric charge but did not understand what caused the phenomenon. Today we have the advantage of knowing that normal matter is made of atoms, and that atoms contain positive and negative charges, usually in equal amounts.

[link] shows a simple model of an atom with negative **electrons** orbiting its positive nucleus. The nucleus is positive due to the presence of positively charged **protons**. Nearly all charge in nature is due to electrons and protons, which are two of the three building blocks of most matter. (The third is the neutron, which is neutral, carrying no charge.) Other charge-carrying particles are observed in cosmic rays and nuclear decay, and are created in

particle accelerators. All but the electron and proton survive only a short time and are quite rare by comparison.



This simplified (and not to scale) view of an atom is called the planetary model of the atom. Negative electrons orbit a much heavier positive nucleus, as the planets orbit the much heavier sun. There the similarity ends, because forces in the atom are electromagnetic, whereas those in the planetary system are gravitational. Normal macroscopic amounts of matter contain immense numbers of atoms and molecules and, hence, even greater numbers of individual

negative and positive
charges.

The charges of electrons and protons are identical in magnitude but opposite in sign. Furthermore, all charged objects in nature are integral multiples of this basic quantity of charge, meaning that all charges are made of combinations of a basic unit of charge. Usually, charges are formed by combinations of electrons and protons. The magnitude of this basic charge is

Equation:

$$\mid q_e \mid = 1.60 \times 10^{-19}\ \text{C}.$$

The symbol q is commonly used for charge and the subscript e indicates the charge of a single electron (or proton).

The SI unit of charge is the coulomb (C). The number of protons needed to make a charge of 1.00 C is

Equation:

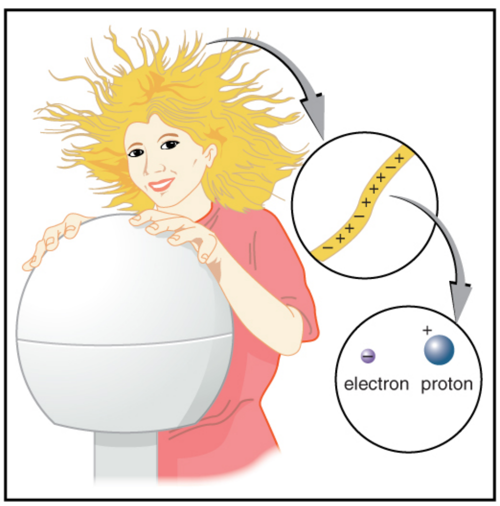
$$1.00\ \text{C} \times \frac{1\ \text{proton}}{1.60 \times 10^{-19}\ \text{C}} = 6.25 \times 10^{18}\ \text{protons}.$$

Similarly, 6.25×10^{18} electrons have a combined charge of −1.00 coulomb. Just as there is a smallest bit of an element (an atom), there is a smallest bit of charge. There is no directly observed charge smaller than $\mid q_e \mid$, and all observed charges are integral multiples of $\mid q_e \mid$.

Note:
Things Great and Small: The Submicroscopic Origin of Charge
With the exception of exotic, short-lived particles, all charge in nature is carried by electrons and protons. Electrons carry the charge we have

named negative. Protons carry an equal-magnitude charge that we call positive. (See [link].) Electron and proton charges are considered fundamental building blocks, since all other charges are integral multiples of those carried by electrons and protons. Electrons and protons are also two of the three fundamental building blocks of ordinary matter. The neutron is the third and has zero total charge.

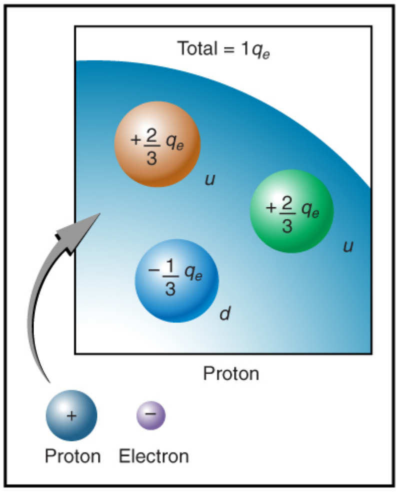
[link] shows a person touching a Van de Graaff generator and receiving excess positive charge. The expanded view of a hair shows the existence of both types of charges but an excess of positive. The repulsion of these positive like charges causes the strands of hair to repel other strands of hair and to stand up. The further blowup shows an artist's conception of an electron and a proton perhaps found in an atom in a strand of hair.



When this person touches a Van de Graaff generator, she receives an excess of positive charge, causing her hair to stand on end. The charges in one hair are shown. An artist's conception of an

electron and a proton illustrate the particles carrying the negative and positive charges. We cannot really see these particles with visible light because they are so small (the electron seems to be an infinitesimal point), but we know a great deal about their measurable properties, such as the charges they carry.

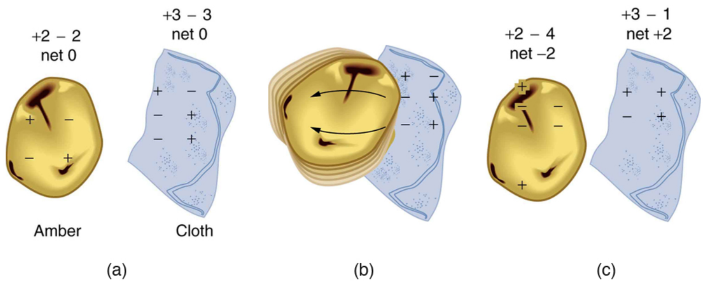
The electron seems to have no substructure; in contrast, when the substructure of protons is explored by scattering extremely energetic electrons from them, it appears that there are point-like particles inside the proton. These sub-particles, named quarks, have never been directly observed, but they are believed to carry fractional charges as seen in [link]. Charges on electrons and protons and all other directly observable particles are unitary, but these quark substructures carry charges of either $-\frac{1}{3}$ or $+\frac{2}{3}$. There are continuing attempts to observe fractional charge directly and to learn of the properties of quarks, which are perhaps the ultimate substructure of matter.



Artist's conception of fractional quark charges inside a proton. A group of three quark charges add up to the single positive charge on the proton: $-\frac{1}{3}q_e + \frac{2}{3}q_e + \frac{2}{3}q_e = +1q_e$.

Separation of Charge in Atoms

Charges in atoms and molecules can be separated—for example, by rubbing materials together. Some atoms and molecules have a greater affinity for electrons than others and will become negatively charged by close contact in rubbing, leaving the other material positively charged. (See [link].) Positive charge can similarly be induced by rubbing. Methods other than rubbing can also separate charges. Batteries, for example, use combinations of substances that interact in such a way as to separate charges. Chemical interactions may transfer negative charge from one substance to the other, making one battery terminal negative and leaving the first one positive.



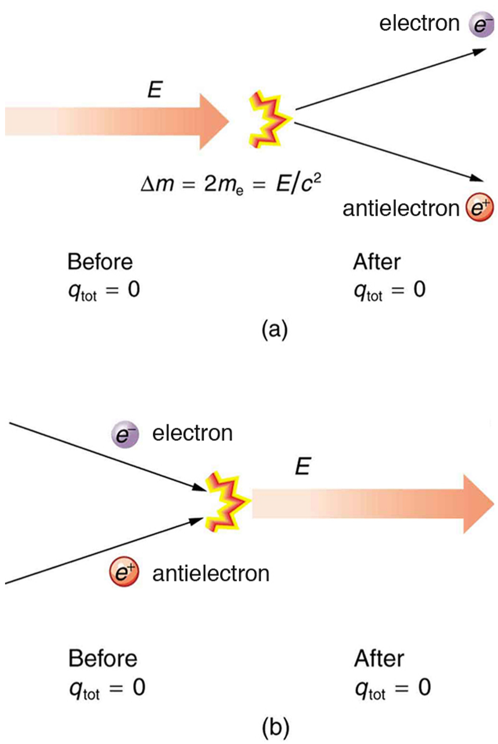
When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal.

No charge is actually created or destroyed when charges are separated as we have been discussing. Rather, existing charges are moved about. In fact, in all situations the total amount of charge is always constant. This universally obeyed law of nature is called the **law of conservation of charge**.

Note:
Law of Conservation of Charge
Total charge is constant in any process.

In more exotic situations, such as in particle accelerators, mass, Δm, can be created from energy in the amount $\Delta m = \frac{E}{c^2}$. Sometimes, the created mass is charged, such as when an electron is created. Whenever a charged particle is created, another having an opposite charge is always created along with it, so that the total charge created is zero. Usually, the two particles are “matter-antimatter” counterparts. For example, an antielectron would usually be created at the same time as an electron. The antielectron has a positive charge (it is called a positron), and so the total charge created is zero. (See [link].) All particles have antimatter counterparts with opposite signs. When matter and antimatter counterparts are brought together, they completely annihilate one another. By annihilate, we mean that the mass of the two particles is converted to energy E, again obeying the relationship $\Delta m = \frac{E}{c^2}$. Since the two particles have equal and opposite charge, the total charge is zero before and after the annihilation; thus, total charge is conserved.

Note:
Making Connections: Conservation Laws
Only a limited number of physical quantities are universally conserved. Charge is one—energy, momentum, and angular momentum are others. Because they are conserved, these physical quantities are used to explain more phenomena and form more connections than other, less basic quantities. We find that conserved quantities give us great insight into the rules followed by nature and hints to the organization of nature. Discoveries of conservation laws have led to further discoveries, such as the weak nuclear force and the quark substructure of protons and other particles.



(a) When enough energy is present, it can be converted into matter. Here the matter created is an electron–antielectron pair. (m_e is the electron's mass.) The total charge before and after this event is zero. (b) When matter and antimatter collide, they annihilate each other; the total charge is conserved at zero before and after the annihilation.

The law of conservation of charge is absolute—it has never been observed to be violated. Charge, then, is a special physical quantity, joining a very

short list of other quantities in nature that are always conserved. Other conserved quantities include energy, momentum, and angular momentum.

Section Summary

- There are only two types of charge, which we call positive and negative.
- Like charges repel, unlike charges attract, and the force between charges decreases with the square of the distance.
- The vast majority of positive charge in nature is carried by protons, while the vast majority of negative charge is carried by electrons.
- The electric charge of one electron is equal in magnitude and opposite in sign to the charge of one proton.
- An ion is an atom or molecule that has nonzero total charge due to having unequal numbers of electrons and protons.
- The SI unit for charge is the coulomb (C), with protons and electrons having charges of opposite sign but equal magnitude; the magnitude of this basic charge $\mid q_e \mid$ is
  Equation:
  $$\mid q_e \mid = 1.60 \times 10^{-19}\ \text{C}.$$
- Whenever charge is created or destroyed, equal amounts of positive and negative are involved.
- Most often, existing charges are separated from neutral objects to obtain some net charge.
- Both positive and negative charges exist in neutral objects and can be separated by rubbing one object with another. For macroscopic objects, negatively charged means an excess of electrons and positively charged means a depletion of electrons.
- The law of conservation of charge ensures that whenever a charge is created, an equal charge of the opposite sign is created at the same time.

Conceptual Questions

Exercise:

Problem:

There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

Exercise:

Problem:

Why do most objects tend to contain nearly equal numbers of positive and negative charges?

Problems & Exercises

Exercise:

Problem:

Common static electricity involves charges ranging from nanocoulombs to microcoulombs. (a) How many electrons are needed to form a charge of –$2.00\ \text{nC}$ (b) How many electrons must be removed from a neutral object to leave a net charge of $0.500\ \mu\text{C}$?

Solution:

(a) 1.25×10^{10}

(b) 3.13×10^{12}

Exercise:

Problem:

If 1.80×10^{20} electrons move through a pocket calculator during a full day's operation, how many coulombs of charge moved through it?

Exercise:

Problem:

To start a car engine, the car battery moves 3.75×10^{21} electrons through the starter motor. How many coulombs of charge were moved?

Solution:

-600 C

Exercise:

Problem:

A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge $\mid q_e \mid$ is this?

Glossary

electric charge
: a physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electromagnetic force

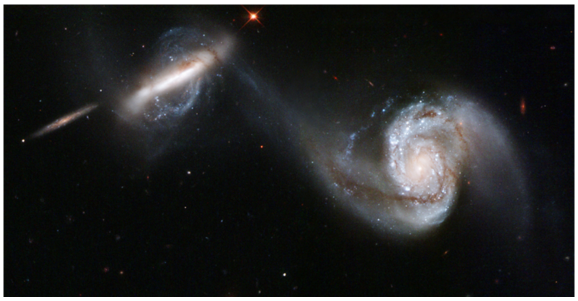
law of conservation of charge
: states that whenever a charge is created, an equal amount of charge with the opposite sign is created simultaneously

electron
: a particle orbiting the nucleus of an atom and carrying the smallest unit of negative charge

proton
: a particle in the nucleus of an atom and carrying a positive charge equal in magnitude and opposite in sign to the amount of negative charge carried by an electron

Coulomb’s Law

- State Coulomb’s law in terms of how the electrostatic force changes with the distance between two objects.
- Calculate the electrostatic force between two charged point forces, such as electrons or protons.
- Compare the electrostatic force to the gravitational attraction for a proton and an electron; for a human and the Earth.



This NASA image of Arp 87 shows the result of a strong gravitational attraction between two galaxies. In contrast, at the subatomic level, the electrostatic attraction between two objects, such as an electron and a proton, is far greater than their mutual attraction due to gravity. (credit: NASA/HST)

Through the work of scientists in the late 18th century, the main features of the **electrostatic force**—the existence of two types of charge, the observation that like charges repel, unlike charges attract, and the decrease of force with distance—were eventually refined, and expressed as a mathematical formula. The mathematical formula for the electrostatic force is called **Coulomb’s law** after the French physicist Charles Coulomb (1736–1806), who performed experiments and first proposed a formula to calculate it.

Note:
Coulomb’s Law
Equation:

$$F = k\frac{|q_1 q_2|}{r^2}.$$

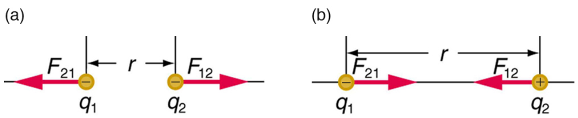
Coulomb's law calculates the magnitude of the force F between two point charges, q_1 and q_2, separated by a distance r. In SI units, the constant k is equal to

Equation:

$$k = 8.988 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}} \approx 8.99 \times 10^9 \frac{\mathrm{N \cdot m^2}}{\mathrm{C^2}}.$$

The electrostatic force is a vector quantity and is expressed in units of newtons. The force is understood to be along the line joining the two charges. (See [link].)

Although the formula for Coulomb's law is simple, it was no mean task to prove it. The experiments Coulomb did, with the primitive equipment then available, were difficult. Modern experiments have verified Coulomb's law to great precision. For example, it has been shown that the force is inversely proportional to distance between two objects squared $\left(F \propto 1/r^2\right)$ to an accuracy of 1 part in 10^{16}. No exceptions have ever been found, even at the small distances within the atom.



The magnitude of the electrostatic force F between point charges q_1 and q_2 separated by a distance r is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on q_1 is equal in magnitude and opposite in direction to the force it exerts on q_2. (a) Like charges. (b) Unlike charges.

Example:

How Strong is the Coulomb Force Relative to the Gravitational Force?

Compare the electrostatic force between an electron and proton separated by 0.530×10^{-10} m with the gravitational force between them. This distance is their average separation in a hydrogen atom.

Strategy

To compare the two forces, we first compute the electrostatic force using Coulomb's law, $F = k\frac{|q_1 q_2|}{r^2}$. We then calculate the gravitational force using Newton's universal law of

gravitation. Finally, we take a ratio to see how the forces compare in magnitude.

Solution

Entering the given and known information about the charges and separation of the electron and proton into the expression of Coulomb's law yields

Equation:

$$F = k\frac{|q_1 q_2|}{r^2}$$

Equation:

$$= \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \times \frac{(1.60\times10^{-19}\text{ C})(1.60\times10^{-19}\text{ C})}{(0.530\times10^{-10}\text{ m})^2}$$

Thus the Coulomb force is

Equation:

$$F = 8.19 \times 10^{-8}\text{ N}.$$

The charges are opposite in sign, so this is an attractive force. This is a very large force for an electron—it would cause an acceleration of $8.99 \times 10^{22}\text{ m/s}^2$(verification is left as an end-of-section problem).The gravitational force is given by Newton's law of gravitation as:

Equation:

$$F_G = G\frac{mM}{r^2},$$

where $G = 6.67 \times 10^{-11}\text{ N} \cdot \text{m}^2/\text{kg}^2$. Here m and M represent the electron and proton masses, which can be found in the appendices. Entering values for the knowns yields

Equation:

$$F_G = (6.67 \times 10^{-11}\text{ N} \cdot \text{m}^2/\text{kg}^2) \times \frac{(9.11 \times 10^{-31}\text{ kg})(1.67 \times 10^{-27}\text{ kg})}{(0.530 \times 10^{-10}\text{ m})^2} = 3.61 \times 10^{-47}\text{ N}$$

This is also an attractive force, although it is traditionally shown as positive since gravitational force is always attractive. The ratio of the magnitude of the electrostatic force to gravitational force in this case is, thus,

Equation:

$$\frac{F}{F_G} = 2.27 \times 10^{39}.$$

Discussion

This is a remarkably large ratio! Note that this will be the ratio of electrostatic force to gravitational force for an electron and a proton at any distance (taking the ratio before entering numerical values shows that the distance cancels). This ratio gives some indication

of just how much larger the Coulomb force is than the gravitational force between two of the most common particles in nature.

As the example implies, gravitational force is completely negligible on a small scale, where the interactions of individual charged particles are important. On a large scale, such as between the Earth and a person, the reverse is true. Most objects are nearly electrically neutral, and so attractive and repulsive **Coulomb forces** nearly cancel. Gravitational force on a large scale dominates interactions between large objects because it is always attractive, while Coulomb forces tend to cancel.

Section Summary

- Frenchman Charles Coulomb was the first to publish the mathematical equation that describes the electrostatic force between two objects.
- Coulomb's law gives the magnitude of the force between point charges. It is
  Equation:

  $$F = k\frac{|q_1 q_2|}{r^2},$$

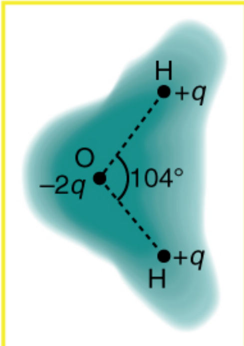
  where q_1 and q_2 are two point charges separated by a distance r, and $k \approx 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$
- This Coulomb force is extremely basic, since most charges are due to point-like particles. It is responsible for all electrostatic effects and underlies most macroscopic forces.
- The Coulomb force is extraordinarily strong compared with the gravitational force, another basic force—but unlike gravitational force it can cancel, since it can be either attractive or repulsive.
- The electrostatic force between two subatomic particles is far greater than the gravitational force between the same two particles.

Conceptual Questions

Exercise:

Problem:

[link] shows the charge distribution in a water molecule, which is called a polar molecule because it has an inherent separation of charge. Given water's polar character, explain what effect humidity has on removing excess charge from objects.



Schematic representation of the outer electron cloud of a neutral water molecule. The electrons spend more time near the oxygen than the hydrogens, giving a permanent charge separation as shown. Water is thus a *polar molecule*. It is more easily affected by electrostatic forces than molecules with uniform charge distributions.

Exercise:

Problem:

Using [link], explain, in terms of Coulomb's law, why a polar molecule (such as in [link]) is attracted by both positive and negative charges.

Exercise:

Problem:

Given the polar character of water molecules, explain how ions in the air form nucleation centers for rain droplets.

Problems & Exercises

Exercise:

Problem:

What is the repulsive force between two pith balls that are 8.00 cm apart and have equal charges of – 30.0 nC?

Exercise:

Problem:

(a) How strong is the attractive force between a glass rod with a $0.700\ \mu\text{C}$ charge and a silk cloth with a $-0.600\ \mu\text{C}$ charge, which are 12.0 cm apart, using the approximation that they act like point charges? (b) Discuss how the answer to this problem might be affected if the charges are distributed over some area and do not act like point charges.

Solution:

(a) 0.263 N

(b) If the charges are distributed over some area, there will be a concentration of charge along the side closest to the oppositely charged object. This effect will increase the net force.

Exercise:

Problem:

Two point charges exert a 5.00 N force on each other. What will the force become if the distance between them is increased by a factor of three?

Exercise:

Problem:

Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?

Solution:

The separation decreased by a factor of 5.

Exercise:

Problem:

How far apart must two point charges of 75.0 nC (typical of static electricity) be to have a force of 1.00 N between them?

Exercise:

Problem:

If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them? You will see that even at a distance as large as 1 km, the repulsive force is substantial because 1 C is a very significant amount of charge.

Exercise:

Problem:

Bare free charges do not remain stationary when close together. To illustrate this, calculate the acceleration of two isolated protons separated by 2.00 nm (a typical distance between gas atoms).

Solution:

$$\begin{aligned} F &= k\frac{|q_1 q_2|}{r^2} = ma \Rightarrow a = \frac{kq^2}{mr^2} \\ &= \frac{\left(9.00\times10^{9}\ \text{N·m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{m}\right)^2}{\left(1.67\times10^{-27}\ \text{kg}\right)\left(2.00\times10^{-9}\ \text{m}\right)^2} \\ &= 3.45\times10^{16}\ \text{m/s}^2 \end{aligned}$$

Exercise:

Problem:

(a) By what factor must you change the distance between two point charges to change the force between them by a factor of 10? (b) Explain how the distance can either increase or decrease by this factor and still cause a factor of 10 change in the force.

Solution:

(a) 3.2

(b) If the distance increases by 3.2, then the force will decrease by a factor of 10 ; if the distance decreases by 3.2, then the force will increase by a factor of 10. Either way, the force changes by a factor of 10.

Exercise:

Problem:

Suppose you have a total charge q_{tot} that you can split in any manner. Once split, the separation distance is fixed. How do you split the charge to achieve the greatest force?

Exercise:

Problem:

(a) Common transparent tape becomes charged when pulled from a dispenser. If one piece is placed above another, the repulsive force can be great enough to support the top piece's weight. Assuming equal point charges (only an approximation), calculate the magnitude of the charge if electrostatic force is great enough to support the weight of a 10.0 mg piece of tape held 1.00 cm above another. (b) Discuss whether the magnitude of this charge is consistent with what is typical of static electricity.

Solution:

(a) 1.04×10^{-9} C

(b) This charge is approximately 1 nC, which is consistent with the magnitude of charge typical for static electricity

Exercise:

Problem:

(a) Find the ratio of the electrostatic to gravitational force between two electrons. (b) What is this ratio for two protons? (c) Why is the ratio different for electrons and protons?

Exercise:

Problem:

At what distance is the electrostatic force between two protons equal to the weight of one proton?

Exercise:

Problem:

A certain five cent coin contains 5.00 g of nickel. What fraction of the nickel atoms' electrons, removed and placed 1.00 m above it, would support the weight of this coin? The atomic mass of nickel is 58.7, and each nickel atom contains 28 electrons and 28 protons.

Solution:

1.02×10^{-11}

Exercise:

Problem:

(a) Two point charges totaling $8.00\ \mu C$ exert a repulsive force of 0.150 N on one another when separated by 0.500 m. What is the charge on each? (b) What is the charge on each if the force is attractive?

Glossary

Coulomb's law
: the mathematical equation calculating the electrostatic force vector between two charged particles

Coulomb force
: another term for the electrostatic force

electrostatic force
: the amount and direction of attraction or repulsion between two charged bodies

Electric Field: Concept of a Field Revisited

- Describe a force field and calculate the strength of an electric field due to a point charge.
- Calculate the force exerted on a test charge by an electric field.
- Explain the relationship between electrical force (F) on a test charge and electrical field strength (E).

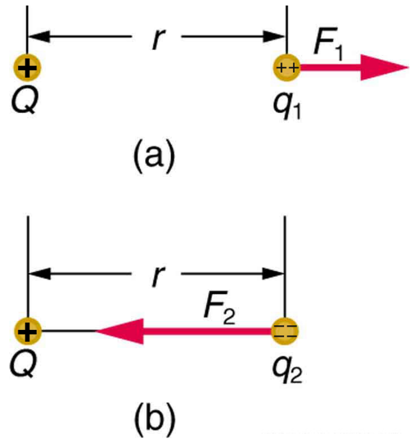
Contact forces, such as between a baseball and a bat, are explained on the small scale by the interaction of the charges in atoms and molecules in close proximity. They interact through forces that include the **Coulomb force**. Action at a distance is a force between objects that are not close enough for their atoms to “touch.” That is, they are separated by more than a few atomic diameters.

For example, a charged rubber comb attracts neutral bits of paper from a distance via the Coulomb force. It is very useful to think of an object being surrounded in space by a **force field**. The force field carries the force to another object (called a test object) some distance away.

Concept of a Field

A field is a way of conceptualizing and mapping the force that surrounds any object and acts on another object at a distance without apparent physical connection. For example, the gravitational field surrounding the earth (and all other masses) represents the gravitational force that would be experienced if another mass were placed at a given point within the field.

In the same way, the Coulomb force field surrounding any charge extends throughout space. Using Coulomb’s law, $F=k|q_1q_2|/r^2$, its magnitude is given by the equation $F=k|\,qQ\,|/r^2$, for a **point charge** (a particle having a charge Q) acting on a **test charge** q at a distance r (see [link]). Both the magnitude and direction of the Coulomb force field depend on Q and the test charge q.



The Coulomb force field due to a positive charge Q is shown acting on two different charges. Both charges are the same distance from Q. (a) Since q_1 is positive, the force F_1 acting on it is repulsive. (b) The charge q_2 is negative and greater in magnitude than q_1, and so the force F_2 acting on it is attractive and stronger than F_1. The Coulomb force field is thus not unique at any point in space, because it depends on the test charges q_1 and q_2

as well as the
charge Q.

To simplify things, we would prefer to have a field that depends only on Q and not on the test charge q. The electric field is defined in such a manner that it represents only the charge creating it and is unique at every point in space. Specifically, the electric field E is defined to be the ratio of the Coulomb force to the test charge:

Equation:

$$\mathbf{E} = \frac{\mathbf{F}}{q},$$

where $\mathbf{F}$ is the electrostatic force (or Coulomb force) exerted on a positive test charge q. It is understood that $\mathbf{E}$ is in the same direction as $\mathbf{F}$. It is also assumed that q is so small that it does not alter the charge distribution creating the electric field. The units of electric field are newtons per coulomb (N/C). If the electric field is known, then the electrostatic force on any charge q is simply obtained by multiplying charge times electric field, or $\mathbf{F} = q\mathbf{E}$. Consider the electric field due to a point charge Q. According to Coulomb's law, the force it exerts on a test charge q is $F = k|\, qQ\, |/r^2$. Thus the magnitude of the electric field, E, for a point charge is

Equation:

$$E = \left|\frac{F}{q}\right| = k\left|\frac{\mathrm{qQ}}{qr^2}\right| = k\frac{|Q|}{r^2}.$$

Since the test charge cancels, we see that

Equation:

$$E = k\frac{|Q|}{r^2}.$$

The electric field is thus seen to depend only on the charge Q and the distance r; it is completely independent of the test charge q.

Example:
Calculating the Electric Field of a Point Charge
Calculate the strength and direction of the electric field E due to a point charge of 2.00 nC (nano-Coulombs) at a distance of 5.00 mm from the charge.
Strategy
We can find the electric field created by a point charge by using the equation $E = \mathrm{kQ}/r^2$.
Solution
Here $Q = 2.00 \times 10^{-9}$ C and $r = 5.00 \times 10^{-3}$ m. Entering those values into the above equation gives
Equation:

$$\begin{aligned} E &= k\frac{Q}{r^2} \\ &= (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}) \times \frac{(2.00 \times 10^{-9}\ \mathrm{C})}{(5.00 \times 10^{-3}\ \mathrm{m})^2} \\ &= 7.19 \times 10^5\ \mathrm{N/C}. \end{aligned}$$

Discussion
This **electric field strength** is the same at any point 5.00 mm away from the charge Q that creates the field. It is positive, meaning that it has a direction pointing away from the charge Q.

Example:
Calculating the Force Exerted on a Point Charge by an Electric Field
What force does the electric field found in the previous example exert on a point charge of –0.250 μC?
Strategy
Since we know the electric field strength and the charge in the field, the force on that charge can be calculated using the definition of electric field

$\mathbf{E} = \mathbf{F}/q$ rearranged to $\mathbf{F} = q\mathbf{E}$.

Solution

The magnitude of the force on a charge $q = -0.250\ \mu\text{C}$ exerted by a field of strength $E = 7.20 \times 10^5$ N/C is thus,

Equation:

$$\begin{aligned} F &= -qE \\ &= (0.250 \times 10^{-6}\ \text{C})(7.20 \times 10^5\ \text{N/C}) \\ &= 0.180\ \text{N}. \end{aligned}$$

Because q is negative, the force is directed opposite to the direction of the field.

Discussion

The force is attractive, as expected for unlike charges. (The field was created by a positive charge and here acts on a negative charge.) The charges in this example are typical of common static electricity, and the modest attractive force obtained is similar to forces experienced in static cling and similar situations.

Section Summary

- The electrostatic force field surrounding a charged object extends out into space in all directions.
- The electrostatic force exerted by a point charge on a test charge at a distance r depends on the charge of both charges, as well as the distance between the two.
- The electric field $\mathbf{E}$ is defined to be

  Equation:

  $$\mathbf{E} = \frac{\mathbf{F}}{q},$$

  where $\mathbf{F}$ is the Coulomb or electrostatic force exerted on a small positive test charge q. $\mathbf{E}$ has units of N/C.

- The magnitude of the electric field $\mathbf{E}$ created by a point charge Q is
  Equation:

  $$\mathbf{E} = k\frac{|Q|}{r^2}.$$

  where r is the distance from Q. The electric field $\mathbf{E}$ is a vector and fields due to multiple charges add like vectors.

Conceptual Questions

Exercise:

Problem:

Why must the test charge q in the definition of the electric field be vanishingly small?

Exercise:

Problem:

Are the direction and magnitude of the Coulomb force unique at a given point in space? What about the electric field?

Problem Exercises

Exercise:

Problem:

What is the magnitude and direction of an electric field that exerts a 2.00×10^{-5} N upward force on a $-1.75\ \mu\text{C}$ charge?

Exercise:

Problem:

What is the magnitude and direction of the force exerted on a $3.50\ \mu\text{C}$ charge by a 250 N/C electric field that points due east?

Solution:

$8.75 \times 10^{-4}\ \text{N}$

Exercise:

Problem:

Calculate the magnitude of the electric field 2.00 m from a point charge of 5.00 mC (such as found on the terminal of a Van de Graaff).

Exercise:

Problem:

(a) What magnitude point charge creates a 10,000 N/C electric field at a distance of 0.250 m? (b) How large is the field at 10.0 m?

Solution:

(a) $6.94 \times 10^{-8}\ \text{C}$

(b) $6.25\ \text{N/C}$

Exercise:

Problem:

Calculate the initial (from rest) acceleration of a proton in a $5.00 \times 10^{6}\ \text{N/C}$ electric field (such as created by a research Van de Graaff). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

Exercise:

Problem:

(a) Find the direction and magnitude of an electric field that exerts a 4.80×10^{-17} N westward force on an electron. (b) What magnitude and direction force does this field exert on a proton?

Solution:

(a) 300 N/C (east)

(b) $4.80 \times 10^{-17} \text{ N (east)}$

Glossary

field
: a map of the amount and direction of a force acting on other objects, extending out into space

point charge
: A charged particle, designated Q, generating an electric field

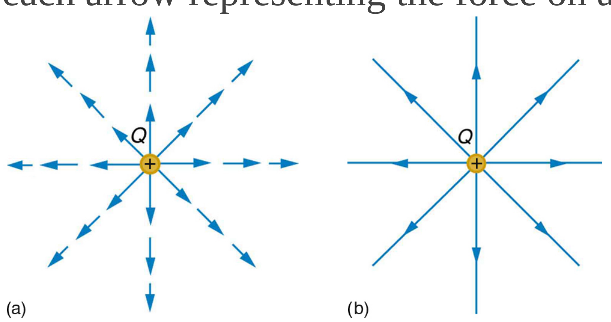
test charge
: A particle (designated q) with either a positive or negative charge set down within an electric field generated by a point charge

Electric Field Lines

- Calculate the total force (magnitude and direction) exerted on a test charge from more than one charge
- Describe an electric field diagram of a positive point charge; of a negative point charge with twice the magnitude of positive charge
- Draw the electric field lines between two points of the same charge; between two points of opposite charge.

Drawings using lines to represent **electric fields** around charged objects are very useful in visualizing field strength and direction. Since the electric field has both magnitude and direction, it is a vector. Like all **vectors**, the electric field can be represented by an arrow that has length proportional to its magnitude and that points in the correct direction. (We have used arrows extensively to represent force vectors, for example.)

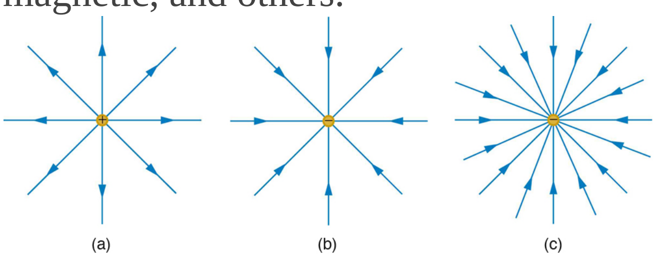
[link] shows two pictorial representations of the same electric field created by a positive point charge Q. [link] (b) shows the standard representation using continuous lines. [link] (a) shows numerous individual arrows with each arrow representing the force on a test charge q.



Two equivalent representations of the electric field due to a positive charge Q. (a) Arrows representing the electric field's magnitude and direction. (b) In the standard representation, the arrows are replaced by continuous field lines having the same direction at any point as the electric field. The

closeness of the lines is directly related to the strength of the electric field. A test charge placed anywhere will feel a force in the direction of the field line; this force will have a strength proportional to the density of the lines (being greater near the charge, for example).

Note that the electric field is defined for a positive test charge q, so that the field lines point away from a positive charge and toward a negative charge. (See [link].) The electric field strength is exactly proportional to the number of field lines per unit area, since the magnitude of the electric field for a point charge is $E = k|Q|/r^2$ and area is proportional to r^2. This pictorial representation, in which field lines represent the direction and their closeness (that is, their areal density or the number of lines crossing a unit area) represents strength, is used for all fields: electrostatic, gravitational, magnetic, and others.



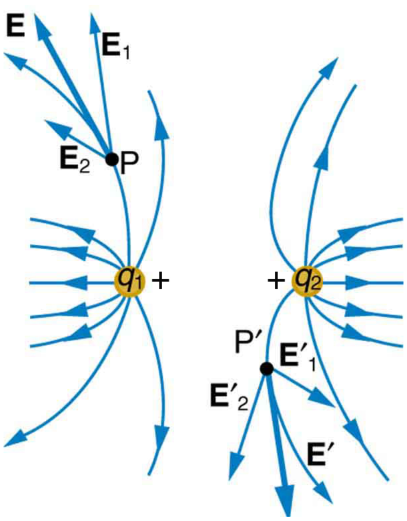
The electric field surrounding three different point charges. (a) A positive charge. (b) A negative charge of equal magnitude. (c) A larger negative charge.

In many situations, there are multiple charges. The total electric field created by multiple charges is the vector sum of the individual fields created by each charge.

[link] shows how the electric field from two point charges can be drawn by finding the total field at representative points and drawing electric field lines consistent with those points. While the electric fields from multiple charges are more complex than those of single charges, some simple features are easily noticed.

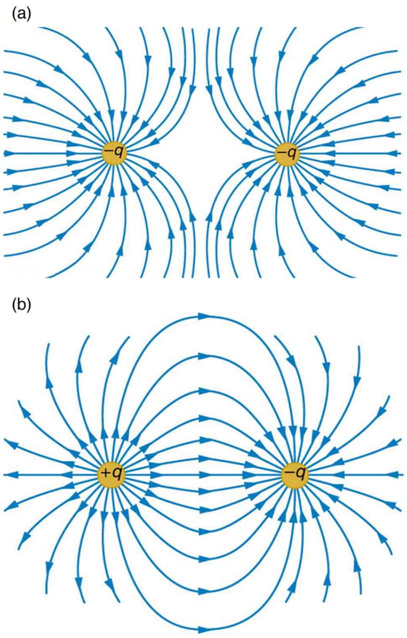
For example, the field is weaker between like charges, as shown by the lines being farther apart in that region. (This is because the fields from each charge exert opposing forces on any charge placed between them.) (See [link] and [link](a).) Furthermore, at a great distance from two like charges, the field becomes identical to the field from a single, larger charge.

[link](b) shows the electric field of two unlike charges. The field is stronger between the charges. In that region, the fields from each charge are in the same direction, and so their strengths add. The field of two unlike charges is weak at large distances, because the fields of the individual charges are in opposite directions and so their strengths subtract. At very large distances, the field of two unlike charges looks like that of a smaller single charge.



Two positive point charges q_1 and q_2 produce the

resultant electric field shown. The field is calculated at representative points and then smooth field lines drawn following the rules outlined in the text.



(a) Two negative charges produce the fields shown. It is very similar to the field produced by two positive charges, except that the directions are reversed. The field is clearly weaker between the

charges. The individual forces on a test charge in that region are in opposite directions. (b) Two opposite charges produce the field shown, which is stronger in the region between the charges.

We use electric field lines to visualize and analyze electric fields (the lines are a pictorial tool, not a physical entity in themselves). The properties of electric field lines for any charge distribution can be summarized as follows:

1. Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
2. The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
3. The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
4. The direction of the electric field is tangent to the field line at any point in space.
5. Field lines can never cross.

The last property means that the field is unique at any point. The field line represents the direction of the field; so if they crossed, the field would have two directions at that location (an impossibility if the field is unique).

Section Summary

- Drawings of electric field lines are useful visual tools. The properties of electric field lines for any charge distribution are that:
- Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
- The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
- The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
- The direction of the electric field is tangent to the field line at any point in space.
- Field lines can never cross.

Conceptual Questions

Exercise:

Problem:

Compare and contrast the Coulomb force field and the electric field. To do this, make a list of five properties for the Coulomb force field analogous to the five properties listed for electric field lines. Compare each item in your list of Coulomb force field properties with those of the electric field—are they the same or different? (For example, electric field lines cannot cross. Is the same true for Coulomb field lines?)

Problem Exercises

Exercise:

Problem:

(a) Sketch the electric field lines near a point charge $+q$. (b) Do the same for a point charge $-3.00q$.

Exercise:

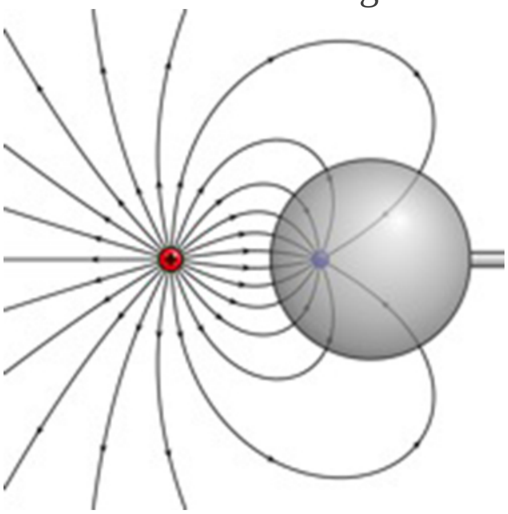
Problem:

Sketch the electric field lines a long distance from the charge distributions shown in [link] (a) and (b)

Exercise:

Problem:

[link] shows the electric field lines near two charges q_1 and q_2. What is the ratio of their magnitudes? (b) Sketch the electric field lines a long distance from the charges shown in the figure.



The electric field near two charges.

Exercise:

Problem:

Sketch the electric field lines in the vicinity of two opposite charges, where the negative charge is three times greater in magnitude than the positive. (See [link] for a similar situation).

Glossary

electric field
: a three-dimensional map of the electric force extended out into space from a point charge

electric field lines
: a series of lines drawn from a point charge representing the magnitude and direction of force exerted by that charge

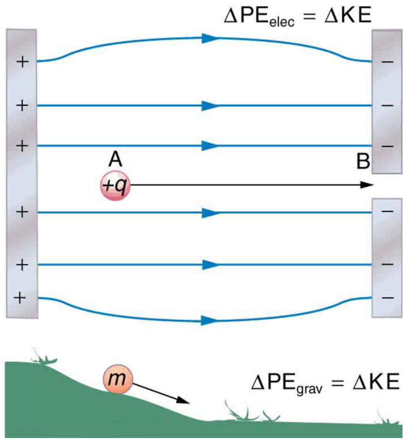
vector
: a quantity with both magnitude and direction

vector addition
: mathematical combination of two or more vectors, including their magnitudes, directions, and positions

Electric Potential and Potential Energy

- Define electric potential and electric potential energy.
- Describe the relationship between electric potential difference and electric field.
- Describe the relationship between electric potential and electrical potential energy.
- Explain electron volt and its usage in submicroscopic process.

When a free positive charge *q* is accelerated by an electric field, such as shown in [link], it is given kinetic energy. The process is analogous to an object being accelerated by a gravitational field. It is as if the charge is going down an electrical hill where its electric potential energy is converted to kinetic energy. Let us explore the work done on a charge *q* by the electric field in this process, so that we may develop a definition of electric potential energy.



A charge accelerated by an electric field is analogous to a mass going down a hill. In both cases potential energy is converted to another form. Work is done by a force, but since this force

is conservative, we can write $W = -\Delta\mathrm{PE}$.

The electrostatic or Coulomb force is conservative, which means that the work done on q is independent of the path taken. This is exactly analogous to the gravitational force in the absence of dissipative forces such as friction. When a force is conservative, it is possible to define a potential energy associated with the force, and it is usually easier to deal with the potential energy (because it depends only on position) than to calculate the work done directly from force ($W = \langle \boldsymbol{F}, \boldsymbol{d} \rangle$, where d is displacement and F is the force).

We use the letters PE to denote electric potential energy, which has units of joules (J). The change in potential energy, $\Delta\mathrm{PE}$, is crucial, since the work done by a conservative force is the negative of the change in potential energy; that is, $W = -\Delta\mathrm{PE}$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative $\Delta\mathrm{PE}$. There must be a minus sign in front of $\Delta\mathrm{PE}$ to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Note:
Potential Energy
$W = -\Delta\mathrm{PE}$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative $\Delta\mathrm{PE}$. There must be a minus sign in front of $\Delta\mathrm{PE}$ to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Gravitational potential energy and electric potential energy are quite analogous. Potential energy accounts for work done by a conservative force

and gives added insight regarding energy and energy transformation without the necessity of dealing with the force directly. It is much more common, for example, to use the concept of voltage (related to electric potential energy) than to deal with the electric field (related to Coulomb force) directly.

Given some conservative force $\boldsymbol{F}$ and displacement $\boldsymbol{d}$ under the force, the work done and the change in potential energy can be calculated as, $W = \langle \boldsymbol{F}, \boldsymbol{d} \rangle$ and $\Delta\text{PE} = -\text{W} = \langle -\mathbf{F}, \boldsymbol{d} \rangle$. For electric force, the force is given by the product of electric charge and the electric field, $\boldsymbol{F} = q\boldsymbol{E}$, where q is the charge experiencing the force and $\boldsymbol{E}$ is the electric field at the location of the charge. So the potential energy change due to work done by electric force is $\Delta\text{PE} = q(\langle -\mathbf{E}, \boldsymbol{d} \rangle)$. If we define change in **electric potential** V as $\mathit{\Delta} V = \langle -\mathbf{E}, \boldsymbol{d} \rangle$, then the electric potential energy PE is simply expressed in terms of electric potential, $\text{PE} = qV$, or,
Equation:

$$V = \frac{\text{PE}}{q},$$

electric potential energy per charge.

Note:
Electric Potential
Electric potential is the electric potential energy per unit charge.
Equation:

$$V = \frac{\text{PE}}{q}$$

With potential energy, the case often is that its value at a single point has no significant meaning but what is important is the difference in potential

energy. From the difference in potential energy, we are able to calculate other quantities, such as change in kinetic energy (if no force other than the conservative force acts) or work needing to be done by other forces (if other forces act). So likewise, rather than the electric potential itself, we are often interested in difference in electric potential ΔV between two points, where,
Equation:

$$\Delta V = V_{\text{B}} - V_{\text{A}} = \frac{\Delta \text{PE}}{q}.$$

The **potential difference** between points A and B, $V_{\text{B}} - V_{\text{A}}$, is thus defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.
Equation:

$$1 \text{ V} = 1 \frac{\text{J}}{\text{C}}$$

Note:
Potential Difference
The potential difference between points A and B, $V_{\text{B}} - V_{\text{A}}$, is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.
Equation:

$$1 \text{ V} = 1 \frac{\text{J}}{\text{C}}$$

The familiar term **voltage** is the common name for potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor.

In summary, the relationship between potential difference (or voltage) and electrical potential energy is given by

Equation:

$$\Delta V = \frac{\Delta \mathrm{PE}}{q} \text{ and } \Delta \mathrm{PE} = q\Delta V.$$

Note:

Potential Difference and Electrical Potential Energy

The relationship between potential difference (or voltage) and electrical potential energy is given by

Equation:

$$\Delta V = \frac{\Delta \mathrm{PE}}{q} \text{ and } \Delta \mathrm{PE} = q\Delta V.$$

The second equation is equivalent to the first.

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other since $\Delta \mathrm{PE} = q\Delta V$. The car battery can move more charge than the motorcycle battery, although both are 12 V batteries.

Example:
Calculating Energy

Suppose you have a 12.0 V motorcycle battery that can move 5000 C of charge, and a 12.0 V car battery that can move 60,000 C of charge. How much energy does each deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)

Strategy

To say we have a 12.0 V battery means that its terminals have a 12.0 V potential difference. When such a battery moves charge, it puts the charge through a potential difference of 12.0 V, and the charge is given a change in potential energy equal to $\Delta\text{PE} = q\Delta V$.

So to find the energy output, we multiply the charge moved by the potential difference.

Solution

For the motorcycle battery, $q = 5000\text{ C}$ and $\Delta V = 12.0\text{ V}$. The total energy delivered by the motorcycle battery is

Equation:

$$\begin{aligned}\Delta\text{PE}_{\text{cycle}} &= (5000\text{ C})(12.0\text{ V})\\ &= (5000\text{ C})(12.0\text{ J/C})\\ &= 6.00\times10^{4}\text{ J.}\end{aligned}$$

Similarly, for the car battery, $q = 60{,}000\text{ C}$ and

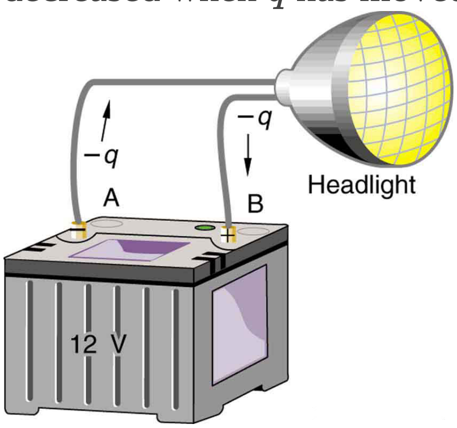
Equation:

$$\begin{aligned}\Delta\text{PE}_{\text{car}} &= (60{,}000\text{ C})(12.0\text{ V})\\ &= 7.20\times10^{5}\text{ J.}\end{aligned}$$

Discussion

While voltage and energy are related, they are not the same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a low car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (A) through whatever circuitry is involved and attract them to their positive terminals (B) as shown in [link]. The change in potential is $\Delta V = V_{\text{B}} - V_{\text{A}} = +12\text{ V}$ and the charge q is negative, so that $\Delta\text{PE} = q\Delta V$ is negative, meaning the potential energy of the battery has decreased when q has moved from A to B.



A battery moves negative charge from its negative terminal through a headlight to its positive terminal. Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the

positive terminal is at a higher voltage than the negative. Inside the battery, both positive and negative charges move.

Example:

How Many Electrons Move through a Headlight Each Second?

When a 12.0 V car battery runs a single 30.0 W headlight, how many electrons pass through it each second?

Strategy

To find the number of electrons, we must first find the charge that moved in 1.00 s. The charge moved is related to voltage and energy through the equation $\Delta \mathrm{PE} = q\Delta V$. A 30.0 W lamp uses 30.0 joules per second. Since the battery loses energy, we have $\Delta \mathrm{PE} = –30.0\ \mathrm{J}$ and, since the electrons are going from the negative terminal to the positive, we see that $\Delta V = +12.0\ \mathrm{V}$.

Solution

To find the charge q moved, we solve the equation $\Delta \mathrm{PE} = q\Delta V$:

Equation:

$$q = \frac{\Delta \mathrm{PE}}{\Delta V}.$$

Entering the values for $\Delta \mathrm{PE}$ and ΔV, we get

Equation:

$$q = \frac{–30.0\ \mathrm{J}}{+12.0\ \mathrm{V}} = \frac{–30.0\ \mathrm{J}}{+12.0\ \mathrm{J/C}} = –2.50\ \mathrm{C}.$$

The number of electrons $\mathrm{n_e}$ is the total charge divided by the charge per electron. That is,

Equation:

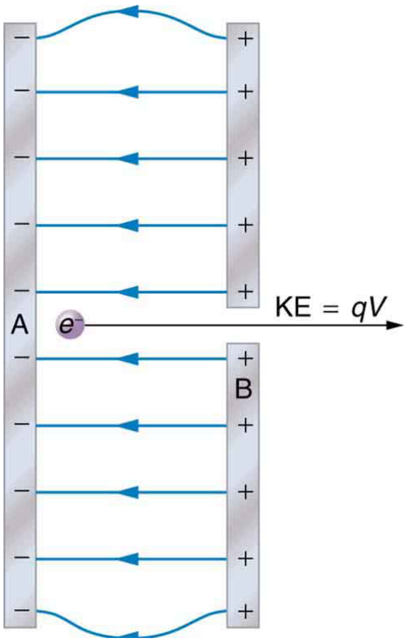
$$n_e = \frac{-2.50\ \mathrm{C}}{-1.60\times10^{-19}\ \mathrm{C/e^-}} = 1.56\times10^{19}\ \text{electrons.}$$

Discussion

This is a very large number. It is no wonder that we do not ordinarily observe individual electrons with so many being present in ordinary systems. In fact, electricity had been in use for many decades before it was determined that the moving charges in many circumstances were negative. Positive charge moving in the opposite direction of negative charge often produces identical effects; this makes it difficult to determine which is moving or whether both are moving.

The Electron Volt

The energy per electron is very small in macroscopic situations like that in the previous example—a tiny fraction of a joule. But on a submicroscopic scale, such energy per particle (electron, proton, or ion) can be of great importance. For example, even a tiny fraction of a joule can be great enough for these particles to destroy organic molecules and harm living tissue. The particle may do its damage by direct collision, or it may create harmful X-rays, which can also inflict damage. It is useful to have an energy unit related to submicroscopic effects. [link] shows a situation related to the definition of such an energy unit. An electron is accelerated between two charged metal plates as it might be in an old-model television tube or oscilloscope. The electron is given kinetic energy that is later converted to another form—light in the television tube, for example. (Note that downhill for the electron is uphill for a positive charge.) Since energy is related to voltage by $\Delta\mathrm{PE} = q\Delta V$, we can think of the joule as a coulomb-volt.



A typical electron gun accelerates electrons using a potential difference between two metal plates. The energy of the electron in electron volts is numerically the same as the voltage between the plates. For example, a 5000 V potential difference produces 5000 eV electrons.

Note:
Electron Volt

On the submicroscopic scale, it is more convenient to define an energy unit called the electron volt (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

Equation:

$$\begin{aligned} 1\text{ eV} &= \left(1.60 \times 10^{-19}\text{ C}\right)(1\text{ V}) = \left(1.60 \times 10^{-19}\text{ C}\right)(1\text{ J/C}) \\ &= 1.60 \times 10^{-19}\text{ J.} \end{aligned}$$

An electron accelerated through a potential difference of 1 V is given an energy of 1 eV. It follows that an electron accelerated through 50 V is given 50 eV. A potential difference of 100,000 V (100 kV) will give an electron an energy of 100,000 eV (100 keV), and so on. Similarly, an ion with a double positive charge accelerated through 100 V will be given 200 eV of energy. These simple relationships between accelerating voltage and particle charges make the electron volt a simple and convenient energy unit in such circumstances.

Note:

Connections: Energy Units

The electron volt (eV) is the most common energy unit for submicroscopic processes. This will be particularly noticeable in the chapters on modern physics. Energy is so important to so many subjects that there is a tendency to define a special energy unit for each major topic. There are, for example, calories for food energy, and kilowatt-hours for electrical energy.

The electron volt is commonly employed in submicroscopic processes—chemical valence energies and molecular and nuclear binding energies are among the quantities often expressed in electron volts. For example, about 5 eV of energy is required to break up certain organic molecules. If a proton is accelerated from rest through a potential difference of 30 kV, it is given an energy of 30 keV (30,000 eV) and it can break up as many as 6000 of

these molecules ($30{,}000 \text{ eV} \div 5 \text{ eV per molecule} = 6000 \text{ molecules}$). Nuclear decay energies are on the order of 1 MeV (1,000,000 eV) per event and can, thus, produce significant biological damage.

Conservation of Energy

The total energy of a system is conserved if there is no net addition (or subtraction) of work or heat transfer. For conservative forces, such as the electrostatic force, conservation of energy states that mechanical energy is a constant.

Mechanical energy is the sum of the kinetic energy and potential energy of a system; that is, $\text{KE} + \text{PE} = \text{constant}$. A loss of PE of a charged particle becomes an increase in its KE. Here PE is the electric potential energy. Conservation of energy is stated in equation form as

Equation:

$$\text{KE} + \text{PE} = \text{constant}$$

or

Equation:

$$\text{KE}_\text{i} + \text{PE}_\text{i} = \text{KE}_\text{f} + \text{PE}_\text{f},$$

where i and f stand for initial and final conditions. As we have found many times before, considering energy can give us insights and facilitate problem solving.

Example:
Electrical Potential Energy Converted to Kinetic Energy
Calculate the final speed of a free electron accelerated from rest through a potential difference of 100 V. (Assume that this numerical value is accurate to three significant figures.)
Strategy

We have a system with only conservative forces. Assuming the electron is accelerated in a vacuum, and neglecting the gravitational force (we will check on this assumption later), all of the electrical potential energy is converted into kinetic energy. We can identify the initial and final forms of energy to be $\mathrm{KE_i} = 0$, $\mathrm{KE_f} = ½\, mv^2$, $\mathrm{PE_i} = qV$, and $\mathrm{PE_f} = 0$.

Solution

Conservation of energy states that

Equation:

$$\mathrm{KE_i} + \mathrm{PE_i} = \mathrm{KE_f} + \mathrm{PE_f}.$$

Entering the forms identified above, we obtain

Equation:

$$qV = \frac{mv^2}{2}.$$

We solve this for v:

Equation:

$$v = \sqrt{\frac{\mathrm{2qV}}{m}}.$$

Entering values for q, V, and m gives

Equation:

$$\begin{aligned} v &= \sqrt{\frac{2\left(-1.60\times10^{-19}\ \mathrm{C}\right)\left(-100\ \mathrm{J/C}\right)}{9.11\times10^{-31}\ \mathrm{kg}}} \\ &= 5.93 \times 10^{6}\ \mathrm{m/s}. \end{aligned}$$

Discussion

Note that both the charge and the initial voltage are negative, as in [link]. We know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because

of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns.

Section Summary

- Electric potential is potential energy per unit charge.
- The potential difference between points A and B, $V_B - V_A$, defined to be the change in potential energy of a charge q moved from A to B, is equal to the change in potential energy divided by the charge, Potential difference is commonly called voltage, represented by the symbol ΔV.
  Equation:

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta \text{PE} = q\Delta V.$$

- An electron volt is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,
  Equation:

$$\begin{aligned} 1\text{ eV} &= \left(1.60 \times 10^{-19}\text{ C}\right)(1\text{ V}) = \left(1.60 \times 10^{-19}\text{ C}\right)(1\text{ J/C}) \\ &= 1.60 \times 10^{-19}\text{ J}. \end{aligned}$$

- Mechanical energy is the sum of the kinetic energy and potential energy of a system, that is, $\text{KE} + \text{PE}$. This sum is a constant.

Conceptual Questions

Exercise:

Problem:

Voltage is the common word for potential difference. Which term is more descriptive, voltage or potential difference?

Exercise:

Problem:

If the voltage between two points is zero, can a test charge be moved between them with zero net work being done? Can this necessarily be done without exerting a force? Explain.

Exercise:

Problem:

What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential energy?

Exercise:

Problem: Voltages are always measured between two points. Why?

Exercise:

Problem:

How are units of volts and electron volts related? How do they differ?

Problems & Exercises

Exercise:

Problem:

Find the ratio of speeds of an electron and a negative hydrogen ion (one having an extra electron) accelerated through the same voltage, assuming non-relativistic final speeds. Take the mass of the hydrogen ion to be 1.67×10^{-27} kg.

Solution:

42.8

Exercise:

Problem:

An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce X-rays. Non-relativistically, what would be the maximum speed of these electrons?

Exercise:

Problem:

A bare helium nucleus has two positive charges and a mass of 6.64×10^{-27} kg. (a) Calculate its kinetic energy in joules at 2.00% of the speed of light. (b) What is this in electron volts? (c) What voltage would be needed to obtain this energy?

Exercise:

Problem: **Integrated Concepts**

Singly charged gas ions are accelerated from rest through a voltage of 13.0 V. At what temperature will the average kinetic energy of gas molecules be the same as that given these ions?

Solution:

1.00×10^{5} K

Exercise:

Problem: **Integrated Concepts**

The temperature near the center of the Sun is thought to be 15 million degrees Celsius $\left(1.5 \times 10^{7}\ {}^{\circ}\text{C}\right)$. Through what voltage must a singly charged ion be accelerated to have the same energy as the average kinetic energy of ions at this temperature?

Exercise:

Problem: **Integrated Concepts**

(a) What is the average power output of a heart defibrillator that dissipates 400 J of energy in 10.0 ms? (b) Considering the high-power output, why doesn't the defibrillator produce serious burns?

Solution:

(a) $4\times10^4\ \text{W}$

(b) A defibrillator does not cause serious burns because the skin conducts electricity well at high voltages, like those used in defibrillators. The gel used aids in the transfer of energy to the body, and the skin doesn't absorb the energy, but rather lets it pass through to the heart.

Exercise:

Problem: **Integrated Concepts**

A lightning bolt strikes a tree, moving 20.0 C of charge through a potential difference of $1.00\times10^2\ \text{MV}$. (a) What energy was dissipated? (b) What mass of water could be raised from 15^{o}C to the boiling point and then boiled by this energy? (c) Discuss the damage that could be caused to the tree by the expansion of the boiling steam.

Exercise:

Problem: **Integrated Concepts**

A 12.0 V battery-operated bottle warmer heats 50.0 g of glass, 2.50×10^2 g of baby formula, and 2.00×10^2 g of aluminum from 20.0^{o}C to 90.0^{o}C. (a) How much charge is moved by the battery? (b) How many electrons per second flow if it takes 5.00 min to warm the formula? (Hint: Assume that the specific heat of baby formula is about the same as the specific heat of water.)

Solution:

(a) $7.40\times10^3\ \text{C}$

(b) 1.54×10^{20} electrons per second

Exercise:

Problem: **Integrated Concepts**

A battery-operated car utilizes a 12.0 V system. Find the charge the batteries must be able to move in order to accelerate the 750 kg car from rest to 25.0 m/s, make it climb a $2.00 \times 10^{2}\ \text{m}$ high hill, and then cause it to travel at a constant 25.0 m/s by exerting a $5.00 \times 10^{2}\ \text{N}$ force for an hour.

Solution:

$3.89\times10^{6}\ \text{C}$

Exercise:

Problem: **Integrated Concepts**

Fusion probability is greatly enhanced when appropriate nuclei are brought close together, but mutual Coulomb repulsion must be overcome. This can be done using the kinetic energy of high-temperature gas ions or by accelerating the nuclei toward one another. (a) Calculate the potential energy of two singly charged nuclei separated by $1.00\times10^{-12}\ \text{m}$ by finding the voltage of one at that distance and multiplying by the charge of the other. (b) At what temperature will atoms of a gas have an average kinetic energy equal to this needed electrical potential energy?

Glossary

electric potential
: potential energy per unit charge

potential difference (or voltage)

change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt

electron volt
: the energy given to a fundamental charge accelerated through a potential difference of one volt

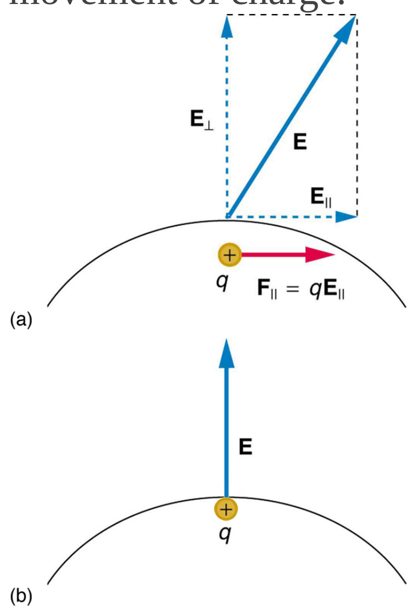
mechanical energy
: sum of the kinetic energy and potential energy of a system; this sum is a constant

Conductors and Applications of Electrostatics

- List the three properties of a conductor in electrostatic equilibrium.
- Explain why the electric field is zero inside a conductor in electrostatic equilibrium.
- Describe how a lightning rod works.
- Name several real-world applications of the study of electrostatics.

Conductors contain **free charges** that move easily. When excess charge is placed on a conductor or the conductor is put into a static electric field, charges in the conductor quickly respond to reach a steady state called **electrostatic equilibrium**.

[link] shows the effect of an electric field on free charges in a conductor. The free charges move until the field is perpendicular to the conductor's surface. There can be no component of the field parallel to the surface in electrostatic equilibrium, since, if there were, it would produce further movement of charge.

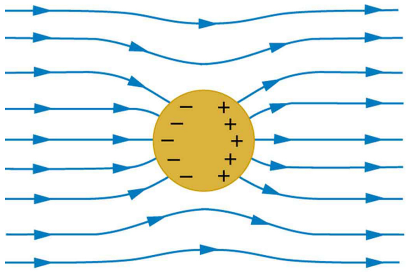


When an electric field $\mathbf{E}$ is applied to a conductor, free charges inside the conductor move

until the field is perpendicular to the surface. (a) The electric field is a vector quantity, with both parallel and perpendicular components. The parallel component ($\mathbf{E}_{\parallel}$) exerts a force ($\mathbf{F}_{\parallel}$) on the free charge q, which moves the charge until $\mathbf{F}_{\parallel} = 0$. (b) The resulting field is perpendicular to the surface. The free charge has been brought to the conductor's surface, leaving electrostatic forces in equilibrium. A positive free charge is shown in the figure for the example, but the result holds similarly for a negative free charge.

A conductor placed in an **electric field** will be **polarized**. [link] shows the result of placing a neutral conductor in an originally uniform electric field.

The field becomes stronger near the conductor but entirely disappears inside it.



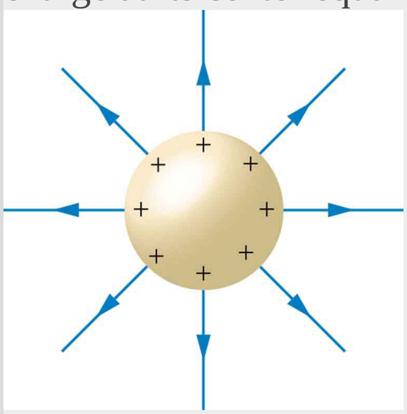
This illustration shows a spherical conductor in static equilibrium with an originally uniform electric field. Free charges move within the conductor, polarizing it, until the electric field lines are perpendicular to the surface. The field lines end on excess negative charge on one section of the surface and begin again on excess positive charge on the opposite side. No electric field exists inside the conductor, since free charges in the conductor would continue moving in response to any

field until it was neutralized.

Note:

Misconception Alert: Electric Field inside a Conductor

Excess charges placed on a spherical conductor repel and move until they are evenly distributed, as shown in [link]. Excess charge is forced to the surface until the field inside the conductor is zero. Outside the conductor, the field is exactly the same as if the conductor were replaced by a point charge at its center equal to the excess charge.



The mutual repulsion of excess positive charges on a spherical conductor distributes them uniformly on its surface. The resulting electric field is perpendicular to the surface and zero inside. Outside the

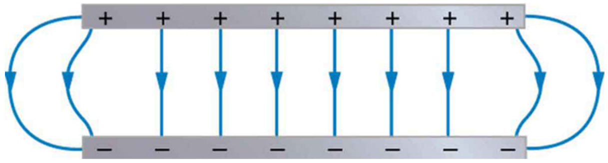
conductor, the field is identical to that of a point charge at the center equal to the excess charge.

Note:
Properties of a Conductor in Electrostatic Equilibrium

1. The electric field is zero inside a conductor.
2. Just outside a conductor, the electric field lines are perpendicular to its surface, ending or beginning on charges on the surface.
3. Any excess charge resides entirely on the surface or surfaces of a conductor.

The properties of a conductor are consistent with the situations already discussed and can be used to analyze any conductor in electrostatic equilibrium. This can lead to some interesting new insights, such as described below.

How can a very uniform electric field be created? Consider a system of two metal plates with opposite charges on them, as shown in [link]. The properties of conductors in electrostatic equilibrium indicate that the electric field between the plates will be uniform in strength and direction. Except near the edges (which can be ignored when plates are close to each other), the excess charges distribute themselves uniformly, producing field lines that are uniformly spaced (hence uniform in strength) and perpendicular to the surfaces (hence uniform in direction, since the plates are flat).



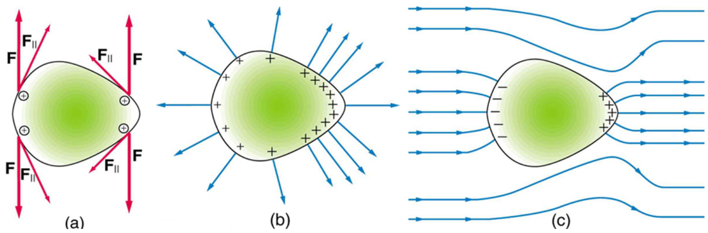
Two metal plates with equal, but opposite, excess charges. The field between them is uniform in strength and direction except near the edges. One use of such a field is to produce uniform acceleration of charges between the plates, such as in the electron gun of a TV tube.

In fact, this is a particular example of electric circuit elements called **capacitors**. This particular example is called a **parallel-plate capacitor** and can be used to analyze properties of capacitors in general, such as how many charges can be stored using two nearby conductors set at a certain electric potential difference (voltage) apart. Capacitors find wide use in electric circuits alongside the resistors and inductors that you will learn about later.

Electric Fields on Uneven Surfaces

So far we have considered excess charges on a smooth, symmetrical conductor surface. What happens if a conductor has sharp corners or is pointed? Excess charges on a nonuniform conductor become concentrated at the sharpest points. Additionally, excess charge may move on or off the conductor at the sharpest points. To see how and why this happens, consider the charged conductor in [link]. The electrostatic repulsion of like charges is most effective in moving them apart on the flattest surface, and so they become least concentrated there. The same effect is produced on a conductor by an externally applied electric field, as seen in [link] (c). Since

the field lines must be perpendicular to the surface, more of them are concentrated on the most curved parts.



Excess charge on a nonuniform conductor becomes most concentrated at the location of greatest curvature. (a) The forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surface are different. It is $\mathbf{F}_{||}$ that moves the charges apart once they have reached the surface. (b) $\mathbf{F}_{||}$ is smallest at the more pointed end, the charges are left closer together, producing the electric field shown. (c) An uncharged conductor in an originally uniform electric field is polarized, with the most concentrated charge at its most pointed end.

Applications of Conductors

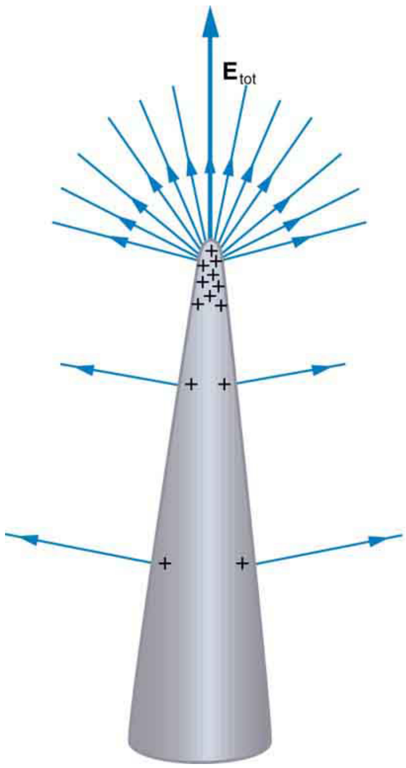
On a very sharply curved surface, such as shown in [link], the charges are so concentrated at the point that the resulting electric field can be great enough to remove them from the surface. This can be useful.

Lightning rods work best when they are most pointed. The large charges created in storm clouds induce an opposite charge on a building that can result in a lightning bolt hitting the building. The induced charge is bled away continually by a lightning rod, preventing the more dramatic lightning strike.

Of course, we sometimes wish to prevent the transfer of charge rather than to facilitate it. In that case, the conductor should be very smooth and have as large a radius of curvature as possible (see [link], which shows a Van de Graaff generator). Smooth surfaces are used on high-voltage transmission lines, for example, to avoid leakage of charge into the air.

Another device that makes use of some of these principles is a **Faraday cage**. This is a metal shield that encloses a volume. All electrical charges will reside on the outside surface of this shield, and there will be no electrical field inside. A Faraday cage is used to prohibit stray electrical fields in the environment from interfering with sensitive measurements, such as the electrical signals inside a nerve cell.

During electrical storms if you are driving a car, it is best to stay inside the car as its metal body acts as a Faraday cage with zero electrical field inside. If in the vicinity of a lightning strike, its effect is felt on the outside of the car and the inside is unaffected, provided you remain totally inside. This is also true if an active (“hot”) electrical wire was broken (in a storm or an accident) and fell on your car.



A very pointed conductor has a large charge concentration at the point. The electric field is very strong at the point and can exert a force large enough to transfer charge on or off the conductor. Lightning rods are used to prevent the buildup of large excess charges on structures and, thus, are pointed.

(a) (b)

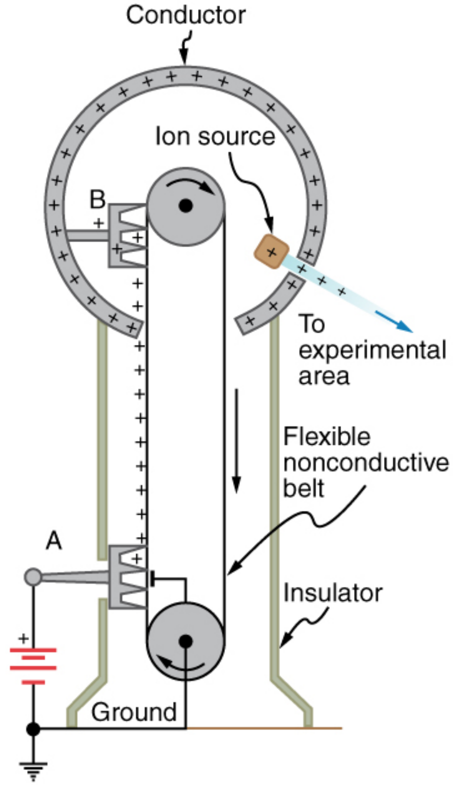
(a) A lightning rod is pointed to facilitate the transfer of charge. (credit: Romaine, Wikimedia Commons) (b) This Van de Graaff generator has a smooth surface with a large radius of curvature to prevent the transfer of charge and allow a large voltage to be generated. The mutual repulsion of like charges is evident in the person’s hair while touching the metal sphere. (credit: Jon ‘ShakataGaNai’ Davis/Wikimedia Commons).

Below are additional applications of **electrostatics**, the study of electricity in electrostatic equilibrium, many of them using conductors for their properties just discussed above.

The Van de Graaff Generator

Van de Graaff generators (or Van de Graaffs) are not only spectacular devices used to demonstrate high voltage due to static electricity—they are also used for serious research. The first was built by Robert Van de Graaff in 1931 (based on original suggestions by Lord Kelvin) for use in nuclear physics research. [link] shows a schematic of a large research version. Van de Graaffs utilize both smooth and pointed surfaces, and conductors and insulators to generate large static charges and, hence, large voltages.

A very large excess charge can be deposited on the sphere, because it moves quickly to the outer surface. Practical limits arise because the large electric fields polarize and eventually ionize surrounding materials, creating free charges that neutralize excess charge or allow it to escape. Nevertheless, voltages of 15 million volts are well within practical limits.



Schematic of Van de Graaff generator. A battery (A) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (B) on top in the large

sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not remain inside the conducting sphere but moves to its outside surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities.

Note:
Take-Home Experiment: Electrostatics and Humidity
Rub a comb through your hair and use it to lift pieces of paper. It may help to tear the pieces of paper rather than cut them neatly. Repeat the exercise in your bathroom after you have had a long shower and the air in the bathroom is moist. Is it easier to get electrostatic effects in dry or moist air? Why would torn paper be more attractive to the comb than cut paper? Explain your observations.

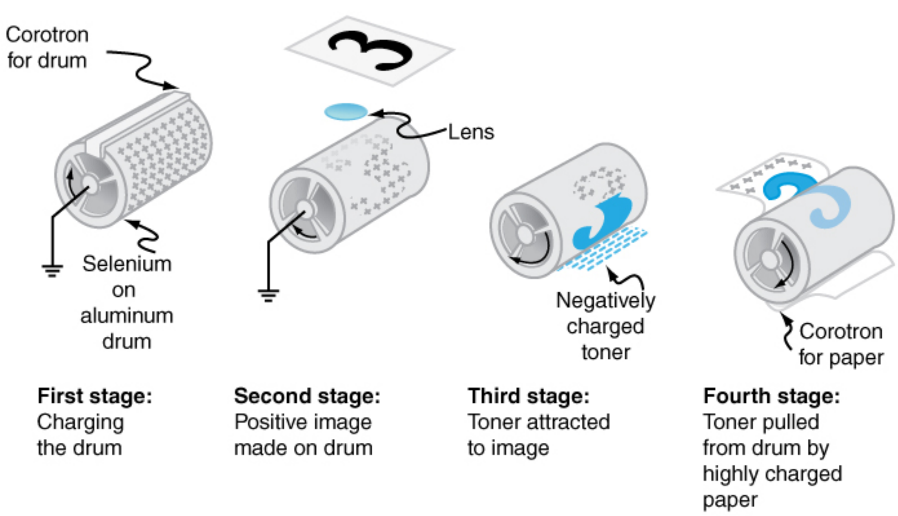
Xerography

Most copy machines use an electrostatic process called **xerography**—a word coined from the Greek words *xeros* for dry and *graphos* for writing. The heart of the process is shown in simplified form in [link].

A selenium-coated aluminum drum is sprayed with positive charge from points on a device called a corotron. Selenium is a substance with an interesting property—it is a **photoconductor**. That is, selenium is an insulator when in the dark and a conductor when exposed to light.

In the first stage of the xerography process, the conducting aluminum drum is **grounded** so that a negative charge is induced under the thin layer of uniformly positively charged selenium. In the second stage, the surface of the drum is exposed to the image of whatever is to be copied. Where the image is light, the selenium becomes conducting, and the positive charge is neutralized. In dark areas, the positive charge remains, and so the image has been transferred to the drum.

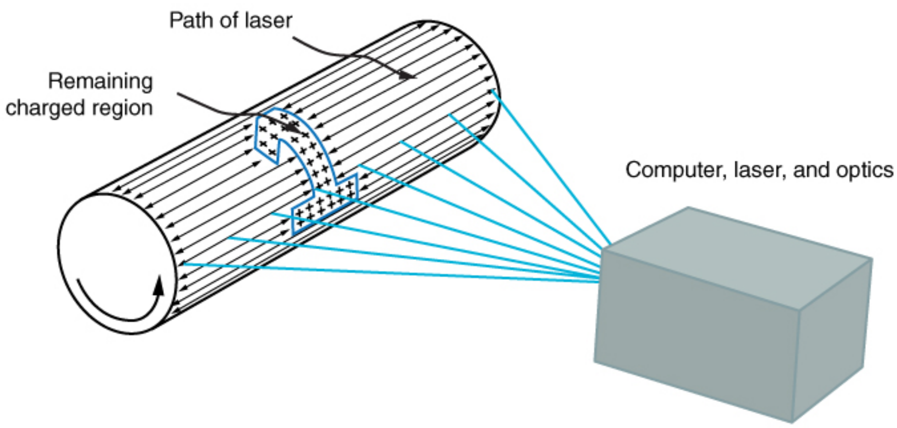
The third stage takes a dry black powder, called toner, and sprays it with a negative charge so that it will be attracted to the positive regions of the drum. Next, a blank piece of paper is given a greater positive charge than on the drum so that it will pull the toner from the drum. Finally, the paper and electrostatically held toner are passed through heated pressure rollers, which melt and permanently adhere the toner within the fibers of the paper.



Xerography is a dry copying process based on electrostatics. The major steps in the process are the charging of the photoconducting drum, transfer of an image creating a positive charge duplicate, attraction of toner to the charged parts of the drum, and transfer of toner to the paper. Not shown are heat treatment of the paper and cleansing of the drum for the next copy.

Laser Printers

Laser printers use the xerographic process to make high-quality images on paper, employing a laser to produce an image on the photoconducting drum as shown in [link]. In its most common application, the laser printer receives output from a computer, and it can achieve high-quality output because of the precision with which laser light can be controlled. Many laser printers do significant information processing, such as making sophisticated letters or fonts, and may contain a computer more powerful than the one giving them the raw data to be printed.



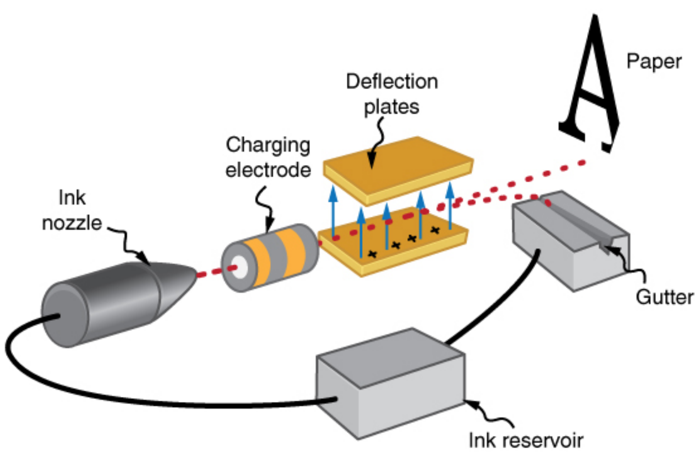
In a laser printer, a laser beam is scanned across a photoconducting drum, leaving a positive charge image. The other steps for charging the drum and

transferring the image to paper are the same as in xerography. Laser light can be very precisely controlled, enabling laser printers to produce high-quality images.

Ink Jet Printers and Electrostatic Painting

The **ink jet printer**, commonly used to print computer-generated text and graphics, also employs electrostatics. A nozzle makes a fine spray of tiny ink droplets, which are then given an electrostatic charge. (See [link].)

Once charged, the droplets can be directed, using pairs of charged plates, with great precision to form letters and images on paper. Ink jet printers can produce color images by using a black jet and three other jets with primary colors, usually cyan, magenta, and yellow, much as a color television produces color. (This is more difficult with xerography, requiring multiple drums and toners.)



The nozzle of an ink-jet printer produces small ink droplets, which are sprayed with electrostatic charge. Various computer-driven devices are

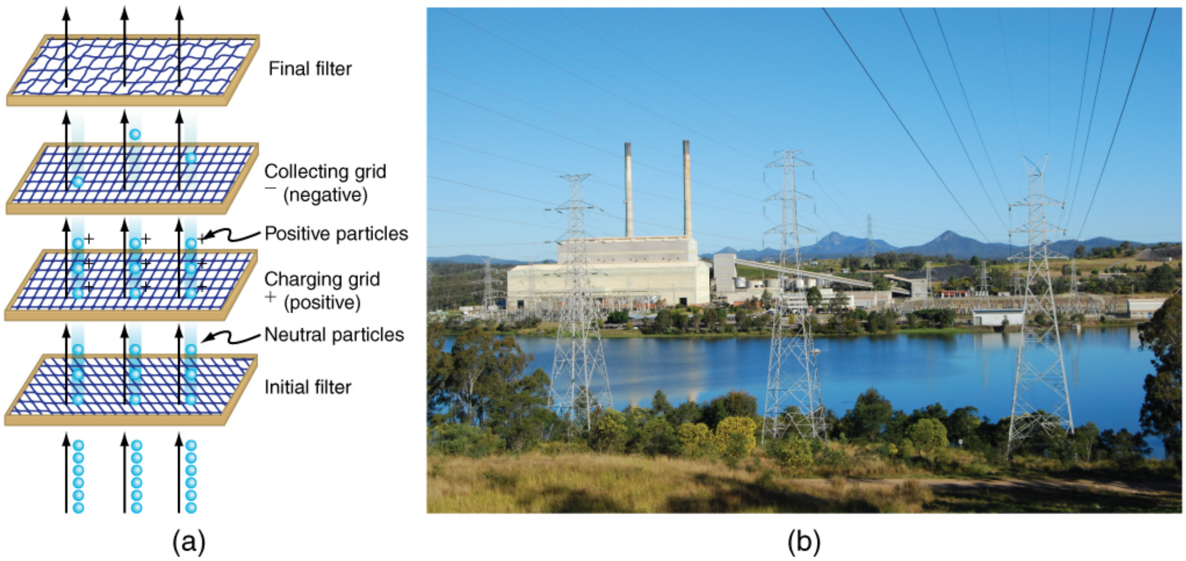
then used to direct the droplets to the correct positions on a page.

Electrostatic painting employs electrostatic charge to spray paint onto odd-shaped surfaces. Mutual repulsion of like charges causes the paint to fly away from its source. Surface tension forms drops, which are then attracted by unlike charges to the surface to be painted. Electrostatic painting can reach those hard-to-get at places, applying an even coat in a controlled manner. If the object is a conductor, the electric field is perpendicular to the surface, tending to bring the drops in perpendicularly. Corners and points on conductors will receive extra paint. Felt can similarly be applied.

Smoke Precipitators and Electrostatic Air Cleaning

Another important application of electrostatics is found in air cleaners, both large and small. The electrostatic part of the process places excess (usually positive) charge on smoke, dust, pollen, and other particles in the air and then passes the air through an oppositely charged grid that attracts and retains the charged particles. (See [link].)

Large **electrostatic precipitators** are used industrially to remove over 99% of the particles from stack gas emissions associated with the burning of coal and oil. Home precipitators, often in conjunction with the home heating and air conditioning system, are very effective in removing polluting particles, irritants, and allergens.



(a) Schematic of an electrostatic precipitator. Air is passed through grids of opposite charge. The first grid charges airborne particles, while the second attracts and collects them. (b) The dramatic effect of electrostatic precipitators is seen by the absence of smoke from this power plant. (credit: Cmdalgleish, Wikimedia Commons)

Section Summary

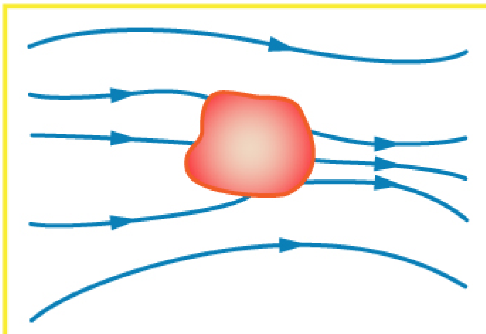
- The electrical forces around a conductor will cause free charges to move around inside the conductor until static equilibrium is reached.
- Any excess charge will collect along the surface of a conductor.
- Conductors with sharp corners or points will collect more charge at those points.
- A lightning rod is a conductor with sharply pointed ends that collect excess charge on the building caused by an electrical storm and allow it to dissipate back into the air.
- A Faraday cage acts like a shield around an object, preventing electric charge from penetrating inside.
- In addition to research using equipment such as a Van de Graaff generator, many practical applications of electrostatics exist, including photocopiers, laser printers, ink-jet printers and electrostatic air filters.

Conceptual Questions

Exercise:

Problem:

Is the object in [link] a conductor or an insulator? Justify your answer.



Exercise:

Problem:

If the electric field lines in the figure above were perpendicular to the object, would it necessarily be a conductor? Explain.

Exercise:

Problem:

The discussion of the electric field between two parallel conducting plates, in this module states that edge effects are less important if the plates are close together. What does close mean? That is, is the actual plate separation crucial, or is the ratio of plate separation to plate area crucial?

Exercise:

Problem:

Would the self-created electric field at the end of a pointed conductor, such as a lightning rod, remove positive or negative charge from the conductor? Would the same sign charge be removed from a neutral pointed conductor by the application of a similar externally created electric field? (The answers to both questions have implications for charge transfer utilizing points.)

Exercise:

Problem:

Why is a golfer with a metal club over her shoulder vulnerable to lightning in an open fairway? Would she be any safer under a tree?

Exercise:

Problem:

Can the belt of a Van de Graaff accelerator be a conductor? Explain.

Exercise:

Problem:

Are you relatively safe from lightning inside an automobile? Give two reasons.

Exercise:

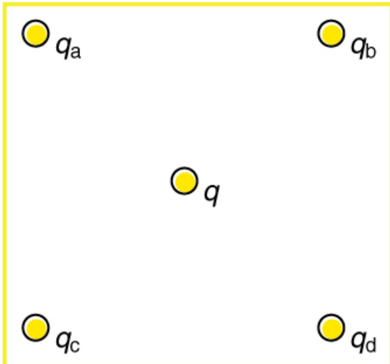
Problem:

Discuss pros and cons of a lightning rod being grounded versus simply being attached to a building.

Exercise:

Problem:

Using the symmetry of the arrangement, show that the net Coulomb force on the charge q at the center of the square below ([link]) is zero if the charges on the four corners are exactly equal.



Four point charges q_a, q_b, q_c, and q_d lie on the corners of a square and q is located at its center.

Exercise:

Problem:

(a) Using the symmetry of the arrangement, show that the electric field at the center of the square in [link] is zero if the charges on the four corners are exactly equal. (b) Show that this is also true for any combination of charges in which $q_a = q_d$ and $q_b = q_c$

Exercise:

Problem:

(a) What is the direction of the total Coulomb force on q in [link] if q is negative, $q_a = q_c$ and both are negative, and $q_b = q_c$ and both are positive? (b) What is the direction of the electric field at the center of the square in this situation?

Exercise:

Problem:

Considering [link], suppose that $q_a = q_d$ and $q_b = q_c$. First show that q is in static equilibrium. (You may neglect the gravitational force.) Then discuss whether the equilibrium is stable or unstable, noting that this may depend on the signs of the charges and the direction of displacement of q from the center of the square.

Exercise:

Problem:

If $q_a = 0$ in [link], under what conditions will there be no net Coulomb force on q?

Exercise:

Problem:

In regions of low humidity, one develops a special “grip” when opening car doors, or touching metal door knobs. This involves placing as much of the hand on the device as possible, not just the ends of one’s fingers. Discuss the induced charge and explain why this is done.

Exercise:

Problem:

Tollbooth stations on roadways and bridges usually have a piece of wire stuck in the pavement before them that will touch a car as it approaches. Why is this done?

Exercise:

Problem:

Suppose a woman carries an excess charge. To maintain her charged status can she be standing on ground wearing just any pair of shoes? How would you discharge her? What are the consequences if she simply walks away?

Glossary

conductor
: an object with properties that allow charges to move about freely within it

free charge
: an electrical charge (either positive or negative) which can move about separately from its base molecule

electrostatic equilibrium
: an electrostatically balanced state in which all free electrical charges have stopped moving about

polarized
: a state in which the positive and negative charges within an object have collected in separate locations

Faraday cage
: a metal shield which prevents electric charge from penetrating its surface

capacitor
: an arrangement of conductors designed to store charge using voltage difference

parallel-plate capacitor
: an example of capacitor using arrangement of two parallel conducting plates placed near each other

Van de Graaff generator
: a machine that produces a large amount of excess charge, used for experiments with high voltage

electrostatics
: the study of electricity in electrostatic equilibrium

photoconductor

a substance that is an insulator until it is exposed to light, when it becomes a conductor

xerography
: a dry copying process based on electrostatics

grounded
: connected to the ground with a conductor, so that charge flows freely to and from the Earth to the grounded object

laser printer
: uses a laser to create a photoconductive image on a drum, which attracts dry ink particles that are then rolled onto a sheet of paper to print a high-quality copy of the image

ink-jet printer
: small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper

electrostatic precipitators
: filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream

Current

- Define electric current, ampere, and drift velocity
- Describe the direction of charge flow in conventional current.

Electric Current

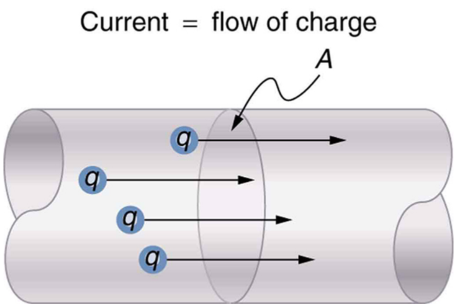
Electric current is defined to be the rate at which charge flows. A large current, such as that used to start a truck engine, moves a large amount of charge in a small time, whereas a small current, such as that used to operate a hand-held calculator, moves a small amount of charge over a long period of time. In equation form, **electric current** I is defined to be
Equation:

$$I=\frac{\Delta Q}{\Delta t},$$

where ΔQ is the amount of charge passing through a given area in time Δt. (As in previous chapters, initial time is often taken to be zero, in which case $\Delta t=t$.) (See [link].) The SI unit for current is the **ampere** (A), named for the French physicist André-Marie Ampère (1775–1836). Since $I=\Delta Q/\Delta t$, we see that an ampere is one coulomb per second:
Equation:

$$1\text{ A}=1\text{ C/s}$$

Not only are fuses and circuit breakers rated in amperes (or amps), so are many electrical appliances.



The rate of flow of charge is current. An ampere is the flow of one coulomb through an area in one second.

Example:

Calculating Currents: Current in a Truck Battery and a Handheld Calculator

(a) What is the current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine? (b) How long does it take 1.00 C of charge to flow through a handheld calculator if a 0.300-mA current is flowing?

Strategy

We can use the definition of current in the equation $I = \Delta Q / \Delta t$ to find the current in part (a), since charge and time are given. In part (b), we rearrange the definition of current and use the given values of charge and current to find the time required.

Solution for (a)

Entering the given values for charge and time into the definition of current gives

Equation:

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} = \frac{720\text{ C}}{4.00\text{ s}} = 180\text{ C/s} \\ &= 180\text{ A}. \end{aligned}$$

Discussion for (a)

This large value for current illustrates the fact that a large charge is moved in a small amount of time. The currents in these "starter motors" are fairly large because large frictional forces need to be overcome when setting something in motion.

Solution for (b)

Solving the relationship $I = \Delta Q/\Delta t$ for time Δt, and entering the known values for charge and current gives

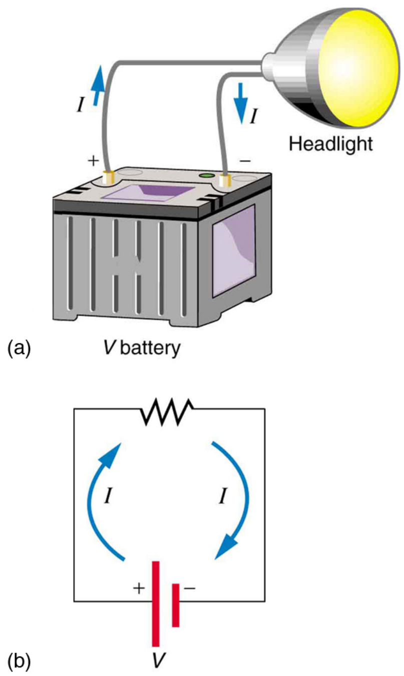
Equation:

$$\begin{array}{lll}\Delta t & = & \frac{\Delta Q}{I} = \frac{1.00\text{ C}}{0.300\times 10^{-3}\text{ C/s}}\\ & = & 3.33\times 10^{3}\text{ s.}\end{array}$$

Discussion for (b)

This time is slightly less than an hour. The small current used by the hand-held calculator takes a much longer time to move a smaller charge than the large current of the truck starter. So why can we operate our calculators only seconds after turning them on? It's because calculators require very little energy. Such small current and energy demands allow handheld calculators to operate from solar cells or to get many hours of use out of small batteries. Remember, calculators do not have moving parts in the same way that a truck engine has with cylinders and pistons, so the technology requires smaller currents.

[link] shows a simple circuit and the standard schematic representation of a battery, conducting path, and load (a resistor). Schematics are very useful in visualizing the main features of a circuit. A single schematic can represent a wide variety of situations. The schematic in [link] (b), for example, can represent anything from a truck battery connected to a headlight lighting the street in front of the truck to a small battery connected to a penlight lighting a keyhole in a door. Such schematics are useful because the analysis is the same for a wide variety of situations. We need to understand a few schematics to apply the concepts and analysis to many more situations.



(a) A simple electric circuit. A closed path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by the two parallel red lines, conducting wires are shown as straight lines, and the zigzag represents the load. The schematic represents a wide

variety of similar circuits.

Note that the direction of current flow in [link] is from positive to negative. *The direction of conventional current is the direction that positive charge would flow*. Depending on the situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This is also true in nerve cells. A Van de Graaff generator used for nuclear research can produce a current of pure positive charges, such as protons. [link] illustrates the movement of charged particles that compose a current. The fact that conventional current is taken to be in the direction that positive charge would flow can be traced back to American politician and scientist Benjamin Franklin in the 1700s. He named the type of charge associated with electrons negative, long before they were known to carry current in so many situations. Franklin, in fact, was totally unaware of the small-scale structure of electricity.

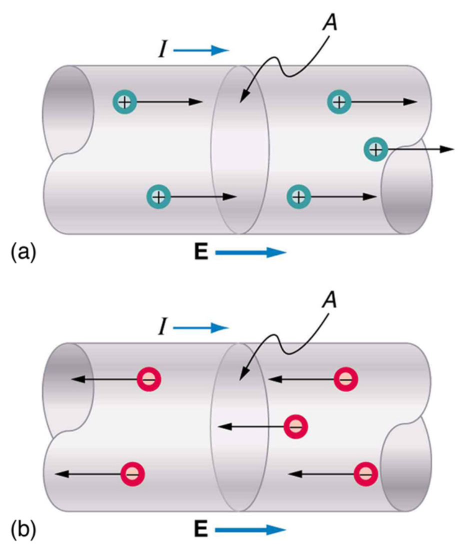
It is important to realize that there is an electric field in conductors responsible for producing the current, as illustrated in [link]. Unlike static electricity, where a conductor in equilibrium cannot have an electric field in it, conductors carrying a current have an electric field and are not in static equilibrium. An electric field is needed to supply energy to move the charges.

Note:

Making Connections: Take-Home Investigation—Electric Current Illustration

Find a straw and little peas that can move freely in the straw. Place the straw flat on a table and fill the straw with peas. When you pop one pea in at one end, a different pea should pop out the other end. This demonstration is an analogy for an electric current. Identify what compares

to the electrons and what compares to the supply of energy. What other analogies can you find for an electric current?
Note that the flow of peas is based on the peas physically bumping into each other; electrons flow due to mutually repulsive electrostatic forces.



Current I is the rate at which charge moves through an area A, such as the cross-section of a wire. Conventional current is defined to move in the direction of the electric field. (a) Positive charges move in the direction of the electric field and the same direction as conventional current. (b) Negative charges move in the direction opposite to the electric field. Conventional

current is in the direction opposite to the movement of negative charge. The flow of electrons is sometimes referred to as electronic flow.

Example:

Calculating the Number of Electrons that Move through a Calculator

If the 0.300-mA current through the calculator mentioned in the [link] example is carried by electrons, how many electrons per second pass through it?

Strategy

The current calculated in the previous example was defined for the flow of positive charge. For electrons, the magnitude is the same, but the sign is opposite, $I_{\text{electrons}} = -0.300 \times 10^{-3}\,\text{C/s}$ .Since each electron (e^{-}) has a charge of -1.60×10^{-19} C, we can convert the current in coulombs per second to electrons per second.

Solution

Starting with the definition of current, we have

Equation:

$$I_{\text{electrons}} = \frac{\Delta Q_{\text{electrons}}}{\Delta t} = \frac{-0.300 \times 10^{-3}\ \text{C}}{\text{s}}.$$

We divide this by the charge per electron, so that

Equation:

$$\begin{aligned}\frac{e^{-}}{\text{s}} &= \frac{-0.300\times10^{-3}\ \text{C}}{\text{s}} \times \frac{1\ e^{-}}{-1.60\times10^{-19}\ \text{C}}\\ &= 1.88\times10^{15}\ \frac{e^{-}}{\text{s}}.\end{aligned}$$

Discussion

There are so many charged particles moving, even in small currents, that individual charges are not noticed, just as individual water molecules are not noticed in water flow. Even more amazing is that they do not always keep moving forward like soldiers in a parade. Rather they are like a crowd of people with movement in different directions but a general trend to move forward. There are lots of collisions with atoms in the metal wire and, of course, with other electrons.

Section Summary

- Electric current I is the rate at which charge flows, given by
  Equation:
  $$I = \frac{\Delta Q}{\Delta t},$$
  where ΔQ is the amount of charge passing through an area in time Δt.
- The direction of conventional current is taken as the direction in which positive charge moves.
- The SI unit for current is the ampere (A), where $1\ \text{A} = 1\ \text{C/s}$.
- Current is the flow of free charges, such as electrons and ions.

Conceptual Questions

Exercise:

Problem:

Can a wire carry a current and still be neutral—that is, have a total charge of zero? Explain.

Exercise:

Problem:

Car batteries are rated in ampere-hours ($A \cdot h$). To what physical quantity do ampere-hours correspond (voltage, charge, . . .), and what relationship do ampere-hours have to energy content?

Exercise:

Problem:

Why are two conducting paths from a voltage source to an electrical device needed to operate the device?

Exercise:

Problem:

In cars, one battery terminal is connected to the metal body. How does this allow a single wire to supply current to electrical devices rather than two wires?

Exercise:

Problem:

Why isn't a bird sitting on a high-voltage power line electrocuted? Contrast this with the situation in which a large bird hits two wires simultaneously with its wings.

Problems & Exercises

Exercise:

Problem:

What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?

Solution:

0.278 mA

Exercise:

Problem:

A total of 600 C of charge passes through a flashlight in 0.500 h. What is the average current?

Exercise:

Problem:

What is the current when a typical static charge of $0.250\ \mu\text{C}$ moves from your finger to a metal doorknob in $1.00\ \mu\text{s}$?

Solution:

0.250 A

Exercise:

Problem:

Find the current when 2.00 nC jumps between your comb and hair over a $0.500\text{ - }\mu\text{s}$ time interval.

Exercise:

Problem:

A large lightning bolt had a 20,000-A current and moved 30.0 C of charge. What was its duration?

Solution:

1.50ms

Exercise:

Problem:

The 200-A current through a spark plug moves 0.300 mC of charge. How long does the spark last?

Exercise:

Problem:

(a) A defibrillator passes 12.0 A of current through the torso of a person for 0.0100 s. How much charge moves? (b) How many electrons pass through the wires connected to the patient? (See figure two problems earlier.)

Solution:

(a) 0.120 C

(b) $7.50 \times 10^{17} \text{electrons}$

Exercise:

Problem:

A clock battery wears out after moving 10,000 C of charge through the clock at a rate of 0.500 mA. (a) How long did the clock run? (b) How many electrons per second flowed?

Exercise:

Problem:

The batteries of a submerged non-nuclear submarine supply 1000 A at full speed ahead. How long does it take to move Avogadro’s number (6.02×10^{23}) of electrons at this rate?

Solution:

96.3 s

Exercise:

Problem:

Electron guns are used in X-ray tubes. The electrons are accelerated through a relatively large voltage and directed onto a metal target, producing X-rays. (a) How many electrons per second strike the target if the current is 0.500 mA? (b) What charge strikes the target in 0.750 s?

Exercise:

Problem:

A large cyclotron directs a beam of He^{++} nuclei onto a target with a beam current of 0.250 mA. (a) How many He^{++} nuclei per second is this? (b) How long does it take for 1.00 C to strike the target? (c) How long before 1.00 mol of He^{++} nuclei strike the target?

Solution:

(a) $7.81 \times 10^{14}\ \text{He}^{++}\ \text{nuclei/s}$

(b) $4.00 \times 10^{3}\ \text{s}$

(c) $7.71 \times 10^{8}\ \text{s}$

Glossary

electric current
: the rate at which charge flows, $I = \Delta Q/\Delta t$

ampere
: (amp) the SI unit for current; 1 A = 1 C/s

Ohm’s Law: Resistance and Simple Circuits

- Explain the origin of Ohm’s law.
- Calculate voltages, currents, or resistances with Ohm’s law.
- Explain what an ohmic material is.
- Describe a simple circuit.

What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—which are necessary to maintain a current. All such devices create a potential difference and are loosely referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electric field. The electric field in turn exerts force on charges, causing current.

Ohm’s Law

The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is *directly proportional to the voltage applied*:
Equation:

$$I \propto V.$$

This important relationship is known as **Ohm’s law**. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn’t always occur.

Resistance and Simple Circuits

If voltage drives current, what impedes it? The electric property that impedes current (crudely similar to friction and air resistance) is called **resistance** R. Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. Resistance is defined as inversely proportional to current, or

Equation:

$$I \propto \frac{1}{R}.$$

Thus, for example, current is cut in half if resistance doubles. Combining the relationships of current to voltage and current to resistance gives

Equation:

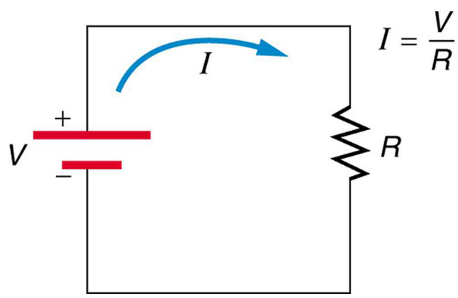
$$I = \frac{V}{R}.$$

This relationship is also called Ohm's law. Ohm's law in this form really defines resistance for certain materials. Ohm's law (like Hooke's law) is not universally valid. The many substances for which Ohm's law holds are called **ohmic**. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances. Ohmic materials have a resistance R that is independent of voltage V and current I. An object that has simple resistance is called a *resistor*, even if its resistance is small. The unit for resistance is an **ohm** and is given the symbol Ω (upper case Greek omega). Rearranging $I = \mathrm{V/R}$ gives $R = \mathrm{V/I}$, and so the units of resistance are 1 ohm = 1 volt per ampere:

Equation:

$$1\ \Omega = 1\frac{V}{A}.$$

[link] shows the schematic for a simple circuit. A **simple circuit** has a single voltage source and a single resistor. The wires connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in R.



A simple electric circuit in which a closed path for current to flow is supplied by conductors (usually metal wires) connecting a load to the terminals of a battery, represented by the red parallel lines. The zigzag symbol represents the single resistor and includes any resistance in the connections to the voltage source.

Example:
Calculating Resistance: An Automobile Headlight
What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?
Strategy
We can rearrange Ohm's law as stated by $I = \mathrm{V/R}$ and use it to find the resistance.
Solution
Rearranging $I = \mathrm{V/R}$ and substituting known values gives

Equation:

$$R = \frac{V}{I} = \frac{12.0 \text{ V}}{2.50 \text{ A}} = 4.80 \ \Omega.$$

Discussion

This is a relatively small resistance, but it is larger than the cold resistance of the headlight. Resistance usually increases with temperature, and so the bulb has a lower resistance when it is first switched on and will draw considerably more current during its brief warm-up period.

Resistances range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of $10^{12}\ \Omega$ or more. A dry person may have a hand-to-foot resistance of $10^{5}\ \Omega$, whereas the resistance of the human heart is about $10^{3}\ \Omega$. A meter-long piece of large-diameter copper wire may have a resistance of $10^{-5}\ \Omega$, and superconductors have no resistance at all (they are non-ohmic). Resistance is related to the shape of an object and the material of which it is composed.

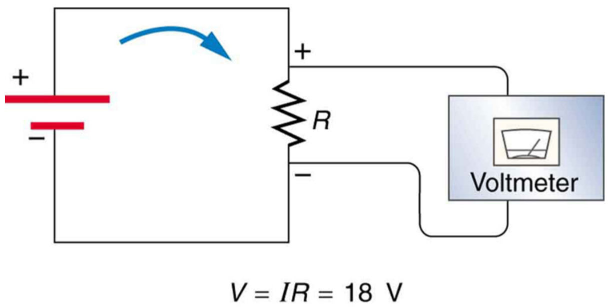
Additional insight is gained by solving $I = \text{V}/\text{R}$ for V, yielding

Equation:

$$V = \text{IR}.$$

This expression for V can be interpreted as the *voltage drop across a resistor produced by the flow of current* I. The phrase IR *drop* is often used for this voltage. For instance, the headlight in [link] has an IR drop of 12.0 V. If voltage is measured at various points in a circuit, it will be seen to increase at the voltage source and decrease at the resistor. Voltage is similar to fluid pressure. The voltage source is like a pump, creating a pressure difference, causing current—the flow of charge. The resistor is like a pipe that reduces pressure and limits flow because of its resistance. Conservation of energy has important consequences here. The voltage source supplies energy (causing an electric field and a current), and the resistor converts it to another form (such as thermal energy). In a simple circuit (one with a

single simple resistor), the voltage supplied by the source equals the voltage drop across the resistor, since $\mathrm{PE} = q\Delta V$, and the same q flows through each. Thus the energy supplied by the voltage source and the energy converted by the resistor are equal. (See [link].)



The voltage drop across a resistor in a simple circuit equals the voltage output of the battery.

Note:
Making Connections: Conservation of Energy
In a simple electrical circuit, the sole resistor converts energy supplied by the source into another form. Conservation of energy is evidenced here by the fact that all of the energy supplied by the source is converted to another form by the resistor alone. We will find that conservation of energy has other important applications in circuits and is a powerful tool in circuit analysis.

Section Summary

- A simple circuit *is* one in which there is a single voltage source and a single resistance.
- One statement of Ohm's law gives the relationship between current I, voltage V, and resistance R in a simple circuit to be $I = \frac{V}{R}$.
- Resistance has units of ohms (Ω), related to volts and amperes by $1\ \Omega = 1\ \text{V}/\text{A}$.
- There is a voltage or IR drop across a resistor, caused by the current flowing through it, given by $V = \text{IR}$.

Conceptual Questions

Exercise:

Problem:

The IR drop across a resistor means that there is a change in potential or voltage across the resistor. Is there any change in current as it passes through a resistor? Explain.

Exercise:

Problem:

How is the IR drop in a resistor similar to the pressure drop in a fluid flowing through a pipe?

Problems & Exercises

Exercise:

Problem:

What current flows through the bulb of a 3.00-V flashlight when its hot resistance is $3.60\ \Omega$?

Solution:

0.833 A

Exercise:

Problem:

Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.

Exercise:

Problem:

What is the effective resistance of a car's starter motor when 150 A flows through it as the car battery applies 11.0 V to the motor?

Solution:

$7.33 \times 10^{-2}\ \Omega$

Exercise:

Problem:

How many volts are supplied to operate an indicator light on a DVD player that has a resistance of $140\ \Omega$, given that 25.0 mA passes through it?

Exercise:

Problem:

(a) Find the voltage drop in an extension cord having a $0.0600\text{-}\Omega$ resistance and through which 5.00 A is flowing. (b) A cheaper cord utilizes thinner wire and has a resistance of $0.300\ \Omega$. What is the voltage drop in it when 5.00 A flows? (c) Why is the voltage to whatever appliance is being used reduced by this amount? What is the effect on the appliance?

Solution:

(a) 0.300 V

(b) 1.50 V

(c) The voltage supplied to whatever appliance is being used is reduced because the total voltage drop from the wall to the final output of the appliance is fixed. Thus, if the voltage drop across the extension cord is large, the voltage drop across the appliance is significantly decreased, so the power output by the appliance can be significantly decreased, reducing the ability of the appliance to work properly.

Exercise:

Problem:

A power transmission line is hung from metal towers with glass insulators having a resistance of $1.00\times10^{9}\ \Omega$. What current flows through the insulator if the voltage is 200 kV? (Some high-voltage lines are DC.)

Glossary

Ohm’s law
: an empirical relation stating that the current *I* is proportional to the potential difference *V*. It is often written as *I* = *V/R*, where *R* is the resistance

resistance
: the electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, *R* = *V/I*

ohm
: the unit of resistance, given by 1Ω = 1 V/A

ohmic
: a type of a material for which Ohm's law is valid

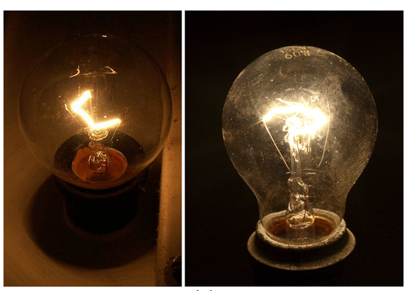
simple circuit
: a circuit with a single voltage source and a single resistor

Electric Power and Energy

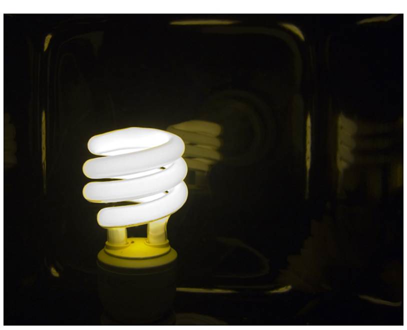
- Calculate the power dissipated by a resistor and power supplied by a power supply.
- Calculate the cost of electricity under various circumstances.

Power in Electric Circuits

Power is associated by many people with electricity. Knowing that power is the rate of energy use or energy conversion, what is the expression for **electric power**? Power transmission lines might come to mind. We also think of lightbulbs in terms of their power ratings in watts. Let us compare a 25-W bulb with a 60-W bulb. (See [link](a).) Since both operate on the same voltage, the 60-W bulb must draw more current to have a greater power rating. Thus the 60-W bulb's resistance must be lower than that of a 25-W bulb. If we increase voltage, we also increase power. For example, when a 25-W bulb that is designed to operate on 120 V is connected to 240 V, it briefly glows very brightly and then burns out. Precisely how are voltage, current, and resistance related to electric power?



(a)



(b)

(a) Which of these lightbulbs, the 25-W bulb (upper left) or the 60-W bulb (upper right), has the higher resistance? Which draws more current? Which uses the most energy? Can you tell from the color that the 25-W filament is cooler? Is the brighter bulb a different color and if so why? (credits: Dickbauch, Wikimedia Commons; Greg Westfall, Flickr) (b) This compact fluorescent light (CFL) puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. (credit: dbgg1979, Flickr)

Electric energy depends on both the voltage involved and the charge moved. This is expressed most simply as $\mathrm{PE} = \mathrm{qV}$, where q is the charge moved and V is the voltage (or more precisely, the potential difference the

charge moves through). Power is the rate at which energy is moved, and so electric power is

Equation:

$$P = \frac{\text{PE}}{t} = \frac{\text{qV}}{t}.$$

Recognizing that current is $I = q/t$ (note that $\Delta t = t$ here), the expression for power becomes

Equation:

$$P = \text{IV}.$$

Electric power (P) is simply the product of current times voltage. Power has familiar units of watts. Since the SI unit for potential energy (PE) is the joule, power has units of joules per second, or watts. Thus, $1\ \text{A} \cdot \text{V} = 1\ \text{W}$. For example, cars often have one or more auxiliary power outlets with which you can charge a cell phone or other electronic devices. These outlets may be rated at 20 A, so that the circuit can deliver a maximum power $P = \text{IV} = (20\ \text{A})(12\ \text{V}) = 240\ \text{W}$. In some applications, electric power may be expressed as volt-amperes or even kilovolt-amperes ($1\ \text{kA} \cdot \text{V} = 1\ \text{kW}$).

To see the relationship of power to resistance, we combine Ohm's law with $P = \text{IV}$. Substituting $I = \text{V}/\text{R}$ gives $P = (V/R)V = V^2/R$. Similarly, substituting $V = \text{IR}$ gives $P = I(\text{IR}) = I^2R$. Three expressions for electric power are listed together here for convenience:

Equation:

$$P = \text{IV}$$

Equation:

$$P = \frac{V^2}{R}$$

Equation:

$$P = I^2 R.$$

Note that the first equation is always valid, whereas the other two can be used only for resistors. In a simple circuit, with one voltage source and a single resistor, the power supplied by the voltage source and that dissipated by the resistor are identical. (In more complicated circuits, P can be the power dissipated by a single device and not the total power in the circuit.)

Different insights can be gained from the three different expressions for electric power. For example, $P = V^2/R$ implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in $P = V^2/R$, the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature its resistance is higher, too.

Example:
Calculating Power Dissipation and Current
Consider the example given in "Ohm's Law: Resistance and Simple Circuits." Then find the power dissipated by the car headlight.
Strategy
For the headlight, we know voltage and current, so we can use $P = \text{IV}$ to find the power.
Solution
Entering the known values of current and voltage for the hot headlight, we obtain
Equation:

$$P = \text{IV} = (2.50\text{ A})(12.0\text{ V}) = 30.0\text{ W}.$$

Discussion
The 30 W dissipated by the hot headlight is typical.

The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since $P = E/t$, we see that

Equation:

$$E = \text{Pt}$$

is the energy used by a device using power P for a time interval t. For example, the more lightbulbs burning, the greater P used; the longer they are on, the greater t is. The energy unit on electric bills is the kilowatt-hour ($\text{kW} \cdot \text{h}$), consistent with the relationship $E = \text{Pt}$. It is easy to estimate the cost of operating electric appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted to joules. You can prove to yourself that $1\ \text{kW} \cdot \text{h} = 3.6 \times 10^6\ \text{J}$.

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This will not only reduce the cost, but it will also result in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home's use of energy goes to lighting, while the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFL). (See [link](b).) Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.) The heat transfer from these CFLs is less, and they last up to 10 times longer. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED

bulbs) are even more efficient (twice that of CFLs) and last 5 times longer than CFLs. However, their cost is still high.

Note:
Making Connections: Energy, Power, and Time
The relationship $E = \mathrm{Pt}$ is one that you will find useful in many different contexts. The energy your body uses in exercise is related to the power level and duration of your activity, for example. The amount of heating by a power source is related to the power level and time it is applied. Even the radiation dose of an X-ray image is related to the power and time of exposure.

Example:
Calculating the Cost Effectiveness of Compact Fluorescent Lights (CFL)
If the cost of electricity in your area is 12 cents per kWh, what is the total cost (capital plus operation) of using a 60-W incandescent bulb for 1000 hours (the lifetime of that bulb) if the bulb cost 25 cents? (b) If we replace this bulb with a compact fluorescent light that provides the same light output, but at one-quarter the wattage, and which costs $1.50 but lasts 10 times longer (10,000 hours), what will that total cost be?
Strategy
To find the operating cost, we first find the energy used in kilowatt-hours and then multiply by the cost per kilowatt-hour.
Solution for (a)
The energy used in kilowatt-hours is found by entering the power and time into the expression for energy:
Equation:

$$E = \mathrm{Pt} = (60\ \mathrm{W})(1000\ \mathrm{h}) = 60{,}000\ \mathrm{W} \cdot \mathrm{h}.$$

In kilowatt-hours, this is
Equation:

$$E = 60.0\ \text{kW} \cdot \text{h}.$$

Now the electricity cost is
Equation:

$$\text{cost} = (60.0\ \text{kW} \cdot \text{h})(\$0.12/\text{kW} \cdot \text{h}) = \$7.20.$$

The total cost will be $7.20 for 1000 hours (about one-half year at 5 hours per day).

Solution for (b)

Since the CFL uses only 15 W and not 60 W, the electricity cost will be $7.20/4 = $1.80. The CFL will last 10 times longer than the incandescent, so that the investment cost will be 1/10 of the bulb cost for that time period of use, or 0.1($1.50) = $0.15. Therefore, the total cost will be $1.95 for 1000 hours.

Discussion

Therefore, it is much cheaper to use the CFLs, even though the initial investment is higher. The increased cost of labor that a business must include for replacing the incandescent bulbs more often has not been figured in here.

Note:

Making Connections: Take-Home Experiment—Electrical Energy Use Inventory

1) Make a list of the power ratings on a range of appliances in your home or room. Explain why something like a toaster has a higher rating than a digital clock. Estimate the energy consumed by these appliances in an average day (by estimating their time of use). Some appliances might only state the operating current. If the household voltage is 120 V, then use $P = \text{IV}$. 2) Check out the total wattage used in the rest rooms of your school's floor or building. (You might need to assume the long fluorescent lights in use are rated at 32 W.) Suppose that the building was closed all weekend and that these lights were left on from 6 p.m. Friday until 8 a.m. Monday. What would this oversight cost? How about for an entire year of weekends?

Section Summary

- Electric power P is the rate (in watts) that energy is supplied by a source or dissipated by a device.
- Three expressions for electrical power are
  Equation:
  $$P = \text{IV},$$
  Equation:
  $$P = \frac{V^2}{R},$$
  and
  Equation:
  $$P = I^2 R.$$
- The energy used by a device with a power P over a time t is $E = \text{Pt}$.

Conceptual Questions

Exercise:

Problem:

Why do incandescent lightbulbs grow dim late in their lives, particularly just before their filaments break?

Exercise:

Problem:

The power dissipated in a resistor is given by $P = V^2/R$, which means power decreases if resistance increases. Yet this power is also given by $P = I^2 R$, which means power increases if resistance increases. Explain why there is no contradiction here.

Problem Exercises

Exercise:

Problem:

What is the power of a 1.00×10^2 MV lightning bolt having a current of 2.00×10^4 A?

Solution:

2.00×10^{12} W

Exercise:

Problem:

What power is supplied to the starter motor of a large truck that draws 250 A of current from a 24.0-V battery hookup?

Exercise:

Problem:

A charge of 4.00 C of charge passes through a pocket calculator's solar cells in 4.00 h. What is the power output, given the calculator's voltage output is 3.00 V? (See [link].)

The strip of solar

cells just above the keys of this calculator convert light to electricity to supply its energy needs. (credit: Evan-Amos, Wikimedia Commons)

Exercise:

Problem:

How many watts does a flashlight that has 6.00×10^2 C pass through it in 0.500 h use if its voltage is 3.00 V?

Exercise:

Problem:

Find the power dissipated in each of these extension cords: (a) an extension cord having a $0.0600 - \Omega$ resistance and through which 5.00 A is flowing; (b) a cheaper cord utilizing thinner wire and with a resistance of $0.300\ \Omega$.

Solution:

(a) 1.50 W

(b) 7.50 W

Exercise:

Problem:

Verify that the units of a volt-ampere are watts, as implied by the equation $P = \text{IV}$.

Exercise:

Problem:

Show that the units $1\ \mathrm{V}^2/\Omega = 1\mathrm{W}$, as implied by the equation $P = V^2/R$.

Solution:

$$\frac{V^2}{\Omega} = \frac{V^2}{\mathrm{V/A}} = \mathrm{AV} = \left(\frac{C}{s}\right)\left(\frac{J}{C}\right) = \frac{J}{s} = 1\ \mathrm{W}$$

Exercise:

Problem:

Show that the units $1\ \mathrm{A}^2 \cdot \Omega = 1\ \mathrm{W}$, as implied by the equation $P = I^2 R$.

Exercise:

Problem:

Verify the energy unit equivalence that $1\ \mathrm{kW} \cdot \mathrm{h} = 3.60{\times}10^6\ \mathrm{J}$.

Solution:

$$1\ \mathrm{kW} \cdot \mathrm{h}{=}\left(\frac{1{\times}10^3\ \mathrm{J}}{1\ \mathrm{s}}\right)(1\ \mathrm{h})\left(\frac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right) = 3.60 \times 10^6\ \mathrm{J}$$

Exercise:

Problem:

Electrons in an X-ray tube are accelerated through $1.00 \times 10^2\ \mathrm{kV}$ and directed toward a target to produce X-rays. Calculate the power of the electron beam in this tube if it has a current of 15.0 mA.

Exercise:

Problem:

An electric water heater consumes 5.00 kW for 2.00 h per day. What is the cost of running it for one year if electricity costs $12.0\ \text{cents/kW}\cdot\text{h}$? See [link].

On-demand electric hot water heater. Heat is supplied to water only when needed. (credit: aviddavid, Flickr)

Solution:

\$438/y

Exercise:

Problem:

With a 1200-W toaster, how much electrical energy is needed to make a slice of toast (cooking time = 1 minute)? At $9.0\ \text{cents/kW}\cdot\text{h}$, how much does this cost?

Exercise:

Problem:

Some makes of older cars have 6.00-V electrical systems. (a) What is the hot resistance of a 30.0-W headlight in such a car? (b) What current flows through it?

Exercise:

Problem:

Alkaline batteries have the advantage of putting out constant voltage until very nearly the end of their life. How long will an alkaline battery rated at $1.00\text{ A}\cdot\text{h}$ and 1.58 V keep a 1.00-W flashlight bulb burning?

Solution:

1.58 h

Exercise:

Problem:

A cauterizer, used to stop bleeding in surgery, puts out 2.00 mA at 15.0 kV. (a) What is its power output? (b) What is the resistance of the path?

Exercise:

Problem:

The average television is said to be on 6 hours per day. Estimate the yearly cost of electricity to operate 100 million TVs, assuming their power consumption averages 150 W and the cost of electricity averages $12.0\text{ cents/kW}\cdot\text{h}$.

Solution:

$3.94 billion/year

Glossary

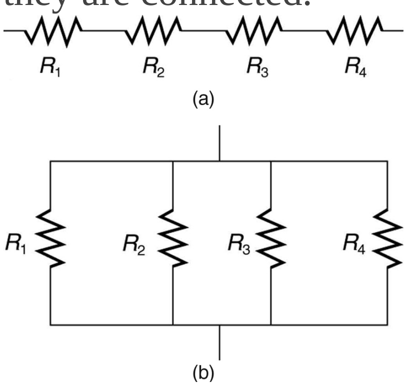
electric power

the rate at which electrical energy is supplied by a source or dissipated by a device; it is the product of current times voltage

Resistors in Series and Parallel

- Draw a circuit with resistors in parallel and in series.
- Calculate the voltage drop of a current across a resistor using Ohm’s law.
- Contrast the way total resistance is calculated for resistors in series and in parallel.
- Explain why total resistance of a parallel circuit is less than the smallest resistance of any of the resistors in that circuit.
- Calculate total resistance of a circuit that contains a mixture of resistors connected in series and in parallel.

Most circuits have more than one component, called a **resistor** that limits the flow of charge in the circuit. A measure of this limit on charge flow is called **resistance**. The simplest combinations of resistors are the series and parallel connections illustrated in [link]. The total resistance of a combination of resistors depends on both their individual values and how they are connected.

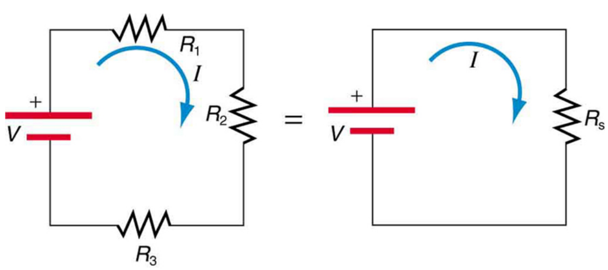


(a) A series connection of resistors. (b) A parallel connection of resistors.

Resistors in Series

When are resistors in **series**? Resistors are in series whenever the flow of charge, called the **current**, must flow through devices sequentially. For example, if current flows through a person holding a screwdriver and into the Earth, then R_1 in [link](a) could be the resistance of the screwdriver's shaft, R_2 the resistance of its handle, R_3 the person's body resistance, and R_4 the resistance of her shoes.

[link] shows resistors in series connected to a **voltage** source. It seems reasonable that the total resistance is the sum of the individual resistances, considering that the current has to pass through each resistor in sequence. (This fact would be an advantage to a person wishing to avoid an electrical shock, who could reduce the current by wearing high-resistance rubber-soled shoes. It could be a disadvantage if one of the resistances were a faulty high-resistance cord to an appliance that would reduce the operating current.)



Three resistors connected in series to a battery (left) and the equivalent single or series resistance (right).

To verify that resistances in series do indeed add, let us consider the loss of electrical power, called a **voltage drop**, in each resistor in [link].

According to **Ohm's law**, the voltage drop, V, across a resistor when a current flows through it is calculated using the equation $V = \mathrm{IR}$, where I equals the current in amps (A) and R is the resistance in ohms (Ω). Another

way to think of this is that V is the voltage necessary to make a current I flow through a resistance R.

So the voltage drop across R_1 is $V_1 = IR_1$, that across R_2 is $V_2 = IR_2$, and that across R_3 is $V_3 = IR_3$. The sum of these voltages equals the voltage output of the source; that is,

Equation:

$$V = V_1 + V_2 + V_3.$$

This equation is based on the conservation of energy and conservation of charge. Electrical potential energy can be described by the equation $\mathrm{PE} = \mathrm{qV}$, where q is the electric charge and V is the voltage. Thus the energy supplied by the source is qV, while that dissipated by the resistors is

Equation:

$$qV_1 + qV_2 + qV_3.$$

Note:
Connections: Conservation Laws
The derivations of the expressions for series and parallel resistance are based on the laws of conservation of energy and conservation of charge, which state that total charge and total energy are constant in any process. These two laws are directly involved in all electrical phenomena and will be invoked repeatedly to explain both specific effects and the general behavior of electricity.

These energies must be equal, because there is no other source and no other destination for energy in the circuit. Thus, $qV = qV_1 + qV_2 + qV_3$. The charge q cancels, yielding $V = V_1 + V_2 + V_3$, as stated. (Note that the same amount of charge passes through the battery and each resistor in a given amount of time, since there is no capacitance to store charge, there is no place for charge to leak, and charge is conserved.)

Now substituting the values for the individual voltages gives
Equation:

$$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3).$$

Note that for the equivalent single series resistance R_s, we have
Equation:

$$V = IR_\mathrm{s}.$$

This implies that the total or equivalent series resistance R_s of three resistors is $R_\mathrm{s} = R_1 + R_2 + R_3$.

This logic is valid in general for any number of resistors in series; thus, the total resistance R_s of a series connection is
Equation:

$$R_\mathrm{s} = R_1 + R_2 + R_3 + ...,$$

as proposed. Since all of the current must pass through each resistor, it experiences the resistance of each, and resistances in series simply add up.

Example:
Calculating Resistance, Current, Voltage Drop, and Power Dissipation: Analysis of a Series Circuit
Suppose the voltage output of the battery in [link] is $12.0\ \mathrm{V}$, and the resistances are $R_1 = 1.00\ \Omega$, $R_2 = 6.00\ \Omega$, and $R_3 = 13.0\ \Omega$. (a) What is the total resistance? (b) Find the current. (c) Calculate the voltage drop in each resistor, and show these add to equal the voltage output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.
Strategy and Solution for (a)

The total resistance is simply the sum of the individual resistances, as given by this equation:

Equation:

$$\begin{aligned} R_s &= R_1 + R_2 + R_3 \\ &= 1.00\ \Omega + 6.00\ \Omega + 13.0\ \Omega \\ &= 20.0\ \Omega. \end{aligned}$$

Strategy and Solution for (b)

The current is found using Ohm's law, $V = \mathrm{IR}$. Entering the value of the applied voltage and the total resistance yields the current for the circuit:

Equation:

$$I = \frac{V}{R_s} = \frac{12.0\ \mathrm{V}}{20.0\ \Omega} = 0.600\ \mathrm{A}.$$

Strategy and Solution for (c)

The voltage—or IR drop—in a resistor is given by Ohm's law. Entering the current and the value of the first resistance yields

Equation:

$$V_1 = IR_1 = (0.600\ \mathrm{A})(1.0\ \Omega) = 0.600\ \mathrm{V}.$$

Similarly,

Equation:

$$V_2 = IR_2 = (0.600\ \mathrm{A})(6.0\ \Omega) = 3.60\ \mathrm{V}$$

and

Equation:

$$V_3 = IR_3 = (0.600\ \mathrm{A})(13.0\ \Omega) = 7.80\ \mathrm{V}.$$

Discussion for (c)

The three IR drops add to 12.0 V, as predicted:

Equation:

$$V_1 + V_2 + V_3 = (0.600 + 3.60 + 7.80)\ \mathrm{V} = 12.0\ \mathrm{V}.$$

Strategy and Solution for (d)

The easiest way to calculate power in watts (W) dissipated by a resistor in a DC circuit is to use **Joule's law**, $P = \mathrm{IV}$, where P is electric power. In this case, each resistor has the same full current flowing through it. By substituting Ohm's law $V = \mathrm{IR}$ into Joule's law, we get the power dissipated by the first resistor as

Equation:

$$P_1 = I^2 R_1 = (0.600\ \mathrm{A})^2 (1.00\ \Omega) = 0.360\ \mathrm{W}.$$

Similarly,

Equation:

$$P_2 = I^2 R_2 = (0.600\ \mathrm{A})^2 (6.00\ \Omega) = 2.16\ \mathrm{W}$$

and

Equation:

$$P_3 = I^2 R_3 = (0.600\ \mathrm{A})^2 (13.0\ \Omega) = 4.68\ \mathrm{W}.$$

Discussion for (d)

Power can also be calculated using either $P = \mathrm{IV}$ or $P = \frac{V^2}{R}$, where V is the voltage drop across the resistor (not the full voltage of the source). The same values will be obtained.

Strategy and Solution for (e)

The easiest way to calculate power output of the source is to use $P = \mathrm{IV}$, where V is the source voltage. This gives

Equation:

$$P = (0.600\ \mathrm{A})(12.0\ \mathrm{V}) = 7.20\ \mathrm{W}.$$

Discussion for (e)

Note, coincidentally, that the total power dissipated by the resistors is also 7.20 W, the same as the power put out by the source. That is,

Equation:

$$P_1 + P_2 + P_3 = (0.360 + 2.16 + 4.68)\ \mathrm{W} = 7.20\ \mathrm{W}.$$

Power is energy per unit time (watts), and so conservation of energy requires the power output of the source to be equal to the total power dissipated by the resistors.

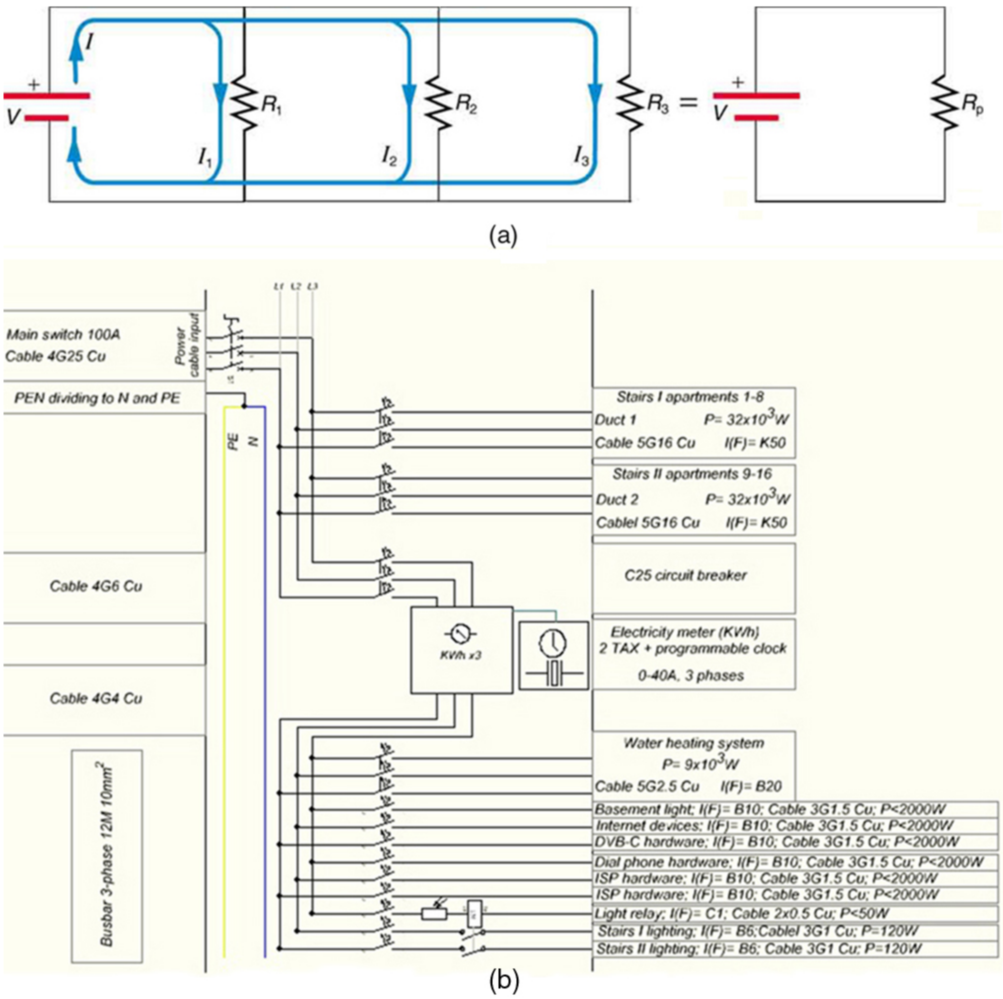
Note:
Major Features of Resistors in Series

1. Series resistances add: $R_s = R_1 + R_2 + R_3 + ...$
2. The same current flows through each resistor in series.
3. Individual resistors in series do not get the total source voltage, but divide it.

Resistors in Parallel

[link] shows resistors in **parallel**, wired to a voltage source. Resistors are in parallel when each resistor is connected directly to the voltage source by connecting wires having negligible resistance. Each resistor thus has the full voltage of the source applied to it.

Each resistor draws the same current it would if it alone were connected to the voltage source (provided the voltage source is not overloaded). For example, an automobile’s headlights, radio, and so on, are wired in parallel, so that they utilize the full voltage of the source and can operate completely independently. The same is true in your house, or any building. (See [link] (b).)



(a) Three resistors connected in parallel to a battery and the equivalent single or parallel resistance. (b) Electrical power setup in a house. (credit: Dmitry G, Wikimedia Commons)

To find an expression for the equivalent parallel resistance R_{p}, let us consider the currents that flow and how they are related to resistance. Since each resistor in the circuit has the full voltage, the currents flowing through the individual resistors are $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$, and $I_3 = \frac{V}{R_3}$. Conservation of charge implies that the total current I produced by the source is the sum of these currents:

Equation:

$$I=I_1+I_2+I_3.$$

Substituting the expressions for the individual currents gives
Equation:

$$I=\frac{V}{R_1}+\frac{V}{R_2}+\frac{V}{R_3}=V\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right).$$

Note that Ohm’s law for the equivalent single resistance gives
Equation:

$$I=\frac{V}{R_\mathrm{p}}=V\left(\frac{1}{R_\mathrm{p}}\right).$$

The terms inside the parentheses in the last two equations must be equal. Generalizing to any number of resistors, the total resistance R_p of a parallel connection is related to the individual resistances by
Equation:

$$\frac{1}{R_\mathrm{p}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_{.3}}+....$$

This relationship results in a total resistance R_p that is less than the smallest of the individual resistances. (This is seen in the next example.) When resistors are connected in parallel, more current flows from the source than would flow for any of them individually, and so the total resistance is lower.

Example:
Calculating Resistance, Current, Power Dissipation, and Power Output: Analysis of a Parallel Circuit
Let the voltage output of the battery and resistances in the parallel connection in [link] be the same as the previously considered series

connection: $V = 12.0\text{ V}$, $R_1 = 1.00\ \Omega$, $R_2 = 6.00\ \Omega$, and $R_3 = 13.0\ \Omega$. (a) What is the total resistance? (b) Find the total current. (c) Calculate the currents in each resistor, and show these add to equal the total current output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance for a parallel combination of resistors is found using the equation below. Entering known values gives

Equation:

$$\frac{1}{R_{\text{p}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.00\ \Omega} + \frac{1}{6.00\ \Omega} + \frac{1}{13.0\ \Omega}.$$

Thus,

Equation:

$$\frac{1}{R_{\text{p}}} = \frac{1.00}{\Omega} + \frac{0.1667}{\Omega} + \frac{0.07692}{\Omega} = \frac{1.2436}{\Omega}.$$

(Note that in these calculations, each intermediate answer is shown with an extra digit.)

We must invert this to find the total resistance R_{p}. This yields

Equation:

$$R_{\text{p}} = \frac{1}{1.2436}\Omega = 0.8041\ \Omega.$$

The total resistance with the correct number of significant digits is $R_{\text{p}} = 0.804\ \Omega$.

Discussion for (a)

R_{p} is, as predicted, less than the smallest individual resistance.

Strategy and Solution for (b)

The total current can be found from Ohm’s law, substituting R_{p} for the total resistance. This gives

Equation:

$$I = \frac{V}{R_{\mathrm{p}}} = \frac{12.0\ \mathrm{V}}{0.8041\ \Omega} = 14.92\ \mathrm{A}.$$

Discussion for (b)
Current I for each device is much larger than for the same devices connected in series (see the previous example). A circuit with parallel connections has a smaller total resistance than the resistors connected in series.
Strategy and Solution for (c)
The individual currents are easily calculated from Ohm’s law, since each resistor gets the full voltage. Thus,
Equation:

$$I_1 = \frac{V}{R_1} = \frac{12.0\ \mathrm{V}}{1.00\,\Omega} = 12.0\ \mathrm{A}.$$

Similarly,
Equation:

$$I_2 = \frac{V}{R_2} = \frac{12.0\ \mathrm{V}}{6.00\,\Omega} = 2.00\ \mathrm{A}$$

and
Equation:

$$I_3 = \frac{V}{R_3} = \frac{12.0\ \mathrm{V}}{13.0\,\Omega} = 0.92\ \mathrm{A}.$$

Discussion for (c)
The total current is the sum of the individual currents:
Equation:

$$I_1 + I_2 + I_3 = 14.92\ \mathrm{A}.$$

This is consistent with conservation of charge.
Strategy and Solution for (d)
The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three

are known. Let us use $P = \frac{V^2}{R}$, since each resistor gets full voltage. Thus,
Equation:

$$P_1 = \frac{V^2}{R_1} = \frac{(12.0\ \text{V})^2}{1.00\ \Omega} = 144\ \text{W}.$$

Similarly,
Equation:

$$P_2 = \frac{V^2}{R_2} = \frac{(12.0\ \text{V})^2}{6.00\ \Omega} = 24.0\ \text{W}$$

and
Equation:

$$P_3 = \frac{V^2}{R_3} = \frac{(12.0\ \text{V})^2}{13.0\ \Omega} = 11.1\ \text{W}.$$

Discussion for (d)
The power dissipated by each resistor is considerably higher in parallel than when connected in series to the same voltage source.
Strategy and Solution for (e)
The total power can also be calculated in several ways. Choosing $P = \text{IV}$, and entering the total current, yields
Equation:

$$P = \text{IV} = (14.92\ \text{A})(12.0\ \text{V}) = 179\ \text{W}.$$

Discussion for (e)
Total power dissipated by the resistors is also 179 W:
Equation:

$$P_1 + P_2 + P_3 = 144\ \text{W} + 24.0\ \text{W} + 11.1\ \text{W} = 179\ \text{W}.$$

This is consistent with the law of conservation of energy.
Overall Discussion
Note that both the currents and powers in parallel connections are greater than for the same devices in series.

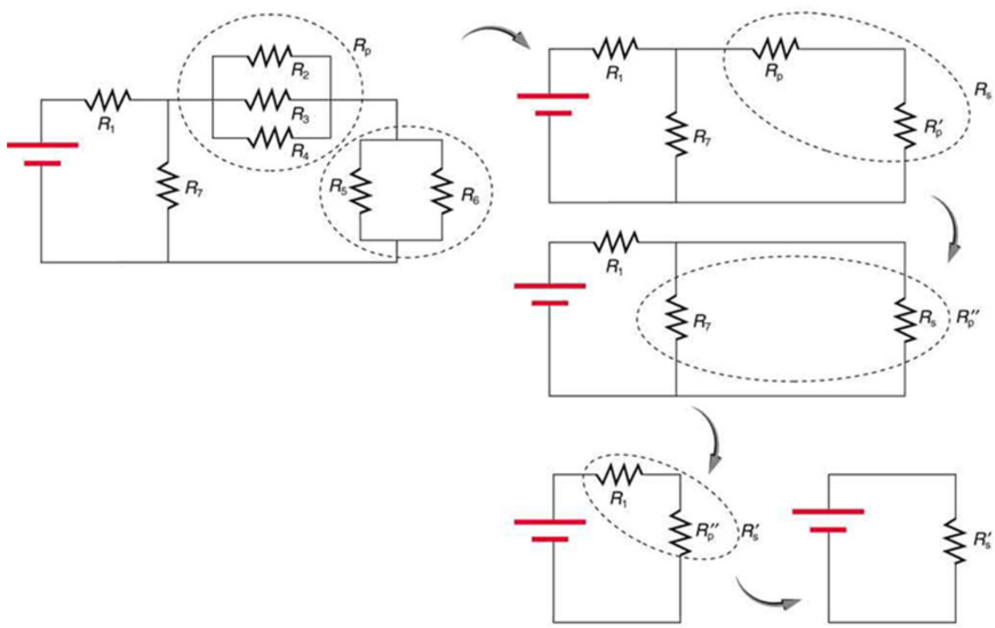
Note:
Major Features of Resistors in Parallel

1. Parallel resistance is found from $\frac{1}{R_{\text{p}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$, and it is smaller than any individual resistance in the combination.
2. Each resistor in parallel has the same full voltage of the source applied to it. (Power distribution systems most often use parallel connections to supply the myriad devices served with the same voltage and to allow them to operate independently.)
3. Parallel resistors do not each get the total current; they divide it.

Combinations of Series and Parallel

More complex connections of resistors are sometimes just combinations of series and parallel. These are commonly encountered, especially when wire resistance is considered. In that case, wire resistance is in series with other resistances that are in parallel.

Combinations of series and parallel can be reduced to a single equivalent resistance using the technique illustrated in [link]. Various parts are identified as either series or parallel, reduced to their equivalents, and further reduced until a single resistance is left. The process is more time consuming than difficult.

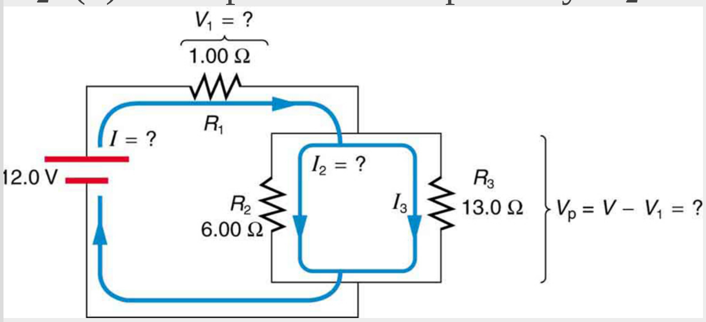


This combination of seven resistors has both series and parallel parts. Each is identified and reduced to an equivalent resistance, and these are further reduced until a single equivalent resistance is reached.

The simplest combination of series and parallel resistance, shown in [link], is also the most instructive, since it is found in many applications. For example, R_1 could be the resistance of wires from a car battery to its electrical devices, which are in parallel. R_2 and R_3 could be the starter motor and a passenger compartment light. We have previously assumed that wire resistance is negligible, but, when it is not, it has important effects, as the next example indicates.

Example:
Calculating Resistance, IR Drop, Current, and Power Dissipation: Combining Series and Parallel Circuits
[link] shows the resistors from the previous two examples wired in a different way—a combination of series and parallel. We can consider R_1 to be the resistance of wires leading to R_2 and R_3. (a) Find the total

resistance. (b) What is the IR drop in R_1? (c) Find the current I_2 through R_2. (d) What power is dissipated by R_2?



These three resistors are connected to a voltage source so that R_2 and R_3 are in parallel with one another and that combination is in series with R_1.

Strategy and Solution for (a)

To find the total resistance, we note that R_2 and R_3 are in parallel and their combination R_p is in series with R_1. Thus the total (equivalent) resistance of this combination is

Equation:

$$R_\mathrm{tot} = R_1 + R_\mathrm{p}.$$

First, we find R_p using the equation for resistors in parallel and entering known values:

Equation:

$$\frac{1}{R_\mathrm{p}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6.00\ \Omega} + \frac{1}{13.0\ \Omega} = \frac{0.2436}{\Omega}.$$

Inverting gives

Equation:

$$R_\mathrm{p} = \frac{1}{0.2436}\Omega = 4.11\ \Omega.$$

So the total resistance is

Equation:

$$R_{\text{tot}} = R_1 + R_{\text{p}} = 1.00\ \Omega + 4.11\ \Omega = 5.11\ \Omega.$$

Discussion for (a)

The total resistance of this combination is intermediate between the pure series and pure parallel values ($20.0\ \Omega$ and $0.804\ \Omega$, respectively) found for the same resistors in the two previous examples.

Strategy and Solution for (b)

To find the IR drop in R_1, we note that the full current I flows through R_1. Thus its IR drop is

Equation:

$$V_1 = \mathrm{IR}_1.$$

We must find I before we can calculate V_1. The total current I is found using Ohm's law for the circuit. That is,

Equation:

$$I = \frac{V}{R_{\text{tot}}} = \frac{12.0\ \text{V}}{5.11\ \Omega} = 2.35\ \text{A}.$$

Entering this into the expression above, we get

Equation:

$$V_1 = \mathrm{IR}_1 = (2.35\ \text{A})(1.00\ \Omega) = 2.35\ \text{V}.$$

Discussion for (b)

The voltage applied to R_2 and R_3 is less than the total voltage by an amount V_1. When wire resistance is large, it can significantly affect the operation of the devices represented by R_2 and R_3.

Strategy and Solution for (c)

To find the current through R_2, we must first find the voltage applied to it. We call this voltage V_{p}, because it is applied to a parallel combination of resistors. The voltage applied to both R_2 and R_3 is reduced by the amount V_1, and so it is

Equation:

$$V_{\mathrm{p}} = V - V_1 = 12.0\ \mathrm{V} - 2.35\ \mathrm{V} = 9.65\ \mathrm{V}.$$

Now the current I_2 through resistance R_2 is found using Ohm's law:
Equation:

$$I_2 = \frac{V_{\mathrm{p}}}{R_2} = \frac{9.65\ \mathrm{V}}{6.00\ \Omega} = 1.61\ \mathrm{A}.$$

Discussion for (c)
The current is less than the 2.00 A that flowed through R_2 when it was connected in parallel to the battery in the previous parallel circuit example.
Strategy and Solution for (d)
The power dissipated by R_2 is given by
Equation:

$$P_2 = (I_2)^2 R_2 = (1.61\ \mathrm{A})^2 (6.00\ \Omega) = 15.5\ \mathrm{W}.$$

Discussion for (d)
The power is less than the 24.0 W this resistor dissipated when connected in parallel to the 12.0-V source.

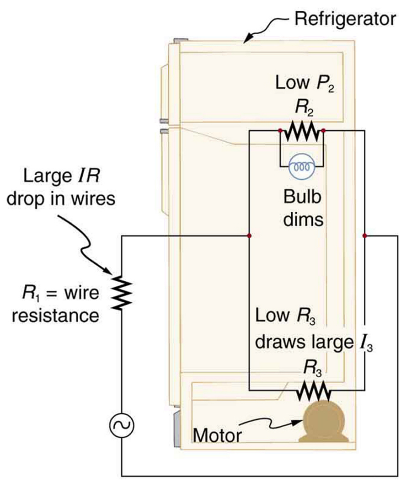
Practical Implications

One implication of this last example is that resistance in wires reduces the current and power delivered to a resistor. If wire resistance is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant. If a large current is drawn, the IR drop in the wires can also be significant.

For example, when you are rummaging in the refrigerator and the motor comes on, the refrigerator light dims momentarily. Similarly, you can see the passenger compartment light dim when you start the engine of your car (although this may be due to resistance inside the battery itself).

What is happening in these high-current situations is illustrated in [link]. The device represented by R_3 has a very low resistance, and so when it is

switched on, a large current flows. This increased current causes a larger IR drop in the wires represented by R_1, reducing the voltage across the light bulb (which is R_2), which then dims noticeably.



Why do lights dim when a large appliance is switched on? The answer is that the large current the appliance motor draws causes a significant IR drop in the wires and reduces the voltage across the light.

Exercise:
Check Your Understanding

Problem:

Can any arbitrary combination of resistors be broken down into series and parallel combinations? See if you can draw a circuit diagram of resistors that cannot be broken down into combinations of series and parallel.

Solution:

No, there are many ways to connect resistors that are not combinations of series and parallel, including loops and junctions. In such cases Kirchhoff's rules, to be introduced in Kirchhoff's Rules, will allow you to analyze the circuit.

Note:
Problem-Solving Strategies for Series and Parallel Resistors

1. Draw a clear circuit diagram, labeling all resistors and voltage sources. This step includes a list of the knowns for the problem, since they are labeled in your circuit diagram.
2. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
3. Determine whether resistors are in series, parallel, or a combination of both series and parallel. Examine the circuit diagram to make this assessment. Resistors are in series if the same current must pass sequentially through them.
4. Use the appropriate list of major features for series or parallel connections to solve for the unknowns. There is one list for series and another for parallel. If your problem has a combination of series and parallel, reduce it in steps by considering individual groups of series or parallel connections, as done in this module and the examples. Special note: When finding R_p, the reciprocal must be taken with care.
5. Check to see whether the answers are reasonable and consistent. Units and numerical results must be reasonable. Total series resistance

should be greater, whereas total parallel resistance should be smaller, for example. Power should be greater for the same devices in parallel compared with series, and so on.

Section Summary

- The total resistance of an electrical circuit with resistors wired in a series is the sum of the individual resistances: $R_{\mathrm{s}} = R_1 + R_2 + R_3 + ...$
- Each resistor in a series circuit has the same amount of current flowing through it.
- The voltage drop, or power dissipation, across each individual resistor in a series is different, and their combined total adds up to the power source input.
- The total resistance of an electrical circuit with resistors wired in parallel is less than the lowest resistance of any of the components and can be determined using the formula:
  Equation:

$$\frac{1}{R_{\mathrm{p}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$$

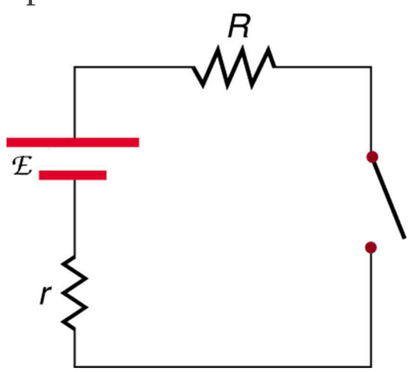
- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.
- If a more complex connection of resistors is a combination of series and parallel, it can be reduced to a single equivalent resistance by identifying its various parts as series or parallel, reducing each to its equivalent, and continuing until a single resistance is eventually reached.

Conceptual Questions

Exercise:

Problem:

A switch has a variable resistance that is nearly zero when closed and extremely large when open, and it is placed in series with the device it controls. Explain the effect the switch in [link] has on current when open and when closed.



A switch is ordinarily in series with a resistance and voltage source. Ideally, the switch has nearly zero resistance when closed but has an extremely large resistance when open. (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)

Exercise:

Problem: What is the voltage across the open switch in [link]?

Exercise:

Problem:

There is a voltage across an open switch, such as in [link]. Why, then, is the power dissipated by the open switch small?

Exercise:

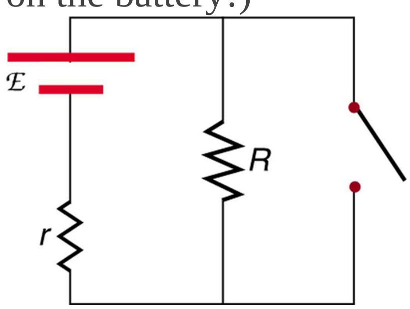
Problem:

Why is the power dissipated by a closed switch, such as in [link], small?

Exercise:

Problem:

A student in a physics lab mistakenly wired a light bulb, battery, and switch as shown in [link]. Explain why the bulb is on when the switch is open, and off when the switch is closed. (Do not try this—it is hard on the battery!)



A wiring mistake put this switch in parallel with the device represented by R. (Note that in this diagram, the script E represents the voltage (or

electromotive force) of the battery.)

Exercise:

Problem:

Knowing that the severity of a shock depends on the magnitude of the current through your body, would you prefer to be in series or parallel with a resistance, such as the heating element of a toaster, if shocked by it? Explain.

Exercise:

Problem:

Would your headlights dim when you start your car’s engine if the wires in your automobile were superconductors? (Do not neglect the battery’s internal resistance.) Explain.

Exercise:

Problem:

Some strings of holiday lights are wired in series to save wiring costs. An old version utilized bulbs that break the electrical connection, like an open switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 40 identical bulbs, what is the normal operating voltage of each? Newer versions use bulbs that short circuit, like a closed switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 39 remaining identical bulbs, what is then the operating voltage of each?

Exercise:

Problem:

If two household lightbulbs rated 60 W and 100 W are connected in series to household power, which will be brighter? Explain.

Exercise:

Problem:

Suppose you are doing a physics lab that asks you to put a resistor into a circuit, but all the resistors supplied have a larger resistance than the requested value. How would you connect the available resistances to attempt to get the smaller value asked for?

Exercise:

Problem:

Before World War II, some radios got power through a “resistance cord” that had a significant resistance. Such a resistance cord reduces the voltage to a desired level for the radio’s tubes and the like, and it saves the expense of a transformer. Explain why resistance cords become warm and waste energy when the radio is on.

Exercise:

Problem:

Some light bulbs have three power settings (not including zero), obtained from multiple filaments that are individually switched and wired in parallel. What is the minimum number of filaments needed for three power settings?

Problem Exercises

Note: Data taken from figures can be assumed to be accurate to three significant digits.
Exercise:

Problem:

(a) What is the resistance of ten $275\text{-}\Omega$ resistors connected in series? (b) In parallel?

Solution:

(a) $2.75\ \text{k}\Omega$

(b) $27.5\ \Omega$

Exercise:

Problem:

(a) What is the resistance of a 1.00×10^2 $\text{-}\Omega$, a $2.50\text{-k}\Omega$, and a $4.00\text{-k}\Omega$ resistor connected in series? (b) In parallel?

Exercise:

Problem:

What are the largest and smallest resistances you can obtain by connecting a $36.0\text{-}\Omega$, a $50.0\text{-}\Omega$, and a $700\text{-}\Omega$ resistor together?

Solution:

(a) $786\ \Omega$

(b) $20.3\ \Omega$

Exercise:

Problem:

An 1800-W toaster, a 1400-W electric frying pan, and a 75-W lamp are plugged into the same outlet in a 15-A, 120-V circuit. (The three devices are in parallel when plugged into the same socket.). (a) What current is drawn by each device? (b) Will this combination blow the 15-A fuse?

Exercise:

Problem:

Your car's 30.0-W headlight and 2.40-kW starter are ordinarily connected in parallel in a 12.0-V system. What power would one headlight and the starter consume if connected in series to a 12.0-V battery? (Neglect any other resistance in the circuit and any change in resistance in the two devices.)

Solution:

29.6 W

Exercise:

Problem:

(a) Given a 48.0-V battery and $24.0\text{-}\Omega$ and $96.0\text{-}\Omega$ resistors, find the current and power for each when connected in series. (b) Repeat when the resistances are in parallel.

Exercise:

Problem:

Referring to the example combining series and parallel circuits and [link], calculate I_3 in the following two different ways: (a) from the known values of I and I_2; (b) using Ohm's law for R_3. In both parts explicitly show how you follow the steps in the Problem-Solving Strategies for Series and Parallel Resistors.

Solution:

(a) 0.74 A

(b) 0.742 A

Exercise:

Problem:

Referring to [link]: (a) Calculate P_3 and note how it compares with P_3 found in the first two example problems in this module. (b) Find the total power supplied by the source and compare it with the sum of the powers dissipated by the resistors.

Exercise:

Problem:

Refer to [link] and the discussion of lights dimming when a heavy appliance comes on. (a) Given the voltage source is 120 V, the wire resistance is $0.400\ \Omega$, and the bulb is nominally 75.0 W, what power will the bulb dissipate if a total of 15.0 A passes through the wires when the motor comes on? Assume negligible change in bulb resistance. (b) What power is consumed by the motor?

Solution:

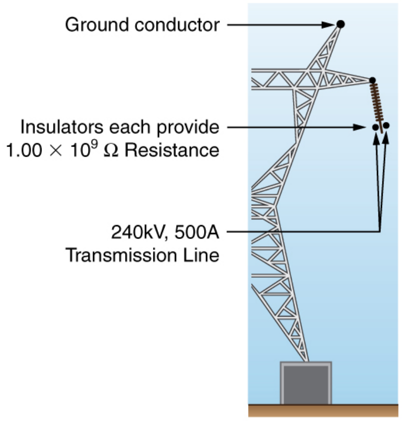
(a) 60.8 W

(b) 3.18 kW

Exercise:

Problem:

A 240-kV power transmission line carrying $5.00 \times 10^2\ \text{A}$ is hung from grounded metal towers by ceramic insulators, each having a $1.00{\times}10^9$ -Ω resistance. [link]. (a) What is the resistance to ground of 100 of these insulators? (b) Calculate the power dissipated by 100 of them. (c) What fraction of the power carried by the line is this? Explicitly show how you follow the steps in the Problem-Solving Strategies for Series and Parallel Resistors.



High-voltage (240-kV) transmission line carrying 5.00×10^2 A is hung from a grounded metal transmission tower. The row of ceramic insulators provide $1.00 \times 10^9 \Omega$ of resistance each.

Exercise:

Problem:

Show that if two resistors R_1 and R_2 are combined and one is much greater than the other ($R_1 >> R_2$): (a) Their series resistance is very nearly equal to the greater resistance R_1. (b) Their parallel resistance is very nearly equal to smaller resistance R_2.

Solution:

(a) $$\begin{aligned} R_\text{s} &= R_1 + R_2 \\ \Rightarrow R_\text{s} &\approx R_1 (R_1 >> R_2) \end{aligned}$$

(b) $\frac{1}{R_{\text{p}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2}$,

so that

$$R_{\text{p}} = \frac{R_1 R_2}{R_1 + R_2} \approx \frac{R_1 R_2}{R_1} = R_2 (R_1 >> R_2).$$

Exercise:

Problem: Unreasonable Results

Two resistors, one having a resistance of $145\ \Omega$, are connected in parallel to produce a total resistance of $150\ \Omega$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Exercise:

Problem: Unreasonable Results

Two resistors, one having a resistance of $900\ \text{k}\Omega$, are connected in series to produce a total resistance of $0.500\ \text{M}\Omega$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) $-400\ \text{k}\Omega$

(b) Resistance cannot be negative.

(c) Series resistance is said to be less than one of the resistors, but it must be greater than any of the resistors.

Glossary

series

a sequence of resistors or other components wired into a circuit one after the other

resistor
: a component that provides resistance to the current flowing through an electrical circuit

resistance
: causing a loss of electrical power in a circuit

Ohm's law
: the relationship between current, voltage, and resistance within an electrical circuit: $V = \mathrm{IR}$

voltage
: the electrical potential energy per unit charge; electric pressure created by a power source, such as a battery

voltage drop
: the loss of electrical power as a current travels through a resistor, wire or other component

current
: the flow of charge through an electric circuit past a given point of measurement

Joule's law
: the relationship between potential electrical power, voltage, and resistance in an electrical circuit, given by: $P_e = \mathrm{IV}$

parallel
: the wiring of resistors or other components in an electrical circuit such that each component receives an equal voltage from the power source; often pictured in a ladder-shaped diagram, with each component on a rung of the ladder

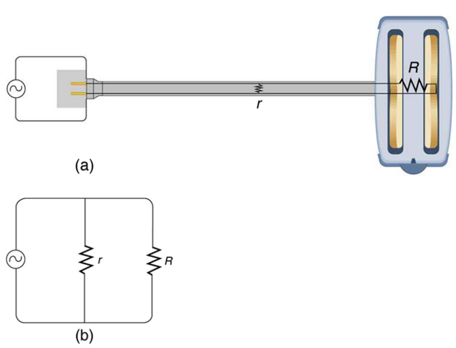
Electric Hazards and the Human Body

- Define thermal hazard, shock hazard, and short circuit.
- Explain what effects various levels of current have on the human body.

There are two known hazards of electricity—thermal and shock. A **thermal hazard** is one where excessive electric power causes undesired thermal effects, such as starting a fire in the wall of a house. A **shock hazard** occurs when electric current passes through a person. Shocks range in severity from painful, but otherwise harmless, to heart-stopping lethality. This section considers these hazards and the various factors affecting them in a quantitative manner.

Thermal Hazards

Electric power causes undesired heating effects whenever electric energy is converted to thermal energy at a rate faster than it can be safely dissipated. A classic example of this is the **short circuit**, a low-resistance path between terminals of a voltage source. An example of a short circuit is shown in [link]. Insulation on wires leading to an appliance has worn through, allowing the two wires to come into contact. Such an undesired contact with a high voltage is called a *short*. Since the resistance of the short, r, is very small, the power dissipated in the short, $P = V^2/r$, is very large. For example, if V is 120 V and r is $0.100\ \Omega$, then the power is 144 kW, *much* greater than that used by a typical household appliance. Thermal energy delivered at this rate will very quickly raise the temperature of surrounding materials, melting or perhaps igniting them.

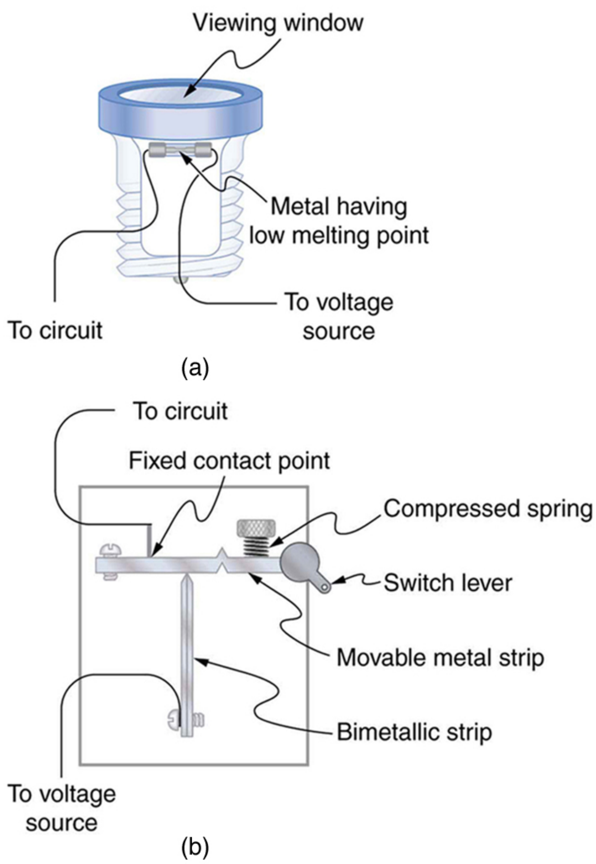


A short circuit is an undesired low-resistance path across a voltage source. (a) Worn insulation on the wires of a toaster allow them to come into contact with a low resistance r. Since $P = V^2/r$, thermal power is created so rapidly that the cord melts or burns. (b) A schematic of the short circuit.

One particularly insidious aspect of a short circuit is that its resistance may actually be decreased due to the increase in temperature. This can happen if the short creates ionization. These charged atoms and molecules are free to move and, thus, lower the resistance r. Since $P = V^2/r$, the power dissipated in the short rises, possibly causing more ionization, more power, and so on. High voltages, such as the 480-V AC used in some industrial applications, lend themselves to this hazard, because higher voltages create higher initial power production in a short.

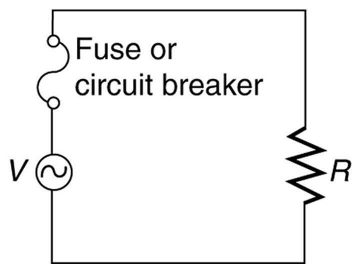
Another serious, but less dramatic, thermal hazard occurs when wires supplying power to a user are overloaded with too great a current. As discussed in the previous section, the power dissipated in the supply wires is $P = I^2 R_{\mathrm{w}}$, where R_{w} is the resistance of the wires and I the current flowing through them. If either I or R_{w} is too large, the wires overheat. For example, a worn appliance cord (with some of its braided wires broken) may have $R_{\mathrm{w}} = 2.00\ \Omega$ rather than the $0.100\ \Omega$ it should be. If 10.0 A of current passes through the cord, then $P = I^2 R_{\mathrm{w}} = 200$ W is dissipated in the cord—much more than is safe. Similarly, if a wire with a $0.100\text{ - }\Omega$

resistance is meant to carry a few amps, but is instead carrying 100 A, it will severely overheat. The power dissipated in the wire will in that case be $P = 1000\text{ W}$. Fuses and circuit breakers are used to limit excessive currents. (See [link] and [link].) Each device opens the circuit automatically when a sustained current exceeds safe limits.



(a) A fuse has a metal strip with a low melting point that, when overheated by an excessive current, permanently breaks the connection of a circuit to a voltage source. (b) A circuit breaker is an automatic but restorable electric switch. The one shown here has a bimetallic strip that bends to the right and

into the notch if overheated. The spring then forces the metal strip downward, breaking the electrical connection at the points.



Schematic of a circuit with a fuse or circuit breaker in it. Fuses and circuit breakers act like automatic switches that open when sustained current exceeds desired limits.

Fuses and circuit breakers for typical household voltages and currents are relatively simple to produce, but those for large voltages and currents experience special problems. For example, when a circuit breaker tries to interrupt the flow of high-voltage electricity, a spark can jump across its points that ionizes the air in the gap and allows the current to continue

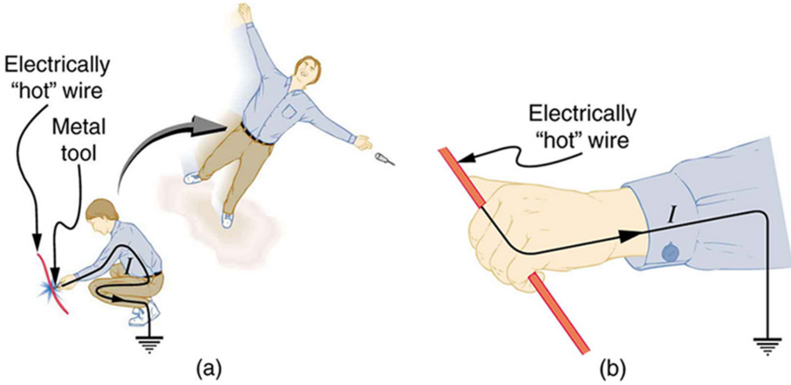
flowing. Large circuit breakers found in power-distribution systems employ insulating gas and even use jets of gas to blow out such sparks. Here AC is safer than DC, since AC current goes through zero 120 times per second, giving a quick opportunity to extinguish these arcs.

Shock Hazards

Electrical currents through people produce tremendously varied effects. An electrical current can be used to block back pain. The possibility of using electrical current to stimulate muscle action in paralyzed limbs, perhaps allowing paraplegics to walk, is under study. TV dramatizations in which electrical shocks are used to bring a heart attack victim out of ventricular fibrillation (a massively irregular, often fatal, beating of the heart) are more than common. Yet most electrical shock fatalities occur because a current put the heart into fibrillation. A pacemaker uses electrical shocks to stimulate the heart to beat properly. Some fatal shocks do not produce burns, but warts can be safely burned off with electric current (though freezing using liquid nitrogen is now more common). Of course, there are consistent explanations for these disparate effects. The major factors upon which the effects of electrical shock depend are

1. The amount of current I
2. The path taken by the current
3. The duration of the shock
4. The frequency f of the current ($f = 0$ for DC)

[link] gives the effects of electrical shocks as a function of current for a typical accidental shock. The effects are for a shock that passes through the trunk of the body, has a duration of 1 s, and is caused by 60-Hz power.



An electric current can cause muscular contractions with varying effects. (a) The victim is "thrown" backward by involuntary muscle contractions that extend the legs and torso. (b) The victim can't let go of the wire that is stimulating all the muscles in the hand. Those that close the fingers are stronger than those that open them.

Current (mA)	**Effect**
1	Threshold of sensation
5	Maximum harmless current
10–20	Onset of sustained muscular contraction; cannot let go for duration of shock; contraction of chest muscles may stop breathing during shock

Current (mA)	Effect
50	Onset of pain
100–300+	Ventricular fibrillation possible; often fatal
300	Onset of burns depending on concentration of current
6000 (6 A)	Onset of sustained ventricular contraction and respiratory paralysis; both cease when shock ends; heartbeat may return to normal; used to defibrillate the heart

Effects of Electrical Shock as a Function of Current[footnote]
For an average male shocked through trunk of body for 1 s by 60-Hz AC. Values for females are 60–80% of those listed.

Our bodies are relatively good conductors due to the water in our bodies. Given that larger currents will flow through sections with lower resistance (to be further discussed in the next chapter), electric currents preferentially flow through paths in the human body that have a minimum resistance in a direct path to earth. The earth is a natural electron sink. Wearing insulating shoes, a requirement in many professions, prohibits a pathway for electrons by providing a large resistance in that path. Whenever working with high-power tools (drills), or in risky situations, ensure that you do not provide a pathway for current flow (especially through the heart).

Very small currents pass harmlessly and unfelt through the body. This happens to you regularly without your knowledge. The threshold of sensation is only 1 mA and, although unpleasant, shocks are apparently harmless for currents less than 5 mA. A great number of safety rules take the 5-mA value for the maximum allowed shock. At 10 to 20 mA and above, the current can stimulate sustained muscular contractions much as regular nerve impulses do. People sometimes say they were knocked across the room by a shock, but what really happened was that certain muscles

contracted, propelling them in a manner not of their own choosing. (See [link](a).) More frightening, and potentially more dangerous, is the “can’t let go” effect illustrated in [link](b). The muscles that close the fingers are stronger than those that open them, so the hand closes involuntarily on the wire shocking it. This can prolong the shock indefinitely. It can also be a danger to a person trying to rescue the victim, because the rescuer’s hand may close about the victim’s wrist. Usually the best way to help the victim is to give the fist a hard knock/blow/jar with an insulator or to throw an insulator at the fist. Modern electric fences, used in animal enclosures, are now pulsed on and off to allow people who touch them to get free, rendering them less lethal than in the past.

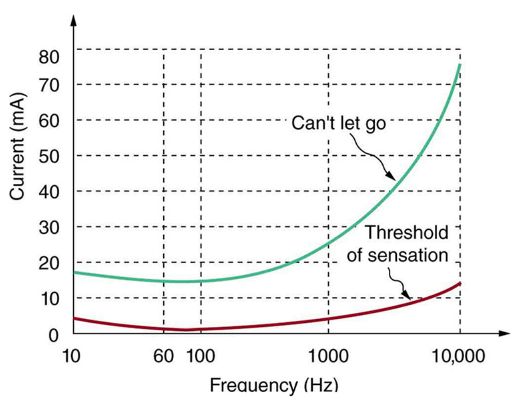
Greater currents may affect the heart. Its electrical patterns can be disrupted, so that it beats irregularly and ineffectively in a condition called “ventricular fibrillation.” This condition often lingers after the shock and is fatal due to a lack of blood circulation. The threshold for ventricular fibrillation is between 100 and 300 mA. At about 300 mA and above, the shock can cause burns, depending on the concentration of current—the more concentrated, the greater the likelihood of burns.

Very large currents cause the heart and diaphragm to contract for the duration of the shock. Both the heart and breathing stop. Interestingly, both often return to normal following the shock. The electrical patterns on the heart are completely erased in a manner that the heart can start afresh with normal beating, as opposed to the permanent disruption caused by smaller currents that can put the heart into ventricular fibrillation. The latter is something like scribbling on a blackboard, whereas the former completely erases it. TV dramatizations of electric shock used to bring a heart attack victim out of ventricular fibrillation also show large paddles. These are used to spread out current passed through the victim to reduce the likelihood of burns.

Current is the major factor determining shock severity (given that other conditions such as path, duration, and frequency are fixed, such as in the table and preceding discussion). A larger voltage is more hazardous, but since $I = \mathrm{V/R}$, the severity of the shock depends on the combination of voltage and resistance. For example, a person with dry skin has a resistance

of about $200\ \text{k}\Omega$. If he comes into contact with 120-V AC, a current $I=(120\ \text{V})/(200\ \text{k}\Omega)=0.6\ \text{mA}$ passes harmlessly through him. The same person soaking wet may have a resistance of $10.0\ \text{k}\Omega$ and the same 120 V will produce a current of 12 mA—above the "can't let go" threshold and potentially dangerous.

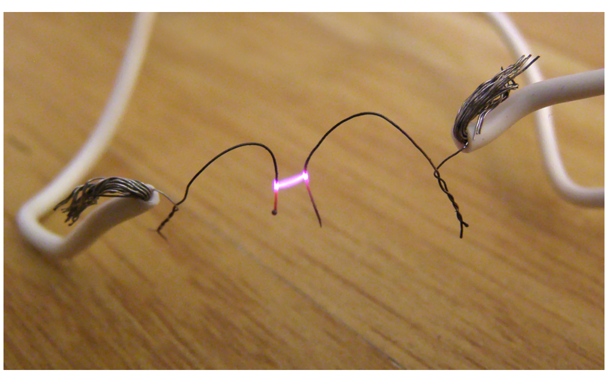
Most of the body's resistance is in its dry skin. When wet, salts go into ion form, lowering the resistance significantly. The interior of the body has a much lower resistance than dry skin because of all the ionic solutions and fluids it contains. If skin resistance is bypassed, such as by an intravenous infusion, a catheter, or exposed pacemaker leads, a person is rendered **microshock sensitive**. In this condition, currents about 1/1000 those listed in [link] produce similar effects. During open-heart surgery, currents as small as $20\ \mu\text{A}$ can be used to still the heart. Stringent electrical safety requirements in hospitals, particularly in surgery and intensive care, are related to the doubly disadvantaged microshock-sensitive patient. The break in the skin has reduced his resistance, and so the same voltage causes a greater current, and a much smaller current has a greater effect.



Graph of average values for the threshold of sensation and the "can't let go" current as a function of frequency. The lower the value, the

more sensitive the body is at that frequency.

Factors other than current that affect the severity of a shock are its path, duration, and AC frequency. Path has obvious consequences. For example, the heart is unaffected by an electric shock through the brain, such as may be used to treat manic depression. And it is a general truth that the longer the duration of a shock, the greater its effects. [link] presents a graph that illustrates the effects of frequency on a shock. The curves show the minimum current for two different effects, as a function of frequency. The lower the current needed, the more sensitive the body is at that frequency. Ironically, the body is most sensitive to frequencies near the 50- or 60-Hz frequencies in common use. The body is slightly less sensitive for DC ($f = 0$), mildly confirming Edison's claims that AC presents a greater hazard. At higher and higher frequencies, the body becomes progressively less sensitive to any effects that involve nerves. This is related to the maximum rates at which nerves can fire or be stimulated. At very high frequencies, electrical current travels only on the surface of a person. Thus a wart can be burned off with very high frequency current without causing the heart to stop. (Do not try this at home with 60-Hz AC!) Some of the spectacular demonstrations of electricity, in which high-voltage arcs are passed through the air and over people's bodies, employ high frequencies and low currents. (See [link].)



Is this electric arc dangerous? The answer depends on the AC

frequency and the power involved. (credit: Khimich Alex, Wikimedia Commons)

Section Summary

- The two types of electric hazards are thermal (excessive power) and shock (current through a person).
- Shock severity is determined by current, path, duration, and AC frequency.
- [link] lists shock hazards as a function of current.
- [link] graphs the threshold current for two hazards as a function of frequency.

Conceptual Questions

Exercise:

Problem:

Using an ohmmeter, a student measures the resistance between various points on his body. He finds that the resistance between two points on the same finger is about the same as the resistance between two points on opposite hands—both are several hundred thousand ohms. Furthermore, the resistance decreases when more skin is brought into contact with the probes of the ohmmeter. Finally, there is a dramatic drop in resistance (to a few thousand ohms) when the skin is wet. Explain these observations and their implications regarding skin and internal resistance of the human body.

Exercise:

Problem: What are the two major hazards of electricity?

Exercise:

Problem: Why isn’t a short circuit a shock hazard?

Exercise:

Problem:

What determines the severity of a shock? Can you say that a certain voltage is hazardous without further information?

Exercise:

Problem:

An electrified needle is used to burn off warts, with the circuit being completed by having the patient sit on a large butt plate. Why is this plate large?

Exercise:

Problem:

Some surgery is performed with high-voltage electricity passing from a metal scalpel through the tissue being cut. Considering the nature of electric fields at the surface of conductors, why would you expect most of the current to flow from the sharp edge of the scalpel? Do you think high- or low-frequency AC is used?

Exercise:

Problem:

Some devices often used in bathrooms, such as hairdryers, often have safety messages saying “Do not use when the bathtub or basin is full of water.” Why is this so?

Exercise:

Problem:

We are often advised to not flick electric switches with wet hands, dry your hand first. We are also advised to never throw water on an electric fire. Why is this so?

Exercise:

Problem:

Before working on a power transmission line, linemen will touch the line with the back of the hand as a final check that the voltage is zero. Why the back of the hand?

Exercise:

Problem:

Why is the resistance of wet skin so much smaller than dry, and why do blood and other bodily fluids have low resistances?

Exercise:

Problem:

Could a person on intravenous infusion (an IV) be microshock sensitive?

Exercise:

Problem:

In view of the small currents that cause shock hazards and the larger currents that circuit breakers and fuses interrupt, how do they play a role in preventing shock hazards?

Problem Exercises

Exercise:

Problem:

(a) How much power is dissipated in a short circuit of 240-V AC through a resistance of $0.250\ \Omega$? (b) What current flows?

Solution:

(a) 230 kW

(b) 960 A

Exercise:

Problem:

What voltage is involved in a 1.44-kW short circuit through a 0.100 - Ω resistance?

Exercise:

Problem:

Find the current through a person and identify the likely effect on her if she touches a 120-V AC source: (a) if she is standing on a rubber mat and offers a total resistance of $300\ \text{k}\Omega$; (b) if she is standing barefoot on wet grass and has a resistance of only $4000\ \text{k}\Omega$.

Solution:

(a) 0.400 mA, no effect

(b) 26.7 mA, muscular contraction for duration of the shock (can't let go)

Exercise:

Problem:

While taking a bath, a person touches the metal case of a radio. The path through the person to the drainpipe and ground has a resistance of $4000\ \Omega$. What is the smallest voltage on the case of the radio that could cause ventricular fibrillation?

Exercise:

Problem:

Foolishly trying to fish a burning piece of bread from a toaster with a metal butter knife, a man comes into contact with 120-V AC. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?

Solution:

$1.20 \times 10^5\ \Omega$

Exercise:

Problem:

(a) During surgery, a current as small as $20.0\ \mu A$ applied directly to the heart may cause ventricular fibrillation. If the resistance of the exposed heart is $300\ \Omega$, what is the smallest voltage that poses this danger? (b) Does your answer imply that special electrical safety precautions are needed?

Exercise:

Problem:

(a) What is the resistance of a 220-V AC short circuit that generates a peak power of 96.8 kW? (b) What would the average power be if the voltage was 120 V AC?

Solution:

(a) $1.00\ \Omega$

(b) 14.4 kW

Exercise:

Problem:

A heart defibrillator passes 10.0 A through a patient's torso for 5.00 ms in an attempt to restore normal beating. (a) How much charge passed? (b) What voltage was applied if 500 J of energy was dissipated? (c) What was the path's resistance? (d) Find the temperature increase caused in the 8.00 kg of affected tissue.

Exercise:

Problem: **Integrated Concepts**

A short circuit in a 120-V appliance cord has a $0.500\text{-}\Omega$ resistance. Calculate the temperature rise of the 2.00 g of surrounding materials, assuming their specific heat capacity is $0.200 \text{ cal/g} \cdot^{\text{o}} \text{C}$ and that it takes 0.0500 s for a circuit breaker to interrupt the current. Is this likely to be damaging?

Solution:

Temperature increases 860^{o} C. It is very likely to be damaging.

Exercise:

Problem: **Construct Your Own Problem**

Consider a person working in an environment where electric currents might pass through her body. Construct a problem in which you calculate the resistance of insulation needed to protect the person from harm. Among the things to be considered are the voltage to which the person might be exposed, likely body resistance (dry, wet, …), and acceptable currents (safe but sensed, safe and unfelt, …).

Glossary

thermal hazard
: a hazard in which electric current causes undesired thermal effects

shock hazard
: when electric current passes through a person

short circuit
: also known as a “short,” a low-resistance path between terminals of a voltage source

microshock sensitive
: a condition in which a person’s skin resistance is bypassed, possibly by a medical procedure, rendering the person vulnerable to electrical shock at currents about 1/1000 the normally required level

Introduction to Magnetism
class="introduction"

The magnificent spectacle of the Aurora Borealis, or northern lights, glows in the northern sky above Bear Lake near Eielson Air Force Base, Alaska. Shaped by the Earth’s magnetic field, this light is produced by radiation spewed from solar storms. (credit: Senior Airman Joshua Strang, via Flickr)

One evening, an Alaskan sticks a note to his refrigerator with a small magnet. Through the kitchen window, the Aurora Borealis glows in the night sky. This grand spectacle is shaped by the same force that holds the note to the refrigerator.

People have been aware of magnets and magnetism for thousands of years. The earliest records date to well before the time of Christ, particularly in a region of Asia Minor called Magnesia (the name of this region is the source of words like *magnetic*). Magnetic rocks found in Magnesia, which is now part of western Turkey, stimulated interest during ancient times. A practical application for magnets was found later, when they were employed as navigational compasses. The use of magnets in compasses resulted not only in improved long-distance sailing, but also in the names of “north” and “south” being given to the two types of magnetic poles.

Today magnetism plays many important roles in our lives. Physicists’ understanding of magnetism has enabled the development of technologies that affect our everyday lives. The iPod in your purse or backpack, for example, wouldn’t have been possible without the applications of magnetism and electricity on a small scale.

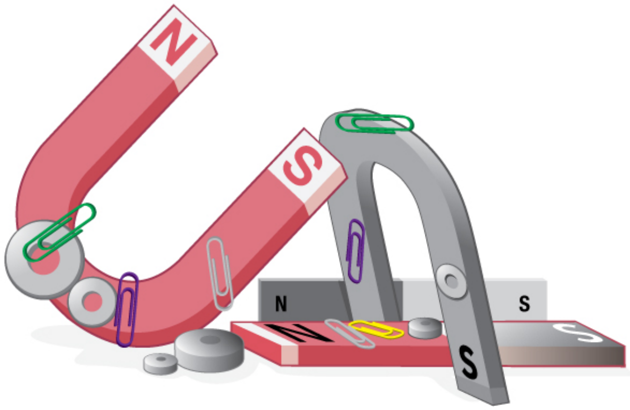
The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology. The 2007 Nobel Prize in Physics went to Albert Fert from France and Peter Grunberg from Germany for this discovery of *giant magnetoresistance* and its applications to computer memory.

All electric motors, with uses as diverse as powering refrigerators, starting cars, and moving elevators, contain magnets. Generators, whether producing hydroelectric power or running bicycle lights, use magnetic fields. Recycling facilities employ magnets to separate iron from other refuse. Hundreds of millions of dollars are spent annually on magnetic containment of fusion as a future energy source. Magnetic resonance imaging (MRI) has become an important diagnostic tool in the field of medicine, and the use of magnetism to explore brain activity is a subject of contemporary research and development. The list of applications also includes computer hard drives, tape recording, detection of inhaled asbestos, and levitation of high-speed trains. Magnetism is used to explain atomic energy levels, cosmic rays, and charged particles trapped in the Van Allen belts. Once again, we will find all these disparate phenomena are linked by a small number of underlying physical principles.

Engineering of technology like iPods would not be possible without a deep understanding magnetism. (credit: Jesse! S?, Flickr)

Magnets

- Describe the difference between the north and south poles of a magnet.
- Describe how magnetic poles interact with each other.

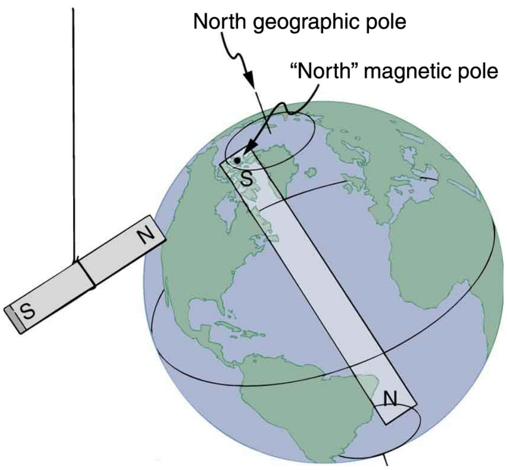


Magnets come in various shapes, sizes, and strengths. All have both a north pole and a south pole. There is never an isolated pole (a monopole).

All magnets attract iron, such as that in a refrigerator door. However, magnets may attract or repel other magnets. Experimentation shows that all magnets have two poles. If freely suspended, one pole will point toward the north. The two poles are thus named the **north magnetic pole** and the **south magnetic pole** (or more properly, north-seeking and south-seeking poles, for the attractions in those directions).

Note:
Universal Characteristics of Magnets and Magnetic Poles
It is a universal characteristic of all magnets that *like poles repel and unlike poles attract*. (Note the similarity with electrostatics: unlike charges attract and like charges repel.)

Further experimentation shows that it is *impossible to separate north and south poles* in the manner that + and − charges can be separated.



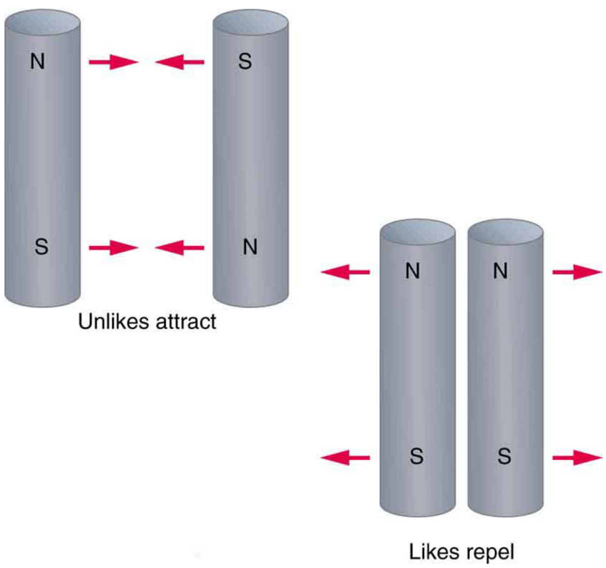
One end of a bar magnet is suspended from a thread that points toward north. The magnet's two poles are labeled N and S for north-seeking and south-seeking poles, respectively.

Note:

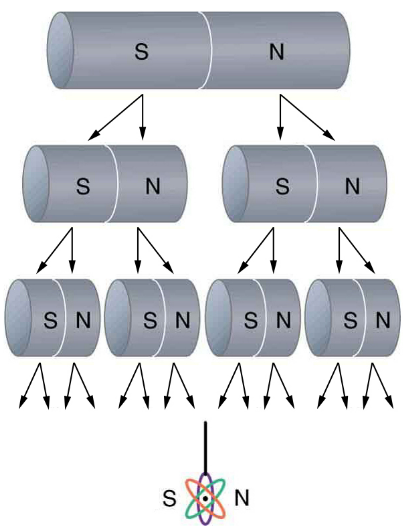
Misconception Alert: Earth's Geographic North Pole Hides an S

The Earth acts like a very large bar magnet with its south-seeking pole near the geographic North Pole. That is why the north pole of your compass is attracted toward the geographic north pole of the Earth—because the magnetic pole that is near the geographic North Pole is actually a south magnetic pole! Confusion arises because the geographic term "North Pole" has come to be used (incorrectly) for the magnetic pole that is near the

North Pole. Thus, “North magnetic pole” is actually a misnomer—it should be called the South magnetic pole.



Unlike poles attract, whereas like poles repel.



North and south poles always occur in pairs. Attempts

to separate them result in more pairs of poles. If we continue to split the magnet, we will eventually get down to an iron atom with a north pole and a south pole—these, too, cannot be separated.

The fact that magnetic poles always occur in pairs of north and south is true from the very large scale—for example, sunspots always occur in pairs that are north and south magnetic poles—all the way down to the very small scale. Magnetic atoms have both a north pole and a south pole, as do many types of subatomic particles, such as electrons, protons, and neutrons.

Note:
Making Connections: Take-Home Experiment—Refrigerator Magnets
We know that like magnetic poles repel and unlike poles attract. See if you can show this for two refrigerator magnets. Will the magnets stick if you turn them over? Why do they stick to the door anyway? What can you say about the magnetic properties of the door next to the magnet? Do refrigerator magnets stick to metal or plastic spoons? Do they stick to all types of metal?

Section Summary

- Magnetism is a subject that includes the properties of magnets, the effect of the magnetic force on moving charges and currents, and the

creation of magnetic fields by currents.

- There are two types of magnetic poles, called the north magnetic pole and south magnetic pole.
- North magnetic poles are those that are attracted toward the Earth’s geographic north pole.
- Like poles repel and unlike poles attract.
- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.

Conceptual Questions

Exercise:

Problem:

Volcanic and other such activity at the mid-Atlantic ridge extrudes material to fill the gap between separating tectonic plates associated with continental drift. The magnetization of rocks is found to reverse in a coordinated manner with distance from the ridge. What does this imply about the Earth’s magnetic field and how could the knowledge of the spreading rate be used to give its historical record?

Glossary

north magnetic pole
: the end or the side of a magnet that is attracted toward Earth’s geographic north pole

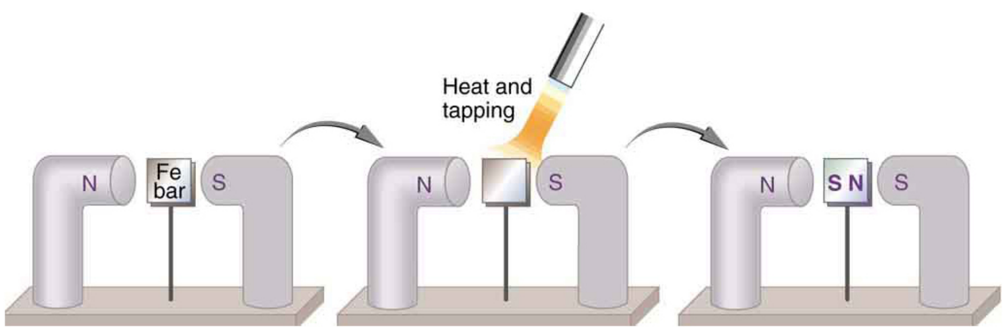
south magnetic pole
: the end or the side of a magnet that is attracted toward Earth’s geographic south pole

Ferromagnets and Electromagnets

- Define ferromagnet.
- Describe the role of magnetic domains in magnetization.
- Explain the significance of the Curie temperature.
- Describe the relationship between electricity and magnetism.

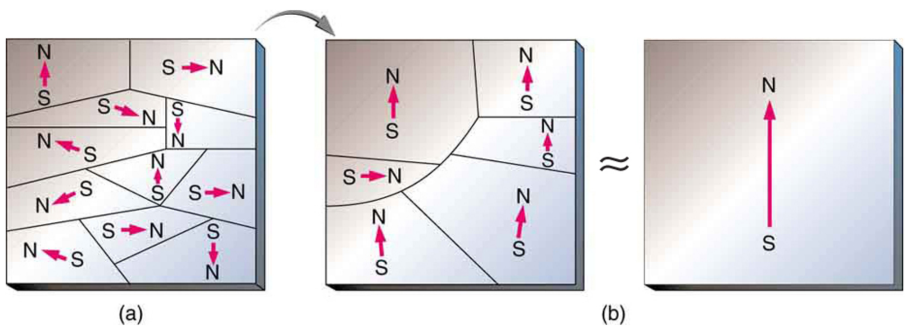
Ferromagnets

Only certain materials, such as iron, cobalt, nickel, and gadolinium, exhibit strong magnetic effects. Such materials are called **ferromagnetic**, after the Latin word for iron, *ferrum*. A group of materials made from the alloys of the rare earth elements are also used as strong and permanent magnets; a popular one is neodymium. Other materials exhibit weak magnetic effects, which are detectable only with sensitive instruments. Not only do ferromagnetic materials respond strongly to magnets (the way iron is attracted to magnets), they can also be **magnetized** themselves—that is, they can be induced to be magnetic or made into permanent magnets.



An unmagnetized piece of iron is placed between two magnets, heated, and then cooled, or simply tapped when cold. The iron becomes a permanent magnet with the poles aligned as shown: its south pole is adjacent to the north pole of the original magnet, and its north pole is adjacent to the south pole of the original magnet. Note that there are attractive forces between the magnets.

When a magnet is brought near a previously unmagnetized ferromagnetic material, it causes local magnetization of the material with unlike poles closest, as in [link]. (This results in the attraction of the previously unmagnetized material to the magnet.) What happens on a microscopic scale is illustrated in [link]. The regions within the material called **domains** act like small bar magnets. Within domains, the poles of individual atoms are aligned. Each atom acts like a tiny bar magnet. Domains are small and randomly oriented in an unmagnetized ferromagnetic object. In response to an external magnetic field, the domains may grow to millimeter size, aligning themselves as shown in [link](b). This induced magnetization can be made permanent if the material is heated and then cooled, or simply tapped in the presence of other magnets.



(a) An unmagnetized piece of iron (or other ferromagnetic material) has randomly oriented domains. (b) When magnetized by an external field, the domains show greater alignment, and some grow at the expense of others. Individual atoms are aligned within domains; each atom acts like a tiny bar magnet.

Conversely, a permanent magnet can be demagnetized by hard blows or by heating it in the absence of another magnet. Increased thermal motion at higher temperature can disrupt and randomize the orientation and the size of

the domains. There is a well-defined temperature for ferromagnetic materials, which is called the **Curie temperature**, above which they cannot be magnetized. The Curie temperature for iron is 1043 K $(770^{\circ}C)$, which is well above room temperature. There are several elements and alloys that have Curie temperatures much lower than room temperature and are ferromagnetic only below those temperatures.

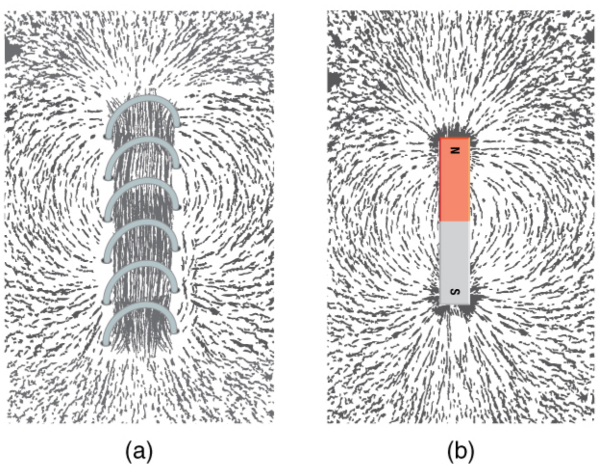
Electromagnets

Early in the 19th century, it was discovered that electrical currents cause magnetic effects. The first significant observation was by the Danish scientist Hans Christian Oersted (1777–1851), who found that a compass needle was deflected by a current-carrying wire. This was the first significant evidence that the movement of charges had any connection with magnets. **Electromagnetism** is the use of electric current to make magnets. These temporarily induced magnets are called **electromagnets**. Electromagnets are employed for everything from a wrecking yard crane that lifts scrapped cars to controlling the beam of a 90-km-circumference particle accelerator to the magnets in medical imaging machines (See [link]).

Instrument for magnetic resonance imaging (MRI). The device uses a superconducting

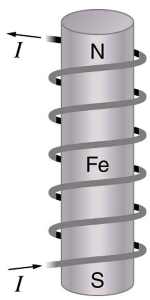
cylindrical coil for the main magnetic field. The patient goes into this “tunnel” on the gurney. (credit: Bill McChesney, Flickr)

[link] shows that the response of iron filings to a current-carrying coil and to a permanent bar magnet. The patterns are similar. In fact, electromagnets and ferromagnets have the same basic characteristics—for example, they have north and south poles that cannot be separated and for which like poles repel and unlike poles attract.



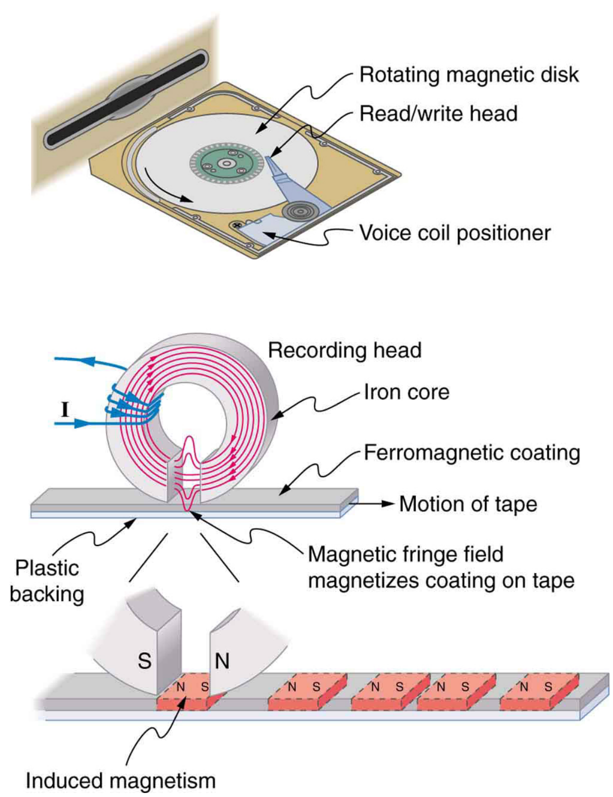
Iron filings near (a) a current-carrying coil and (b) a magnet act like tiny compass needles, showing the shape of their fields. Their response to a current-carrying coil and a permanent magnet is seen to be very similar, especially near the ends of the coil and the magnet.

Combining a ferromagnet with an electromagnet can produce particularly strong magnetic effects. (See [link].) Whenever strong magnetic effects are needed, such as lifting scrap metal, or in particle accelerators, electromagnets are enhanced by ferromagnetic materials. Limits to how strong the magnets can be made are imposed by coil resistance (it will overheat and melt at sufficiently high current), and so superconducting magnets may be employed. These are still limited, because superconducting properties are destroyed by too great a magnetic field.



An electromagnet with a ferromagnetic core can produce very strong magnetic effects. Alignment of domains in the core produces a magnet, the poles of which are aligned with the electromagnet.

[link] shows a few uses of combinations of electromagnets and ferromagnets. Ferromagnetic materials can act as memory devices, because the orientation of the magnetic fields of small domains can be reversed or erased. Magnetic information storage on videotapes and computer hard drives are among the most common applications. This property is vital in our digital world.



An electromagnet induces regions of permanent magnetism on a floppy disk coated with a ferromagnetic material. The information stored here is digital (a region is either magnetic or not); in other applications, it can be analog

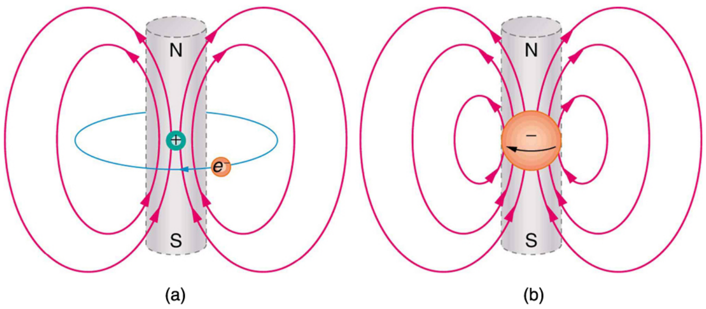
(with a varying strength), such as on audiotapes.

Current: The Source of All Magnetism

An electromagnet creates magnetism with an electric current. In later sections we explore this more quantitatively, finding the strength and direction of magnetic fields created by various currents. But what about ferromagnets? [link] shows models of how electric currents create magnetism at the submicroscopic level. (Note that we cannot directly observe the paths of individual electrons about atoms, and so a model or visual image, consistent with all direct observations, is made. We can directly observe the electron's orbital angular momentum, its spin momentum, and subsequent magnetic moments, all of which are explained with electric-current-creating subatomic magnetism.) Currents, including those associated with other submicroscopic particles like protons, allow us to explain ferromagnetism and all other magnetic effects. Ferromagnetism, for example, results from an internal cooperative alignment of electron spins, possible in some materials but not in others.

Crucial to the statement that electric current is the source of all magnetism is the fact that it is impossible to separate north and south magnetic poles. (This is far different from the case of positive and negative charges, which are easily separated.) A current loop always produces a magnetic dipole—that is, a magnetic field that acts like a north pole and south pole pair. Since isolated north and south magnetic poles, called **magnetic monopoles**, are not observed, currents are used to explain all magnetic effects. If magnetic monopoles did exist, then we would have to modify this underlying connection that all magnetism is due to electrical current. There is no known reason that magnetic monopoles should not exist—they are simply never observed—and so searches at the subnuclear level continue. If they do *not* exist, we would like to find out why not. If they *do* exist, we would like to see evidence of them.

Note:
Electric Currents and Magnetism
Electric current is the source of all magnetism.



(a) In the planetary model of the atom, an electron orbits a nucleus, forming a closed-current loop and producing a magnetic field with a north pole and a south pole. (b) Electrons have spin and can be crudely pictured as rotating charge, forming a current that produces a magnetic field with a north pole and a south pole. Neither the planetary model nor the image of a spinning electron is completely consistent with modern physics. However, they do provide a useful way of understanding phenomena.

Section Summary

- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.
- All magnetism is created by electric current.

- Ferromagnetic materials, such as iron, are those that exhibit strong magnetic effects.
- The atoms in ferromagnetic materials act like small magnets (due to currents within the atoms) and can be aligned, usually in millimeter-sized regions called domains.
- Domains can grow and align on a larger scale, producing permanent magnets. Such a material is magnetized, or induced to be magnetic.
- Above a material’s Curie temperature, thermal agitation destroys the alignment of atoms, and ferromagnetism disappears.
- Electromagnets employ electric currents to make magnetic fields, often aided by induced fields in ferromagnetic materials.

Glossary

ferromagnetic
: materials, such as iron, cobalt, nickel, and gadolinium, that exhibit strong magnetic effects

magnetized
: to be turned into a magnet; to be induced to be magnetic

domains
: regions within a material that behave like small bar magnets

Curie temperature
: the temperature above which a ferromagnetic material cannot be magnetized

electromagnetism
: the use of electrical currents to induce magnetism

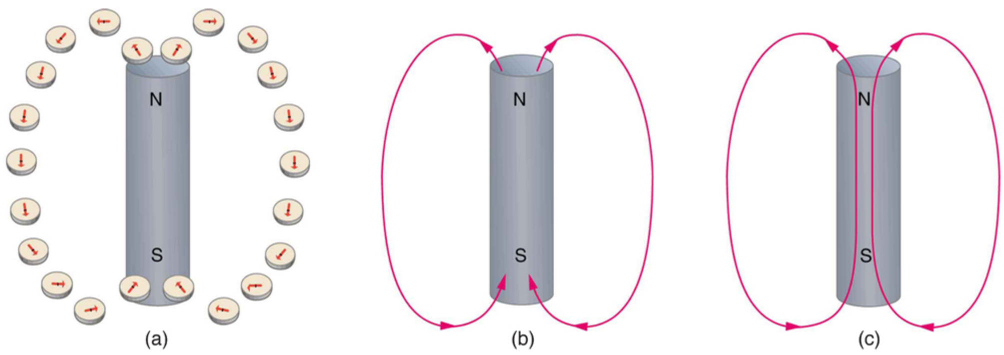
electromagnet
: an object that is temporarily magnetic when an electrical current is passed through it

magnetic monopoles
: an isolated magnetic pole; a south pole without a north pole, or vice versa (no magnetic monopole has ever been observed)

Magnetic Fields and Magnetic Field Lines

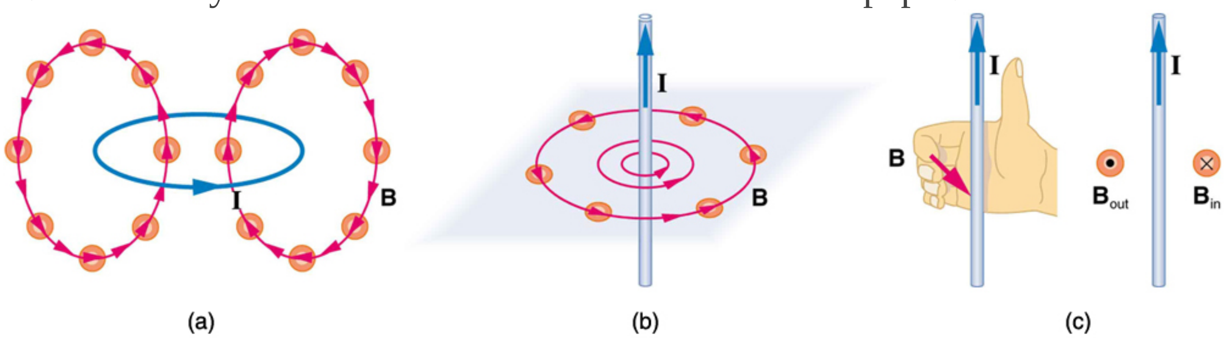
- Define magnetic field and describe the magnetic field lines of various magnetic fields.

Einstein is said to have been fascinated by a compass as a child, perhaps musing on how the needle felt a force without direct physical contact. His ability to think deeply and clearly about action at a distance, particularly for gravitational, electric, and magnetic forces, later enabled him to create his revolutionary theory of relativity. Since magnetic forces act at a distance, we define a **magnetic field** to represent magnetic forces. The pictorial representation of **magnetic field lines** is very useful in visualizing the strength and direction of the magnetic field. As shown in [link], the **direction of magnetic field lines** is defined to be the direction in which the north end of a compass needle points. The magnetic field is traditionally called the ***B*-field**.



Magnetic field lines are defined to have the direction that a small compass points when placed at a location. (a) If small compasses are used to map the magnetic field around a bar magnet, they will point in the directions shown: away from the north pole of the magnet, toward the south pole of the magnet. (Recall that the Earth's north magnetic pole is really a south pole in terms of definitions of poles on a bar magnet.) (b) Connecting the arrows gives continuous magnetic field lines. The strength of the field is proportional to the closeness (or density) of the lines. (c) If the interior of the magnet could be probed, the field lines would be found to form continuous closed loops.

Small compasses used to test a magnetic field will not disturb it. (This is analogous to the way we tested electric fields with a small test charge. In both cases, the fields represent only the object creating them and not the probe testing them.) [link] shows how the magnetic field appears for a current loop and a long straight wire, as could be explored with small compasses. A small compass placed in these fields will align itself parallel to the field line at its location, with its north pole pointing in the direction of *B*. Note the symbols used for field into and out of the paper.



Small compasses could be used to map the fields shown here. (a) The magnetic field of a circular current loop is similar to that of a bar magnet. (b) A long and straight wire creates a field with magnetic field lines forming circular loops. (c) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note that the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward (like the tip of an arrow).

Note:
Making Connections: Concept of a Field
A field is a way of mapping forces surrounding any object that can act on another object at a distance without apparent physical connection. The field represents the object generating it. Gravitational fields map

gravitational forces, electric fields map electrical forces, and magnetic fields map magnetic forces.

Extensive exploration of magnetic fields has revealed a number of hard-and-fast rules. We use magnetic field lines to represent the field (the lines are a pictorial tool, not a physical entity in and of themselves). The properties of magnetic field lines can be summarized by these rules:

1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.
2. The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).
3. Magnetic field lines can never cross, meaning that the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without beginning or end. They go from the north pole to the south pole.

The last property is related to the fact that the north and south poles cannot be separated. It is a distinct difference from electric field lines, which begin and end on the positive and negative charges. If magnetic monopoles existed, then magnetic field lines would begin and end on them.

Section Summary

- Magnetic fields can be pictorially represented by magnetic field lines, the properties of which are as follows:

1. The field is tangent to the magnetic field line.
2. Field strength is proportional to the line density.
3. Field lines cannot cross.
4. Field lines are continuous loops.

Conceptual Questions

Exercise:

Problem:

Explain why the magnetic field would not be unique (that is, not have a single value) at a point in space where magnetic field lines might cross. (Consider the direction of the field at such a point.)

Exercise:

Problem:

List the ways in which magnetic field lines and electric field lines are similar. For example, the field direction is tangent to the line at any point in space. Also list the ways in which they differ. For example, electric force is parallel to electric field lines, whereas magnetic force on moving charges is perpendicular to magnetic field lines.

Exercise:

Problem:

Noting that the magnetic field lines of a bar magnet resemble the electric field lines of a pair of equal and opposite charges, do you expect the magnetic field to rapidly decrease in strength with distance from the magnet? Is this consistent with your experience with magnets?

Exercise:

Problem:

Is the Earth’s magnetic field parallel to the ground at all locations? If not, where is it parallel to the surface? Is its strength the same at all locations? If not, where is it greatest?

Glossary

magnetic field
: the representation of magnetic forces

B-field
another term for magnetic field

magnetic field lines
the pictorial representation of the strength and the direction of a magnetic field

direction of magnetic field lines
the direction that the north end of a compass needle points

Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

- Describe the effects of magnetic fields on moving charges.
- Use the right hand rule 1 to determine the velocity of a charge, the direction of the magnetic field, and the direction of the magnetic force on a moving charge.
- Calculate the magnetic force on a moving charge.

What is the mechanism by which one magnet exerts a force on another? The answer is related to the fact that all magnetism is caused by current, the flow of charge. *Magnetic fields exert forces on moving charges*, and so they exert forces on other magnets, all of which have moving charges.

Magnetic Force on a Moving Charge

The magnetic force on a moving charge is one of the most fundamental known. Magnetic force is as important as the electrostatic or Coulomb force. Yet the magnetic force is more complex, in both the number of factors that affects it and in its direction, than the relatively simple Coulomb force.

The magnitude of the magnetic force F on a charge depends on: the quantity of charge q, its speed v, the strength of magnetic field B, and **the direction of motion relative to the magnetic field's direction**. Motion, and its direction, are critical.

The **maximum** force occurs when the direction of motion and the magnetic field's direction are *perpendicular* to one another (i.e. ninety degree angle between directions).

$$\mathbf{v} \perp \mathbf{B}$$

In that situation, the magnitude of the magnetic force is

$$F = qvB$$

The **minimum** force occurs when the direction of motion and the magnetic field's direction are *parallel* to one another (i.e. zero or 180 degree angle

between directions).

$$\mathbf{v} \parallel \mathbf{B}$$

In that situation, the magnitude of the magnetic force is

$$F = 0$$

We define the magnetic field strength B in terms of the force on a charged particle moving in a magnetic field. The SI unit for magnetic field strength B is called the **tesla** (T) after the eccentric but brilliant inventor Nikola Tesla (1856–1943). To determine how the tesla relates to other SI units, we solve for the magnetic field strength.
Equation:

$$B = \frac{F}{qv}$$

So, the tesla is
Equation:

$$1\ \mathrm{T} = \frac{1\ \mathrm{N}}{\mathrm{C} \cdot \mathrm{m/s}} = \frac{1\ \mathrm{N}}{\mathrm{A} \cdot \mathrm{m}}$$

(note that C/s = A).

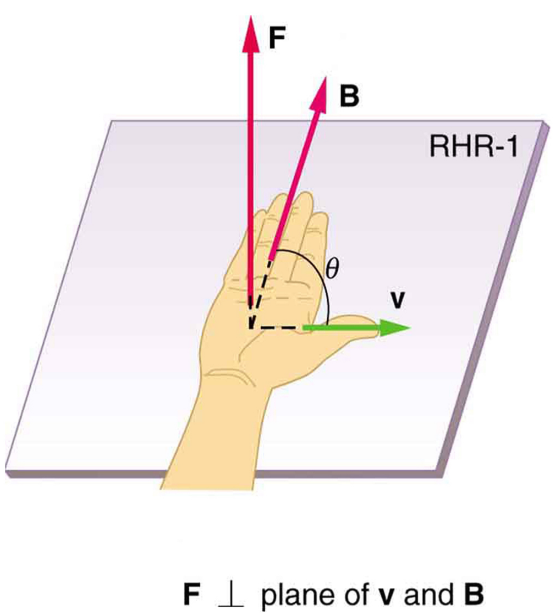
Another smaller unit, called the **gauss** (G), where $1\ \mathrm{G} = 10^{-4}\ \mathrm{T}$, is sometimes used. The strongest permanent magnets have fields near 2 T; superconducting electromagnets may attain 10 T or more. The Earth's magnetic field on its surface is only about $5 \times 10^{-5}\ \mathrm{T}$, or 0.5 G.

Note:
Making Connections: Charges and Magnets
There is no magnetic force on static charges. However, there is a magnetic force on moving charges. When charges are stationary, their electric fields

do not affect magnets. But, when charges move, they produce magnetic fields that exert forces on other magnets. When there is relative motion, a connection between electric and magnetic fields emerges—each affects the other.

Direction of Force: Right Hand Rule 1

The *direction* of the magnetic force $\mathbf{F}$ is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$, as determined by the **right hand rule 1** (or RHR-1), which is illustrated in [link]. RHR-1 states that, to determine the direction of the magnetic force on a positive moving charge, you point the thumb of the right hand in the direction of $\mathbf{v}$, the fingers in the direction of $\mathbf{B}$, and a perpendicular to the palm points in the direction of $\mathbf{F}$. One way to remember this is that there is one velocity, and so the thumb represents it. There are many field lines, and so the fingers represent them. The force is in the direction you would push with your palm. The force on a negative charge is in exactly the opposite direction to that on a positive charge.



Magnetic fields exert forces on moving charges. This

force is one of the most basic known. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$ and follows right hand rule–1 (RHR-1) as shown. The magnitude of the force is proportional to q, v, B, and depends on the angle between $\mathbf{v}$ and $\mathbf{B}$.

Section Summary

- The maximum force a magnetic field can exert on a moving charge is

$$F = qvB$$

- The SI unit for magnetic field strength B is the tesla (T), which is related to other units by
  Equation:

$$1\ \mathrm{T} = \frac{1\ \mathrm{N}}{\mathrm{C} \cdot \mathrm{m/s}} = \frac{1\ \mathrm{N}}{\mathrm{A} \cdot \mathrm{m}}.$$

- The *direction* of the force on a moving charge is given by right hand rule 1 (RHR-1): Point the thumb of the right hand in the direction of v, the fingers in the direction of B, and a perpendicular to the palm points in the direction of F.
- The force is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$. Since the force is zero if $\mathbf{v}$ is parallel to $\mathbf{B}$, charged particles often follow magnetic field lines rather than cross them.

Conceptual Questions

Exercise:

Problem:

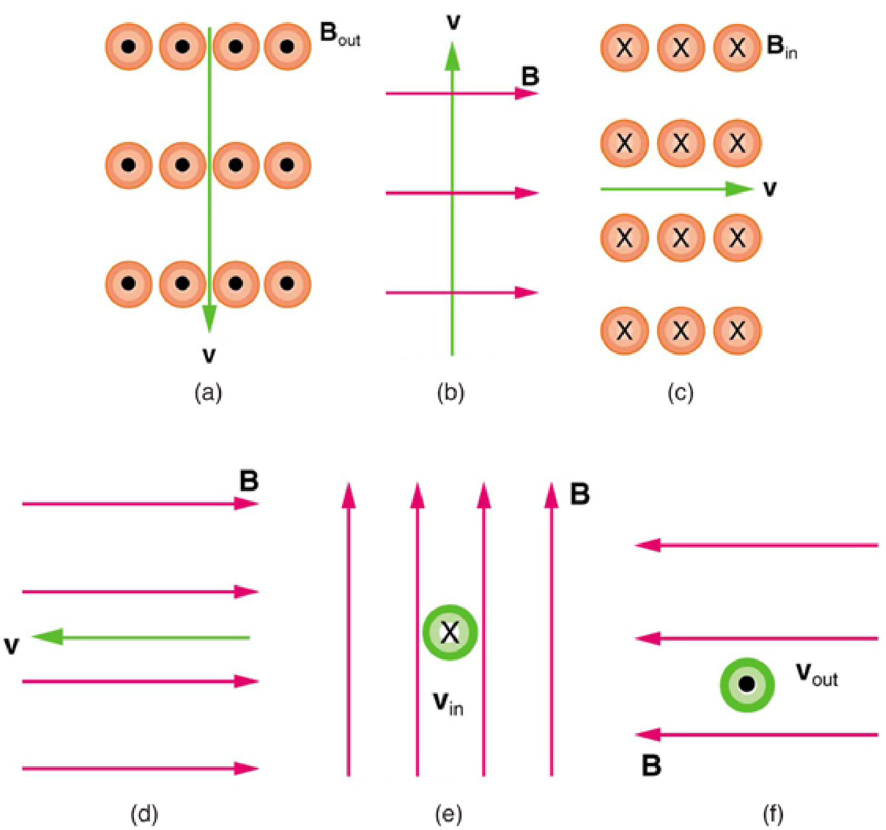
If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in that region is necessarily zero?

Problems & Exercises

Exercise:

Problem:

What is the direction of the magnetic force on a positive charge that moves as shown in each of the six cases shown in [link]? Note that ⊙ indicates "coming out of the page" and ⊗ means "going into the page."



Solution:

(a) Left (West)

(b) Into the page

(c) Up (North)

(d) No force

(e) Right (East)

(f) Down (South)

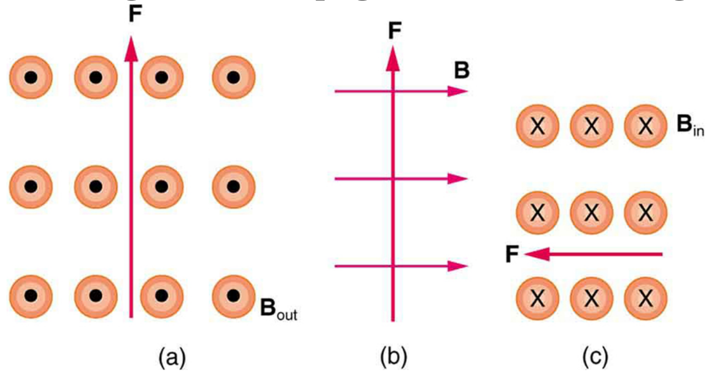
Exercise:

Problem: Repeat [link] for a negative charge.

Exercise:

Problem:

What is the direction of the velocity of a negative charge that experiences the magnetic force shown in each of the three cases in [link], assuming it moves perpendicular to $\mathbf{B}$? Note that $\odot$ indicates "coming out of the page" and $\otimes$ means "going into the page."



Solution:

(a) East (right)

(b) Into page

(c) South (down)

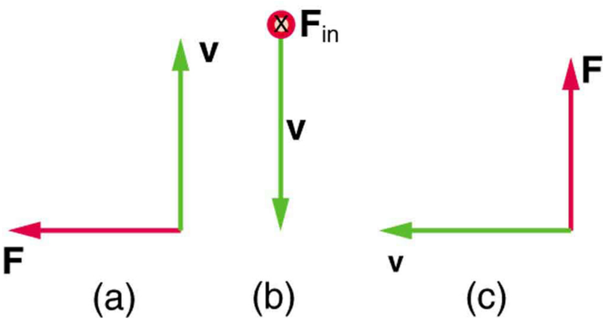
Exercise:

Problem: Repeat [link] for a positive charge.

Exercise:

Problem:

What is the direction of the magnetic field that produces the magnetic force on a positive charge as shown in each of the three cases in the figure below, assuming $\mathbf{B}$ is perpendicular to $\mathbf{v}$? Note that $\otimes$ means "going into the page."



Solution:

(a) Into page

(b) West (left)

(c) Out of page

Exercise:

Problem: Repeat [link] for a negative charge.

Exercise:

Problem:

What is the maximum force on an aluminum rod with a $0.100\text{-}\mu\text{C}$ charge that you pass between the poles of a 1.50-T permanent magnet at a speed of 5.00 m/s? In what direction is the force?

Solution:

7.50×10^{-7} N perpendicular to both the magnetic field lines and the velocity

Exercise:

Problem:

(a) Aircraft sometimes acquire small static charges. Suppose a supersonic jet has a $0.500\text{-}\mu\text{C}$ charge and flies due west at a speed of 660 m/s over the Earth's south magnetic pole, where the $8.00 \times 10^{-5}\text{-T}$ magnetic field points straight up. What are the direction and the magnitude of the magnetic force on the plane? (b) Discuss whether the value obtained in part (a) implies this is a significant or negligible effect.

Glossary

right hand rule 1 (RHR-1)
: the rule to determine the direction of the magnetic force on a positive moving charge: when the thumb of the right hand points in the direction of the charge's velocity **v** and the fingers point in the direction of the magnetic field **B**, then the force on the charge is perpendicular and away from the palm; the force on a negative charge is perpendicular and into the palm

tesla
: T, the SI unit of the magnetic field strength; $1\text{ T} = \frac{1\text{ N}}{\text{A}\cdot\text{m}}$

magnetic force

the force on a charge produced by its motion through a magnetic field

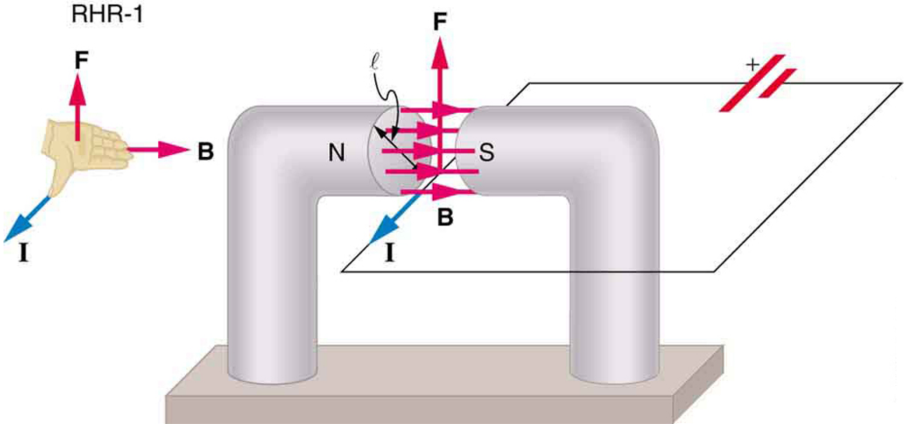
gauss

G, the unit of the magnetic field strength; $1\ \mathrm{G} = 10^{-4}\ \mathrm{T}$

Magnetic Force on a Current-Carrying Conductor

- Describe the effects of a magnetic force on a current-carrying conductor.
- Calculate the magnetic force on a current-carrying conductor.

Because charges ordinarily cannot escape a conductor, the magnetic force on charges moving in a conductor is transmitted to the conductor itself.



The magnetic field exerts a force on a current-carrying wire in a direction given by the right hand rule 1 (the same direction as that on the individual moving charges). This force can easily be large enough to move the wire, since typical currents consist of very large numbers of moving charges.

The maximum force on a current-carrying conductor occurs when the current direction and the magnetic field's direction are perpendicular to one another (i.e. ninety degree angle between directions). We can derive an expression for the maximum magnetic force on a current by taking a sum of the magnetic forces on individual charges. (The forces add because they are in the same direction.) The force on an individual charge moving at the drift velocity vd is given by $F = qv_dB$. Taking B to be uniform over a length of wire l and zero elsewhere, the total magnetic force on the wire is then $F = (qv_dB)(N)$, where N is the number of charge carriers in the section of wire of length l. Now, $N = nV$, where n is the number of charge

carriers per unit volume and V is the volume of wire in the field. Noting that $V = Al$, where A is the cross-sectional area of the wire, then the force on the wire is $F = (qv_d B)(nAl)$. Gathering terms,

Equation:

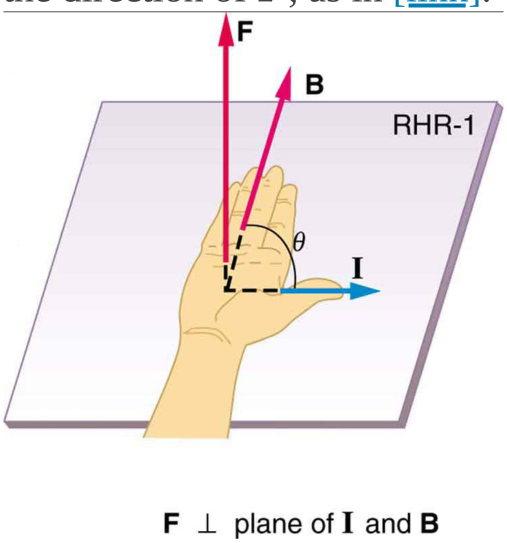
$$F = (nqAv_d)(lB).$$

Because $nqAv_\mathrm{d} = I$,

Equation:

$$F = IlB$$

is the equation for *maximum magnetic force on a length l of wire carrying a current I in a uniform magnetic field B*, as shown in [link]. If we divide both sides of this expression by l, we find that the magnetic force per unit length of wire in a uniform field is $\frac{F}{l} = IB$. The direction of this force is given by RHR-1, with the thumb in the direction of the current I. Then, with the fingers in the direction of B, a perpendicular to the palm points in the direction of F, as in [link].



The force on a current-carrying wire in a magnetic field is

$F = IlB$. Its direction is given by RHR-1.

Example:
Calculating Magnetic Force on a Current-Carrying Wire: A Strong Magnetic Field

Calculate the force on the wire shown in [link], given $B = 1.50\text{ T}$, $l = 5.00\text{ cm}$, and $I = 20.0\text{ A}$.

Strategy

The force can be found with the given information by using $F = IlB$ because the angle between I and B is 90°.

Solution

Entering the given values into $F = IlB$ yields

Equation:

$$F = IlB = (20.0\text{ A})(0.0500\text{ m})(1.50\text{ T}).$$

The units for tesla are $1\text{ T} = \frac{\text{N}}{\text{A}\cdot\text{m}}$; thus,

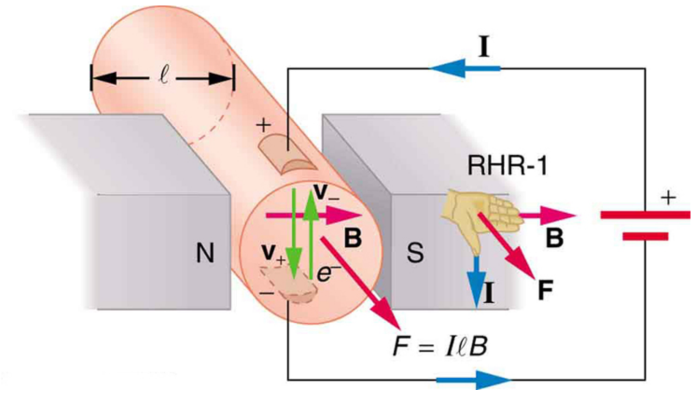
Equation:

$$F = 1.50\text{ N}.$$

Discussion

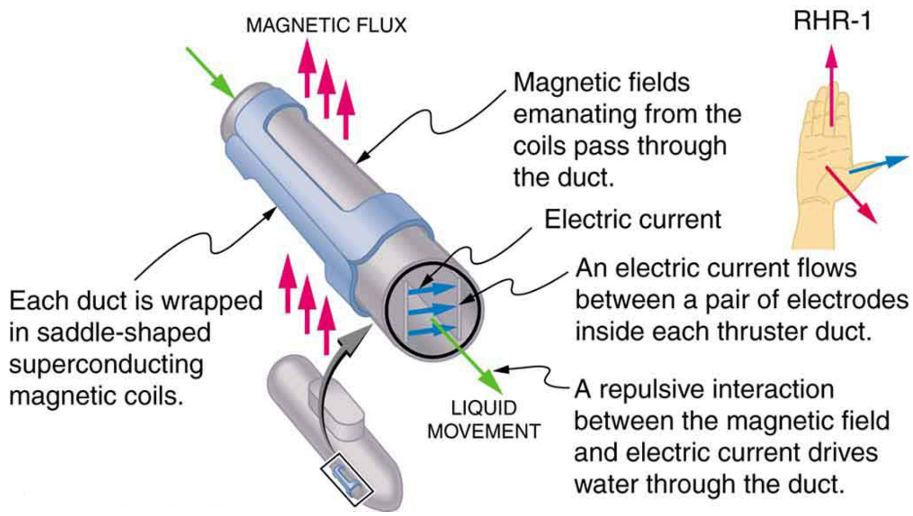
This large magnetic field creates a significant force on a small length of wire.

Magnetic force on current-carrying conductors is used to convert electric energy to work. (Motors are a prime example—they employ loops of wire and are considered in the next section.) Magnetohydrodynamics (MHD) is the technical name given to a clever application where magnetic force pumps fluids without moving mechanical parts. (See [link].)



Magnetohydrodynamics. The magnetic force on the current passed through this fluid can be used as a nonmechanical pump.

A strong magnetic field is applied across a tube and a current is passed through the fluid at right angles to the field, resulting in a force on the fluid parallel to the tube axis as shown. The absence of moving parts makes this attractive for moving a hot, chemically active substance, such as the liquid sodium employed in some nuclear reactors. Experimental artificial hearts are testing with this technique for pumping blood, perhaps circumventing the adverse effects of mechanical pumps. (Cell membranes, however, are affected by the large fields needed in MHD, delaying its practical application in humans.) MHD propulsion for nuclear submarines has been proposed, because it could be considerably quieter than conventional propeller drives. The deterrent value of nuclear submarines is based on their ability to hide and survive a first or second nuclear strike. As we slowly disassemble our nuclear weapons arsenals, the submarine branch will be the last to be decommissioned because of this ability (See [link].) Existing MHD drives are heavy and inefficient—much development work is needed.



An MHD propulsion system in a nuclear submarine could produce significantly less turbulence than propellers and allow it to run more silently. The development of a silent drive submarine was dramatized in the book and the film *The Hunt for Red October*.

Section Summary

- The magnetic force on current-carrying conductors (when current direction and magnetic field direction are perpendicular) is given by **Equation:**

$$F = IlB,$$

where I is the current, l is the length of a straight conductor in a uniform magnetic field B, and $I \perp B$. The force follows RHR-1 with the thumb in the direction of I.

Conceptual Questions

Exercise:

Problem:

Draw a sketch of the situation in [link] showing the direction of electrons carrying the current, and use RHR-1 to verify the direction of the force on the wire.

Exercise:

Problem:

Verify that the direction of the force in an MHD drive, such as that in [link], does not depend on the sign of the charges carrying the current across the fluid.

Exercise:

Problem:

Why would a magnetohydrodynamic drive work better in ocean water than in fresh water? Also, why would superconducting magnets be desirable?

Exercise:

Problem:

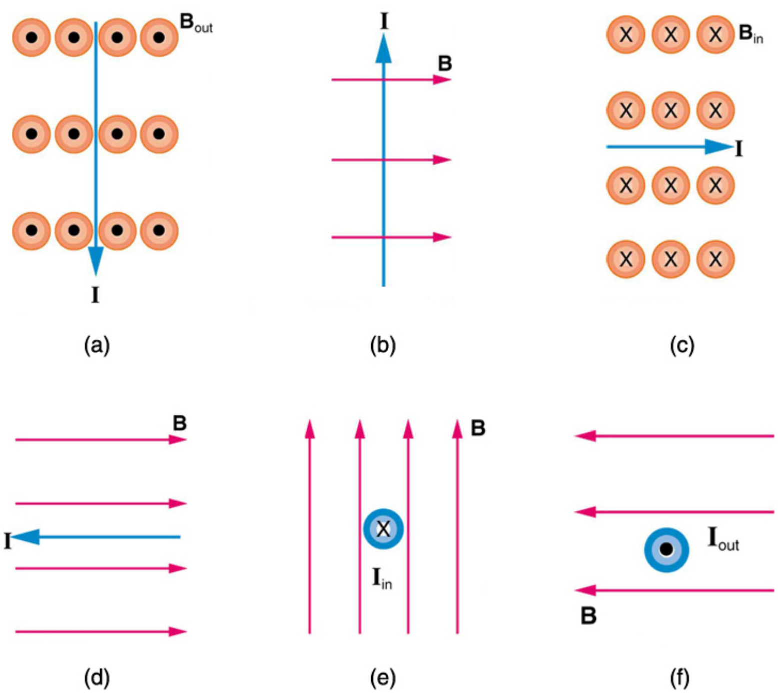
Which is more likely to interfere with compass readings, AC current in your refrigerator or DC current when you start your car? Explain.

Problems & Exercises

Exercise:

Problem:

What is the direction of the magnetic force on the current in each of the six cases in [link]? Note that $\odot$ indicates "coming out of the page" and $\otimes$ means "going into the page."



Solution:

(a) west (left)

(b) into page

(c) north (up)

(d) no force

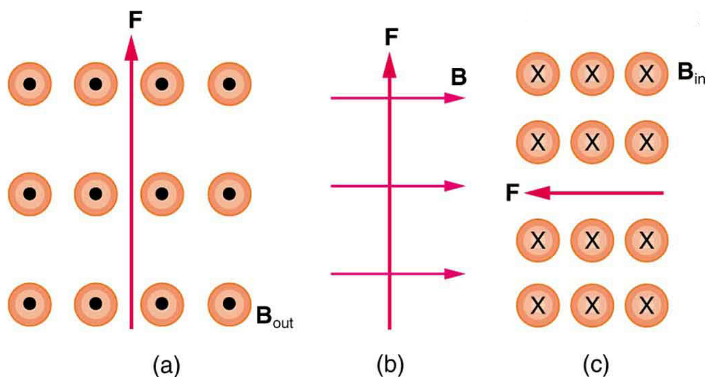
(e) east (right)

(f) south (down)

Exercise:

Problem:

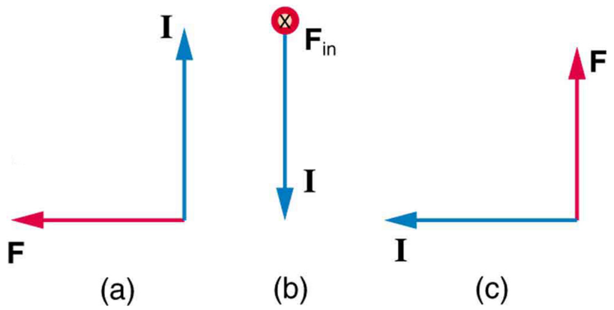
What is the direction of a current that experiences the magnetic force shown in each of the three cases in [link], assuming the current runs perpendicular to B? Note that $\odot$ indicates "coming out of the page" and $\otimes$ means "going into the page."



Exercise:

Problem:

What is the direction of the magnetic field that produces the magnetic force shown on the currents in each of the three cases in [link], assuming $\mathbf{B}$ is perpendicular to $\mathbf{I}$? Note that $\otimes$ means "going into the page."



Solution:

(a) into page

(b) west (left)

(c) out of page

Exercise:

Problem:

(a) What is the force per meter on a lightning bolt at the equator that carries 20,000 A perpendicular to the Earth's $3.00 \times 10^{-5}\text{-T}$ field? (b) What is the direction of the force if the current is straight up and the Earth's field direction is due north, parallel to the ground?

Exercise:

Problem:

(a) A DC power line for a light-rail system carries 1000 A. If Earth's magnetic field at this location is 5.00×10^{-5} T, what is the maximum possible magnetic force on a 100-m section of this line? (b) Discuss practical concerns this presents, if any.

Solution:

(a) 5.00 N

(b) This is about a pound of force per 100 m of wire, which is much less than the weight of the wire itself. Therefore, it does not cause any special concerns.

Exercise:

Problem:

What force is exerted on the water in an MHD drive utilizing a 25.0-cm-diameter tube, if 100-A current is passed across the tube that is perpendicular to a 2.00-T magnetic field? (The relatively small size of this force indicates the need for very large currents and magnetic fields to make practical MHD drives.)

Exercise:

Problem:

A wire carrying a 30.0-A current passes between the poles of a strong magnet that is perpendicular to its field and experiences a 2.16-N force on the 4.00 cm of wire in the field. What is the average field strength?

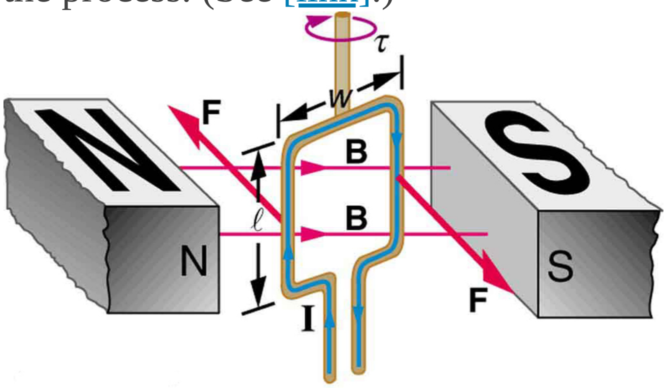
Solution:

1.80 T

Motors and Meters

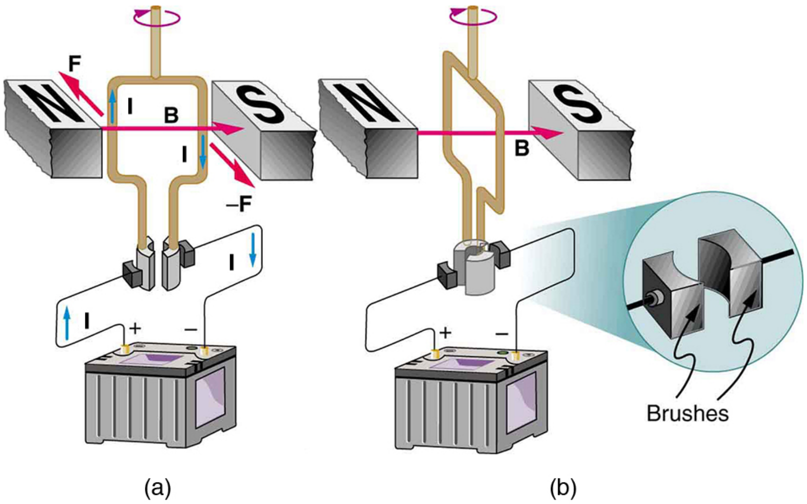
- Describe how motors and meters work in terms of force on a current loop.

Motors are the most common application of magnetic force on current-carrying wires. Motors have loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts force on the loops, which rotates a shaft. Electrical energy is converted to mechanical work in the process. (See [link].)



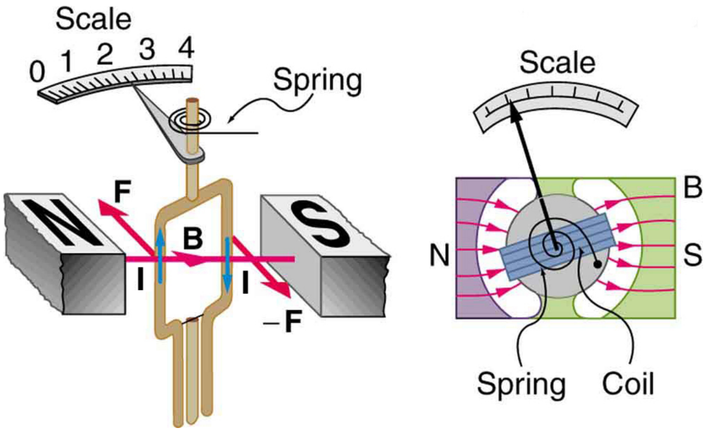
Force on a current loop. A current-carrying loop of wire attached to a vertically rotating shaft feels magnetic forces that produce a clockwise rotation as viewed from above.

As the coil rotates, the force decreases to zero at $\theta = 0$. The force then *reverses* its direction once the coil rotates past $\theta = 0$. This means that, unless we do something, the coil will oscillate back and forth about equilibrium at $\theta = 0$. To get the coil to continue rotating in the same direction, we can reverse the current as it passes through $\theta = 0$ with automatic switches called *brushes*. (See [link].)



(a) As the momentum of the coil carries it through $\theta = 0$, the brushes reverse the current to keep the motion clockwise. (b) The coil will rotate continuously in the clockwise direction, with the current reversing each half revolution to maintain the motion.

Meters, such as those in analog fuel gauges on a car, are another common application of magnetic force on a current-carrying loop. [link] shows that a meter is very similar in construction to a motor. The meter in the figure has its magnets shaped to limit the effect of θ by making B perpendicular to the loop over a large angular range. A linear spring exerts a counter-force that balances the current-produced force. This makes the needle deflection proportional to I. If an exact proportionality cannot be achieved, the gauge reading can be calibrated. To produce a galvanometer for use in analog voltmeters and ammeters that have a low resistance and respond to small currents, we use a large loop area A, high magnetic field B, and low-resistance coils.



Meters are very similar to motors but only rotate through a part of a revolution. The magnetic poles of this meter are shaped to keep the component of B perpendicular to the loop constant, so that the force does not depend on θ and the deflection against the return spring is proportional only to the current I.

Glossary

motor
: loop of wire in a magnetic field; when current is passed through the loops, the magnetic field exerts force on the loops, which rotates a shaft; electrical energy is converted to mechanical work in the process

meter
: common application of magnetic force on a current-carrying loop that is very similar in construction to a motor; by design, the force is proportional to I and not θ, so the needle deflection is proportional to the current

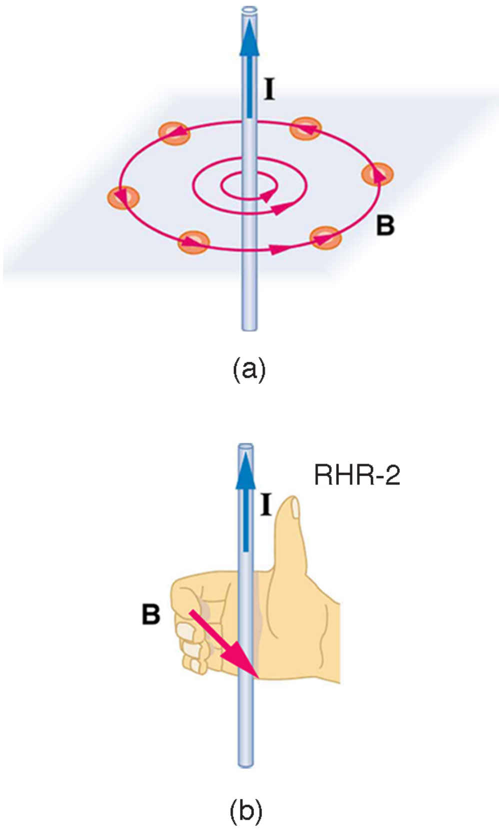
Magnetic Fields Produced by Currents: Ampere’s Law

- Calculate current that produces a magnetic field.
- Use the right hand rule 2 to determine the direction of current or the direction of magnetic field loops.

How much current is needed to produce a significant magnetic field, perhaps as strong as the Earth’s field? Surveyors will tell you that overhead electric power lines create magnetic fields that interfere with their compass readings. Indeed, when Oersted discovered in 1820 that a current in a wire affected a compass needle, he was not dealing with extremely large currents. How does the shape of wires carrying current affect the shape of the magnetic field created? We noted earlier that a current loop created a magnetic field similar to that of a bar magnet, but what about a straight wire or a toroid (doughnut)? How is the direction of a current-created field related to the direction of the current? Answers to these questions are explored in this section, together with a brief discussion of the law governing the fields created by currents.

Magnetic Field Created by a Long Straight Current-Carrying Wire: Right Hand Rule 2

Magnetic fields have both direction and magnitude. As noted before, one way to explore the direction of a magnetic field is with compasses, as shown for a long straight current-carrying wire in [link]. Hall probes can determine the magnitude of the field. The field around a long straight wire is found to be in circular loops. The **right hand rule 2** (RHR-2) emerges from this exploration and is valid for any current segment—*point the thumb in the direction of the current, and the fingers curl in the direction of the magnetic field loops* created by it.



(a) Compasses placed near a long straight current-carrying wire indicate that field lines form circular loops centered on the wire. (b) Right hand rule 2 states that, if the right hand thumb points in the direction of the current, the fingers curl in the direction of the field. This rule is consistent with the field mapped for the long straight wire and is valid for any current segment.

The **magnetic field strength (magnitude) produced by a long straight current-carrying wire** is found by experiment to be

Equation:

$$B = \frac{\mu_0 I}{2\pi r} \text{ (long straight wire)},$$

where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ is the **permeability of free space**. (μ_0 is one of the basic constants in nature. We will see later that μ_0 is related to the speed of light.) Since the wire is very long, the magnitude of the field depends only on distance from the wire r, not on position along the wire.

Example:

Calculating Current that Produces a Magnetic Field

Find the current in a long straight wire that would produce a magnetic field twice the strength of the Earth's at a distance of 5.0 cm from the wire.

Strategy

The Earth's field is about $5.0 \times 10^{-5}\ \mathrm{T}$, and so here B due to the wire is taken to be $1.0 \times 10^{-4}\ \mathrm{T}$. The equation $B = \frac{\mu_0 I}{2\pi r}$ can be used to find I, since all other quantities are known.

Solution

Solving for I and entering known values gives

Equation:

$$\begin{aligned} I &= \frac{2\pi r B}{\mu_0} = \frac{2\pi\left(5.0 \times 10^{-2}\ \mathrm{m}\right)\left(1.0 \times 10^{-4}\ \mathrm{T}\right)}{4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}} \\ &= 25\ \mathrm{A}. \end{aligned}$$

Discussion

So a moderately large current produces a significant magnetic field at a distance of 5.0 cm from a long straight wire. Note that the answer is stated to only two digits, since the Earth's field is specified to only two digits in this example.

Ampere’s Law and Others

The magnetic field of a long straight wire has more implications than you might at first suspect. *Each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment.* The formal statement of the direction and magnitude of the field due to each segment is called the **Biot-Savart law**. Integral calculus is needed to sum the field for an arbitrary shape current. This results in a more complete law, called **Ampere’s law**, which relates magnetic field and current in a general way. Ampere’s law in turn is a part of **Maxwell’s equations**, which give a complete theory of all electromagnetic phenomena. Considerations of how Maxwell’s equations appear to different observers led to the modern theory of relativity, and the realization that electric and magnetic fields are different manifestations of the same thing. Most of this is beyond the scope of this text in both mathematical level, requiring calculus, and in the amount of space that can be devoted to it. But for the interested student, and particularly for those who continue in physics, engineering, or similar pursuits, delving into these matters further will reveal descriptions of nature that are elegant as well as profound. In this text, we shall keep the general features in mind, such as RHR-2 and the rules for magnetic field lines listed in Magnetic Fields and Magnetic Field Lines, while concentrating on the fields created in certain important situations.

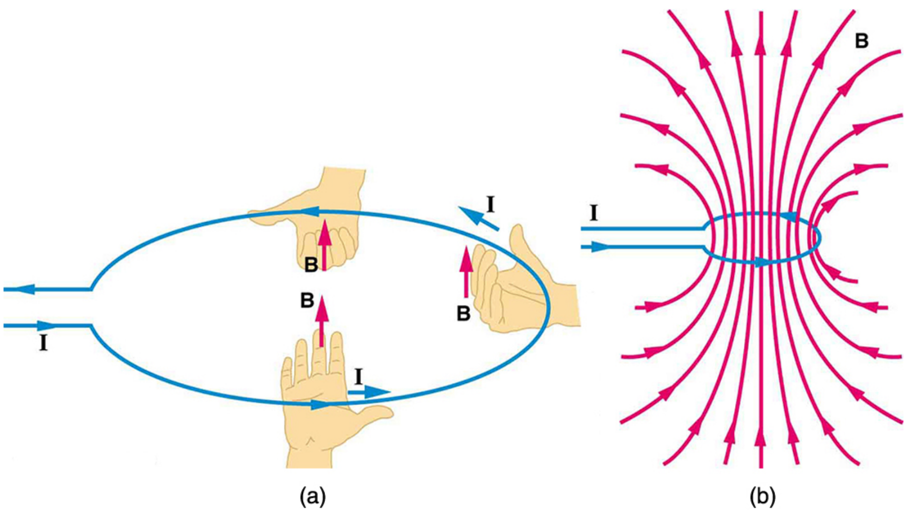
Magnetic Field Produced by a Current-Carrying Circular Loop

The magnetic field near a current-carrying loop of wire is shown in [link]. Both the direction and the magnitude of the magnetic field produced by a current-carrying loop are complex. RHR-2 can be used to give the direction of the field near the loop, but mapping with compasses and the rules about field lines given in "Magnetic Fields and Magnetic Field Lines" are needed for more detail. There is a simple formula for the **magnetic field strength at the center of a circular loop**. It is

Equation:

$$B = \frac{\mu_0 I}{2R} \text{ (at center of loop)},$$

where R is the radius of the loop. This equation is very similar to that for a straight wire, but it is valid *only* at the center of a circular loop of wire. The similarity of the equations does indicate that similar field strength can be obtained at the center of a loop. One way to get a larger field is to have N loops; then, the field is $B = N\mu_0 I/(2R)$. Note that the larger the loop, the smaller the field at its center, because the current is farther away.

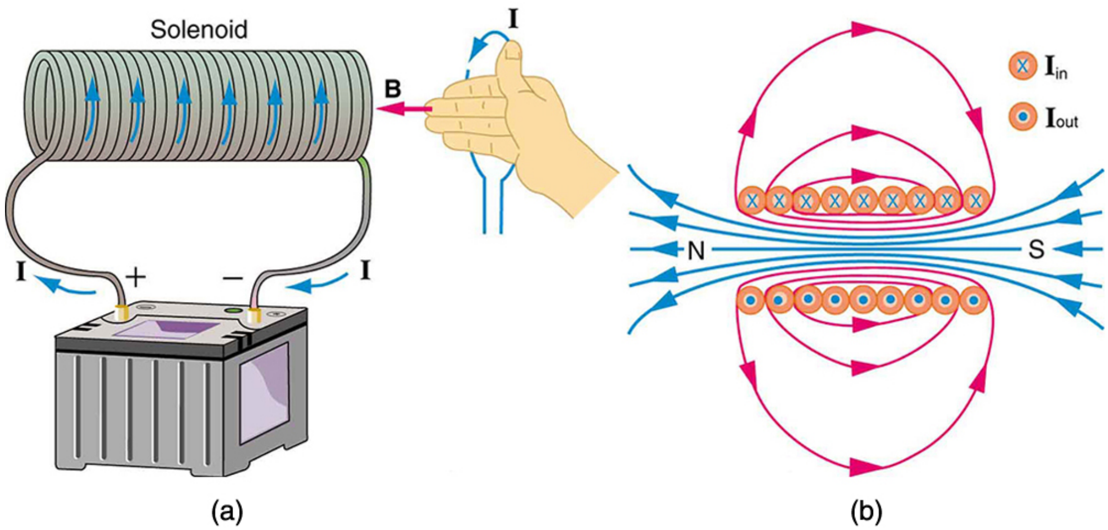


(a) RHR-2 gives the direction of the magnetic field inside and outside a current-carrying loop. (b) More detailed mapping with compasses or with a Hall probe completes the picture. The field is similar to that of a bar magnet.

Magnetic Field Produced by a Current-Carrying Solenoid

A **solenoid** is a long coil of wire (with many turns or loops, as opposed to a flat loop). Because of its shape, the field inside a solenoid can be very

uniform, and also very strong. The field just outside the coils is nearly zero. [link] shows how the field looks and how its direction is given by RHR-2.



(a) Because of its shape, the field inside a solenoid of length l is remarkably uniform in magnitude and direction, as indicated by the straight and uniformly spaced field lines. The field outside the coils is nearly zero. (b) This cutaway shows the magnetic field generated by the current in the solenoid.

The magnetic field inside of a current-carrying solenoid is very uniform in direction and magnitude. Only near the ends does it begin to weaken and change direction. The field outside has similar complexities to flat loops and bar magnets, but the **magnetic field strength inside a solenoid** is simply

Equation:

$$B = \mu_0 \text{nI} \ \ (\text{inside a solenoid}),$$

where n is the number of loops per unit length of the solenoid ($n = N/l$, with N being the number of loops and l the length). Note that B is the field

strength anywhere in the uniform region of the interior and not just at the center. Large uniform fields spread over a large volume are possible with solenoids, as [link] implies.

Example:
Calculating Field Strength inside a Solenoid
What is the field inside a 2.00-m-long solenoid that has 2000 loops and carries a 1600-A current?

Strategy
To find the field strength inside a solenoid, we use $B=\mu_0 \mathrm{nI}$. First, we note the number of loops per unit length is

Equation:

$$n^{-1}=\frac{N}{l}=\frac{2000}{2.00\text{ m}}=1000\text{ m}^{-1}=10\text{ cm}^{-1}.$$

Solution
Substituting known values gives

Equation:

$$\begin{aligned} B &= \mu_0 \mathrm{nI}=\left(4\pi\times 10^{-7}\text{ T}\cdot\text{m/A}\right)\left(1000\text{ m}^{-1}\right)(1600\text{ A}) \\ &= 2.01\text{ T}. \end{aligned}$$

Discussion
This is a large field strength that could be established over a large-diameter solenoid, such as in medical uses of magnetic resonance imaging (MRI). The very large current is an indication that the fields of this strength are not easily achieved, however. Such a large current through 1000 loops squeezed into a meter's length would produce significant heating. Higher currents can be achieved by using superconducting wires, although this is expensive. There is an upper limit to the current, since the superconducting state is disrupted by very large magnetic fields.

There are interesting variations of the flat coil and solenoid. For example, the toroidal coil used to confine the reactive particles in tokamaks is much like a solenoid bent into a circle. The field inside a toroid is very strong but circular. Charged particles travel in circles, following the field lines, and collide with one another, perhaps inducing fusion. But the charged particles do not cross field lines and escape the toroid. A whole range of coil shapes are used to produce all sorts of magnetic field shapes. Adding ferromagnetic materials produces greater field strengths and can have a significant effect on the shape of the field. Ferromagnetic materials tend to trap magnetic fields (the field lines bend into the ferromagnetic material, leaving weaker fields outside it) and are used as shields for devices that are adversely affected by magnetic fields, including the Earth's magnetic field.

Section Summary

- The strength of the magnetic field created by current in a long straight wire is given by
  Equation:
  $$B = \frac{\mu_0 I}{2\pi r}(\text{long straight wire}),$$
  where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}$ is the permeability of free space.
- The direction of the magnetic field created by a long straight wire is given by right hand rule 2 (RHR-2): *Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops* created by it.
- The magnetic field created by current following any path is the sum (or integral) of the fields due to segments along the path (magnitude and direction as for a straight wire), resulting in a general relationship between current and field known as Ampere's law.
- The magnetic field strength at the center of a circular loop is given by
  Equation:
  $$B = \frac{\mu_0 I}{2R}(\text{at center of loop}),$$

where R is the radius of the loop. This equation becomes $B=\mu_0\mathrm{nI}/(2R)$ for a flat coil of N loops. RHR-2 gives the direction of the field about the loop. A long coil is called a solenoid.

- The magnetic field strength inside a solenoid is
  Equation:

  $$B=\mu_0\mathrm{nI}\ \ (\text{inside a solenoid}),$$

  where n is the number of loops per unit length of the solenoid. The field inside is very uniform in magnitude and direction.

Glossary

right hand rule 2 (RHR-2)
: a rule to determine the direction of the magnetic field induced by a current-carrying wire: Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops

magnetic field strength (magnitude) produced by a long straight current-carrying wire
: defined as $B=\frac{\mu_0 I}{2\pi r}$, where I is the current, r is the shortest distance to the wire, and μ_0 is the permeability of free space

permeability of free space
: the measure of the ability of a material, in this case free space, to support a magnetic field; the constant $\mu_0=4\pi\times10^{-7}\ \mathrm{T\cdot m/A}$

magnetic field strength at the center of a circular loop
: defined as $B=\frac{\mu_0 I}{2R}$ where R is the radius of the loop

solenoid
: a thin wire wound into a coil that produces a magnetic field when an electric current is passed through it

magnetic field strength inside a solenoid

defined as $B = \mu_0 n I$ where n is the number of loops per unit length of the solenoid ($n = N/l$, with N being the number of loops and l the length)

Biot-Savart law
: a physical law that describes the magnetic field generated by an electric current in terms of a specific equation

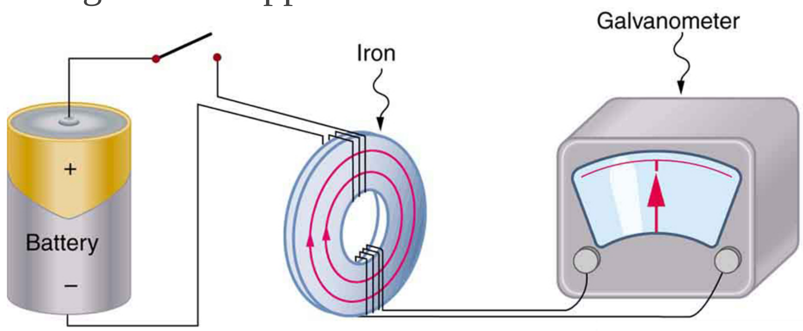
Ampere’s law
: the physical law that states that the magnetic field around an electric current is proportional to the current; each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment

Maxwell’s equations
: a set of four equations that describe electromagnetic phenomena

Induced Voltage and Magnetic Flux

- Describe methods to produce an induced voltage with a magnetic field or magnet and a loop of wire.

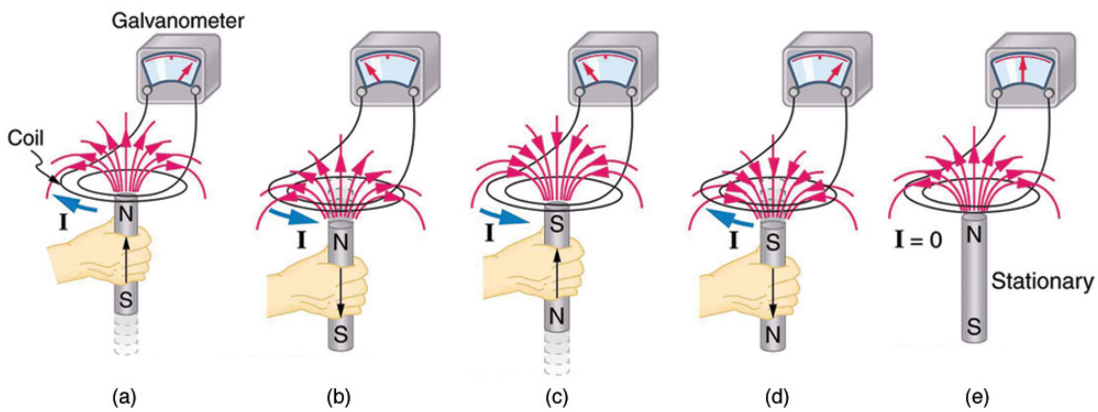
The apparatus used by Faraday to demonstrate that magnetic fields can create currents is illustrated in [link]. When the switch is closed, a magnetic field is produced in the coil on the top part of the iron ring and transmitted to the coil on the bottom part of the ring. The galvanometer is used to detect any current induced in the coil on the bottom. It was found that each time the switch is closed, the galvanometer detects a current in one direction in the coil on the bottom. Each time the switch is opened, the galvanometer detects a current in the opposite direction. Interestingly, if the switch remains closed or open for any length of time, there is no current through the galvanometer. *Closing and opening the switch* induces the current. It is the *change* in magnetic field that creates the current in the following way: *a changing magnetic field induces an electric field*, which results in the induced voltage. When this induced voltage occurs over a conducting path, as in this example, the induced voltage causes a current to flow. As a shorthand, we call the resulting current **induced current**; the changing magnetic field does not induce the current directly but through the induced voltage and an application of Ohm's law.



Faraday's apparatus for demonstrating that a magnetic field can produce a current. A change in the field produced by the top coil induces an voltage and, hence, a current in the bottom coil. When the switch is opened and closed, the galvanometer registers currents in opposite directions. No current

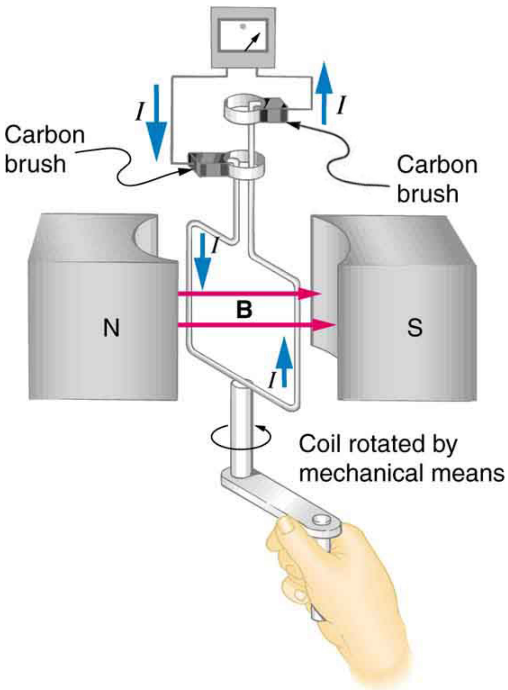
flows through the galvanometer when the switch remains closed or open.

An experiment easily performed and often done in physics labs is illustrated in [link]. A voltage is induced in the coil when a bar magnet is pushed in and out of it. Voltages of opposite signs are produced by motion in opposite directions, and the voltages are also reversed by reversing poles. The same results are produced if the coil is moved rather than the magnet—it is the relative motion that is important. The faster the motion, the greater the voltage, and there is no voltage when the magnet is stationary relative to the coil.



Movement of a magnet relative to a coil produces voltage as shown. The same voltages are produced if the coil is moved relative to the magnet. The greater the speed, the greater the magnitude of the voltage, and the voltage is zero when there is no motion.

The method of inducing a voltage used in most electric generators is shown in [link]. A coil is rotated in a magnetic field, producing an alternating voltage (and current), which depends on rotation rate and other factors that will be explored in later sections. Note that the generator is remarkably similar in construction to a motor.



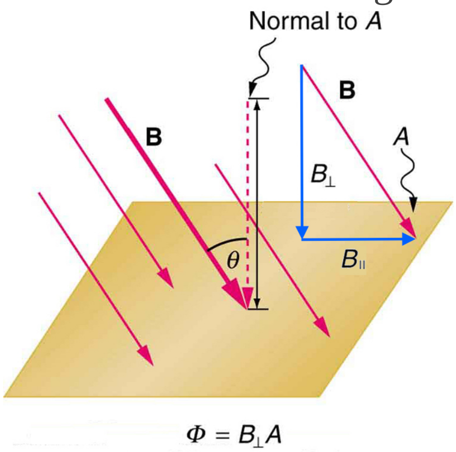
Rotation of a coil in a magnetic field produces a voltage. This is the basic construction of a generator, where work done to turn the coil is converted to electric energy. Note the generator is very similar in construction to a motor.

So we see that changing the magnitude or direction of a magnetic field produces a voltage. Experiments revealed that there is a crucial quantity called the **magnetic flux**, Φ , given by

Equation:

$$\Phi = B_{\perp}A,$$

where B is the magnetic field strength over an area A, at an angle θ with the perpendicular to the area as shown in [link]. **Any change in magnetic flux Φ induces a voltage.** This process is defined to be **electromagnetic induction**. Units of magnetic flux Φ are $\mathrm{T \cdot m^2}$.



Magnetic flux Φ is related to the magnetic field and the area A over which it exists. Only the portion of the magnetic field that is perpendicular to the area ($B_\perp$) contributes to the flux. The flux $\Phi = B_\perp A$ is related to induction; any change in Φ induces a voltage.

All induction, including the examples given so far, arises from some change in magnetic flux Φ. For example, Faraday changed B and hence Φ when opening and closing the switch in his apparatus (shown in [link]). This is also true for the bar magnet and coil shown in [link]. When rotating the coil of a generator, the angle θ and, hence, Φ is changed. Just how great a voltage and what direction it takes depend on the change in Φ and how rapidly the change is made, as examined in the next section.

Section Summary

- The crucial quantity in induction is magnetic flux Φ, defined to be $\Phi = B_{\perp}A$, where $B_{\perp}$ is the magnetic field strength perpendicular to the area A.
- Units of magnetic flux Φ are $\mathrm{T} \cdot \mathrm{m}^2$.
- Any change in magnetic flux Φ induces a voltage—the process is defined to be electromagnetic induction.

Conceptual Questions

Exercise:

Problem:

How do the multiple-loop coils and iron ring in the version of Faraday’s apparatus shown in [link] enhance the observation of induced voltage?

Exercise:

Problem:

When a magnet is thrust into a coil as in [link](a), what is the direction of the force exerted by the coil on the magnet? Draw a diagram showing the direction of the current induced in the coil and the magnetic field it produces, to justify your response. How does the magnitude of the force depend on the resistance of the galvanometer?

Exercise:

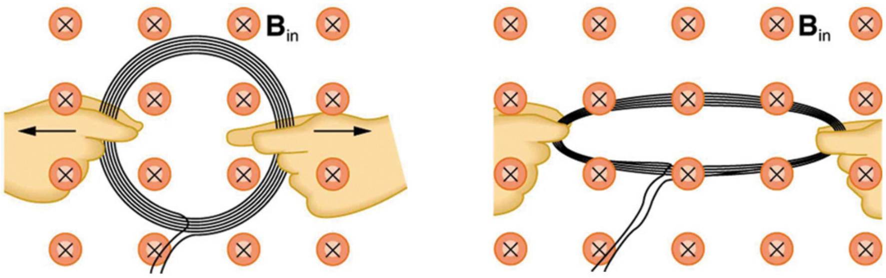
Problem:

Explain how magnetic flux can be zero when the magnetic field is not zero.

Exercise:

Problem:

Is a voltage induced in the coil in [link] when it is stretched? If so, state why and give the direction of the induced current.



A circular coil of wire is stretched in a magnetic field.

Glossary

induced current
: the current created by a changing magnetic field through voltage induced over a conducting path

magnetic flux
: the amount of magnetic field going through a particular area, calculated with $\Phi = B_{\perp}A$, where $B_{\perp}$ is the magnetic field strength perpendicular to the area A

electromagnetic induction
: the process of inducing a voltage with a change in magnetic flux

Faraday’s Law of Induction: Lenz’s Law

- Calculate voltage, current, and magnetic fields using Faraday’s Law.
- Explain the physical results of Lenz’s Law

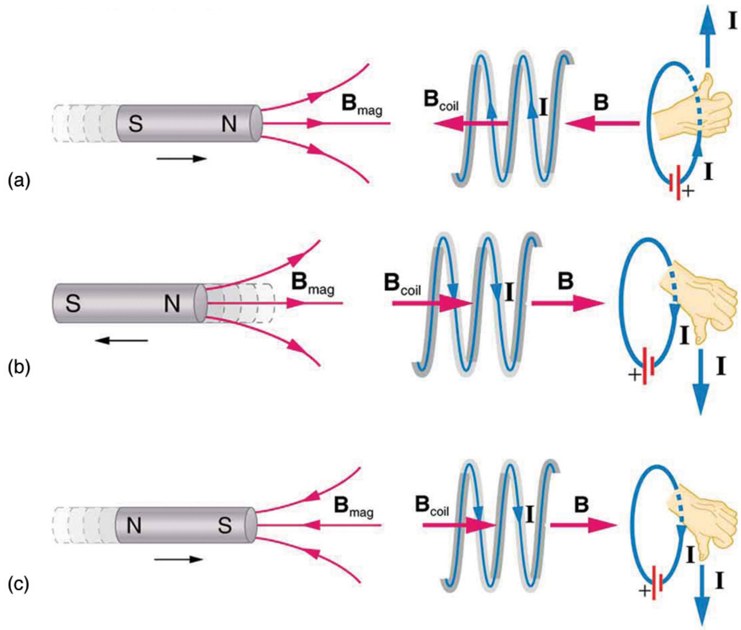
Faraday’s and Lenz’s Law

Faraday’s experiments showed that the voltage induced by a change in magnetic flux depends on only a few factors. First, voltage is directly proportional to the change in flux $\Delta \Phi$. Second, voltage is greatest when the change in time Δt is smallest—that is, voltage is inversely proportional to Δt. Finally, if a coil has N turns, a voltage will be produced that is N times greater than for a single coil, so that voltage is directly proportional to N. The equation for the voltage induced by a change in magnetic flux is
Equation:

$$V = -N\frac{\Delta \Phi}{\Delta t}.$$

This relationship is known as **Faraday’s law of induction**.

The minus sign in Faraday’s law of induction is very important. The minus means that *the induced voltage creates a current I and magnetic field B that oppose the change in flux $\Delta \Phi$—this is known as Lenz’s law*. Faraday was aware of the direction, but Lenz stated it so clearly that he is credited for its discovery. (See [link].)

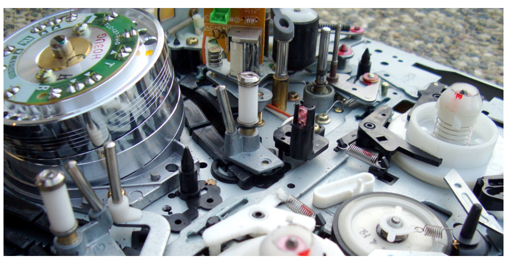


(a) When this bar magnet is thrust into the coil, the strength of the magnetic field increases in the coil. The current induced in the coil creates another field, in the opposite direction of the bar magnet's to oppose the increase. This is one aspect of *Lenz's law—induction opposes any change in flux*. (b) and (c) are two other situations. Verify for yourself that the direction of the induced B_{coil} shown indeed opposes the change in flux and that the current direction shown is consistent with RHR-2.

Applications of Electromagnetic Induction

There are many applications of Faraday's Law of induction, as we will explore in this chapter and others. At this juncture, let us mention several that have to do with data storage and magnetic fields. A very important application has to do with audio and video *recording tapes*. A plastic tape,

coated with iron oxide, moves past a recording head. This recording head is basically a round iron ring about which is wrapped a coil of wire—an electromagnet ([link]). A signal in the form of a varying input current from a microphone or camera goes to the recording head. These signals (which are a function of the signal amplitude and frequency) produce varying magnetic fields at the recording head. As the tape moves past the recording head, the magnetic field orientations of the iron oxide molecules on the tape are changed thus recording the signal. In the playback mode, the magnetized tape is run past another head, similar in structure to the recording head. The different magnetic field orientations of the iron oxide molecules on the tape induces a voltage in the coil of wire in the playback head. This signal then is sent to a loudspeaker or video player.



Recording and playback heads used with audio and video magnetic tapes. (credit: Steve Jurvetson)

Similar principles apply to computer hard drives, except at a much faster rate. Here recordings are on a coated, spinning disk. Read heads historically were made to work on the principle of induction. However, the input information is carried in digital rather than analog form – a series of 0’s or 1’s are written upon the spinning hard drive. Today, most hard drive readout devices do not work on the principle of induction, but use a technique known as *giant magnetoresistance*. (The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of

nanotechnology.) Another application of induction is found on the magnetic stripe on the back of your personal credit card as used at the grocery store or the ATM machine. This works on the same principle as the audio or video tape mentioned in the last paragraph in which a head reads personal information from your card.

Another application of electromagnetic induction is when electrical signals need to be transmitted across a barrier. Consider the *cochlear implant* shown below. Sound is picked up by a microphone on the outside of the skull and is used to set up a varying magnetic field. A current is induced in a receiver secured in the bone beneath the skin and transmitted to electrodes in the inner ear. Electromagnetic induction can be used in other instances where electric signals need to be conveyed across various media.

Electromagnetic induction used in transmitting electric currents across mediums. The device on the baby's head induces an electrical current in a receiver secured in the bone beneath the skin. (credit: Bjorn Knetsch)

Another contemporary area of research in which electromagnetic induction is being successfully implemented (and with substantial potential) is transcranial magnetic simulation. A host of disorders, including depression and hallucinations can be traced to irregular localized electrical activity in the brain. In *transcranial magnetic stimulation*, a rapidly varying and very localized magnetic field is placed close to certain sites identified in the brain. Weak electric currents are induced in the identified sites and can result in recovery of electrical functioning in the brain tissue.

Sleep apnea ("the cessation of breath") affects both adults and infants (especially premature babies and it may be a cause of sudden infant deaths [SID]). In such individuals, breath can stop repeatedly during their sleep. A cessation of more than 20 seconds can be very dangerous. Stroke, heart failure, and tiredness are just some of the possible consequences for a person having sleep apnea. The concern in infants is the stopping of breath for these longer times. One type of monitor to alert parents when a child is not breathing uses electromagnetic induction. A wire wrapped around the infant's chest has an alternating current running through it. The expansion and contraction of the infant's chest as the infant breathes changes the area through the coil. A pickup coil located nearby has an alternating current induced in it due to the changing magnetic field of the initial wire. If the child stops breathing, there will be a change in the induced current, and so a parent can be alerted.

Note:

Making Connections: Conservation of Energy

Lenz's law is a manifestation of the conservation of energy. The induced voltage produces a current that opposes the change in flux, because a change in flux means a change in energy. Energy can enter or leave, but not instantaneously. Lenz's law is a consequence. As the change begins, the law says induction opposes and, thus, slows the change. In fact, if the induced voltage were in the same direction as the change in flux, there would be a positive feedback that would give us free energy from no apparent source—conservation of energy would be violated.

Section Summary

- Faraday's law of induction states that the voltage induced by a change in magnetic flux is
  Equation:
  $$V = -N\frac{\Delta\Phi}{\Delta t}$$
  when flux changes by $\Delta\Phi$ in a time Δt.
- If voltage is induced in a coil, N is its number of turns.
- The minus sign means that the induced voltage creates a current I and magnetic field B that *oppose the change in flux* $\Delta\Phi$ —this opposition is known as Lenz's law.

Conceptual Questions

Exercise:

Problem:

A person who works with large magnets sometimes places her head inside a strong field. She reports feeling dizzy as she quickly turns her head. How might this be associated with induction?

Exercise:

Problem:

A particle accelerator sends high-velocity charged particles down an evacuated pipe. Explain how a coil of wire wrapped around the pipe could detect the passage of individual particles. Sketch a graph of the voltage output of the coil as a single particle passes through it.

Glossary

Faraday's law of induction

the means of calculating the voltage in a coil due to changing magnetic flux, given by $V = -N\frac{\Delta\Phi}{\Delta t}$

Lenz’s law
the minus sign in Faraday’s law, signifying that the voltage induced in a coil opposes the change in magnetic flux

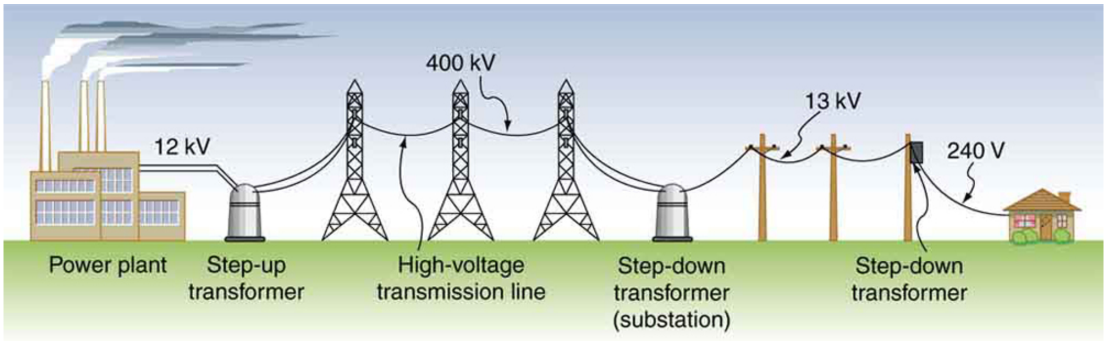
Transformers

- Explain how a transformer works.
- Calculate voltage, current, and/or number of turns given the other quantities.

Transformers do what their name implies—they transform voltages from one value to another. For example, many cell phones, laptops, video games, and power tools and small appliances have a transformer built into their plug-in unit (like that in [link]) that changes 120 V or 240 V AC into whatever voltage the device uses. Transformers are also used at several points in the power distribution systems, such as illustrated in [link]. Power is sent long distances at high voltages, because less current is required for a given amount of power, and this means less line loss, as was discussed previously. But high voltages pose greater hazards, so that transformers are employed to produce lower voltage at the user's location.

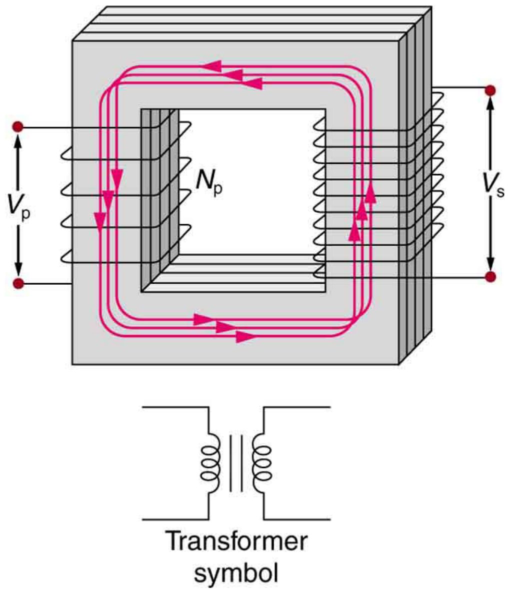
The plug-in transformer has become increasingly familiar with the proliferation of electronic devices that operate on voltages other than common 120 V AC. Most are in the 3 to 12 V range.

(credit: Shop Xtreme)



Transformers change voltages at several points in a power distribution system. Electric power is usually generated at greater than 10 kV, and transmitted long distances at voltages over 200 kV—sometimes as great as 700 kV—to limit energy losses. Local power distribution to neighborhoods or industries goes through a substation and is sent short distances at voltages ranging from 5 to 13 kV. This is reduced to 120, 240, or 480 V for safety at the individual user site.

The type of transformer considered in this text—see [link]—is based on Faraday's law of induction and is very similar in construction to the apparatus Faraday used to demonstrate magnetic fields could cause currents. The two coils are called the *primary* and *secondary coils*. In normal use, the input voltage is placed on the primary, and the secondary produces the transformed output voltage. Not only does the iron core trap the magnetic field created by the primary coil, its magnetization increases the field strength. Since the input voltage is AC, a time-varying magnetic flux is sent to the secondary, inducing its AC output voltage.



A typical construction of a simple transformer has two coils wound on a ferromagnetic core that is laminated to minimize eddy currents. The magnetic field created by the primary is mostly confined to and increased by the core, which transmits it to the secondary coil. Any change in current in the primary induces a current in the secondary.

For the simple transformer shown in [link], the output voltage V_s depends almost entirely on the input voltage V_p and the ratio of the number of loops in the primary and secondary coils. Faraday's law of induction for the secondary coil gives its induced output voltage V_s to be

Equation:

$$V_s = -N_s \frac{\Delta \Phi}{\Delta t},$$

where N_s is the number of loops in the secondary coil and $\Delta\Phi/\ \Delta t$ is the rate of change of magnetic flux. The cross-sectional area of the coils is the same on either side, as is the magnetic field strength, and so $\Delta\Phi/\Delta t$ is the same on either side. The input primary voltage V_p is also related to changing flux by

Equation:

$$V_p = -N_p \frac{\Delta \Phi}{\Delta t}.$$

The reason for this is a little more subtle. Lenz's law tells us that the primary coil opposes the change in flux caused by the input voltage V_p, hence the minus sign. Assuming negligible coil resistance, Kirchhoff's loop rule tells us that the induced voltage exactly equals the input voltage. Taking the ratio of these last two equations yields a useful relationship:

Equation:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

This is known as the **transformer equation**, and it simply states that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils.

The output voltage of a transformer can be less than, greater than, or equal to the input voltage, depending on the ratio of the number of loops in their coils. Some transformers even provide a variable output by allowing connection to be made at different points on the secondary coil. A **step-up transformer** is one that increases voltage, whereas a **step-down transformer** decreases voltage. Assuming, as we have, that resistance is negligible, the electrical power output of a transformer equals its input. This

is nearly true in practice—transformer efficiency often exceeds 99%. Equating the power input and output,

Equation:

$$P_{\mathrm{p}} = I_{\mathrm{p}}V_{\mathrm{p}} = I_{\mathrm{s}}V_{\mathrm{s}} = P_{\mathrm{s}}.$$

Rearranging terms gives

Equation:

$$\frac{V_{\mathrm{s}}}{V_{\mathrm{p}}} = \frac{I_{\mathrm{p}}}{I_{\mathrm{s}}}.$$

Combining this with $\frac{V_{\mathrm{s}}}{V_{\mathrm{p}}} = \frac{N_{\mathrm{s}}}{N_{\mathrm{p}}}$, we find that

Equation:

$$\frac{I_{\mathrm{s}}}{I_{\mathrm{p}}} = \frac{N_{\mathrm{p}}}{N_{\mathrm{s}}}$$

is the relationship between the output and input currents of a transformer. So if voltage increases, current decreases. Conversely, if voltage decreases, current increases.

Example:

Calculating Characteristics of a Step-Up Transformer

A portable x-ray unit has a step-up transformer, the 120 V input of which is transformed to the 100 kV output needed by the x-ray tube. The primary has 50 loops and draws a current of 10.00 A when in use. (a) What is the number of loops in the secondary? (b) Find the current output of the secondary.

Strategy and Solution for (a)

We solve $\frac{V_{\mathrm{s}}}{V_{\mathrm{p}}} = \frac{N_{\mathrm{s}}}{N_{\mathrm{p}}}$ for N_{s}, the number of loops in the secondary, and enter the known values. This gives

Equation:

$$\begin{aligned} N_s &= N_p \frac{V_s}{V_p} \\ &= (50)\frac{100{,}000\ \text{V}}{120\ \text{V}} = 4.17 \times 10^4. \end{aligned}$$

Discussion for (a)

A large number of loops in the secondary (compared with the primary) is required to produce such a large voltage. This would be true for neon sign transformers and those supplying high voltage inside TVs and CRTs.

Strategy and Solution for (b)

We can similarly find the output current of the secondary by solving $\frac{I_s}{I_p} = \frac{N_p}{N_s}$ for I_s and entering known values. This gives

Equation:

$$\begin{aligned} I_s &= I_p \frac{N_p}{N_s} \\ &= (10.00\ \text{A})\,\frac{50}{4.17\times 10^4} = 12.0\ \text{mA}. \end{aligned}$$

Discussion for (b)

As expected, the current output is significantly less than the input. In certain spectacular demonstrations, very large voltages are used to produce long arcs, but they are relatively safe because the transformer output does not supply a large current. Note that the power input here is $P_p = I_p V_p = (10.00\ \text{A})(120\ \text{V}) = 1.20\ \text{kW}$. This equals the power output $P_p = I_s V_s = (12.0\ \text{mA})(100\ \text{kV}) = 1.20\ \text{kW}$, as we assumed in the derivation of the equations used.

Example:

Calculating Characteristics of a Step-Down Transformer

A battery charger meant for a series connection of ten nickel-cadmium batteries needs to have a 15.0 V output to charge the batteries. It uses a step-down transformer with a 200-loop primary and a 120 V input. (a) How many loops should there be in the secondary coil? (b) If the charging current is 16.0 A, what is the input current?

Strategy and Solution for (a)

You would expect the secondary to have a small number of loops. Solving $\frac{V_\mathrm{s}}{V_\mathrm{p}} = \frac{N_\mathrm{s}}{N_\mathrm{p}}$ for N_s and entering known values gives

Equation:

$$\begin{aligned} N_\mathrm{s} &= N_\mathrm{p}\frac{V_\mathrm{s}}{V_\mathrm{p}} \\ &= (200)\frac{15.0\ \mathrm{V}}{120\ \mathrm{V}} = 25. \end{aligned}$$

Strategy and Solution for (b)

The current input can be obtained by solving $\frac{I_\mathrm{s}}{I_\mathrm{p}} = \frac{N_\mathrm{p}}{N_\mathrm{s}}$ for I_p and entering known values. This gives

Equation:

$$\begin{aligned} I_\mathrm{p} &= I_\mathrm{s}\frac{N_\mathrm{s}}{N_\mathrm{p}} \\ &= (16.0\ \mathrm{A})\frac{25}{200} = 2.00\ \mathrm{A}. \end{aligned}$$

Discussion

The number of loops in the secondary is small, as expected for a step-down transformer. We also see that a small input current produces a larger output current in a step-down transformer. When transformers are used to operate large magnets, they sometimes have a small number of very heavy loops in the secondary. This allows the secondary to have low internal resistance and produce large currents. Note again that this solution is based on the assumption of 100% efficiency—or power out equals power in ($P_\mathrm{p} = P_\mathrm{s}$)—reasonable for good transformers. In this case the primary and secondary power is 240 W. (Verify this for yourself as a consistency check.) Note that the Ni-Cd batteries need to be charged from a DC power source (as would a 12 V battery). So the AC output of the secondary coil needs to be converted into DC. This is done using something called a rectifier, which uses devices called diodes that allow only a one-way flow of current.

Section Summary

- Transformers use induction to transform voltages from one value to another.
- For a transformer, the voltages across the primary and secondary coils are related by
  Equation:
  $$\frac{V_s}{V_p} = \frac{N_s}{N_p},$$
  where V_p and V_s are the voltages across primary and secondary coils having N_p and N_s turns.
- The currents I_p and I_s in the primary and secondary coils are related by $\frac{I_s}{I_p} = \frac{N_p}{N_s}$.
- A step-up transformer increases voltage and decreases current, whereas a step-down transformer decreases voltage and increases current.

Problems & Exercises

Exercise:

Problem:

A plug-in transformer, like that in [link], supplies 9.00 V to a video game system. (a) How many turns are in its secondary coil, if its input voltage is 120 V and the primary coil has 400 turns? (b) What is its input current when its output is 1.30 A?

Solution:

(a) 30.0

(b) 9.75×10^{-2} A

Exercise:

Problem:

An American traveler in New Zealand carries a transformer to convert New Zealand's standard 240 V to 120 V so that she can use some small appliances on her trip. (a) What is the ratio of turns in the primary and secondary coils of her transformer? (b) What is the ratio of input to output current? (c) How could a New Zealander traveling in the United States use this same transformer to power her 240 V appliances from 120 V?

Exercise:

Problem:

A digital recorder uses a plug-in transformer to convert 120 V to 12.0 V, with a maximum current output of 200 mA. (a) What is the current input? (b) What is the power input? (c) Is this amount of power reasonable for a small appliance?

Solution:

(a) 20.0 mA

(b) 2.40 W

(c) Yes, this amount of power is quite reasonable for a small appliance.

Exercise:

Problem:

(a) What is the voltage output of a transformer used for rechargeable flashlight batteries, if its primary has 500 turns, its secondary 4 turns, and the input voltage is 120 V? (b) What input current is required to produce a 4.00 A output? (c) What is the power input?

Exercise:

Problem:

(a) The plug-in transformer for a laptop computer puts out 7.50 V and can supply a maximum current of 2.00 A. What is the maximum input current if the input voltage is 240 V? Assume 100% efficiency. (b) If the actual efficiency is less than 100%, would the input current need to be greater or smaller? Explain.

Solution:

(a) 0.063 A

(b) Greater input current needed.

Exercise:

Problem:

A multipurpose transformer has a secondary coil with several points at which a voltage can be extracted, giving outputs of 5.60, 12.0, and 480 V. (a) The input voltage is 240 V to a primary coil of 280 turns. What are the numbers of turns in the parts of the secondary used to produce the output voltages? (b) If the maximum input current is 5.00 A, what are the maximum output currents (each used alone)?

Exercise:

Problem:

A large power plant generates electricity at 12.0 kV. Its old transformer once converted the voltage to 335 kV. The secondary of this transformer is being replaced so that its output can be 750 kV for more efficient cross-country transmission on upgraded transmission lines. (a) What is the ratio of turns in the new secondary compared with the old secondary? (b) What is the ratio of new current output to old output (at 335 kV) for the same power? (c) If the upgraded transmission lines have the same resistance, what is the ratio of new line power loss to old?

Solution:

(a) 2.2

(b) 0.45

(c) 0.20, or 20.0%

Exercise:

Problem:

If the power output in the previous problem is 1000 MW and line resistance is $2.00\ \Omega$, what were the old and new line losses?

Glossary

transformer
: a device that transforms voltages from one value to another using induction

transformer equation
: the equation showing that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils; $\frac{V_{\text{s}}}{V_{\text{p}}} = \frac{N_{\text{s}}}{N_{\text{p}}}$

step-up transformer
: a transformer that increases voltage

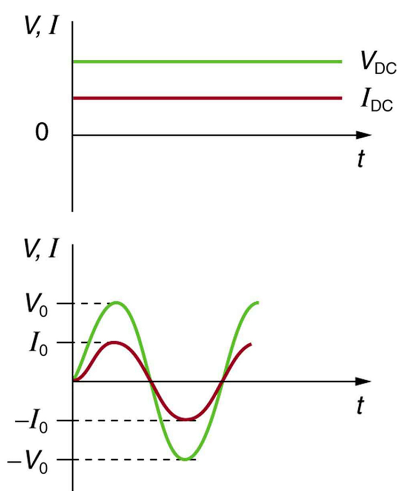
step-down transformer
: a transformer that decreases voltage

Alternating Current versus Direct Current

- Explain the differences and similarities between AC and DC current.
- Describe rms voltage, current, and average power.
- Explain why AC current is used for power transmission.

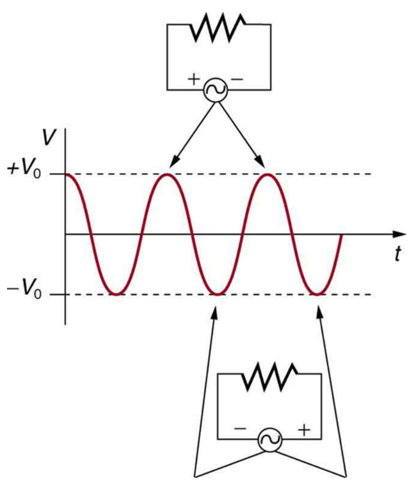
Alternating Current

Most of the examples in electric circuits, and particularly those utilizing batteries, have constant voltage sources. Once the current is established, it is thus also a constant. **Direct current** (DC) is the flow of electric charge in only one direction. It is the steady state of a constant-voltage circuit. Many well-known applications, however, use a time-varying voltage source. **Alternating current** (AC) is the flow of electric charge that periodically reverses direction. If the source varies periodically, particularly sinusoidally, the circuit is known as an alternating current circuit. Examples include the commercial and residential power that serves so many of our needs. [link] shows graphs of voltage and current versus time for typical DC and AC power. The AC voltages and frequencies commonly used in homes and businesses vary around the world. The AC voltages range from 100 V to 240 V; the frequencies range from 50 Hz to 60 Hz.



(a) DC voltage and current are constant

in time, once the current is established. (b) A graph of voltage and current versus time for 60-Hz AC power. The voltage and current are sinusoidal and are in phase for a simple resistance circuit. The frequencies and peak voltages of AC sources differ greatly.



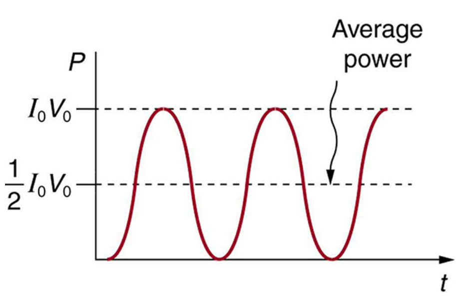
The potential difference V between the terminals of an AC voltage source

fluctuates as shown.

[link] shows a schematic of a simple circuit with an AC voltage source. The voltage between the terminals fluctuates as shown. For this example, the voltage and current are said to be in phase, as seen in [link](b).

Current in the resistor alternates back and forth just like the driving voltage, since $I = \mathrm{V/R}$. If the resistor is a fluorescent light bulb, for example, it brightens and dims 120 times per second as the current repeatedly goes through zero. A 120-Hz flicker is too rapid for your eyes to detect, but if you wave your hand back and forth between your face and a fluorescent light, you will see a stroboscopic effect evidencing AC. The fact that the light output fluctuates means that the power is fluctuating. The power supplied is $P = \mathrm{IV}$.

Note:
Making Connections: Take-Home Experiment—AC/DC Lights
Wave your hand back and forth between your face and a fluorescent light bulb. Do you observe the same thing with the headlights on your car? Explain what you observe. *Warning: Do not look directly at very bright light*.



AC power as a

function of time.

We are most often concerned with average power rather than its fluctuations—that 60-W light bulb in your desk lamp has an average power consumption of 60 W, for example. As illustrated in [link]. One common way to express an average is "root-mean-square," or "**rms**." For example, rms voltage of an AC voltage source is found by first squaring the voltage ("square"), taking an average of this value over one period of oscillation ("mean"), and taking the square root ("root").

It is standard practice to quote I_{rms}, V_{rms}, and P_{ave} rather than the peak values. For example, most household electricity is 120 V AC, which means that V_{rms} is 120 V. The common 10-A circuit breaker will interrupt a sustained I_{rms} greater than 10 A. Your 1.0-kW microwave oven consumes $P_{\text{ave}} = 1.0\text{ kW}$, and so on. You can think of these rms and average values as the equivalent DC values for a simple resistive circuit.

To summarize, when dealing with AC, Ohm’s law and the equations for power are completely analogous to those for DC, but rms and average values are used for AC. Thus, for AC, Ohm’s law is written
Equation:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}.$$

The various expressions for AC power P_{ave} are
Equation:

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}},$$

Equation:

$$P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R},$$

and
Equation:

$$P_{\text{ave}} = I_{\text{rms}}^2 R.$$

Why Use AC for Power Distribution?

Most large power-distribution systems are AC. Moreover, the power is transmitted at much higher voltages than the 120-V AC (240 V in most parts of the world) we use in homes and on the job. Economies of scale make it cheaper to build a few very large electric power-generation plants than to build numerous small ones. This necessitates sending power long distances, and it is obviously important that energy losses en route be minimized. High voltages can be transmitted with much smaller power losses than low voltages, as we shall show below. (See [link].) For safety reasons, the voltage at the user is reduced to familiar values. The crucial factor is that AC voltages can be increased and decreased efficiently with transformers (which uses electromagnetic induction to produce time-varying voltages), while it is more difficult to change DC voltages without power losses. So AC is used in most large power distribution systems.

Power is distributed over large distances at high voltage to reduce power loss in the transmission lines. The

voltages generated at the power plant are stepped up by passive devices called transformers to 330,000 volts (or more in some places worldwide). At the point of use, the transformers reduce the voltage transmitted for safe residential and commercial use. (Credit: GeorgHH, Wikimedia Commons)

Example:

Power Losses Are Less for High-Voltage Transmission

(a) What current is needed to transmit 100 MW of power at 200 kV? (b) What is the power dissipated by the transmission lines if they have a resistance of 1.00 Ω? (c) What percentage of the power is lost in the transmission lines?

Strategy

We are given $P_{\text{ave}} = 100\text{ MW}$, $V_{\text{rms}} = 200\text{ kV}$, and the resistance of the lines is $R = 1.00\ \Omega$. Using these givens, we can find the current flowing (from $P = \text{IV}$) and then the power dissipated in the lines ($P = I^2R$), and we take the ratio to the total power transmitted.

Solution

To find the current, we rearrange the relationship $P_{\text{ave}} = I_{\text{rms}}V_{\text{rms}}$ and substitute known values. This gives

Equation:

$$I_{\text{rms}} = \frac{P_{\text{ave}}}{V_{\text{rms}}} = \frac{100 \times 10^6 \text{ W}}{200 \times 10^3 \text{ V}} = 500 \text{ A}.$$

Solution

Knowing the current and given the resistance of the lines, the power dissipated in them is found from $P_{\text{ave}} = I_{\text{rms}}^2 R$. Substituting the known values gives

Equation:

$$P_{\text{ave}} = I_{\text{rms}}^2 R = (500 \text{ A})^2 (1.00 \ \Omega) = 250 \text{ kW}.$$

Solution

The percent loss is the ratio of this lost power to the total or input power, multiplied by 100:

Equation:

$$\% \text{ loss} = \frac{250 \text{ kW}}{100 \text{ MW}} \times 100 = 0.250 \ \%.$$

Discussion

One-fourth of a percent is an acceptable loss. Note that if 100 MW of power had been transmitted at 25 kV, then a current of 4000 A would have been needed. This would result in a power loss in the lines of 16.0 MW, or 16.0% rather than 0.250%. The lower the voltage, the more current is needed, and the greater the power loss in the fixed-resistance transmission lines. Of course, lower-resistance lines can be built, but this requires larger and more expensive wires. If superconducting lines could be economically produced, there would be no loss in the transmission lines at all. But, as we shall see in a later chapter, there is a limit to current in superconductors, too. In short, high voltages are more economical for transmitting power, and AC voltage is much easier to raise and lower, so that AC is used in most large-scale power distribution systems.

Section Summary

- Ohm’s law for AC is $I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$.
- Expressions for the average power of an AC circuit are $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$, $P_{\text{ave}} = \frac{{V_{\text{rms}}}^2}{R}$, and $P_{\text{ave}} = {I_{\text{rms}}}^2 R$, analogous to the expressions for DC circuits.

Conceptual Questions

Exercise:

Problem:

Give an example of a use of AC power other than in the household. Similarly, give an example of a use of DC power other than that supplied by batteries.

Exercise:

Problem:

Why do voltage, current, and power go through zero 120 times per second for 60-Hz AC electricity?

Exercise:

Problem:

You are riding in a train, gazing into the distance through its window. As close objects streak by, you notice that the nearby fluorescent lights make *dashed* streaks. Explain.

Problem Exercises

Exercise:

Problem:

Military aircraft use 400-Hz AC power, because it is possible to design lighter-weight equipment at this higher frequency. What is the time for one complete cycle of this power?

Solution:

2.50 ms

Exercise:

Problem:

A North American tourist takes his 25.0-W, 120-V AC razor to Europe, finds a special adapter, and plugs it into 240 V AC. Assuming constant resistance, what power does the razor consume as it is ruined?

Exercise:

Problem:

In this problem, you will verify statements made at the end of the power losses for [link]. (a) What current is needed to transmit 100 MW of power at a voltage of 25.0 kV? (b) Find the power loss in a 1.00 - Ω transmission line. (c) What percent loss does this represent?

Solution:

(a) 4.00 kA

(b) 16.0 MW

(c) 16.0%

Exercise:

Problem:

A small office-building air conditioner operates on 408-V AC and consumes 50.0 kW. (a) What is its effective resistance? (b) What is the cost of running the air conditioner during a hot summer month when it is on 8.00 h per day for 30 days and electricity costs $9.00\ \text{cents}/\text{kW}\cdot\text{h}$?

Exercise:

Problem:

What is the average power consumption of a 120-V AC microwave oven that draws 10.0 A?

Solution:

1.20 kW

Exercise:

Problem:

What is the average current through a 500-W room heater that operates on 120-V AC power?

Exercise:

Problem:

Two different electrical devices have the same power consumption, but one is meant to be operated on 120-V AC and the other on 240-V AC. (a) What is the ratio of their resistances? (b) What is the ratio of their currents? (c) Assuming its resistance is unaffected, by what factor will the power increase if a 120-V AC device is connected to 240-V AC?

Solution:

(a) 4.0

(b) 0.50

(c) 4.0

Exercise:

Problem:

Nichrome wire is used in some radiative heaters. Find the resistance needed if the average power output is to be 1.00 kW utilizing 120-V AC.

Glossary

direct current
: (DC) the flow of electric charge in only one direction

alternating current
: (AC) the flow of electric charge that periodically reverses direction

AC voltage
: voltage that fluctuates sinusoidally with time.

AC current
: current that fluctuates sinusoidally with time.

rms
: a type of average taken for a time-varying quantity by squaring it, taking the mean of the square, and then taking the square-root of the mean.

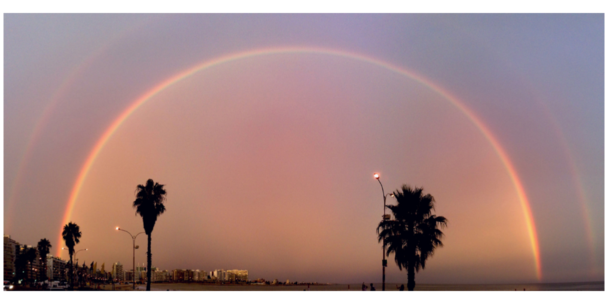
Introduction to Light
class="introduction"

Light

Light from this page or screen is formed into an image by the lens of your eye, much as the lens of the camera that made this photograph. Mirrors, like lenses, can also form images that in turn are captured by your eye.

Image seen as a result of reflection of light on a plane smooth surface. (credit: NASA Goddard Photo and Video, via Flickr)

Our lives are filled with light. Through vision, the most valued of our senses, light can evoke spiritual emotions, such as when we view a magnificent sunset or glimpse a rainbow breaking through the clouds. Light can also simply amuse us in a theater, or warn us to stop at an intersection. It has innumerable uses beyond vision. Light can carry telephone signals through glass fibers or cook a meal in a solar oven. Life itself could not exist without light's energy. From photosynthesis in plants to the sun warming a cold-blooded animal, its supply of energy is vital.



Double Rainbow over the bay

of Pocitos in Montevideo, Uruguay. (credit: Madrax, Wikimedia Commons)

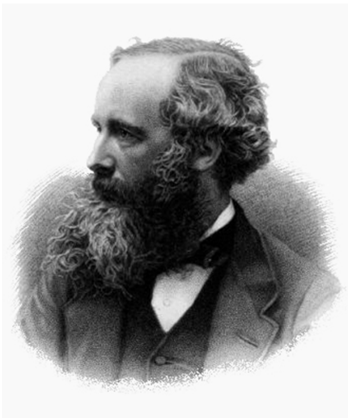
We will start our discussion of visible light as a type of electromagnetic wave. This knowledge will help us answer questions regarding the nature of light and vision. What is color, and how do our eyes detect it? Why do diamonds sparkle? How does light travel? How do lenses and mirrors form images? These are but a few of the questions that are answered by the study of optics. Optics is the branch of physics that deals with the behavior of visible light and other electromagnetic waves.

It is convenient to divide optics into two major parts based on the features we are interested in. The wave characteristics of light, such as frequency and wavelength, relates to the colors we perceive and to how we characterize different types of electromagnetic wave along the electromagnetic spectrum. The wave nature of light is also responsible for phenomena such as diffraction and interference. We call this part of optics "wave optics" or "physical optics." But when light interacts with an object that is several times as large as the light's wavelength, its observable behavior is like that of a ray; it does not prominently display its wave characteristics, and we can look at primarily how it refracts and reflects. We call this part of optics “geometric optics” or "ray optics."

Maxwell’s Equations: Electromagnetic Waves Predicted and Observed

- Restate Maxwell’s equations.

The Scotsman James Clerk Maxwell (1831–1879) is regarded as the greatest theoretical physicist of the 19th century. (See [link].) Although he died young, Maxwell not only formulated a complete electromagnetic theory, represented by **Maxwell’s equations**, he also developed the kinetic theory of gases and made significant contributions to the understanding of color vision and the nature of Saturn’s rings.



James Clerk Maxwell, a 19th-century physicist, developed a theory that explained the relationship between electricity and magnetism and correctly predicted that visible light is caused by electromagnetic

waves. (credit: G. J. Stodart)

Maxwell brought together all the work that had been done by brilliant physicists such as Oersted, Coulomb, Gauss, and Faraday, and added his own insights to develop the overarching theory of electromagnetism. The results of Maxwell’s equations are paraphrased here in words because their mathematical statement is beyond the level of this text. However, the equations illustrate how apparently simple mathematical statements can elegantly unite and express a multitude of concepts—why mathematics is the language of science.

Note:
The Results of Maxwell’s Equations

1. Electric field lines originate on positive charges and terminate on negative charges. The electric field is defined as the force per unit charge on a test charge.
2. Magnetic field lines are continuous, having no beginning or end. No magnetic monopoles are known to exist.
3. A changing magnetic field induces an electric field.
4. Magnetic fields are generated by moving charges or by changing electric fields.

Maxwell’s equations encompass the major laws of electricity and magnetism. What is not so apparent is the symmetry that Maxwell introduced in his mathematical framework. Especially important is his addition of the hypothesis that changing electric fields create magnetic fields. This is exactly analogous (and symmetric) to Faraday’s law of induction and had been suspected for some time, but fits beautifully into Maxwell’s equations.

Note:
Making Connections: Unification of Forces
Maxwell's complete and symmetric theory showed that electric and magnetic forces are not separate, but different manifestations of the same thing—the electromagnetic force. This classical unification of forces is one motivation for current attempts to unify the four basic forces in nature—the gravitational, electrical, strong, and weak nuclear forces.

Since changing electric fields create relatively weak magnetic fields, they could not be easily detected at the time of Maxwell's hypothesis. Maxwell realized, however, that oscillating charges, like those in AC circuits, produce changing electric fields. He predicted that these changing fields would propagate from the source like waves generated on a lake by a jumping fish.

The waves predicted by Maxwell would consist of oscillating electric and magnetic fields—defined to be an electromagnetic wave (EM wave). Electromagnetic waves would be capable of exerting forces on charges great distances from their source, and they might thus be detectable. Maxwell calculated that electromagnetic waves would propagate at the speed of light,

$$c = 3.00 \times 10^{8}\,\mathrm{m/s}.$$

In fact, Maxwell concluded that light is an electromagnetic wave having such wavelengths that it can be detected by the eye.

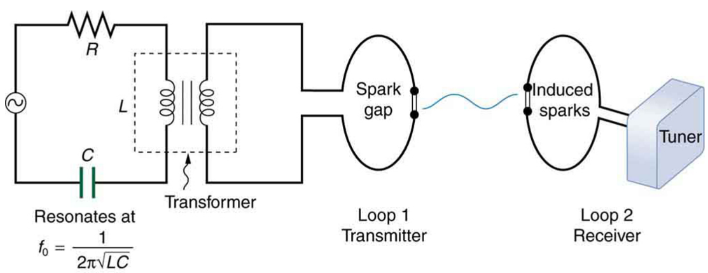
Other wavelengths should exist—it remained to be seen if they did. If so, Maxwell's theory and remarkable predictions would be verified, the greatest triumph of physics since Newton. Experimental verification came within a few years, but not before Maxwell's death.

Hertz's Observations

The German physicist Heinrich Hertz (1857–1894) was the first to generate and detect certain types of electromagnetic waves in the laboratory. Starting in 1887, he performed a series of experiments that not only confirmed the existence of electromagnetic waves, but also verified that they travel at the speed of light.

Hertz used an AC circuit that resonates at a known frequency and connected it to a loop of wire as shown in [link]. High voltages induced across the gap in the loop produced sparks that were visible evidence of the current in the circuit and that helped generate electromagnetic waves.

Across the laboratory, Hertz had another loop attached to another circuit, which could be tuned (as the dial on a radio) to the same resonant frequency as the first and could, thus, be made to receive electromagnetic waves. This loop also had a gap across which sparks were generated, giving solid evidence that electromagnetic waves had been received.



The apparatus used by Hertz in 1887 to generate and detect electromagnetic waves. An AC circuit connected to the first loop caused sparks across a gap in the wire loop and generated electromagnetic waves. Sparks across a gap in the second loop located across the laboratory gave evidence that the waves had been received.

Hertz also studied the reflection, refraction, and interference patterns of the electromagnetic waves he generated, verifying their wave character. He was able to determine wavelength from the interference patterns, and knowing their frequency, he could calculate the propagation speed using the equation $v = f\lambda$ (velocity—or speed—equals frequency times wavelength). Hertz was thus able to prove that electromagnetic waves travel at the speed of light. The SI unit for frequency, the hertz ($1 \text{ Hz} = 1 \text{ cycle/sec}$), is named in his honor.

Section Summary

- Electromagnetic waves consist of oscillating electric and magnetic fields and propagate at the speed of light c. They were predicted by Maxwell, who also showed that
- Maxwell's prediction of electromagnetic waves resulted from his formulation of a complete and symmetric theory of electricity and magnetism, known as Maxwell's equations.
- These four equations are paraphrased in this text, rather than presented numerically, and encompass the major laws of electricity and magnetism. First is Gauss's law for electricity, second is Gauss's law for magnetism, third is Faraday's law of induction, including Lenz's law, and fourth is Ampere's law in a symmetric formulation that adds another source of magnetism—changing electric fields.

Glossary

electromagnetic waves
: radiation in the form of waves of electric and magnetic energy

Maxwell's equations
: a set of four equations that comprise a complete, overarching theory of electromagnetism

hertz
: an SI unit denoting the frequency of an electromagnetic wave, in cycles per second

speed of light

in a vacuum, such as space, the speed of light is a constant 3×10^8 m/s

electric field lines

a pattern of imaginary lines that extend between an electric source and charged objects in the surrounding area, with arrows pointed away from positively charged objects and toward negatively charged objects. The more lines in the pattern, the stronger the electric field in that region

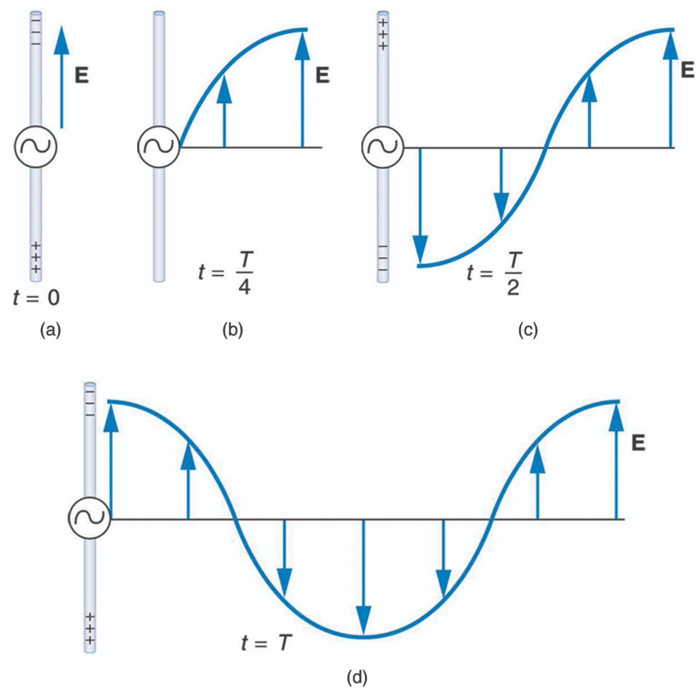
magnetic field lines

a pattern of continuous, imaginary lines that emerge from and enter into opposite magnetic poles. The density of the lines indicates the magnitude of the magnetic field

Production and Properties of Electromagnetic Waves

- Describe the electric and magnetic waves as they move out from a source, such as an AC generator.
- Describe the momentum carried by electromagnetic waves and its relationship to radiation pressure.

We can get a good understanding of **electromagnetic waves** (EM) by considering how they are produced. Whenever a current varies, associated electric and magnetic fields vary, moving out from the source like waves. Perhaps the easiest situation to visualize is a varying current in a long straight wire, produced by an AC generator at its center, as illustrated in [link].



This long straight gray wire with an AC generator at its center becomes a broadcast antenna for electromagnetic waves. Shown here are the charge distributions at four different times. The electric field ($\mathbf{E}$) propagates away from the antenna at the speed of light,

forming part of an electromagnetic wave.

The **electric field** ($\mathbf{E}$) shown surrounding the wire is produced by the charge distribution on the wire. Both the $\mathbf{E}$ and the charge distribution vary as the current changes. The changing field propagates outward at the speed of light.

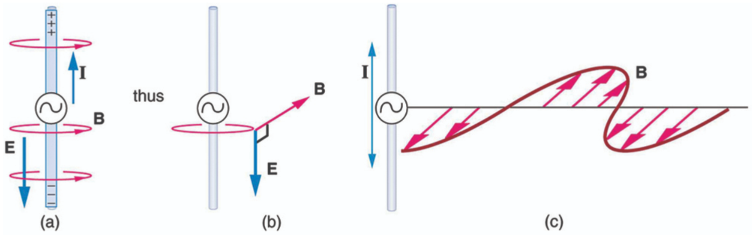
There is an associated **magnetic field** ($\mathbf{B}$) which propagates outward as well (see [link]). The electric and magnetic fields are closely related and propagate as an electromagnetic wave. This is what happens in broadcast antennae such as those in radio and TV stations.

Closer examination of the one complete cycle shown in [link] reveals the periodic nature of the generator-driven charges oscillating up and down in the antenna and the electric field produced. At time $t = 0$, there is the maximum separation of charge, with negative charges at the top and positive charges at the bottom, producing the maximum magnitude of the electric field (or E-field) in the upward direction. One-fourth of a cycle later, there is no charge separation and the field next to the antenna is zero, while the maximum E-field has moved away at speed c.

As the process continues, the charge separation reverses and the field reaches its maximum downward value, returns to zero, and rises to its maximum upward value at the end of one complete cycle. The outgoing wave has an **amplitude** proportional to the maximum separation of charge. Its **wavelength**(λ) is proportional to the period of the oscillation and, hence, is smaller for short periods or high frequencies. (As usual, wavelength and **frequency**(f) are inversely proportional.)

Electric and Magnetic Waves: Moving Together

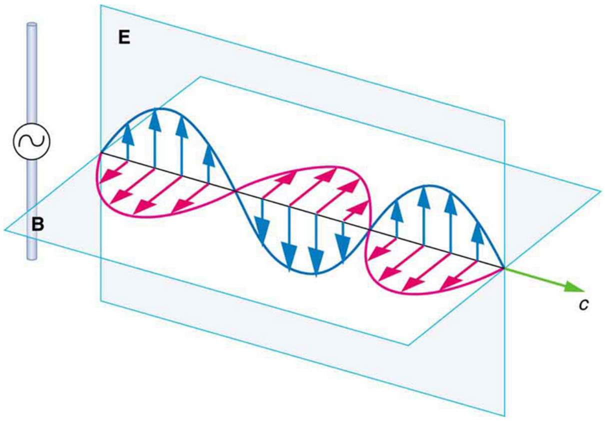
Following Ampere’s law, current in the antenna produces a magnetic field, as shown in [link]. The relationship between $\mathbf{E}$ and $\mathbf{B}$ is shown at one instant in [link] (a). As the current varies, the magnetic field varies in magnitude and direction.



(a) The current in the antenna produces the circular magnetic field lines. The current ($\mathbf{I}$) produces the separation of charge along the wire, which in turn creates the electric field as shown. (b) The electric and magnetic fields ($\mathbf{E}$ and $\mathbf{B}$) near the wire are perpendicular; they are shown here for one point in space. (c) The magnetic field varies with current and propagates away from the antenna at the speed of light.

The magnetic field lines also propagate away from the antenna at the speed of light, forming the other part of the electromagnetic wave, as seen in [link] (b). The magnetic part of the wave has the same period and wavelength as the electric part, since they are both produced by the same movement and separation of charges in the antenna.

The electric and magnetic waves are shown together at one instant in time in [link]. The electric and magnetic fields produced by a long straight wire antenna are exactly in phase. Note that they are perpendicular to one another and to the direction of propagation, making this a **transverse wave**.



A part of the electromagnetic wave sent out from the antenna at one instant in time. The electric and magnetic fields ($\mathbf{E}$ and $\mathbf{B}$) are in phase, and they are perpendicular to one another and the direction of propagation. For clarity, the waves are shown only along one direction, but they propagate out in other directions too.

Electromagnetic waves generally propagate out from a source in all directions, sometimes forming a complex radiation pattern. A linear antenna like this one will not radiate parallel to its length, for example. The wave is shown in one direction from the antenna in [link] to illustrate its basic characteristics.

Instead of the AC generator, the antenna can also be driven by an AC circuit. In fact, charges radiate whenever they are accelerated. But while a current in a circuit needs a complete path, an antenna has a varying charge distribution forming a **standing wave**, driven by the AC. The dimensions of the antenna are critical for determining the frequency of the radiated electromagnetic waves. This is a **resonant** phenomenon and when we tune

radios or TV, we vary electrical properties to achieve appropriate resonant conditions in the antenna.

Receiving Electromagnetic Waves

Electromagnetic waves carry energy away from their source, similar to a sound wave carrying energy away from a standing wave on a guitar string. An antenna for receiving EM signals works in reverse. And like antennas that produce EM waves, receiver antennas are specially designed to resonate at particular frequencies.

An incoming electromagnetic wave accelerates electrons in the antenna, setting up a standing wave. If the radio or TV is switched on, electrical components pick up and amplify the signal formed by the accelerating electrons. The signal is then converted to audio and/or video format. Sometimes big receiver dishes are used to focus the signal onto an antenna.

In fact, charges radiate whenever they are accelerated. When designing circuits, we often assume that energy does not quickly escape AC circuits, and mostly this is true. A broadcast antenna is specially designed to enhance the rate of electromagnetic radiation, and shielding is necessary to keep the radiation close to zero. Some familiar phenomena are based on the production of electromagnetic waves by varying currents. Your microwave oven, for example, sends electromagnetic waves, called microwaves, from a concealed antenna that has an oscillating current imposed on it.

Momentum Carried by Electromagnetic Waves

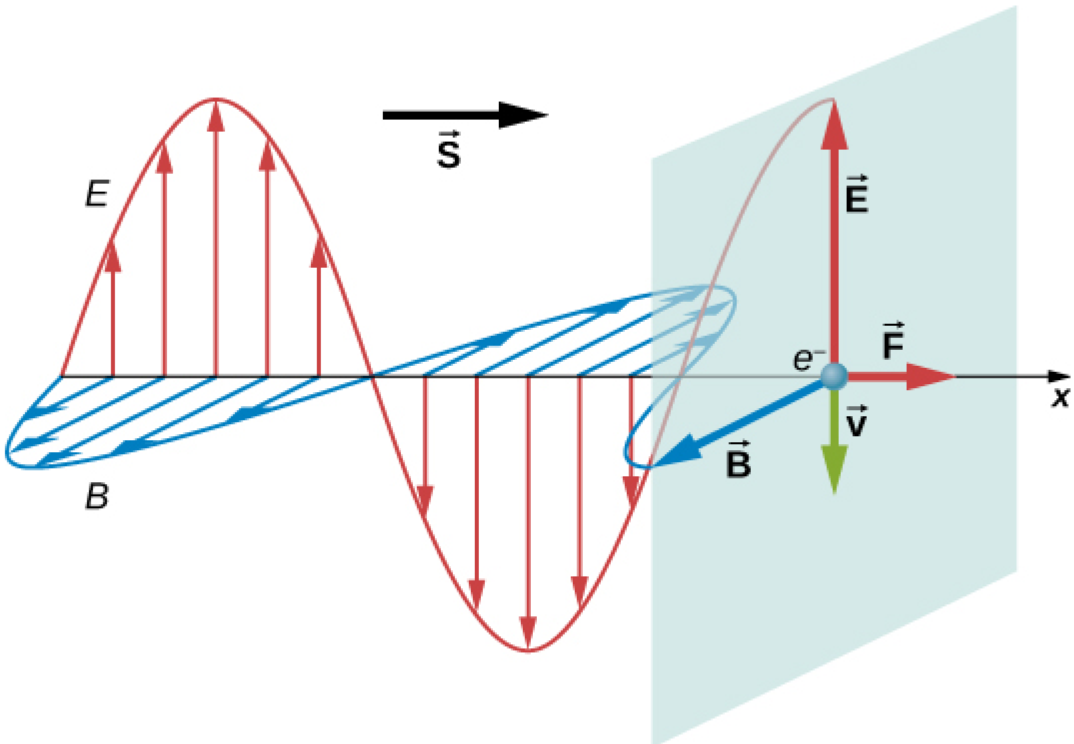
As electromagnetic waves are received, as described above, the forces exerted on the charged particles (see [link] for diagrams) do work on the particles and increase its energy. The energy that sunlight carries is a familiar part of every warm sunny day. A much less familiar feature of electromagnetic radiation is the extremely weak pressure that electromagnetic radiation produces by exerting a force in the direction of the wave. This force occurs because electromagnetic waves contain and transport momentum.

To understand the direction of the force for a very specific case, consider a plane electromagnetic wave incident on a metal. The electrons in the metal move with the velocity that is proportional to the force on it due to applied electric force (in opposite direction to because electron is negatively charged). There are other, frictional forces in the metal that make this motion happen (this is the origin of Ohm's law in electric circuits). Note that there is also a magnetic field in the electromagnetic wave ([link]). Using the right-hand rule (magnetic force is $= q(\ \times\ )$), we can get the direction of force on the electron (remember that electron is negatively charged), which is shown in [link]. This force is in the same direction as the direction of wave propagation, and it represents a transfer of momentum carried by the electromagnetic wave from the wave to the electron. This momentum is directly proportional to the energy carried by the electromagnetic wave and is given by,

Equation:

$$p = \frac{E}{c},$$

where p is the magnitude of momentum of electromagnetic wave, E is the energy of electromagnetic wave, and c is the speed of light.

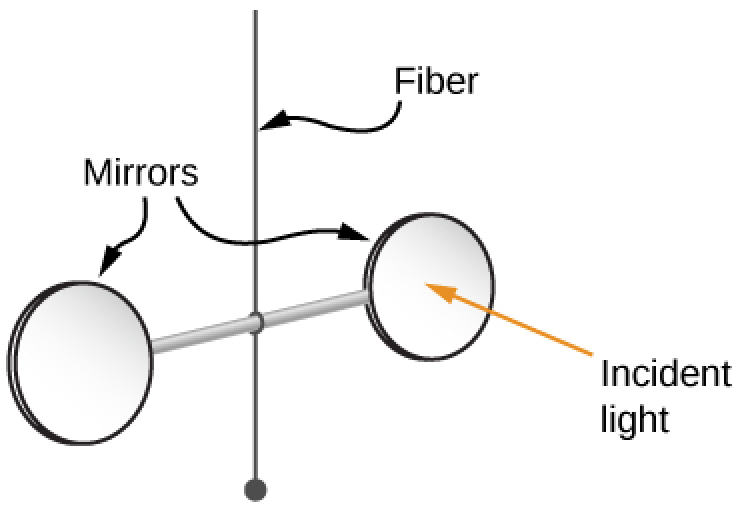


Electric and magnetic fields of an electromagnetic wave can combine to produce a force in the direction of propagation, as illustrated for the special case of electrons whose motion is highly damped by the resistance of a metal.

Maxwell predicted that an electromagnetic wave carries momentum. An object absorbing an electromagnetic wave would experience a force in the direction of propagation of the wave. The force corresponds to **radiation pressure** exerted on the object by the wave. The force would be twice as great if the radiation were reflected rather than absorbed, because the change in the momentum of electromagnetic wave itself is twice as great (and we enforce momentum conservation of the system as a whole).

Maxwell's prediction was confirmed in 1903 by Nichols and Hull by precisely measuring radiation pressures with a torsion balance. The schematic arrangement is shown in [link]. The mirrors suspended from a fiber were housed inside a glass container. Nichols and Hull were able to obtain a small measurable deflection of the mirrors from shining light on

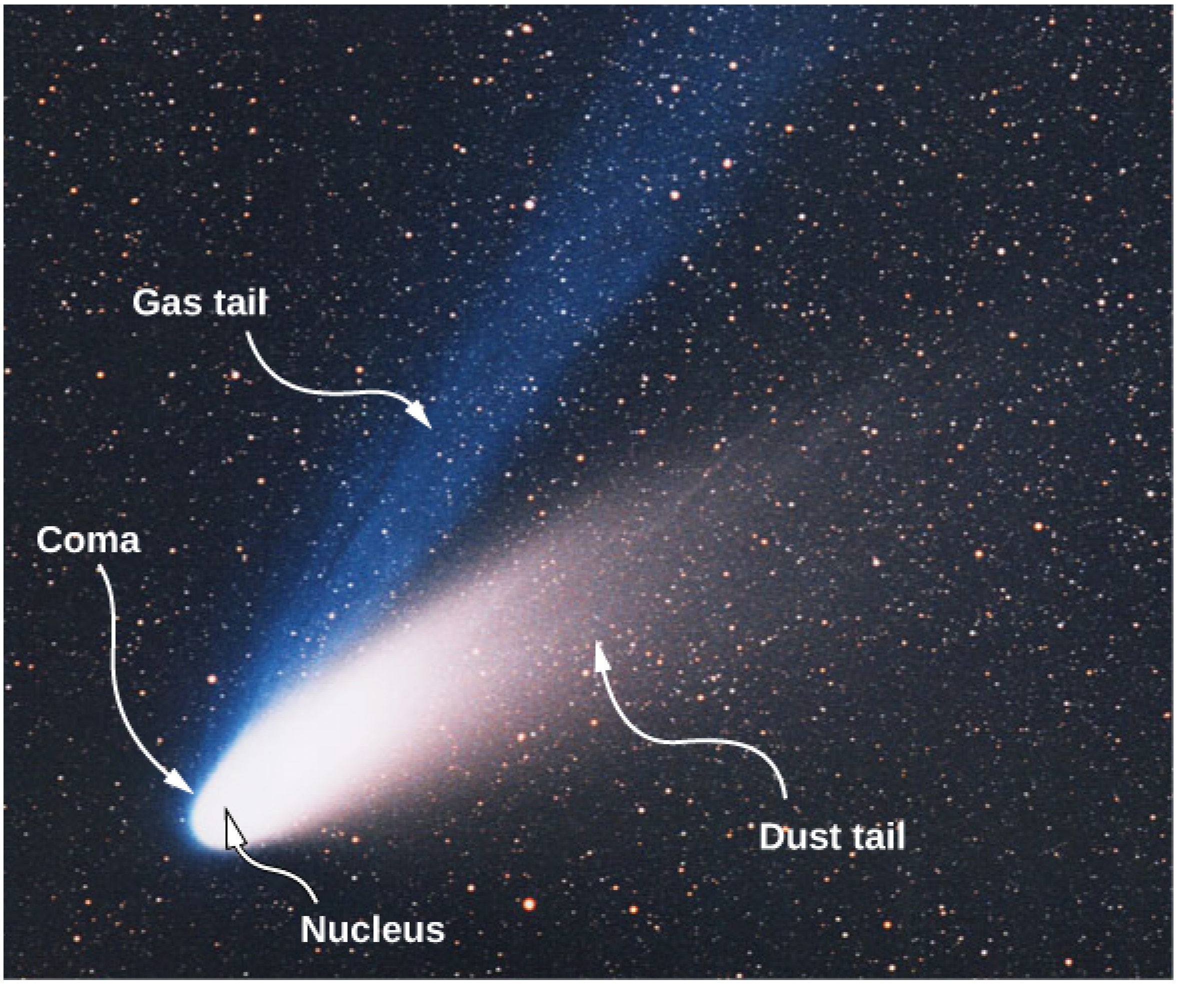
one of them. From the measured deflection, they could calculate the unbalanced force on the mirror, and obtained agreement with the predicted value of the force.



Simplified diagram of the central part of the apparatus Nichols and Hull used to precisely measure radiation pressure and confirm Maxwell's prediction.

Radiation pressure plays a role in explaining many observed astronomical phenomena, including the appearance of comets. Comets are basically chunks of icy material in which frozen gases and particles of rock and dust are embedded. When a comet approaches the Sun, it warms up and its surface begins to evaporate. The *coma* of the comet is the hazy area around it from the gases and dust. Some of the gases and dust form tails when they leave the comet. Notice in [link] that a comet has *two* tails. The *ion tail* (or *gas tail* in [link]) is composed mainly of ionized gases. These ions interact electromagnetically with the solar wind, which is a continuous stream of charged particles emitted by the Sun. The force of the solar wind on the ionized gases is strong enough that the ion tail almost always points directly away from the Sun. The second tail is composed of dust particles. Because

the *dust tail* is electrically neutral, it does not interact with the solar wind. However, this tail is affected by the radiation pressure produced by the light from the Sun. Although quite small, this pressure is strong enough to cause the dust tail to be displaced from the path of the comet.

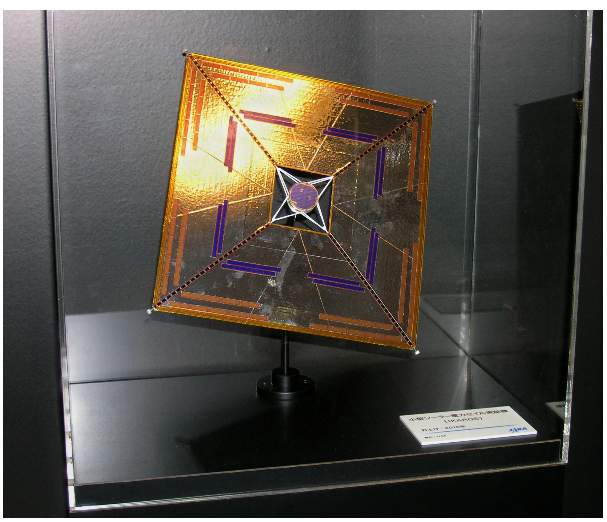


Evaporation of material being warmed by the Sun forms two tails, as shown in this photo of Comet Ison. (credit: modification of work by E. Slawik—ESO)

After Maxwell showed that light carried momentum as well as energy, a novel idea eventually emerged, initially only as science fiction. Perhaps a

spacecraft with a large reflecting solar sail could use radiation pressure for propulsion. Such a vehicle would not have to carry fuel. It would experience a constant but small force from solar radiation, instead of the short bursts from rocket propulsion. It would accelerate slowly, but by being accelerated continuously, it would eventually reach great speeds. A spacecraft with small total mass and a sail with a large area would be necessary to obtain a usable acceleration.

The latest accomplishment on this concept is represented by the IKAROS (Interplanetary Kite-craft Accelerated by Radiation Of the Sun) project by Japan Aerospace Exploration Agency (JAXA), which was launched in May 2010. IKAROS solar sail is currently traveling through the solar system. During its six-month journey to Venus, acceleration and attitude control using the solar sail, rather than thrusters, were successfully tested. Last update from the spacecraft was in May 2014, on a ten-month orbit around the Sun. A scale model of the spacecraft is shown in [link].



A 1:64 scale model of the IKAROS spacecraft at the 61st International Astronautical Congress in Prague, Czech Republic ("IKAROS model" By

Pavel Hrdlička, Wikipedia, CC BY-SA 3.0)

Section Summary

- Electromagnetic waves are created by oscillating charges (which radiate whenever accelerated) and have the same frequency as the oscillation.
- Since the electric and magnetic fields in most electromagnetic waves are perpendicular to the direction in which the wave moves, it is ordinarily a transverse wave.
- Electromagnetic waves carry momentum and exert radiation pressure.
- The momentum of an electromagnetic wave is directly proportional to its energy.

Glossary

electric field
: a vector quantity (**E**); the lines of electric force per unit charge, moving radially outward from a positive charge and in toward a negative charge

electric field strength
: the magnitude of the electric field, denoted *E*-field

magnetic field
: a vector quantity (**B**); can be used to determine the magnetic force on a moving charged particle

magnetic field strength
: the magnitude of the magnetic field, denoted *B*-field

transverse wave
: a wave, such as an electromagnetic wave, which oscillates perpendicular to the axis along the line of travel

standing wave
: a wave that oscillates in place, with nodes where no motion happens

wavelength
: the distance from one peak to the next in a wave

amplitude
: the height, or magnitude, of an electromagnetic wave

frequency
: the number of complete wave cycles (up-down-up) passing a given point within one second (cycles/second)

resonant
: a system that displays enhanced oscillation when subjected to a periodic disturbance of the same frequency as its natural frequency

oscillate
: to fluctuate back and forth in a steady beat

radiation pressure
: pressure exerted by an electromagnetic wave on a surface

solar sail
: a spacecraft that utilizes radiation pressure due to solar radiation in its propulsion

The Electromagnetic Spectrum: an Overview

- List three “rules of thumb” that apply to the different frequencies along the electromagnetic spectrum.
- Explain why the higher the frequency, the shorter the wavelength of an electromagnetic wave.
- Draw a simplified electromagnetic spectrum, indicating the relative positions, frequencies, and spacing of the different types of radiation bands.

In this module we examine how electromagnetic waves are classified into categories such as radio, infrared, ultraviolet, and so on, so that we can understand some of their similarities as well as some of their differences. We will also find that there are many connections with previously discussed topics, such as wavelength and resonance. A brief overview of the production and utilization of electromagnetic waves is found in [link]. Note that the vast majority of the different types of electromagnetic waves originate from atomic and/or molecular electron transitions—that is, from electrons changing their energy levels within atoms or molecules.

Type of EM wave	Production	Applications	Life sciences aspect	Issues
Radio and TV	Accelerating charges	Communications, Remote controls	MRI	Requires controls for band use
Microwaves	Accelerating charges and thermal agitation	Communications, Ovens, Radar	Deep heating	Cell phone use
Infrared	Thermal agitations and atomic/molecular electron transitions	Thermal imaging, Heating	Absorbed by atmosphere	Greenhouse effect
Visible light	Thermal agitations and atomic/molecular electron transitions	All pervasive	Photosynthesis, Human vision	
Ultraviolet	Thermal agitations and atomic/molecular electron transitions	Sterilization, Cancer control	Vitamin D production	Ozone depletion, Cancer causing

Type of EM wave	Production	Applications	Life sciences aspect	Issues
X-rays	Inner atomic electron transitions and fast collisions	Medical, Security	Medical diagnosis, Cancer therapy	Cancer causing
Gamma rays	Nuclear decay	Nuclear medicine, Security	Medical diagnosis, Cancer therapy	Cancer causing, Radiation damage

Electromagnetic Waves

Note:
Connections: Waves
There are many types of waves, such as water waves and even earthquakes. Among the many shared attributes of waves are propagation speed, frequency, and wavelength. These are always related by the expression $v_W = f\lambda$. This module concentrates on EM waves, but other modules contain examples of all of these characteristics for sound waves and submicroscopic particles.

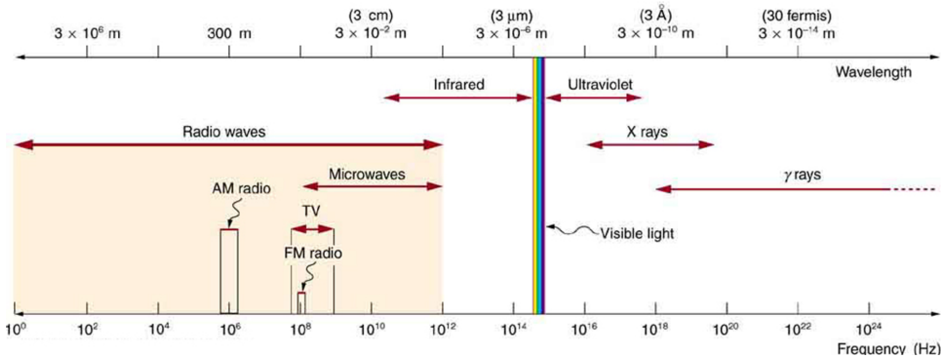
As noted before, an electromagnetic wave has a frequency and a wavelength associated with it and travels at the speed of light, or c. The relationship among these wave characteristics can be described by $v_W = f\lambda$, where v_W is the propagation speed of the wave, f is the frequency, and λ is the wavelength. Here $v_W = c$, so that for all electromagnetic waves,

Equation:

$$c = f\lambda.$$

Thus, for all electromagnetic waves, the greater the frequency, the smaller the wavelength.

[link] shows how the various types of electromagnetic waves are categorized according to their wavelengths and frequencies—that is, it shows the electromagnetic spectrum. Many of the characteristics of the various types of electromagnetic waves are related to their frequencies and wavelengths, as we shall see.



The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap.

Note:
Electromagnetic Spectrum: Rules of Thumb
Three rules that apply to electromagnetic waves in general are as follows:

- High-frequency electromagnetic waves are more energetic and are more able to penetrate than low-frequency waves.
- High-frequency electromagnetic waves can carry more information per unit time than low-frequency waves.
- The shorter the wavelength of any electromagnetic wave probing a material, the smaller the detail it is possible to resolve.

Note that there are exceptions to these rules of thumb.

Transmission, Reflection, and Absorption

What happens when an electromagnetic wave impinges on a material? If the material is transparent to the particular frequency, then the wave can largely be transmitted. If the material is opaque to the frequency, then the wave can be totally reflected. The wave can also be absorbed by the material, indicating that there is some interaction between the wave and the material, such as the thermal agitation of molecules.

Of course it is possible to have partial transmission, reflection, and absorption. We normally associate these properties with visible light, but they do apply to all electromagnetic waves. What is not obvious is that something that is transparent to light may be opaque at other frequencies. For example, ordinary glass is transparent to visible light but largely opaque to ultraviolet radiation. Human skin is opaque to visible light—we cannot see through people—but transparent to X-rays.

Section Summary

- The relationship among the speed of propagation, wavelength, and frequency for any wave is given by $v_{\text{W}} = f\lambda$, so that for electromagnetic waves,
  Equation:
  $$c = f\lambda,$$
  where f is the frequency, λ is the wavelength, and c is the speed of light.
- The electromagnetic spectrum is separated into many categories and subcategories, based on the frequency and wavelength, source, and uses of the electromagnetic waves.

Glossary

electromagnetic spectrum
: the full range of wavelengths or frequencies of electromagnetic radiation

radio waves
: electromagnetic waves with wavelengths in the range from 1 mm to 100 km; they are produced by currents in wires and circuits and by astronomical phenomena

microwaves
: electromagnetic waves with wavelengths in the range from 1 mm to 1 m; they can be produced by currents in macroscopic circuits and devices

infrared radiation (IR)
: a region of the electromagnetic spectrum with a frequency range that extends from just below the red region of the visible light spectrum up to the microwave region, or from 0.74 μm to 300 μm

ultraviolet radiation (UV)
: electromagnetic radiation in the range extending upward in frequency from violet light and overlapping with the lowest X-ray frequencies, with wavelengths from 400 nm down to about 10 nm

visible light
: the narrow segment of the electromagnetic spectrum to which the normal human eye responds

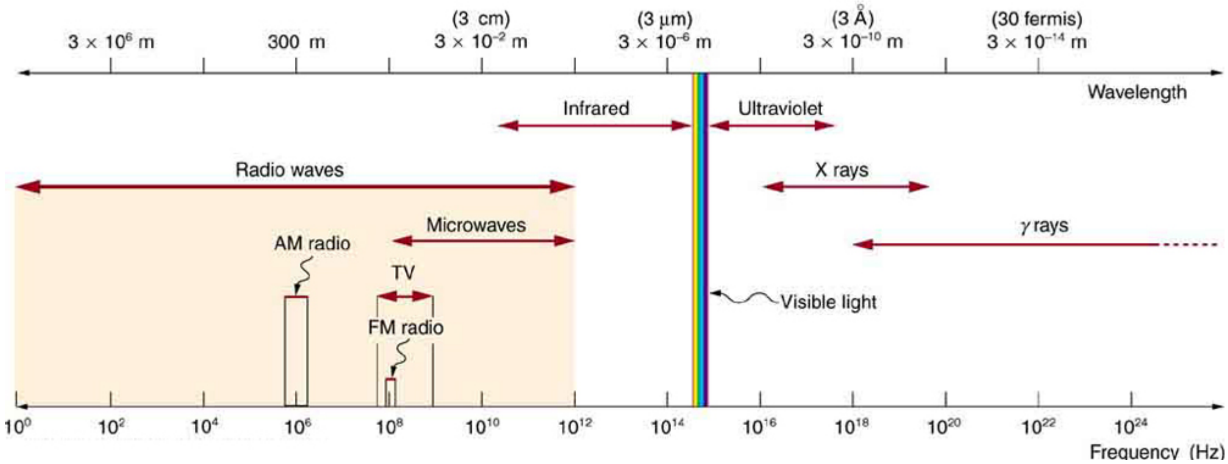
X-ray
: invisible, penetrating form of very high frequency electromagnetic radiation, overlapping both the ultraviolet range and the γ-ray range

gamma ray
: (γ ray); extremely high frequency electromagnetic radiation emitted by the nucleus of an atom, either from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons. The lower end of the γ-ray frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation

The Electromagnetic Spectrum: Application Notes

- List and explain the characteristics and applications of different parts of the electromagnetic spectrum.

In this module, we look at the properties of different types of electromagnetic waves. Again, [link] shows the electromagnetic spectrum. The characteristics of the various types of electromagnetic waves you will read about below are related to their frequencies and wavelengths.



The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap.

Radio and TV Waves

The broad category of **radio waves** is defined to contain any electromagnetic wave produced by currents in wires and circuits. Its name derives from their most common use as a carrier of audio information (i.e., radio). The name is applied to electromagnetic waves of similar frequencies regardless of source. Radio waves from outer space, for example, do not come from alien radio stations. They are created by many astronomical phenomena, and their study has revealed much about nature on the largest scales.

There are many uses for radio waves, and so the category is divided into many subcategories, including microwaves and those electromagnetic waves used for AM and FM radio, cellular telephones, and TV.

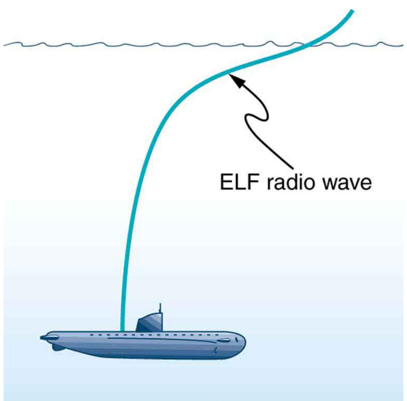
The lowest commonly encountered radio frequencies are produced by high-voltage AC power transmission lines at frequencies of 50 or 60 Hz. (See [link].) These extremely long wavelength electromagnetic waves (about 6000 km!) are one means of energy loss in long-distance power transmission.

This high-voltage traction power line running to Eutingen Railway Substation in Germany radiates electromagnetic waves with very long wavelengths. (credit: Zonk43, Wikimedia Commons)

There is an ongoing controversy regarding potential health hazards associated with exposure to these electromagnetic fields (E-fields). Some people suspect that living near such transmission lines may cause a variety of illnesses, including cancer. But demographic data are either inconclusive

or simply do not support the hazard theory. Recent reports that have looked at many European and American epidemiological studies have found no increase in risk for cancer due to exposure to E-fields.

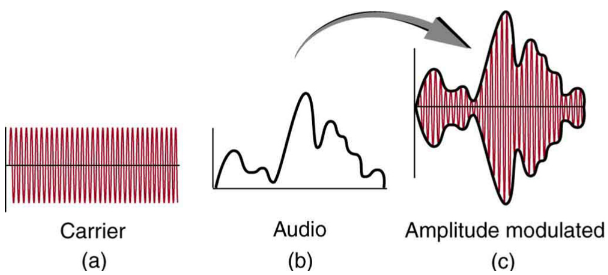
Extremely low frequency (ELF) radio waves of about 1 kHz are used to communicate with submerged submarines. The ability of radio waves to penetrate salt water is related to their wavelength (much like ultrasound penetrating tissue)—the longer the wavelength, the farther they penetrate. Since salt water is a good conductor, radio waves are strongly absorbed by it, and very long wavelengths are needed to reach a submarine under the surface. (See [link].)



Very long wavelength radio waves are needed to reach this submarine, requiring extremely low frequency signals (ELF). Shorter wavelengths do not penetrate to any significant depth.

AM radio waves are used to carry commercial radio signals in the frequency range from 540 to 1600 kHz. The abbreviation AM stands for **amplitude modulation**, which is the method for placing information on these waves. (See [link].) A **carrier wave** having the basic frequency of the radio station, say 1530 kHz, is varied or modulated in amplitude by an audio signal. The resulting wave has a constant frequency, but a varying amplitude.

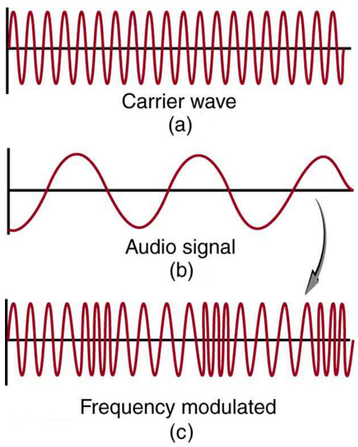
A radio receiver tuned to have the same resonant frequency as the carrier wave can pick up the signal, while rejecting the many other frequencies impinging on its antenna. The receiver’s circuitry is designed to respond to variations in amplitude of the carrier wave to replicate the original audio signal. That audio signal is amplified to drive a speaker or perhaps to be recorded.



Amplitude modulation for AM radio. (a) A carrier wave at the station’s basic frequency. (b) An audio signal at much lower audible frequencies. (c) The amplitude of the carrier is modulated by the audio signal without changing its basic frequency.

FM Radio Waves

FM radio waves are also used for commercial radio transmission, but in the frequency range of 88 to 108 MHz. FM stands for **frequency modulation**, another method of carrying information. (See [link].) Here a carrier wave having the basic frequency of the radio station, perhaps 105.1 MHz, is modulated in frequency by the audio signal, producing a wave of constant amplitude but varying frequency.



Frequency modulation for FM radio. (a) A carrier wave at the station's basic frequency. (b) An audio signal at much lower audible frequencies. (c) The frequency of the carrier is modulated by the audio signal without changing its amplitude.

Since audible frequencies range up to 20 kHz (or 0.020 MHz) at most, the frequency of the FM radio wave can vary from the carrier by as much as 0.020 MHz. Thus the carrier frequencies of two different radio stations cannot be closer than 0.020 MHz. An FM receiver is tuned to resonate at the carrier frequency and has circuitry that responds to variations in frequency, reproducing the audio information.

FM radio is inherently less subject to noise from stray radio sources than AM radio. The reason is that amplitudes of waves add. So an AM receiver would interpret noise added onto the amplitude of its carrier wave as part of the information. An FM receiver can be made to reject amplitudes other than that of the basic carrier wave and only look for variations in frequency. It is thus easier to reject noise from FM, since noise produces a variation in amplitude.

Television is also broadcast on electromagnetic waves. Since the waves must carry a great deal of visual as well as audio information, each channel requires a larger range of frequencies than simple radio transmission. TV channels utilize frequencies in the range of 54 to 88 MHz and 174 to 222 MHz. (The entire FM radio band lies between channels 88 MHz and 174 MHz.) These TV channels are called VHF (for **very high frequency**). Other channels called UHF (for **ultra high frequency**) utilize an even higher frequency range of 470 to 1000 MHz.

The TV video signal is AM, while the TV audio is FM. Note that these frequencies are those of free transmission with the user utilizing an old-fashioned roof antenna. Satellite dishes and cable transmission of TV occurs at significantly higher frequencies and is rapidly evolving with the use of the high-definition or HD format.

Example:
Calculating Wavelengths of Radio Waves

Calculate the wavelengths of a 1530-kHz AM radio signal, a 105.1-MHz FM radio signal, and a 1.90-GHz cell phone signal.

Strategy

The relationship between wavelength and frequency is $c = f\lambda$, where $c = 3.00 \times 10^8\ \text{m/s}$ is the speed of light (the speed of light is only very slightly smaller in air than it is in a vacuum). We can rearrange this equation to find the wavelength for all three frequencies.

Solution

Rearranging gives

Equation:

$$\lambda = \frac{c}{f}.$$

(a) For the $f = 1530\ \text{kHz}$ AM radio signal, then,

Equation:

$$\begin{aligned}\lambda &= \frac{3.00 \times 10^8\ \text{m/s}}{1530 \times 10^3\ \text{cycles/s}} \\ &= 196\ \text{m}.\end{aligned}$$

(b) For the $f = 105.1\ \text{MHz}$ FM radio signal,

Equation:

$$\begin{aligned}\lambda &= \frac{3.00 \times 10^8\ \text{m/s}}{105.1 \times 10^6\ \text{cycles/s}} \\ &= 2.85\ \text{m}.\end{aligned}$$

(c) And for the $f = 1.90\ \text{GHz}$ cell phone,

Equation:

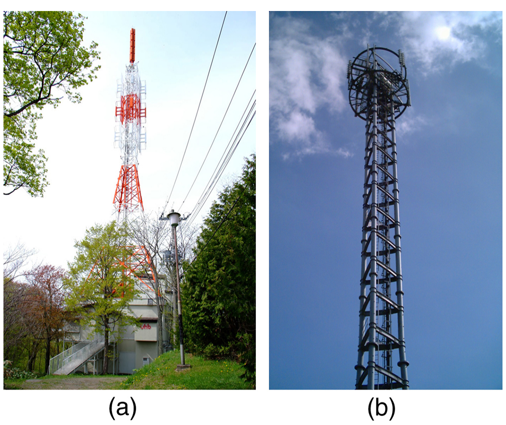
$$\begin{aligned}\lambda &= \frac{3.00 \times 10^8\ \text{m/s}}{1.90 \times 10^9\ \text{cycles/s}} \\ &= 0.158\ \text{m}.\end{aligned}$$

Discussion

These wavelengths are consistent with the spectrum in [link]. The wavelengths are also related to other properties of these electromagnetic waves, as we shall see.

The wavelengths found in the preceding example are representative of AM, FM, and cell phones, and account for some of the differences in how they are broadcast and how well they travel. The most efficient length for a linear antenna, such as discussed in "Production of Electromagnetic Waves", is $\lambda/2$, half the wavelength of the electromagnetic wave. Thus a very large antenna is needed to efficiently broadcast typical AM radio with its carrier wavelengths on the order of hundreds of meters.

One benefit to these long AM wavelengths is that they can go over and around rather large obstacles (like buildings and hills), just as ocean waves can go around large rocks. FM and TV are best received when there is a line of sight between the broadcast antenna and receiver, and they are often sent from very tall structures. FM, TV, and mobile phone antennas themselves are much smaller than those used for AM, but they are elevated to achieve an unobstructed line of sight. (See [link].)



(a) A large tower is used to broadcast TV signals. The actual antennas are small structures on top of the tower—they are placed at great heights to have a clear line of sight over a large broadcast area. (credit: Ozizo, Wikimedia Commons)

(b) The NTT Dokomo mobile phone tower at Tokorozawa City, Japan. (credit: tokoroten, Wikimedia Commons)

Radio Wave Interference

Astronomers and astrophysicists collect signals from outer space using electromagnetic waves. A common problem for astrophysicists is the “pollution” from electromagnetic radiation pervading our surroundings from communication systems in general. Even everyday gadgets like our car keys having the facility to lock car doors remotely and being able to turn TVs on and off using remotes involve radio-wave frequencies. In order to prevent interference between all these electromagnetic signals, strict regulations are drawn up for different organizations to utilize different radio frequency bands.

One reason why we are sometimes asked to switch off our mobile phones (operating in the range of 1.9 GHz) on airplanes and in hospitals is that important communications or medical equipment often uses similar radio frequencies and their operation can be affected by frequencies used in the communication devices.

For example, radio waves used in magnetic resonance imaging (MRI) have frequencies on the order of 100 MHz, although this varies significantly depending on the strength of the magnetic field used and the nuclear type being scanned. MRI is an important medical imaging and research tool, producing highly detailed two- and three-dimensional images. Radio waves are broadcast, absorbed, and reemitted in a resonance process that is sensitive to the density of nuclei (usually protons or hydrogen nuclei).

The wavelength of 100-MHz radio waves is 3 m, yet using the sensitivity of the resonant frequency to the magnetic field strength, details smaller than a millimeter can be imaged. This is a good example of an exception to a rule

of thumb (in this case, the rubric that details much smaller than the probe's wavelength cannot be detected). The intensity of the radio waves used in MRI presents little or no hazard to human health.

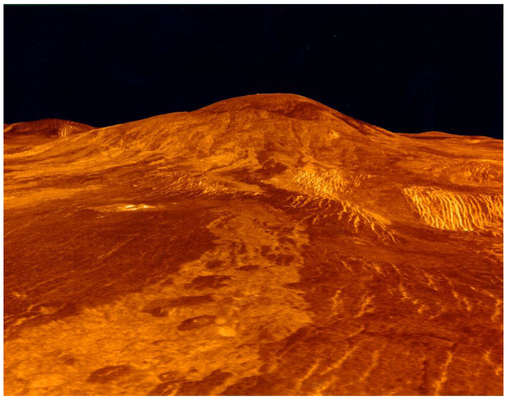
Microwaves

Microwaves are the highest-frequency electromagnetic waves that can be produced by currents in macroscopic circuits and devices. Microwave frequencies range from about 10^9 Hz to the highest practical LC resonance at nearly 10^{12} Hz. Since they have high frequencies, their wavelengths are short compared with those of other radio waves—hence the name "microwave."

Microwaves can also be produced by atoms and molecules. They are, for example, a component of electromagnetic radiation generated by **thermal agitation**. The thermal motion of atoms and molecules in any object at a temperature above absolute zero causes them to emit and absorb radiation.

Since it is possible to carry more information per unit time on high frequencies, microwaves are quite suitable for communications. Most satellite-transmitted information is carried on microwaves, as are land-based long-distance transmissions. A clear line of sight between transmitter and receiver is needed because of the short wavelengths involved.

Radar is a common application of microwaves that was first developed in World War II. By detecting and timing microwave echoes, radar systems can determine the distance to objects as diverse as clouds and aircraft. A Doppler shift in the radar echo can be used to determine the speed of a car or the intensity of a rainstorm. Sophisticated radar systems are used to map the Earth and other planets, with a resolution limited by wavelength. (See [link].) The shorter the wavelength of any probe, the smaller the detail it is possible to observe.



An image of Sif Mons with lava flows on Venus, based on Magellan synthetic aperture radar data combined with radar altimetry to produce a three-dimensional map of the surface. The Venusian atmosphere is opaque to visible light, but not to the microwaves that were used to create this image. (credit: NSSDC, NASA/JPL)

Heating with Microwaves

How does the ubiquitous microwave oven produce microwaves electronically, and why does food absorb them preferentially? Microwaves at a frequency of 2.45 GHz are produced by accelerating electrons. The microwaves are then used to induce an alternating electric field in the oven.

Water and some other constituents of food have a slightly negative charge at one end and a slightly positive charge at one end (called polar molecules). The range of microwave frequencies is specially selected so that the polar

molecules, in trying to keep orienting themselves with the electric field, absorb these energies and increase their temperatures—called dielectric heating.

The energy thereby absorbed results in thermal agitation heating food and not the plate, which does not contain water. Hot spots in the food are related to constructive and destructive interference patterns. Rotating antennas and food turntables help spread out the hot spots.

Another use of microwaves for heating is within the human body. Microwaves will penetrate more than shorter wavelengths into tissue and so can accomplish “deep heating” (called microwave diathermy). This is used for treating muscular pains, spasms, tendonitis, and rheumatoid arthritis.

Note:
Making Connections: Take-Home Experiment—Microwave Ovens

1. Look at the door of a microwave oven. Describe the structure of the door. Why is there a metal grid on the door? How does the size of the holes in the grid compare with the wavelengths of microwaves used in microwave ovens? What is this wavelength?
2. Place a glass of water (about 250 ml) in the microwave and heat it for 30 seconds. Measure the temperature gain (the ΔT). Assuming that the power output of the oven is 1000 W, calculate the efficiency of the heat-transfer process.
3. Remove the rotating turntable or moving plate and place a cup of water in several places along a line parallel with the opening. Heat for 30 seconds and measure the ΔT for each position. Do you see cases of destructive interference?

Microwaves generated by atoms and molecules far away in time and space can be received and detected by electronic circuits. Deep space acts like a blackbody with a 2.7 K temperature, radiating most of its energy in the

microwave frequency range. In 1964, Penzias and Wilson detected this radiation and eventually recognized that it was the radiation of the Big Bang's cooled remnants.

Infrared Radiation

The microwave and infrared regions of the electromagnetic spectrum overlap (see [link]). **Infrared radiation** is generally produced by thermal motion and the vibration and rotation of atoms and molecules. Electronic transitions in atoms and molecules can also produce infrared radiation.

The range of infrared frequencies extends up to the lower limit of visible light, just below red. In fact, infrared means "below red." Frequencies at its upper limit are too high to be produced by accelerating electrons in circuits, but small systems, such as atoms and molecules, can vibrate fast enough to produce these waves.

Water molecules rotate and vibrate particularly well at infrared frequencies, emitting and absorbing them so efficiently that the emissivity for skin is $e = 0.97$ in the infrared. Night-vision scopes can detect the infrared emitted by various warm objects, including humans, and convert it to visible light.

We can examine radiant heat transfer from a house by using a camera capable of detecting infrared radiation. Reconnaissance satellites can detect buildings, vehicles, and even individual humans by their infrared emissions, whose power radiation is proportional to the fourth power of the absolute temperature. More mundanely, we use infrared lamps, some of which are called quartz heaters, to preferentially warm us because we absorb infrared better than our surroundings.

The Sun radiates like a nearly perfect blackbody (that is, it has $e = 1$), with a 6000 K surface temperature. About half of the solar energy arriving at the Earth is in the infrared region, with most of the rest in the visible part of the spectrum, and a relatively small amount in the ultraviolet. On average, 50 percent of the incident solar energy is absorbed by the Earth.

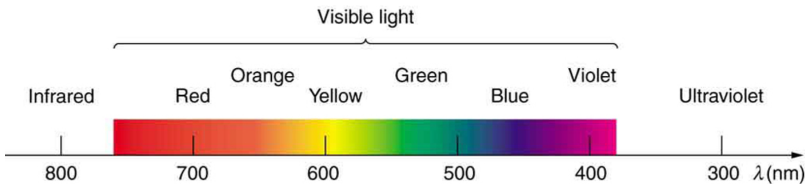
The relatively constant temperature of the Earth is a result of the energy balance between the incoming solar radiation and the energy radiated from the Earth. Most of the infrared radiation emitted from the Earth is absorbed by CO_2 and H_2O in the atmosphere and then radiated back to Earth or into outer space. This radiation back to Earth is known as the greenhouse effect, and it maintains the surface temperature of the Earth about $40^{\circ}C$ higher than it would be if there is no absorption. Some scientists think that the increased concentration of CO_2 and other greenhouse gases in the atmosphere, resulting from increases in fossil fuel burning, has increased global average temperatures.

Visible Light

Visible light is the narrow segment of the electromagnetic spectrum to which the normal human eye responds. Visible light is produced by vibrations and rotations of atoms and molecules, as well as by electronic transitions within atoms and molecules. The receivers or detectors of light largely utilize electronic transitions. We say the atoms and molecules are excited when they absorb and relax when they emit through electronic transitions.

[link] shows this part of the spectrum, together with the colors associated with particular pure wavelengths. We usually refer to visible light as having wavelengths of between 400 nm and 750 nm. (The retina of the eye actually responds to the lowest ultraviolet frequencies, but these do not normally reach the retina because they are absorbed by the cornea and lens of the eye.)

Red light has the lowest frequencies and longest wavelengths, while violet has the highest frequencies and shortest wavelengths. Blackbody radiation from the Sun peaks in the visible part of the spectrum but is more intense in the red than in the violet, making the Sun yellowish in appearance.



A small part of the electromagnetic spectrum that includes its visible components. The divisions between infrared, visible, and ultraviolet are not perfectly distinct, nor are those between the seven rainbow colors.

Living things—plants and animals—have evolved to utilize and respond to parts of the electromagnetic spectrum they are embedded in. Visible light is the most predominant and we enjoy the beauty of nature through visible light. Plants are more selective. Photosynthesis makes use of parts of the visible spectrum to make sugars.

Optics is the study of the behavior of visible light and other forms of electromagnetic waves. Optics falls into two distinct categories. When electromagnetic radiation, such as visible light, interacts with objects that are large compared with its wavelength, its motion can be represented by straight lines like rays. Ray optics is the study of such situations and includes lenses and mirrors.

When electromagnetic radiation interacts with objects about the same size as the wavelength or smaller, its wave nature becomes apparent. For example, observable detail is limited by the wavelength, and so visible light can never detect individual atoms, because they are so much smaller than its wavelength. Physical or wave optics is the study of such situations and includes all wave characteristics.

Note:
Take-Home Experiment: Colors That Match

When you light a match you see largely orange light; when you light a gas stove you see blue light. Why are the colors different? What other colors are present in these?

Ultraviolet Radiation

Ultraviolet means “above violet.” The electromagnetic frequencies of **ultraviolet radiation (UV)** extend upward from violet, the highest-frequency visible light. Ultraviolet is also produced by atomic and molecular motions and electronic transitions. The wavelengths of ultraviolet extend from 400 nm down to about 10 nm at its highest frequencies, which overlap with the lowest X-ray frequencies. It was recognized as early as 1801 by Johann Ritter that the solar spectrum had an invisible component beyond the violet range.

Solar UV radiation is broadly subdivided into three regions: UV-A (320–400 nm), UV-B (290–320 nm), and UV-C (220–290 nm), ranked from long to shorter wavelengths (from smaller to larger energies). Most UV-B and all UV-C is absorbed by ozone (O_3) molecules in the upper atmosphere. Consequently, 99% of the solar UV radiation reaching the Earth’s surface is UV-A.

Human Exposure to UV Radiation

It is largely exposure to UV-B that causes skin cancer. It is estimated that as many as 20% of adults will develop skin cancer over the course of their lifetime. Again, treatment is often successful if caught early. Despite very little UV-B reaching the Earth’s surface, there are substantial increases in skin-cancer rates in countries such as Australia, indicating how important it is that UV-B and UV-C continue to be absorbed by the upper atmosphere.

All UV radiation can damage collagen fibers, resulting in an acceleration of the aging process of skin and the formation of wrinkles. Because there is so little UV-B and UV-C reaching the Earth’s surface, sunburn is caused by large exposures, and skin cancer from repeated exposure. Some studies

indicate a link between overexposure to the Sun when young and melanoma later in life.

The tanning response is a defense mechanism in which the body produces pigments to absorb future exposures in inert skin layers above living cells. Basically UV-B radiation excites DNA molecules, distorting the DNA helix, leading to mutations and the possible formation of cancerous cells.

Repeated exposure to UV-B may also lead to the formation of cataracts in the eyes—a cause of blindness among people living in the equatorial belt where medical treatment is limited. Cataracts, clouding in the eye's lens and a loss of vision, are age related; 60% of those between the ages of 65 and 74 will develop cataracts. However, treatment is easy and successful, as one replaces the lens of the eye with a plastic lens. Prevention is important. Eye protection from UV is more effective with plastic sunglasses than those made of glass.

A major acute effect of extreme UV exposure is the suppression of the immune system, both locally and throughout the body.

Low-intensity ultraviolet is used to sterilize haircutting implements, implying that the energy associated with ultraviolet is deposited in a manner different from lower-frequency electromagnetic waves. (Actually this is true for all electromagnetic waves with frequencies greater than visible light.)

Flash photography is generally not allowed of precious artworks and colored prints because the UV radiation from the flash can cause photo-degradation in the artworks. Often artworks will have an extra-thick layer of glass in front of them, which is especially designed to absorb UV radiation.

UV Light and the Ozone Layer

If all of the Sun's ultraviolet radiation reached the Earth's surface, there would be extremely grave effects on the biosphere from the severe cell damage it causes. However, the layer of ozone (O_3) in our upper

atmosphere (10 to 50 km above the Earth) protects life by absorbing most of the dangerous UV radiation.

Unfortunately, today we are observing a depletion in ozone concentrations in the upper atmosphere. This depletion has led to the formation of an "ozone hole" in the upper atmosphere. The hole is more centered over the southern hemisphere, and changes with the seasons, being largest in the spring. This depletion is attributed to the breakdown of ozone molecules by refrigerant gases called chlorofluorocarbons (CFCs).

The UV radiation helps dissociate the CFC's, releasing highly reactive chlorine (Cl) atoms, which catalyze the destruction of the ozone layer. For example, the reaction of $\mathrm{CFCl_3}$ with a photon of light (hv) can be written as:

Equation:

$$\mathrm{CFCl_3 + h\mathit{v} \rightarrow CFCl_2 + Cl.}$$

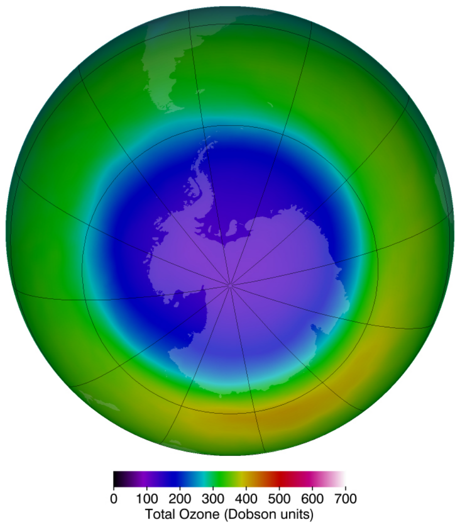
The Cl atom then catalyzes the breakdown of ozone as follows:

Equation:

$$\mathrm{Cl + O_3 \rightarrow ClO + O_2 \text{ and } ClO + O_3 \rightarrow Cl + 2O_2.}$$

A single chlorine atom could destroy ozone molecules for up to two years before being transported down to the surface. The CFCs are relatively stable and will contribute to ozone depletion for years to come. CFCs are found in refrigerants, air conditioning systems, foams, and aerosols.

International concern over this problem led to the establishment of the "Montreal Protocol" agreement (1987) to phase out CFC production in most countries. However, developing-country participation is needed if worldwide production and elimination of CFCs is to be achieved. Probably the largest contributor to CFC emissions today is India. But the protocol seems to be working, as there are signs of an ozone recovery. (See [link].)



This map of ozone concentration over Antarctica in October 2011 shows severe depletion suspected to be caused by CFCs. Less dramatic but more general depletion has been observed over northern latitudes, suggesting the effect is global. With less ozone, more ultraviolet radiation from the Sun reaches the surface, causing more damage. (credit: NASA Ozone Watch)

Benefits of UV Light

Besides the adverse effects of ultraviolet radiation, there are also benefits of exposure in nature and uses in technology. Vitamin D production in the skin (epidermis) results from exposure to UVB radiation, generally from sunlight. A number of studies indicate lack of vitamin D can result in the development of a range of cancers (prostate, breast, colon), so a certain amount of UV exposure is helpful. Lack of vitamin D is also linked to osteoporosis. Exposures (with no sunscreen) of 10 minutes a day to arms, face, and legs might be sufficient to provide the accepted dietary level. However, in the winter time north of about 37° latitude, most UVB gets blocked by the atmosphere.

UV radiation is used in the treatment of infantile jaundice and in some skin conditions. It is also used in sterilizing workspaces and tools, and killing germs in a wide range of applications. It is also used as an analytical tool to identify substances.

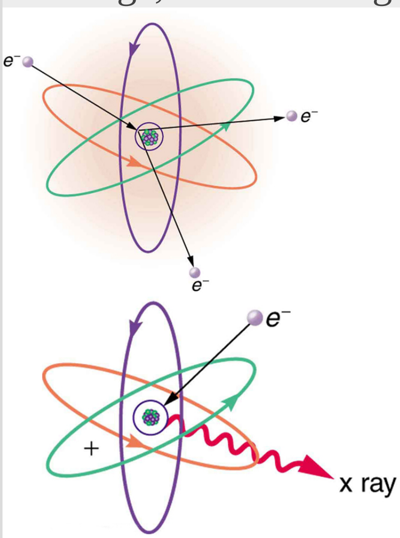
When exposed to ultraviolet, some substances, such as minerals, glow in characteristic visible wavelengths, a process called fluorescence. So-called black lights emit ultraviolet to cause posters and clothing to fluoresce in the visible. Ultraviolet is also used in special microscopes to detect details smaller than those observable with longer-wavelength visible-light microscopes.

X-Rays

In the 1850s, scientists (such as Faraday) began experimenting with high-voltage electrical discharges in tubes filled with rarefied gases. It was later found that these discharges created an invisible, penetrating form of very high frequency electromagnetic radiation. This radiation was called an **X-ray**, because its identity and nature were unknown.

Note:
Things Great and Small: A Submicroscopic View of X-Ray Production
X-rays can be created in a high-voltage discharge. They are emitted in the material struck by electrons in the discharge current. There are two

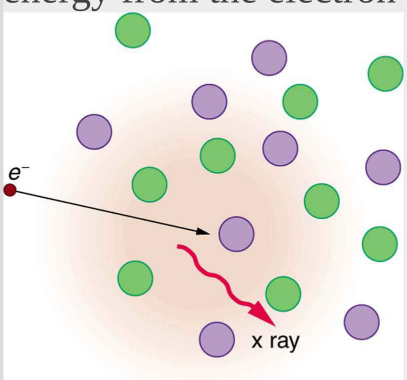
mechanisms by which the electrons create X-rays.
The first method is illustrated in [link]. An electron is accelerated in an evacuated tube by a high positive voltage. The electron strikes a metal plate (e.g., copper) and produces X-rays. Since this is a high-voltage discharge, the electron gains sufficient energy to ionize the atom.



Artist's conception of an electron ionizing an atom followed by the recapture of an electron and emission of an X-ray. An energetic electron strikes an atom and knocks an electron out of one of the orbits closest to the nucleus. Later, the atom captures another electron, and the energy released by its fall into a low orbit generates a

high-energy EM wave called an X-ray.

In the case shown, an inner-shell electron (one in an orbit relatively close to and tightly bound to the nucleus) is ejected. A short time later, another electron is captured and falls into the orbit in a single great plunge. The energy released by this fall is given to an EM wave known as an X-ray. Since the orbits of the atom are unique to the type of atom, the energy of the X-ray is characteristic of the atom, hence the name characteristic X-ray. The second method by which an energetic electron creates an X-ray when it strikes a material is illustrated in [link]. The electron interacts with charges in the material as it penetrates. These collisions transfer kinetic energy from the electron to the electrons and atoms in the material.



Artist’s conception of an electron being slowed by collisions in a material and emitting X-ray radiation. This energetic electron makes numerous collisions with electrons and atoms in a material it penetrates. An

accelerated charge radiates EM waves, a second method by which X-rays are created.

A loss of kinetic energy implies an acceleration, in this case decreasing the electron’s velocity. Whenever a charge is accelerated, it radiates EM waves. Given the high energy of the electron, these EM waves can have high energy. We call them X-rays. Since the process is random, a broad spectrum of X-ray energy is emitted that is more characteristic of the electron energy than the type of material the electron encounters. Such EM radiation is called “bremsstrahlung” (German for “braking radiation”).

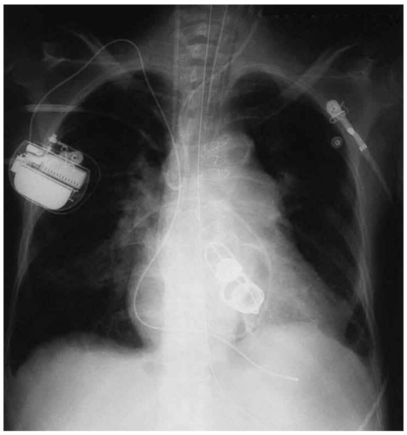
As described above, there are two methods by which X-rays are created—both are submicroscopic processes and can be caused by high-voltage discharges. While the low-frequency end of the X-ray range overlaps with the ultraviolet, X-rays extend to much higher frequencies (and energies).

X-rays have adverse effects on living cells similar to those of ultraviolet radiation, and they have the additional liability of being more penetrating, affecting more than the surface layers of cells. Cancer and genetic defects can be induced by exposure to X-rays. Because of their effect on rapidly dividing cells, X-rays can also be used to treat and even cure cancer.

The widest use of X-rays is for imaging objects that are opaque to visible light, such as the human body or aircraft parts. In humans, the risk of cell damage is weighed carefully against the benefit of the diagnostic information obtained. However, questions have risen in recent years as to accidental overexposure of some people during CT scans—a mistake at least in part due to poor monitoring of radiation dose.

The ability of X-rays to penetrate matter depends on density, and so an X-ray image can reveal very detailed density information. [link] shows an example of the simplest type of X-ray image, an X-ray shadow on film. The

amount of information in a simple X-ray image is impressive, but more sophisticated techniques, such as CT scans, can reveal three-dimensional information with details smaller than a millimeter.



This shadow X-ray image shows many interesting features, such as artificial heart valves, a pacemaker, and the wires used to close the sternum. (credit: P. P. Urone)

The use of X-ray technology in medicine is called radiology—an established and relatively cheap tool in comparison to more sophisticated technologies. Consequently, X-rays are widely available and used extensively in medical diagnostics. During World War I, mobile X-ray units, advocated by Madame Marie Curie, were used to diagnose soldiers.

Because they can have wavelengths less than 0.01 nm, X-rays can be scattered (a process called X-ray diffraction) to detect the shape of molecules and the structure of crystals. X-ray diffraction was crucial to

Crick, Watson, and Wilkins in the determination of the shape of the double-helix DNA molecule.

X-rays are also used as a precise tool for trace-metal analysis in X-ray induced fluorescence, in which the energy of the X-ray emissions are related to the specific types of elements and amounts of materials present.

Gamma Rays

Soon after nuclear radioactivity was first detected in 1896, it was found that at least three distinct types of radiation were being emitted. The most penetrating nuclear radiation was called a **gamma ray (γ ray)** (again a name given because its identity and character were unknown), and it was later found to be an extremely high frequency electromagnetic wave.

In fact, γ rays are any electromagnetic radiation emitted by a nucleus. This can be from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons. The lower end of the $\gamma\text{-ray}$ frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation.

Gamma rays have characteristics identical to X-rays of the same frequency—they differ only in source. At higher frequencies, γ rays are more penetrating and more damaging to living tissue. They have many of the same uses as X-rays, including cancer therapy. Gamma radiation from radioactive materials is used in nuclear medicine.

[link] shows a medical image based on γ rays. Food spoilage can be greatly inhibited by exposing it to large doses of γ radiation, thereby obliterating responsible microorganisms. Damage to food cells through irradiation occurs as well, and the long-term hazards of consuming radiation-preserved food are unknown and controversial for some groups. Both X-ray and $\gamma\text{-ray}$ technologies are also used in scanning luggage at airports.

This is an image of the γ rays emitted by nuclei in a compound that is concentrated in the bones and eliminated through the kidneys. Bone cancer

is evidenced by nonuniform concentration in similar structures. For example, some ribs are darker than others. (credit: P. P. Urone)

Detecting Electromagnetic Waves from Space

A final note on star gazing. The entire electromagnetic spectrum is used by researchers for investigating stars, space, and time. As noted earlier, Penzias and Wilson detected microwaves to identify the background radiation originating from the Big Bang. Radio telescopes such as the Arecibo Radio Telescope in Puerto Rico and Parkes Observatory in Australia were designed to detect radio waves.

Infrared telescopes need to have their detectors cooled by liquid nitrogen to be able to gather useful signals. Since infrared radiation is predominantly from thermal agitation, if the detectors were not cooled, the vibrations of the molecules in the antenna would be stronger than the signal being collected.

The most famous of these infrared sensitive telescopes is the James Clerk Maxwell Telescope in Hawaii. The earliest telescopes, developed in the seventeenth century, were optical telescopes, collecting visible light. Telescopes in the ultraviolet, X-ray, and γ-ray regions are placed outside the atmosphere on satellites orbiting the Earth.

The Hubble Space Telescope (launched in 1990) gathers ultraviolet radiation as well as visible light. In the X-ray region, there is the Chandra X-ray Observatory (launched in 1999), and in the γ-ray region, there is the new Fermi Gamma-ray Space Telescope (launched in 2008—taking the place of the Compton Gamma Ray Observatory, 1991–2000.).

Section Summary

- The electromagnetic spectrum is separated into many categories and subcategories, based on the frequency and wavelength, source, and uses of the electromagnetic waves.
- Any electromagnetic wave produced by currents in wires is classified as a radio wave, the lowest frequency electromagnetic waves. Radio waves are divided into many types, depending on their applications, ranging up to microwaves at their highest frequencies.
- Infrared radiation lies below visible light in frequency and is produced by thermal motion and the vibration and rotation of atoms and molecules. Infrared's lower frequencies overlap with the highest-frequency microwaves.
- Visible light is largely produced by electronic transitions in atoms and molecules, and is defined as being detectable by the human eye. Its colors vary with frequency, from red at the lowest to violet at the highest.
- Ultraviolet radiation starts with frequencies just above violet in the visible range and is produced primarily by electronic transitions in atoms and molecules.
- X-rays are created in high-voltage discharges and by electron bombardment of metal targets. Their lowest frequencies overlap the ultraviolet range but extend to much higher values, overlapping at the high end with gamma rays.
- Gamma rays are nuclear in origin and are defined to include the highest-frequency electromagnetic radiation of any type.

Conceptual Questions

Exercise:

Problem:

If you live in a region that has a particular TV station, you can sometimes pick up some of its audio portion on your FM radio receiver. Explain how this is possible. Does it imply that TV audio is broadcast as FM?

Exercise:

Problem:

Explain why people who have the lens of their eye removed because of cataracts are able to see low-frequency ultraviolet.

Exercise:

Problem:

How do fluorescent soap residues make clothing look “brighter and whiter” in outdoor light? Would this be effective in candlelight?

Exercise:

Problem:

Give an example of resonance in the reception of electromagnetic waves.

Exercise:

Problem:

Illustrate that the size of details of an object that can be detected with electromagnetic waves is related to their wavelength, by comparing details observable with two different types (for example, radar and visible light or infrared and X-rays).

Exercise:

Problem:

Why don’t buildings block radio waves as completely as they do visible light?

Exercise:

Problem:

Make a list of some everyday objects and decide whether they are transparent or opaque to each of the types of electromagnetic waves.

Exercise:

Problem:

Your friend says that more patterns and colors can be seen on the wings of birds if viewed in ultraviolet light. Would you agree with your friend? Explain your answer.

Exercise:

Problem:

The rate at which information can be transmitted on an electromagnetic wave is proportional to the frequency of the wave. Is this consistent with the fact that laser telephone transmission at visible frequencies carries far more conversations per optical fiber than conventional electronic transmission in a wire? What is the implication for ELF radio communication with submarines?

Exercise:

Problem:

Give an example of energy carried by an electromagnetic wave.

Exercise:

Problem:

In an MRI scan, a higher magnetic field requires higher frequency radio waves to resonate with the nuclear type whose density and location is being imaged. What effect does going to a larger magnetic field have on the most efficient antenna to broadcast those radio waves? Does it favor a smaller or larger antenna?

Exercise:

Problem:

Laser vision correction often uses an excimer laser that produces 193-nm electromagnetic radiation. This wavelength is extremely strongly absorbed by the cornea and ablates it in a manner that reshapes the cornea to correct vision defects. Explain how the strong absorption helps concentrate the energy in a thin layer and thus give greater accuracy in shaping the cornea. Also explain how this strong absorption limits damage to the lens and retina of the eye.

Glossary

electromagnetic spectrum
: the full range of wavelengths or frequencies of electromagnetic radiation

radio waves
: electromagnetic waves with wavelengths in the range from 1 mm to 100 km; they are produced by currents in wires and circuits and by astronomical phenomena

microwaves
: electromagnetic waves with wavelengths in the range from 1 mm to 1 m; they can be produced by currents in macroscopic circuits and devices

thermal agitation
: the thermal motion of atoms and molecules in any object at a temperature above absolute zero, which causes them to emit and absorb radiation

radar
: a common application of microwaves. Radar can determine the distance to objects as diverse as clouds and aircraft, as well as determine the speed of a car or the intensity of a rainstorm

infrared radiation (IR)

a region of the electromagnetic spectrum with a frequency range that extends from just below the red region of the visible light spectrum up to the microwave region, or from 0.74 μm to 300 μm

ultraviolet radiation (UV)
: electromagnetic radiation in the range extending upward in frequency from violet light and overlapping with the lowest X-ray frequencies, with wavelengths from 400 nm down to about 10 nm

visible light
: the narrow segment of the electromagnetic spectrum to which the normal human eye responds

amplitude modulation (AM)
: a method for placing information on electromagnetic waves by modulating the amplitude of a carrier wave with an audio signal, resulting in a wave with constant frequency but varying amplitude

extremely low frequency (ELF)
: electromagnetic radiation with wavelengths usually in the range of 0 to 300 Hz, but also about 1kHz

carrier wave
: an electromagnetic wave that carries a signal by modulation of its amplitude or frequency

frequency modulation (FM)
: a method of placing information on electromagnetic waves by modulating the frequency of a carrier wave with an audio signal, producing a wave of constant amplitude but varying frequency

TV
: video and audio signals broadcast on electromagnetic waves

very high frequency (VHF)
: TV channels utilizing frequencies in the two ranges of 54 to 88 MHz and 174 to 222 MHz

ultra-high frequency (UHF)
: TV channels in an even higher frequency range than VHF, of 470 to 1000 MHz

X-ray
: invisible, penetrating form of very high frequency electromagnetic radiation, overlapping both the ultraviolet range and the γ-ray range

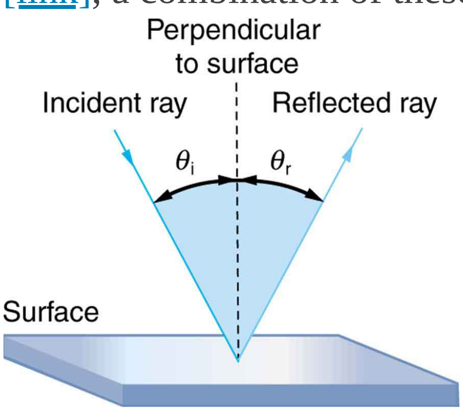
gamma ray
: (γ ray); extremely high frequency electromagnetic radiation emitted by the nucleus of an atom, either from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons. The lower end of the γ-ray frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation

Reflection

- Explain reflection of light from polished and rough surfaces.

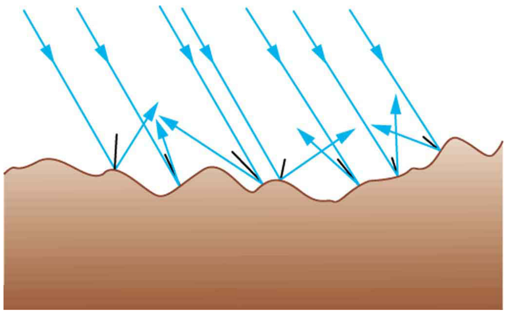
Whenever we look into a mirror, or squint at sunlight glinting from a lake, we are seeing a reflection. When you look at this page, too, you are seeing light reflected from it. Large telescopes use reflection to form an image of stars and other astronomical objects.

The law of reflection is illustrated in [link], which also shows how the angles are measured relative to the perpendicular to the surface at the point where the light ray strikes. We expect to see reflections from smooth surfaces, but [link] illustrates how a rough surface reflects light. Since the light strikes different parts of the surface at different angles, it is reflected in many different directions, or diffused. Diffused light is what allows us to see a sheet of paper from any angle, as illustrated in [link]. Many objects, such as people, clothing, leaves, and walls, have rough surfaces and can be seen from all sides. A mirror, on the other hand, has a smooth surface (compared with the wavelength of light) and reflects light at specific angles, as illustrated in [link]. When the moon reflects from a lake, as shown in [link], a combination of these effects takes place.

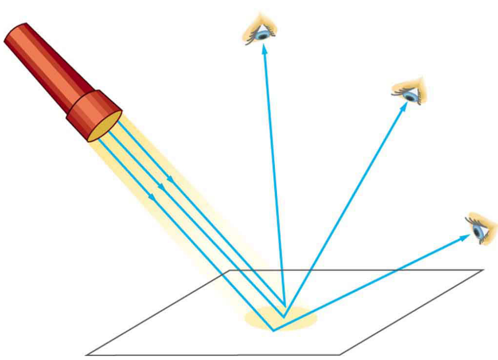


The law of reflection states that the angle of reflection equals the angle of incidence—$\theta_r = \theta_i$. The angles are measured relative to the perpendicular to

the surface at the point where the ray strikes the surface.

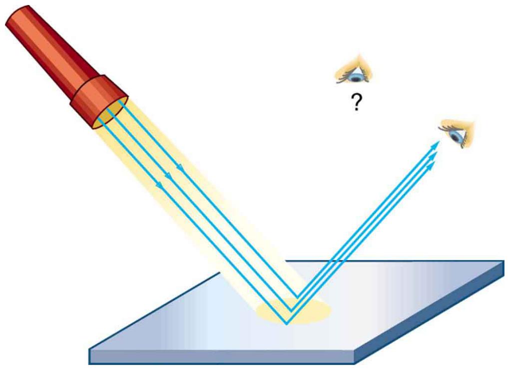


Light is diffused when it reflects from a rough surface. Here many parallel rays are incident, but they are reflected at many different angles since the surface is rough.



When a sheet of paper is illuminated with many parallel incident rays, it can be seen at many different angles, because

its surface is rough and diffuses the light.



A mirror illuminated by many parallel rays reflects them in only one direction, since its surface is very smooth. Only the observer at a particular angle will see the reflected light.

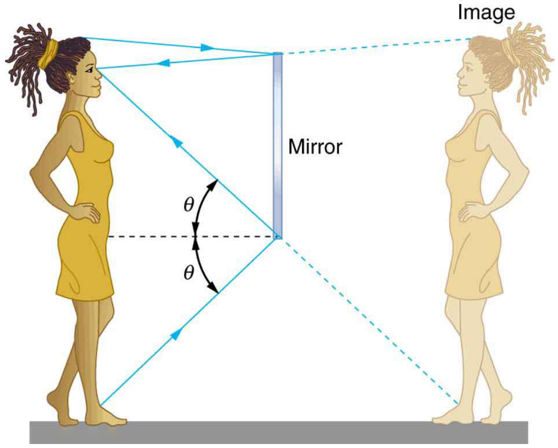
Moonlight is spread out when it is reflected by the lake, since the surface is shiny but uneven. (credit:

Diego Torres Silvestre, Flickr)

The law of reflection is very simple: The angle of reflection equals the angle of incidence.

Note:
The Law of Reflection
The angle of reflection equals the angle of incidence.

When we see ourselves in a mirror, it appears that our image is actually behind the mirror. This is illustrated in [link]. We see the light coming from a direction determined by the law of reflection. The angles are such that our image is exactly the same distance behind the mirror as we stand away from the mirror. If the mirror is on the wall of a room, the images in it are all behind the mirror, which can make the room seem bigger. Although these mirror images make objects appear to be where they cannot be (like behind a solid wall), the images are not figments of our imagination. Mirror images can be photographed and videotaped by instruments and look just as they do with our eyes (optical instruments themselves). The precise manner in which images are formed by mirrors and lenses will be treated in later sections of this chapter.



Our image in a mirror is behind the mirror. The two rays shown are those that strike the mirror at just the correct angles to be reflected into the eyes of the person. The image appears to be in the direction the rays are coming from when they enter the eyes.

Note:

Take-Home Experiment: Law of Reflection

Take a piece of paper and shine a flashlight at an angle at the paper, as shown in [link]. Now shine the flashlight at a mirror at an angle. Do your observations confirm the predictions in [link] and [link]? Shine the flashlight on various surfaces and determine whether the reflected light is diffuse or not. You can choose a shiny metallic lid of a pot or your skin. Using the mirror and flashlight, can you confirm the law of reflection? You will need to draw lines on a piece of paper showing the incident and reflected rays. (This part works even better if you use a laser pencil.)

Section Summary

- The angle of reflection equals the angle of incidence.
- A mirror has a smooth surface and reflects light at specific angles.
- Light is diffused when it reflects from a rough surface.
- Mirror images can be photographed and videotaped by instruments.

Conceptual Questions

Exercise:

Problem:

Using the law of reflection, explain how powder takes the shine off of a person’s nose. What is the name of the optical effect?

Glossary

mirror
: smooth surface that reflects light at specific angles, forming an image of the person or object in front of it

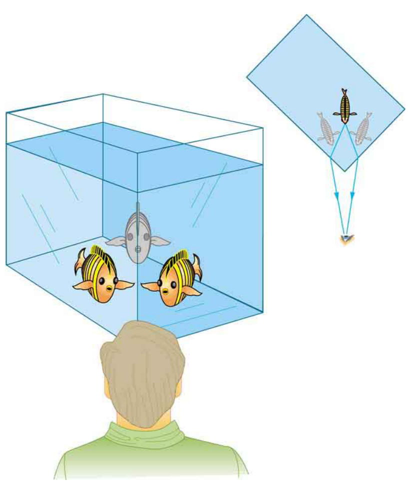
law of reflection
: angle of reflection equals the angle of incidence

Refraction

- Determine the index of refraction, given the speed of light in a medium.

It is easy to notice some odd things when looking into a fish tank. For example, you may see the same fish appearing to be in two different places. (See [link].) This is because light coming from the fish to us changes direction when it leaves the tank, and in this case, it can travel two different paths to get to our eyes. The changing of a light ray’s direction (loosely called bending) when it passes through variations in matter is called **refraction**. Refraction is responsible for a tremendous range of optical phenomena, from the action of lenses to voice transmission through optical fibers.

Note:
Refraction
The changing of a light ray’s direction (loosely called bending) when it passes through variations in matter is called refraction.



Looking at the fish tank as shown, we can see the same

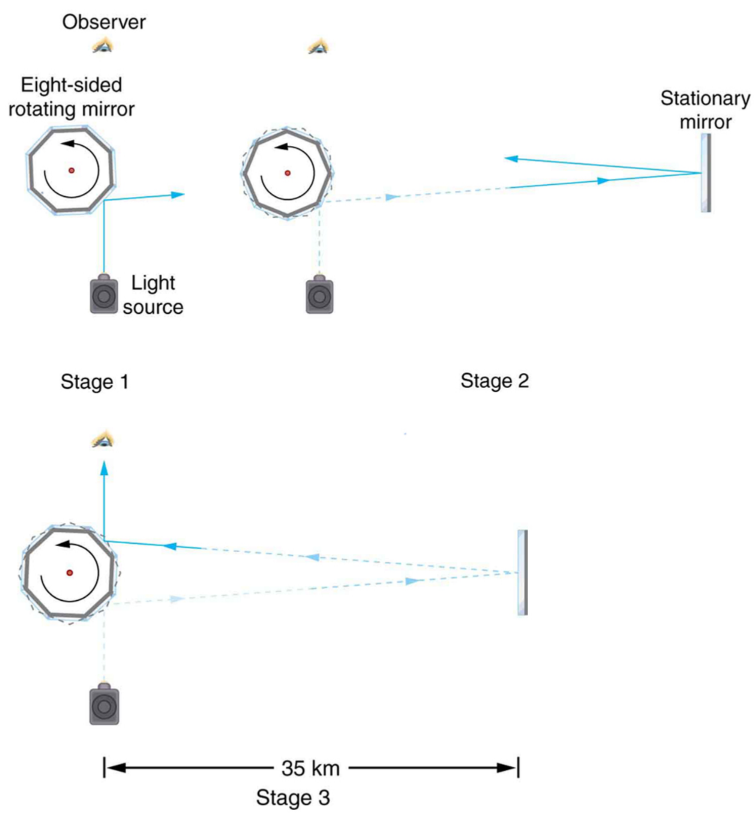
fish in two different locations, because light changes directions when it passes from water to air. In this case, the light can reach the observer by two different paths, and so the fish seems to be in two different places. This bending of light is called refraction and is responsible for many optical phenomena.

Why does light change direction when passing from one material (medium) to another? It is because light changes speed when going from one material to another. So before we study the law of refraction, it is useful to discuss the speed of light and how it varies in different media.

The Speed of Light

Early attempts to measure the speed of light, such as those made by Galileo, determined that light moved extremely fast, perhaps instantaneously. The first real evidence that light traveled at a finite speed came from the Danish astronomer Ole Roemer in the late 17th century. Roemer had noted that the average orbital period of one of Jupiter's moons, as measured from Earth, varied depending on whether Earth was moving toward or away from Jupiter. He correctly concluded that the apparent change in period was due to the change in distance between Earth and Jupiter and the time it took light to travel this distance. From his 1676 data, a value of the speed of light was calculated to be $2.26 \times 10^8 \text{ m/s}$ (only 25% different from today's

accepted value). In more recent times, physicists have measured the speed of light in numerous ways and with increasing accuracy. One particularly direct method, used in 1887 by the American physicist Albert Michelson (1852–1931), is illustrated in [link]. Light reflected from a rotating set of mirrors was reflected from a stationary mirror 35 km away and returned to the rotating mirrors. The time for the light to travel can be determined by how fast the mirrors must rotate for the light to be returned to the observer’s eye.



A schematic of early apparatus used by Michelson and others to determine the speed of light. As the mirrors rotate, the reflected ray is only briefly directed at the stationary mirror. The returning ray will be reflected into the observer's eye only if the next mirror has rotated into the correct position just as the ray

returns. By measuring the correct rotation rate, the time for the round trip can be measured and the speed of light calculated. Michelson’s calculated value of the speed of light was only 0.04% different from the value used today.

The speed of light is now known to great precision. In fact, the speed of light in a vacuum c is so important that it is accepted as one of the basic physical quantities and has the fixed value

Equation:

$$c = 2.9972458 \times 10^8 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s},$$

where the approximate value of $3.00 \times 10^8 \text{ m/s}$ is used whenever three-digit accuracy is sufficient. The speed of light through matter is less than it is in a vacuum, because light interacts with atoms in a material. The speed of light depends strongly on the type of material, since its interaction with different atoms, crystal lattices, and other substructures varies. We define the **index of refraction** n of a material to be

Equation:

$$n = \frac{c}{v},$$

where v is the observed speed of light in the material. Since the speed of light is always less than c in matter and equals c only in a vacuum, the index of refraction is always greater than or equal to one.

Note:
Value of the Speed of Light
Equation:

$$c = 2.9972458 \times 10^8 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s}$$

Note:
Index of Refraction
Equation:

$$n = \frac{c}{v}$$

That is, $n \geq 1$. [link] gives the indices of refraction for some representative substances. The values are listed for a particular wavelength of light, because they vary slightly with wavelength. (This can have important effects, such as colors produced by a prism.) Note that for gases, n is close to 1.0. This seems reasonable, since atoms in gases are widely separated and light travels at c in the vacuum between atoms. It is common to take $n = 1$ for gases unless great precision is needed. Although the speed of light v in a medium varies considerably from its value c in a vacuum, it is still a large speed.

Medium	***n***
Gases at 0°C***, 1 atm***	
Air	1.000293
Carbon dioxide	1.00045

Medium	***n***
Hydrogen	1.000139
Oxygen	1.000271
Liquids at 20°C	
Benzene	1.501
Carbon disulfide	1.628
Carbon tetrachloride	1.461
Ethanol	1.361
Glycerine	1.473
Water, fresh	1.333
Solids at 20°C	
Diamond	2.419
Fluorite	1.434
Glass, crown	1.52
Glass, flint	1.66
Ice at 20°C	1.309
Polystyrene	1.49
Plexiglas	1.51

Medium	***n***
Quartz, crystalline	1.544
Quartz, fused	1.458
Sodium chloride	1.544
Zircon	1.923

Index of Refraction in Various Media

Example:
Speed of Light in Matter
Calculate the speed of light in zircon, a material used in jewelry to imitate diamond.
Strategy
The speed of light in a material, v, can be calculated from the index of refraction n of the material using the equation $n = c/v$.
Solution
The equation for index of refraction states that $n = c/v$. Rearranging this to determine v gives
Equation:

$$v = \frac{c}{n}.$$

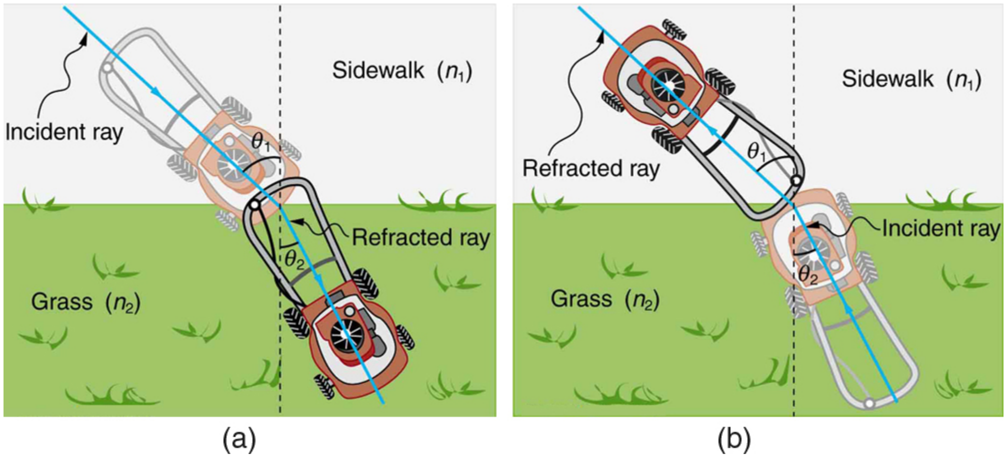
The index of refraction for zircon is given as 1.923 in [link], and c is given in the equation for speed of light. Entering these values in the last expression gives
Equation:

$$\begin{aligned} v &= \frac{3.00 \times 10^8 \text{ m/s}}{1.923} \\ &= 1.56 \times 10^8 \text{ m/s}. \end{aligned}$$

Discussion
This speed is slightly larger than half the speed of light in a vacuum and is still high compared with speeds we normally experience. The only substance listed in [link] that has a greater index of refraction than zircon is diamond. We shall see later that the large index of refraction for zircon makes it sparkle more than glass, but less than diamond.

Law of Refraction

[link] shows how a ray of light changes direction when it passes from one medium to another. As before, the angles are measured relative to a perpendicular to the surface at the point where the light ray crosses it. (Some of the incident light will be reflected from the surface, but for now we will concentrate on the light that is transmitted.) The change in direction of the light ray depends on how the speed of light changes. The change in the speed of light is related to the indices of refraction of the media involved. In the situations shown in [link], medium 2 has a greater index of refraction than medium 1. This means that the speed of light is less in medium 2 than in medium 1. Note that as shown in [link](a), the direction of the ray moves closer to the perpendicular when it slows down. Conversely, as shown in [link](b), the direction of the ray moves away from the perpendicular when it speeds up. The path is exactly reversible. In both cases, you can imagine what happens by thinking about pushing a lawn mower from a footpath onto grass, and vice versa. Going from the footpath to grass, the front wheels are slowed and pulled to the side as shown. This is the same change in direction as for light when it goes from a fast medium to a slow one. When going from the grass to the footpath, the front wheels can move faster and the mower changes direction as shown. This, too, is the same change in direction as for light going from slow to fast.



The change in direction of a light ray depends on how the speed of light changes when it crosses from one medium to another. The speed of light is greater in medium 1 than in medium 2 in the situations shown here. (a) A ray of light moves closer to the perpendicular when it slows down. This is analogous to what happens when a lawn mower goes from a footpath to grass. (b) A ray of light moves away from the perpendicular when it speeds up. This is analogous to what happens when a lawn mower goes from grass to footpath. The paths are exactly reversible.

The amount that a light ray changes its direction depends both on the incident angle and the amount that the speed changes. For a ray at a given incident angle, a large change in speed causes a large change in direction, and thus a large change in angle.

Note:

Take-Home Experiment: A Broken Pencil

A classic observation of refraction occurs when a pencil is placed in a glass half filled with water. Do this and observe the shape of the pencil when

you look at the pencil sideways, that is, through air, glass, water. Explain your observations. Draw ray diagrams for the situation.

Section Summary

- The changing of a light ray's direction when it passes through variations in matter is called refraction.
- The speed of light in vacuum $c = 2.9972458 \times 10^8 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s}$.
- Index of refraction $n = \frac{c}{v}$, where v is the speed of light in the material, c is the speed of light in vacuum, and n is the index of refraction.

Conceptual Questions

Exercise:

Problem:

Diffusion by reflection from a rough surface is described in this chapter. Light can also be diffused by refraction. Describe how this occurs in a specific situation, such as light interacting with crushed ice.

Exercise:

Problem:

Why is the index of refraction always greater than or equal to 1?

Exercise:

Problem:

Does the fact that the light flash from lightning reaches you before its sound prove that the speed of light is extremely large or simply that it is greater than the speed of sound? Discuss how you could use this effect to get an estimate of the speed of light.

Exercise:

Problem:

Will light change direction toward or away from the perpendicular when it goes from air to water? Water to glass? Glass to air?

Exercise:

Problem:

Explain why an object in water always appears to be at a depth shallower than it actually is? Why do people sometimes sustain neck and spinal injuries when diving into unfamiliar ponds or waters?

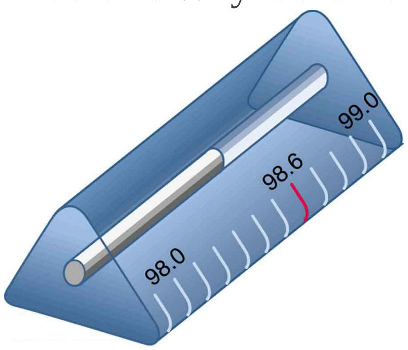
Exercise:

Problem:

Explain why a person's legs appear very short when wading in a pool. Justify your explanation with a ray diagram showing the path of rays from the feet to the eye of an observer who is out of the water.

Exercise:

Problem: Why is the front surface of a thermometer curved as shown?



The curved surface of the thermometer serves a purpose.

Exercise:

Problem:

Suppose light were incident from air onto a material that had a negative index of refraction, say –1.3; where does the refracted light ray go?

Problems & Exercises

Exercise:

Problem: What is the speed of light in water? In glycerine?

Solution:

$2.25 \times 10^8 \text{ m/s}$ in water

$2.04 \times 10^8 \text{ m/s}$ in glycerine

Exercise:

Problem: What is the speed of light in air? In crown glass?

Exercise:

Problem:

Calculate the index of refraction for a medium in which the speed of light is $2.012 \times 10^8 \text{ m/s}$, and identify the most likely substance based on [link].

Solution:

1.490, polystyrene

Exercise:

Problem:

In what substance in [link] is the speed of light $2.290 \times 10^8 \text{ m/s}$?

Exercise:

Problem:

There was a major collision of an asteroid with the Moon in medieval times. It was described by monks at Canterbury Cathedral in England as a red glow on and around the Moon. How long after the asteroid hit the Moon, which is $3.84 \times 10^5 \text{ km}$ away, would the light first arrive on Earth?

Solution:

1.28 s

Exercise:

Problem:

Components of some computers communicate with each other through optical fibers having an index of refraction $n = 1.55$. What time in nanoseconds is required for a signal to travel 0.200 m through such a fiber?

Solution:

1.03 ns

Exercise:

Problem:

On the Moon's surface, lunar astronauts placed a corner reflector, off which a laser beam is periodically reflected. The distance to the Moon is calculated from the round-trip time. What percent correction is needed to account for the delay in time due to the slowing of light in Earth's atmosphere? Assume the distance to the Moon is precisely $3.84 \times 10^8 \text{ m}$, and Earth's atmosphere (which varies in density with altitude) is equivalent to a layer 30.0 km thick with a constant index of refraction $n = 1.000293$.

Glossary

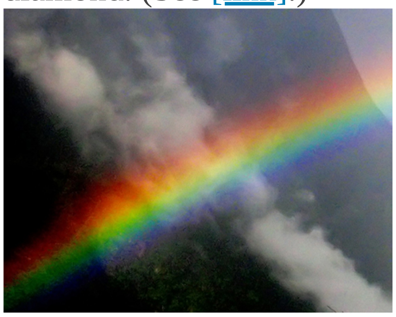
refraction
: changing of a light ray's direction when it passes through variations in matter

index of refraction
: for a material, the ratio of the speed of light in vacuum to that in the material

Dispersion: The Rainbow and Prisms

- Explain the phenomenon of dispersion and discuss its advantages and disadvantages.

Everyone enjoys the spectacle of a rainbow glimmering against a dark stormy sky. How does sunlight falling on clear drops of rain get broken into the rainbow of colors we see? The same process causes white light to be broken into colors by a clear glass prism or a diamond. (See [link].)



(a)

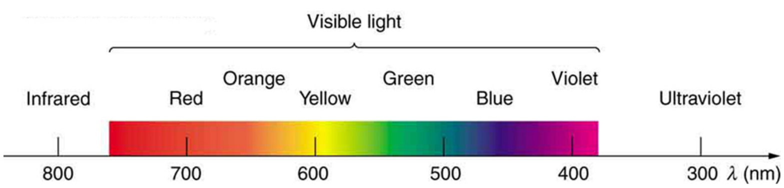
(b)

The colors of the rainbow (a) and those produced by a prism (b) are identical. (credit: Alfredo55, Wikimedia Commons; NASA)

We see about six colors in a rainbow—red, orange, yellow, green, blue, and violet; sometimes indigo is listed, too. Those colors are associated with different wavelengths of light, as shown in [link]. When our eye receives pure-wavelength light, we tend to see only one of the six colors, depending on wavelength. The thousands of other hues we can sense in other situations are our eye’s response to various mixtures of wavelengths. White light, in particular, is a fairly uniform mixture of all visible wavelengths. Sunlight, considered to be white, actually appears to be a bit yellow because of its mixture of wavelengths, but it does contain all visible wavelengths. The sequence of colors in rainbows is the same sequence as the colors plotted versus wavelength in [link]. What this implies is that white light is spread out according to

wavelength in a rainbow. **Dispersion** is defined as the spreading of white light into its full spectrum of wavelengths. More technically, dispersion occurs whenever there is a process that changes the direction of light in a manner that depends on wavelength. Dispersion, as a general phenomenon, can occur for any type of wave and always involves wavelength-dependent processes.

Note:
Dispersion
Dispersion is defined to be the spreading of white light into its full spectrum of wavelengths.



Even though rainbows are associated with seven colors, the rainbow is a continuous distribution of colors according to wavelengths.

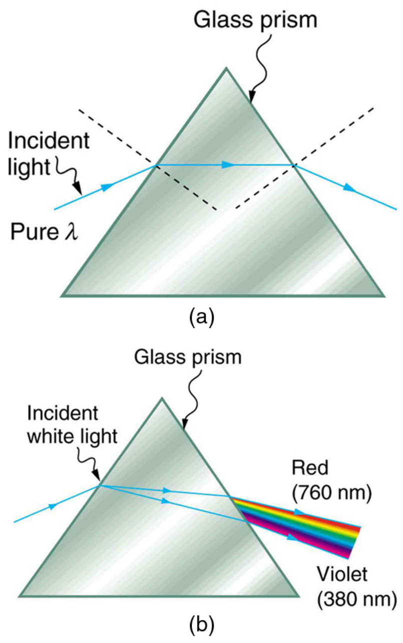
Refraction is responsible for dispersion in rainbows and many other situations. The angle of refraction depends on the index of refraction. We know that the index of refraction n depends on the medium. But for a given medium, n also depends on wavelength. (See [link]. Note that, for a given medium, n increases as wavelength decreases and is greatest for violet light. Thus violet light is bent more than red light, as shown for a prism in [link](b), and the light is dispersed into the same sequence of wavelengths as seen in [link] and [link].

Note:
Making Connections: Dispersion
Any type of wave can exhibit dispersion. Sound waves, all types of electromagnetic waves, and water waves can be dispersed according to wavelength. Dispersion occurs whenever the speed of propagation depends on wavelength, thus separating and spreading out various wavelengths. Dispersion may require special circumstances and can result in spectacular displays such as in the production of a rainbow. This is also

true for sound, since all frequencies ordinarily travel at the same speed. If you listen to sound through a long tube, such as a vacuum cleaner hose, you can easily hear it is dispersed by interaction with the tube. Dispersion, in fact, can reveal a great deal about what the wave has encountered that disperses its wavelengths. The dispersion of electromagnetic radiation from outer space, for example, has revealed much about what exists between the stars—the so-called empty space.

Medium	**Red (660 nm)**	**Orange (610 nm)**	**Yellow (580 nm)**	**Green (550 nm)**	**Blue (470 nm)**	**Violet (410 nm)**
Water	1.331	1.332	1.333	1.335	1.338	1.342
Diamond	2.410	2.415	2.417	2.426	2.444	2.458
Glass, crown	1.512	1.514	1.518	1.519	1.524	1.530
Glass, flint	1.662	1.665	1.667	1.674	1.684	1.698
Polystyrene	1.488	1.490	1.492	1.493	1.499	1.506
Quartz, fused	1.455	1.456	1.458	1.459	1.462	1.468

Index of Refraction n in Selected Media at Various Wavelengths

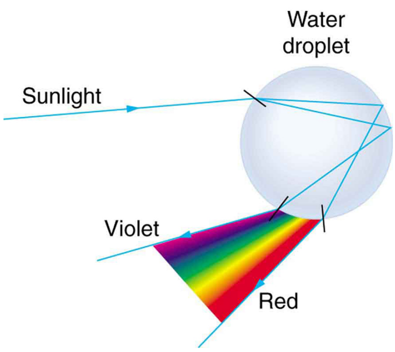


(a) A pure wavelength of light falls onto a prism and is refracted at both surfaces. (b) White light is dispersed by the prism (shown exaggerated). Since the index of refraction varies with wavelength, the angles of refraction vary with wavelength. A sequence of red to violet is produced, because the index of refraction increases steadily with decreasing wavelength.

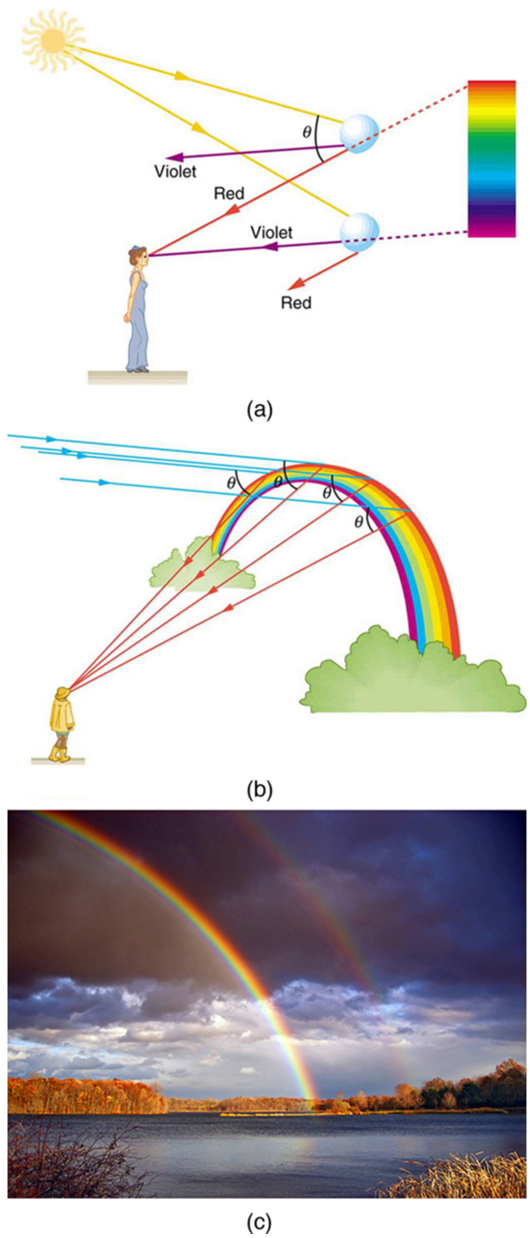
Rainbows are produced by a combination of refraction and reflection. You may have noticed that you see a rainbow only when you look away from the sun. Light enters a drop of water and is reflected from the back of the drop, as shown in [link]. The light is refracted both as it enters and as it leaves the drop. Since the index of refraction of water

varies with wavelength, the light is dispersed, and a rainbow is observed, as shown in [link] (a). (There is no dispersion caused by reflection at the back surface, since the law of reflection does not depend on wavelength.) The actual rainbow of colors seen by an observer depends on the myriad of rays being refracted and reflected toward the observer's eyes from numerous drops of water. The effect is most spectacular when the background is dark, as in stormy weather, but can also be observed in waterfalls and lawn sprinklers. The arc of a rainbow comes from the need to be looking at a specific angle relative to the direction of the sun, as illustrated in [link] (b). (If there are two reflections of light within the water drop, another "secondary" rainbow is produced. This rare event produces an arc that lies above the primary rainbow arc—see [link] (c).)

Note:
Rainbows
Rainbows are produced by a combination of refraction and reflection.



Part of the light falling on this water drop enters and is reflected from the back of the drop. This light is refracted and dispersed both as it enters and as it leaves the drop.



(a) Different colors emerge in different directions, and so you must look at different locations to see the various colors of a rainbow. (b) The arc of a rainbow results from the fact that a line between the observer and any point on the arc must make the correct angle with the parallel rays of sunlight to receive the refracted rays. (c)

Double rainbow. (credit: Nicholas, Wikimedia Commons)

Dispersion may produce beautiful rainbows, but it can cause problems in optical systems. White light used to transmit messages in a fiber is dispersed, spreading out in time and eventually overlapping with other messages. Since a laser produces a nearly pure wavelength, its light experiences little dispersion, an advantage over white light for transmission of information. In contrast, dispersion of electromagnetic waves coming to us from outer space can be used to determine the amount of matter they pass through. As with many phenomena, dispersion can be useful or a nuisance, depending on the situation and our human goals.

Section Summary

- The spreading of white light into its full spectrum of wavelengths is called dispersion.
- Rainbows are produced by a combination of refraction and reflection and involve the dispersion of sunlight into a continuous distribution of colors.
- Dispersion produces beautiful rainbows but also causes problems in certain optical systems.

Problems & Exercises

Exercise:

Problem:

(a) What is the ratio of the speed of red light to violet light in diamond, based on [link]? (b) What is this ratio in polystyrene? (c) Which is more dispersive?

Glossary

dispersion
: spreading of white light into its full spectrum of wavelengths

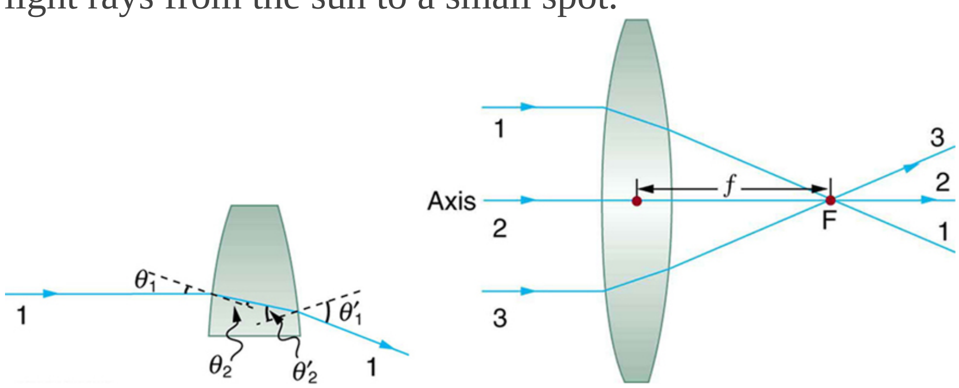
rainbow
: dispersion of sunlight into a continuous distribution of colors according to wavelength, produced by the refraction and reflection of sunlight by water droplets in the sky

Image Formation by Lenses

- List the rules for ray tracking for thin lenses.
- Illustrate the formation of images using the technique of ray tracking.
- Determine power of a lens given the focal length.

Lenses are found in a huge array of optical instruments, ranging from a simple magnifying glass to the eye to a camera's zoom lens. In this section, we will use the law of refraction to explore the properties of lenses and how they form images.

The word *lens* derives from the Latin word for a lentil bean, the shape of which is similar to the convex lens in [link]. The convex lens shown has been shaped so that all light rays that enter it parallel to its axis cross one another at a single point on the opposite side of the lens. (The axis is defined to be a line normal to the lens at its center, as shown in [link].) Such a lens is called a **converging (or convex) lens** for the converging effect it has on light rays. An expanded view of the path of one ray through the lens is shown, to illustrate how the ray changes direction both as it enters and as it leaves the lens. Since the index of refraction of the lens is greater than that of air, the ray moves towards the perpendicular as it enters and away from the perpendicular as it leaves. (This is in accordance with the law of refraction.) Due to the lens's shape, light is thus bent toward the axis at both surfaces. The point at which the rays cross is defined to be the **focal point** F of the lens. The distance from the center of the lens to its focal point is defined to be the **focal length** f of the lens. [link] shows how a converging lens, such as that in a magnifying glass, can converge the nearly parallel light rays from the sun to a small spot.



Rays of light entering a converging lens parallel to its axis converge at its focal point F. (Ray 2 lies on the axis of the lens.) The distance from the center of the lens to the focal point is the lens's focal length f. An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.

Note:
Converging or Convex Lens
The lens in which light rays that enter it parallel to its axis cross one another at a single point on the opposite side with a converging effect is called converging lens.

Note:
Focal Point F
The point at which the light rays cross is called the focal point F of the lens.

Note:
Focal Length f
The distance from the center of the lens to its focal point is called focal length f.

Sunlight focused by a converging magnifying glass can burn paper. Light rays from the sun are nearly parallel and cross at the focal point of the lens. The more powerful the lens, the closer to the lens the rays will cross.

The greater effect a lens has on light rays, the more powerful it is said to be. For example, a powerful converging lens will focus parallel light rays closer to itself and will have a smaller focal length than a weak lens. The light will also focus into a smaller and more intense spot for a more powerful lens. The **power** P of a lens is defined to be the inverse of its focal length. In equation form, this is

Equation:

$$P = \frac{1}{f}.$$

Note:

Power P

The **power** P of a lens is defined to be the inverse of its focal length. In equation form, this is

Equation:

$$P = \frac{1}{f}.$$

where f is the focal length of the lens, which must be given in meters (and not cm or mm). The power of a lens P has the unit diopters (D), provided that the focal length is given in meters. That is, $1\ \text{D} = 1/\text{m}$, or $1\ \text{m}^{-1}$. (Note that this power (optical power, actually) is not the same as power in watts. It is a concept related to the effect of optical devices on light.) Optometrists prescribe common spectacles and contact lenses in units of diopters.

Example:

What is the Power of a Common Magnifying Glass?

Suppose you take a magnifying glass out on a sunny day and you find that it concentrates sunlight to a small spot 8.00 cm away from the lens. What are the focal length and power of the lens?

Strategy

The situation here is the same as those shown in [link] and [link]. The Sun is so far away that the Sun's rays are nearly parallel when they reach Earth. The magnifying glass is a convex (or converging) lens, focusing the nearly parallel rays of sunlight. Thus the focal length of the lens is the distance from the lens to the spot, and its power is the inverse of this distance (in m).

Solution

The focal length of the lens is the distance from the center of the lens to the spot, given to be 8.00 cm. Thus,

Equation:

$$f = 8.00\ \text{cm}.$$

To find the power of the lens, we must first convert the focal length to meters; then, we substitute this value into the equation for power. This gives

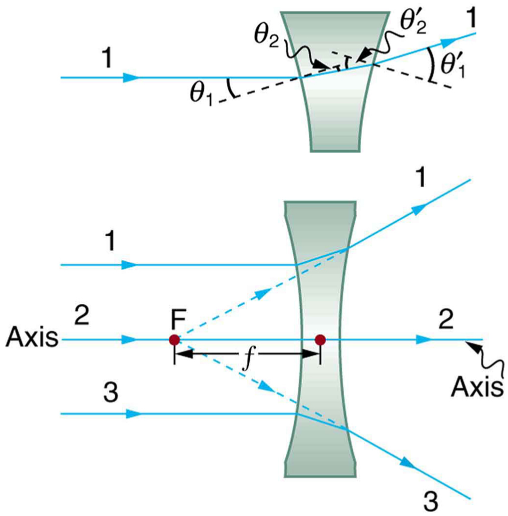
Equation:

$$P = \frac{1}{f} = \frac{1}{0.0800\text{ m}} = 12.5\text{ D}.$$

Discussion

This is a relatively powerful lens. The power of a lens in diopters should not be confused with the familiar concept of power in watts. It is an unfortunate fact that the word "power" is used for two completely different concepts. If you examine a prescription for eyeglasses, you will note lens powers given in diopters. If you examine the label on a motor, you will note energy consumption rate given as a power in watts.

[link] shows a concave lens and the effect it has on rays of light that enter it parallel to its axis (the path taken by ray 2 in the figure is the axis of the lens). The concave lens is a **diverging lens**, because it causes the light rays to bend away (diverge) from its axis. In this case, the lens has been shaped so that all light rays entering it parallel to its axis appear to originate from the same point, F, defined to be the focal point of a diverging lens. The distance from the center of the lens to the focal point is again called the focal length f of the lens. Note that the focal length and power of a diverging lens are defined to be negative. For example, if the distance to F in [link] is 5.00 cm, then the focal length is $f = –5.00\text{ cm}$ and the power of the lens is $P = –20\text{ D}$. An expanded view of the path of one ray through the lens is shown in the figure to illustrate how the shape of the lens, together with the law of refraction, causes the ray to follow its particular path and be diverged.

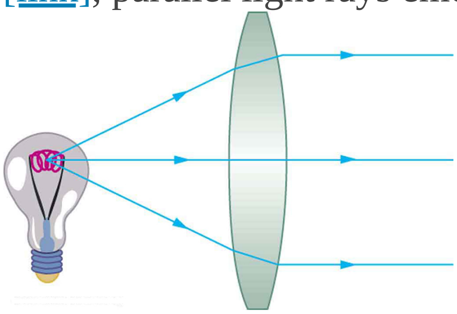


Rays of light entering a diverging lens parallel to its axis are diverged, and all appear to originate at its focal point F. The dashed lines are not rays—they indicate the directions from which the rays appear to come. The focal length f of a diverging lens is negative. An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.

Note:
Diverging Lens

A lens that causes the light rays to bend away from its axis is called a diverging lens.

As noted in the initial discussion of the law of refraction, the paths of light rays are exactly reversible. This means that the direction of the arrows could be reversed for all of the rays in [link] and [link]. For example, if a point light source is placed at the focal point of a convex lens, as shown in [link], parallel light rays emerge from the other side.



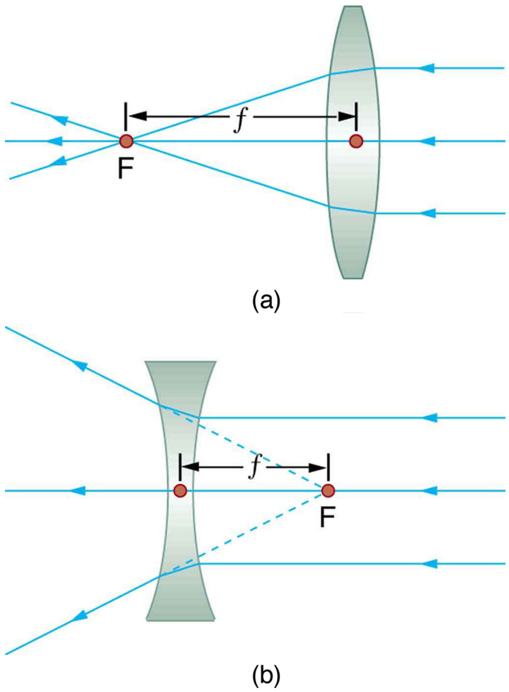
A small light source, like a light bulb filament, placed at the focal point of a convex lens, results in parallel rays of light emerging from the other side. The paths are exactly the reverse of those shown in [link]. This technique is used in lighthouses and sometimes in traffic lights to produce a directional beam of light from a source that emits light in all directions.

Ray Tracing and Thin Lenses

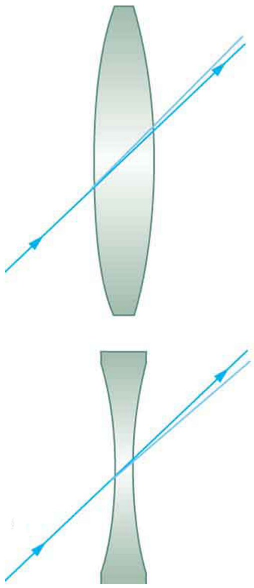
Ray tracing is the technique of determining or following (tracing) the paths that light rays take. For rays passing through matter, the law of refraction is used to trace the paths. Here we use ray tracing to help us understand the action of lenses in situations ranging from forming images on film to magnifying small print to correcting nearsightedness. While ray tracing for complicated lenses, such as those found in sophisticated cameras, may require computer techniques, there is a set of simple rules for tracing rays through thin lenses. A **thin lens** is defined to be one whose thickness allows rays to refract, as illustrated in [link], but does not allow properties such as dispersion and aberrations. An ideal thin lens has two refracting surfaces but the lens is thin enough to assume that light rays bend only once. A thin symmetrical lens has two focal points, one on either side and both at the same distance from the lens. (See [link].) Another important characteristic of a thin lens is that light rays through its center are deflected by a negligible amount, as seen in [link].

> **Note:**
> Thin Lens
> A thin lens is defined to be one whose thickness allows rays to refract but does not allow properties such as dispersion and aberrations.

> **Note:**
> Take-Home Experiment: A Visit to the Optician
> Look through your eyeglasses (or those of a friend) backward and forward and comment on whether they act like thin lenses.



Thin lenses have the same focal length on either side. (a) Parallel light rays entering a converging lens from the right cross at its focal point on the left. (b) Parallel light rays entering a diverging lens from the right seem to come from the focal point on the right.



The light ray through the center of a thin lens is deflected by a negligible amount and is assumed to emerge parallel to its original path (shown as a shaded line).

Using paper, pencil, and a straight edge, ray tracing can accurately describe the operation of a lens. The rules for ray tracing for thin lenses are based on the illustrations already discussed:

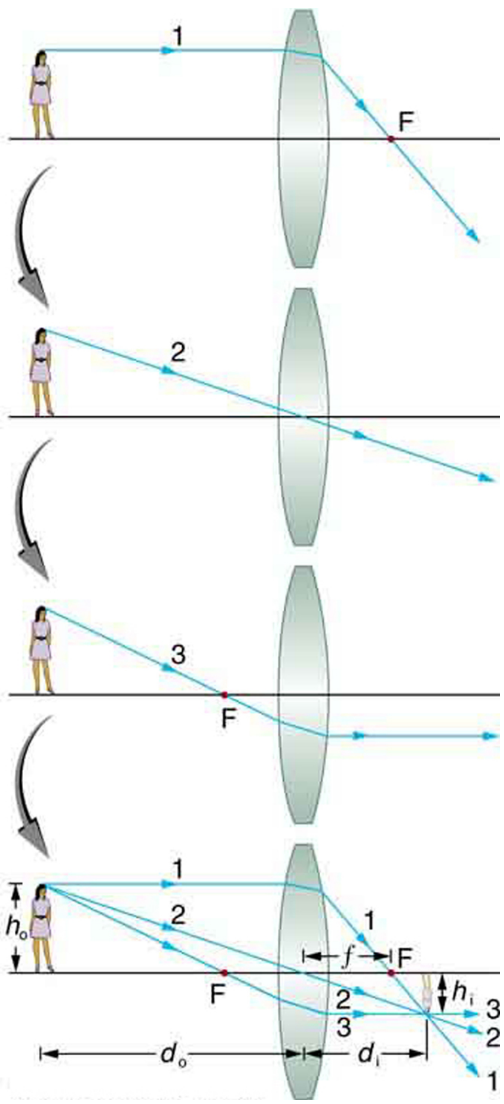
- **Principal Ray 1**: a ray entering a converging lens parallel to its axis passes through the focal point F of the lens on the other side (see rays 1 and 3 in [link]); a ray entering a diverging lens parallel to its axis seems to come from the focal point F (see rays 1 and 3 in [link])
- **Principal Ray 2**: a ray passing through the center of either a converging or a diverging lens does not change direction (see [link], and see ray 2 in [link] and [link])
- **Principal Ray 3**: a ray entering a converging lens through its focal point exits parallel to its axis (the reverse of rays 1 and 3 in [link]); a ray that enters a diverging lens by heading toward the focal point on the opposite side exits parallel to the axis (the reverse of rays 1 and 3 in [link]). *The third principal ray is optional and may be used to verify the accuracy of image location.*

Image Formation by Thin Lenses

In some circumstances, a lens forms an obvious image, such as when a movie projector casts an image onto a screen. In other cases, the image is less obvious. Where, for example, is the image formed by eyeglasses? We will use ray tracing for thin lenses to illustrate how they form images, and we will develop equations to describe the image formation quantitatively.

Consider an object some distance away from a converging lens, as shown in [link]. To find the location and size of the image formed, we trace the paths of principal rays originating from one point on the object, in this case the top of the person’s head. The figure shows the three principal rays from the top of the object as described above. (Note that there are many light rays leaving this point going in many directions, but we concentrate on the three principal rays which can be traced by simple rules.) The first ray is one that enters the lens parallel to its axis and passes through the focal point on the other side (Principal Ray 1). The second ray passes through the center of the lens without changing direction (Principal Ray 2). The third ray passes through the nearer focal point on its way into the lens and leaves the lens

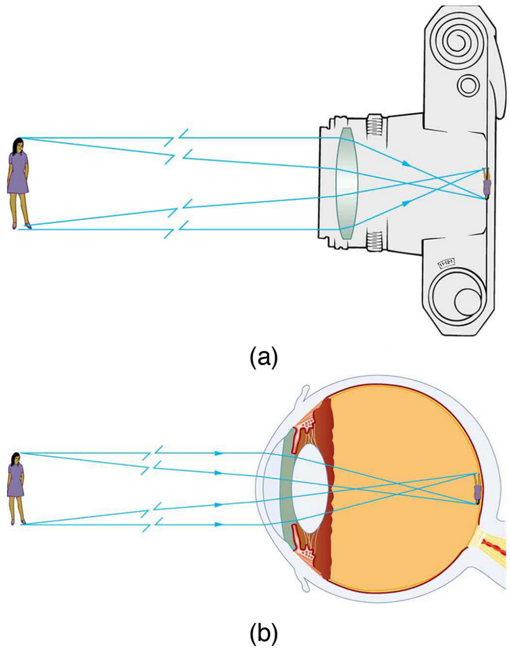
parallel to its axis (Principal Ray 3). If correctly drawn, all three rays cross at the same point on the other side of the lens. The image of the top of the person's head is located at this point. *All* rays, including those that are not principal rays, that come from the same point on the top of the person's head are refracted in such a way as to cross at the point shown; the principal rays are what we use to *find* this point. Rays from another point on the object, such as her belt buckle, will also cross at another common point, forming a complete image, as shown. Although three rays are traced in [link], only two are necessary to locate the image. Before applying ray tracing to other situations, let us consider the example shown in [link] in more detail.



Ray tracing is used to locate the image formed by a lens. Rays originating from the same point on the object are traced—the three chosen rays each follow one of the rules for ray tracing, so that their paths are easy to determine. The image is located at the point where the rays cross. In this case, a real image—one that can be projected on a screen—is formed.

The image formed in [link] is a **real image**, meaning that it can be projected. That is, light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye, for example. [link] shows how such an image would be projected onto film by a camera lens. This figure also shows how a real image is projected onto the retina by the lens of an eye. Note that the image is there whether it is projected onto a screen or not.

Note:
Real Image
The image in which light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye is called a real image.



Real images can be projected. (a) A real image of the person is projected onto film. (b) The converging nature of the multiple surfaces that make up the eye result in the projection of a real image on the retina.

Several important distances appear in [link]. We define d_o to be the object distance, the distance of an object from the center of a lens. **Image distance** d_i is defined to be the distance of the image from the center of a lens. The height of the object and height of the image are given the symbols h_o and h_i, respectively. Images that appear upright relative to the object have heights that are positive and those that are inverted have negative heights. Using the rules of ray tracing and making a scale drawing with paper and pencil, like that in [link], we can accurately describe the location and size of an image. But the real benefit of ray tracing is in visualizing how images are formed in a variety of situations. To obtain numerical information, we use a pair of

equations that can be derived from a geometric analysis of ray tracing for thin lenses. The **thin lens equations** are
Equation:

$$\frac{1}{d_\mathrm{o}} + \frac{1}{d_\mathrm{i}} = \frac{1}{f}$$

and
Equation:

$$\frac{h_\mathrm{i}}{h_\mathrm{o}} = -\frac{d_\mathrm{i}}{d_\mathrm{o}} = m.$$

We define the ratio of image height to object height ($h_\mathrm{i}/h_\mathrm{o}$) to be the **magnification** m. (The minus sign in the equation above will be discussed shortly.) The thin lens equations are broadly applicable to all situations involving thin lenses (and "thin" mirrors, as we will see later). We will explore many features of image formation in the following worked examples.

Note:
Image Distance
The distance of the image from the center of the lens is called image distance.

Note:
Thin Lens Equations and Magnification
Equation:

$$\frac{1}{d_\mathrm{o}} + \frac{1}{d_\mathrm{i}} = \frac{1}{f}$$

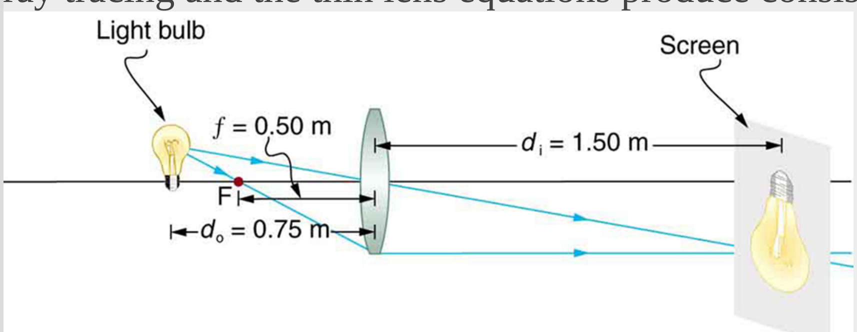
Equation:

$$\frac{h_\text{i}}{h_\text{o}} = -\frac{d_\text{i}}{d_\text{o}} = m$$

Example:

Finding the Image of a Light Bulb Filament by Ray Tracing and by the Thin Lens Equations

A clear glass light bulb is placed 0.750 m from a convex lens having a 0.500 m focal length, as shown in [link]. Use ray tracing to get an approximate location for the image. Then use the thin lens equations to calculate (a) the location of the image and (b) its magnification. Verify that ray tracing and the thin lens equations produce consistent results.



A light bulb placed 0.750 m from a lens having a 0.500 m focal length produces a real image on a poster board as discussed in the example above. Ray tracing predicts the image location and size.

Strategy and Concept

Since the object is placed farther away from a converging lens than the focal length of the lens, this situation is analogous to those illustrated in [link] and [link]. Ray tracing to scale should produce similar results for d_i. Numerical solutions for d_i and m can be obtained using the thin lens equations, noting that $d_\text{o} = 0.750\text{ m}$ and $f = 0.500\text{ m}$.

Solutions (Ray tracing)

The ray tracing to scale in [link] shows two rays from a point on the bulb's filament crossing about 1.50 m on the far side of the lens. Thus the image distance d_i is about 1.50 m. Similarly, the image height based on ray tracing is greater than the object height by about a factor of 2, and the image is inverted. Thus m is about –2. The minus sign indicates that the image is inverted.

The thin lens equations can be used to find d_i from the given information:

Equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}.$$

Rearranging to isolate d_i gives

Equation:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}.$$

Entering known quantities gives a value for $1/d_i$:

Equation:

$$\frac{1}{d_i} = \frac{1}{0.500\text{ m}} - \frac{1}{0.750\text{ m}} = \frac{0.667}{\text{m}}.$$

This must be inverted to find d_i:

Equation:

$$d_i = \frac{\text{m}}{0.667} = 1.50\text{ m}.$$

Note that another way to find d_i is to rearrange the equation:

Equation:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}.$$

This yields the equation for the image distance as:

Equation:

$$d_{\mathrm{i}} = \frac{fd_{\mathrm{o}}}{d_{\mathrm{o}} - f}.$$

Note that there is no inverting here.
The thin lens equations can be used to find the magnification m, since both d_{i} and d_{o} are known. Entering their values gives
Equation:

$$m = -\frac{d_{\mathrm{i}}}{d_{\mathrm{o}}} = -\frac{1.50\ \mathrm{m}}{0.750\ \mathrm{m}} = -2.00.$$

Discussion
Note that the minus sign causes the magnification to be negative when the image is inverted. Ray tracing and the use of the thin lens equations produce consistent results. The thin lens equations give the most precise results, being limited only by the accuracy of the given information. Ray tracing is limited by the accuracy with which you can draw, but it is highly useful both conceptually and visually.

Real images, such as the one considered in the previous example, are formed by converging lenses whenever an object is farther from the lens than its focal length. This is true for movie projectors, cameras, and the eye. We shall refer to these as *case 1* images. A case 1 image is formed when $d_{\mathrm{o}} > f$ and f is positive, as in [link](a). (A summary of the three cases or types of image formation appears at the end of this section.)

A different type of image is formed when an object, such as a person's face, is held close to a convex lens. The image is upright and larger than the object, as seen in [link](b), and so the lens is called a magnifier. If you slowly pull the magnifier away from the face, you will see that the magnification steadily increases until the image begins to blur. Pulling the magnifier even farther away produces an inverted image as seen in [link](a). The distance at which the image blurs, and beyond which it inverts, is the focal length of the lens. To use a convex lens as a magnifier, the object

must be closer to the converging lens than its focal length. This is called a *case 2* image. A case 2 image is formed when $d_o < f$ and f is positive.

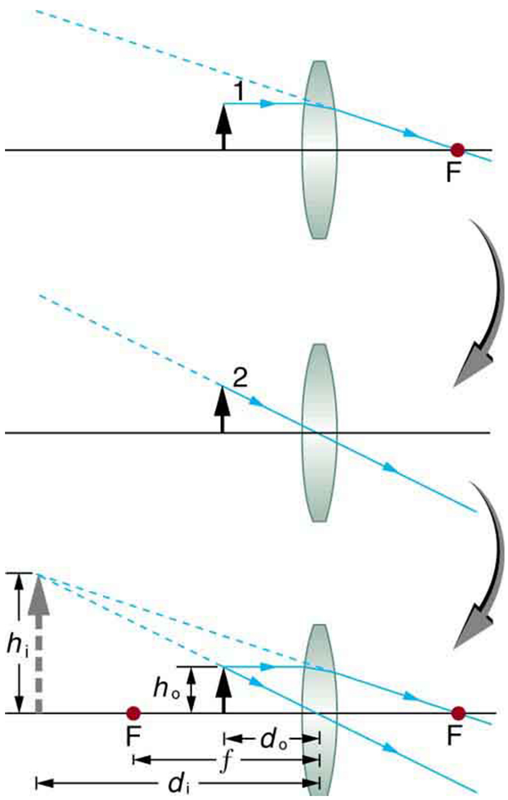
(a)

(b)

(a) When a converging lens is held farther away from the face than the lens's focal length, an inverted image is formed. This is a case 1 image. Note that the image is in focus but the face is not, because the image is much closer to the camera taking this photograph than the face. (credit: DaMongMan, Flickr) (b) A magnified image

of a face is produced by placing it closer to the converging lens than its focal length. This is a case 2 image. (credit: Casey Fleser, Flickr)

[link] uses ray tracing to show how an image is formed when an object is held closer to a converging lens than its focal length. Rays coming from a common point on the object continue to diverge after passing through the lens, but all appear to originate from a point at the location of the image. The image is on the same side of the lens as the object and is farther away from the lens than the object. This image, like all case 2 images, cannot be projected and, hence, is called a **virtual image**. Light rays only appear to originate at a virtual image; they do not actually pass through that location in space. A screen placed at the location of a virtual image will receive only diffuse light from the object, not focused rays from the lens. Additionally, a screen placed on the opposite side of the lens will receive rays that are still diverging, and so no image will be projected on it. We can see the magnified image with our eyes, because the lens of the eye converges the rays into a real image projected on our retina. Finally, we note that a virtual image is upright and larger than the object, meaning that the magnification is positive and greater than 1.



Ray tracing predicts the image location and size for an object held closer to a converging lens than its focal length. Ray 1 enters parallel to the axis and exits through the focal point on the opposite side, while ray 2 passes through the center of the lens without changing path. The two rays continue to diverge on the other side of the lens, but both appear to come from a common point, locating the upright, magnified,

virtual image. This is a case 2 image.

Note:
Virtual Image
An image that is on the same side of the lens as the object and cannot be projected on a screen is called a virtual image.

Example:
Image Produced by a Magnifying Glass
Suppose the book page in [link] (a) is held 7.50 cm from a convex lens of focal length 10.0 cm, such as a typical magnifying glass might have. What magnification is produced?
Strategy and Concept
We are given that $d_{\text{o}} = 7.50 \text{ cm}$ and $f = 10.0 \text{ cm}$, so we have a situation where the object is placed closer to the lens than its focal length. We therefore expect to get a case 2 virtual image with a positive magnification that is greater than 1. Ray tracing produces an image like that shown in [link], but we will use the thin lens equations to get numerical solutions in this example.
Solution
To find the magnification m, we try to use magnification equation, $m = -d_{\text{i}}/d_{\text{o}}$. We do not have a value for d_{i}, so that we must first find the location of the image using lens equation. (The procedure is the same as followed in the preceding example, where d_{o} and f were known.) Rearranging the magnification equation to isolate d_{i} gives
Equation:

$$\frac{1}{d_{\text{i}}} = \frac{1}{f} - \frac{1}{d_{\text{o}}}.$$

Entering known values, we obtain a value for $1/d_{\text{i}}$:
Equation:

$$\frac{1}{d_\mathrm{i}} = \frac{1}{10.0\text{ cm}} - \frac{1}{7.50\text{ cm}} = \frac{-0.0333}{\text{cm}}.$$

This must be inverted to find d_i:

Equation:

$$d_\mathrm{i} = -\frac{\text{cm}}{0.0333} = -30.0\text{ cm}.$$

Now the thin lens equation can be used to find the magnification m, since both d_i and d_o are known. Entering their values gives

Equation:

$$m = -\frac{d_\mathrm{i}}{d_\mathrm{o}} = -\frac{-30.0\text{ cm}}{7.5\text{ cm}} = 4.00.$$

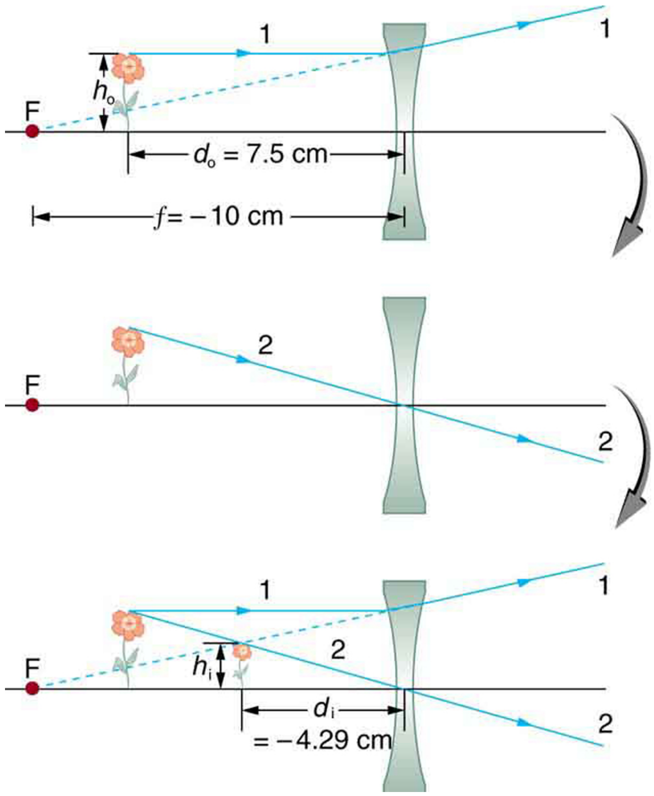
Discussion

A number of results in this example are true of all case 2 images, as well as being consistent with [link]. Magnification is indeed positive (as predicted), meaning the image is upright. The magnification is also greater than 1, meaning that the image is larger than the object—in this case, by a factor of 3. Note that the image distance is negative. This means the image is on the same side of the lens as the object. Thus the image cannot be projected and is virtual. (Negative values of d_i occur for virtual images.) The image is farther from the lens than the object, since the image distance is greater in magnitude than the object distance. The location of the image is not obvious when you look through a magnifier. In fact, since the image is bigger than the object, you may think the image is closer than the object. But the image is farther away, a fact that is useful in correcting farsightedness, as we shall see in a later section.

A third type of image is formed by a diverging or concave lens. Try looking through eyeglasses meant to correct nearsightedness. (See [link].) You will see an image that is upright but smaller than the object. This means that the magnification is positive but less than 1. The ray diagram in [link] shows that the image is on the same side of the lens as the object and, hence,

cannot be projected—it is a virtual image. Note that the image is closer to the lens than the object. This is a *case 3* image, formed for any object by a negative focal length or diverging lens.

A car viewed through a concave or diverging lens looks upright. This is a case 3 image. (credit: Daniel Oines, Flickr)



Ray tracing predicts the image location and size for a concave or diverging lens. Ray 1 enters parallel to the axis and is bent so that it appears to originate from the focal point. Ray 2 passes through the center of the lens without changing path. The two rays appear to come from a common point, locating the upright image. This is a case 3 image, which is closer to the lens than the object and smaller in height.

Example:

Image Produced by a Concave Lens

Suppose an object such as a book page is held 7.50 cm from a concave lens of focal length –10.0 cm. Such a lens could be used in eyeglasses to correct pronounced nearsightedness. What magnification is produced?

Strategy and Concept

This example is identical to the preceding one, except that the focal length is negative for a concave or diverging lens. The method of solution is thus the same, but the results are different in important ways.

Solution

To find the magnification m, we must first find the image distance d_i using thin lens equation

Equation:

$$\frac{1}{d_\mathrm{i}} = \frac{1}{f} - \frac{1}{d_\mathrm{o}},$$

or its alternative rearrangement

Equation:

$$d_i = \frac{f d_\mathrm{o}}{d_\mathrm{o} - f}.$$

We are given that $f =$ –10.0 cm and $d_\mathrm{o} = 7.50$ cm. Entering these yields a value for $1/d_\mathrm{i}$:

Equation:

$$\frac{1}{d_\mathrm{i}} = \frac{1}{-10.0\ \text{cm}} - \frac{1}{7.50\ \text{cm}} = \frac{-0.2333}{\text{cm}}.$$

This must be inverted to find d_i:

Equation:

$$d_\mathrm{i} = -\frac{\text{cm}}{0.2333} = -4.29\ \text{cm}.$$

Or

Equation:

$$d_i = \frac{(7.5)(-10)}{(7.5 - (-10))} = -75/17.5 = -4.29\text{ cm}.$$

Now the magnification equation can be used to find the magnification m, since both d_i and d_o are known. Entering their values gives

Equation:

$$m = -\frac{d_i}{d_o} = -\frac{-4.29\text{ cm}}{7.50\text{ cm}} = 0.571.$$

Discussion

A number of results in this example are true of all case 3 images, as well as being consistent with [link]. Magnification is positive (as predicted), meaning the image is upright. The magnification is also less than 1, meaning the image is smaller than the object—in this case, a little over half its size. The image distance is negative, meaning the image is on the same side of the lens as the object. (The image is virtual.) The image is closer to the lens than the object, since the image distance is smaller in magnitude than the object distance. The location of the image is not obvious when you look through a concave lens. In fact, since the image is smaller than the object, you may think it is farther away. But the image is closer than the object, a fact that is useful in correcting nearsightedness, as we shall see in a later section.

[link] summarizes the three types of images formed by single thin lenses. These are referred to as case 1, 2, and 3 images. Convex (converging) lenses can form either real or virtual images (cases 1 and 2, respectively), whereas concave (diverging) lenses can form only virtual images (always case 3). Real images are always inverted, but they can be either larger or smaller than the object. For example, a slide projector forms an image larger than the slide, whereas a camera makes an image smaller than the object being photographed. Virtual images are always upright and cannot be projected. Virtual images are larger than the object only in case 2, where a convex lens is used. The virtual image produced by a concave lens is

always smaller than the object—a case 3 image. We can see and photograph virtual images only by using an additional lens to form a real image.

Type	Formed when	Image type	d_i	m
Case 1	f positive, $d_o > f$	real	positive	negative
Case 2	f positive, $d_o < f$	virtual	negative	positive, $m > 1$
Case 3	f negative	virtual	negative	positive, $m < 1$

Three Types of Images Formed By Thin Lenses

Note:
Take-Home Experiment: Concentrating Sunlight
Find several lenses and determine whether they are converging or diverging. In general those that are thicker near the edges are diverging and those that are thicker near the center are converging. On a bright sunny day take the converging lenses outside and try focusing the sunlight onto a piece of paper. Determine the focal lengths of the lenses. Be careful because the paper may start to burn, depending on the type of lens you have selected.

Section Summary

- Light rays entering a converging lens parallel to its axis cross one another at a single point on the opposite side.
- For a converging lens, the focal point is the point at which converging light rays cross; for a diverging lens, the focal point is the point from which diverging light rays appear to originate.
- The distance from the center of the lens to its focal point is called the focal length f.
- Power P of a lens is defined to be the inverse of its focal length, $P = \frac{1}{f}$.
- A lens that causes the light rays to bend away from its axis is called a diverging lens.
- Ray tracing is the technique of graphically determining the paths that light rays take.
- The image in which light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye is called a real image.
- Thin lens equations are $\frac{1}{d_\text{o}} + \frac{1}{d_\text{i}} = \frac{1}{f}$ and $\frac{h_\text{i}}{h_\text{o}} = -\frac{d_\text{i}}{d_\text{o}} = m$ (magnification).
- The distance of the image from the center of the lens is called image distance.
- An image that is on the same side of the lens as the object and cannot be projected on a screen is called a virtual image.

Conceptual Questions

Exercise:

Problem:

It can be argued that a flat piece of glass, such as in a window, is like a lens with an infinite focal length. If so, where does it form an image? That is, how are d_i and d_o related?

Exercise:

Problem:

You can often see a reflection when looking at a sheet of glass, particularly if it is darker on the other side. Explain why you can often see a double image in such circumstances.

Exercise:

Problem:

When you focus a camera, you adjust the distance of the lens from the film. If the camera lens acts like a thin lens, why can it not be a fixed distance from the film for both near and distant objects?

Exercise:

Problem:

A thin lens has two focal points, one on either side, at equal distances from its center, and should behave the same for light entering from either side. Look through your eyeglasses (or those of a friend) backward and forward and comment on whether they are thin lenses.

Exercise:

Problem:

Will the focal length of a lens change when it is submerged in water? Explain.

Glossary

converging lens
: a convex lens in which light rays that enter it parallel to its axis converge at a single point on the opposite side

diverging lens
: a concave lens in which light rays that enter it parallel to its axis bend away (diverge) from its axis

focal point

for a converging lens or mirror, the point at which converging light rays cross; for a diverging lens or mirror, the point from which diverging light rays appear to originate

focal length

distance from the center of a lens or curved mirror to its focal point

magnification

ratio of image height to object height

power

inverse of focal length

real image

image that can be projected

virtual image

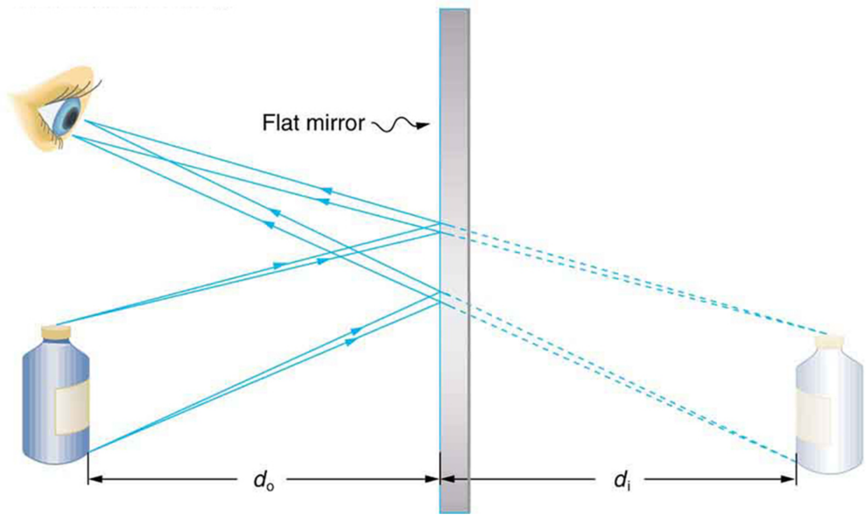
image that cannot be projected

Image Formation by Mirrors

- Illustrate image formation in a flat mirror.
- Explain with ray diagrams the formation of an image using spherical mirrors.
- Determine focal length and magnification given radius of curvature, distance of object and image.

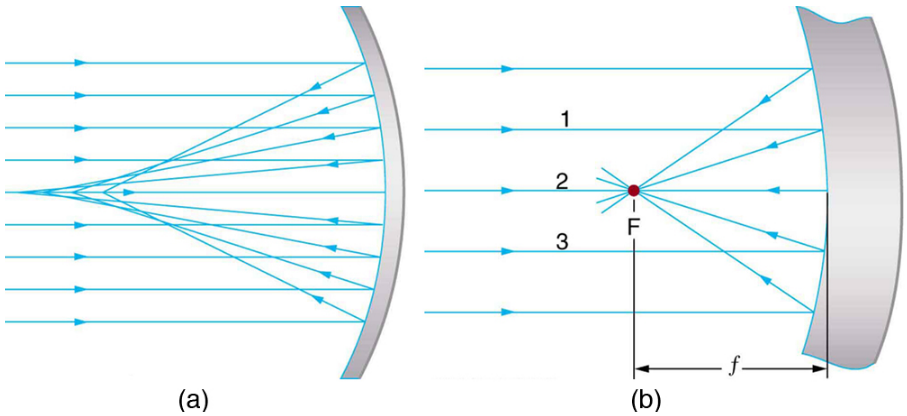
We only have to look as far as the nearest bathroom to find an example of an image formed by a mirror. Images in flat mirrors are the same size as the object and are located behind the mirror. Like lenses, mirrors can form a variety of images. For example, dental mirrors may produce a magnified image, just as makeup mirrors do. Security mirrors in shops, on the other hand, form images that are smaller than the object. We will use the law of reflection to understand how mirrors form images, and we will find that mirror images are analogous to those formed by lenses.

[link] helps illustrate how a flat mirror forms an image. Two rays are shown emerging from the same point, striking the mirror, and being reflected into the observer's eye. The rays can diverge slightly, and both still get into the eye. If the rays are extrapolated backward, they seem to originate from a common point behind the mirror, locating the image. (The paths of the reflected rays into the eye are the same as if they had come directly from that point behind the mirror.) Using the law of reflection—the angle of reflection equals the angle of incidence—we can see that the image and object are the same distance from the mirror. This is a virtual image, since it cannot be projected—the rays only appear to originate from a common point behind the mirror. Obviously, if you walk behind the mirror, you cannot see the image, since the rays do not go there. But in front of the mirror, the rays behave exactly as if they had come from behind the mirror, so that is where the image is situated.



Two sets of rays from common points on an object are reflected by a flat mirror into the eye of an observer. The reflected rays seem to originate from behind the mirror, locating the virtual image.

Now let us consider the focal length of a mirror—for example, the concave spherical mirrors in [link]. Rays of light that strike the surface follow the law of reflection. For a mirror that is large compared with its radius of curvature, as in [link](a), we see that the reflected rays do not cross at the same point, and the mirror does not have a well-defined focal point. If the mirror had the shape of a parabola, the rays would all cross at a single point, and the mirror would have a well-defined focal point. But parabolic mirrors are much more expensive to make than spherical mirrors. The solution is to use a mirror that is small compared with its radius of curvature, as shown in [link](b). (This is the mirror equivalent of the thin lens approximation.) To a very good approximation, this mirror has a well-defined focal point at F that is the focal distance f from the center of the mirror. The focal length f of a concave mirror is positive, since it is a converging mirror.



(a) Parallel rays reflected from a large spherical mirror do not all cross at a common point. (b) If a spherical mirror is small compared with its radius of curvature, parallel rays are focused to a common point. The distance of the focal point from the center of the mirror is its focal length f. Since this mirror is converging, it has a positive focal length.

Just as for lenses, the shorter the focal length, the more powerful the mirror; thus, $P = 1/f$ for a mirror, too. A more strongly curved mirror has a shorter focal length and a greater power. Using the law of reflection and some simple trigonometry, it can be shown that the focal length is half the radius of curvature, or

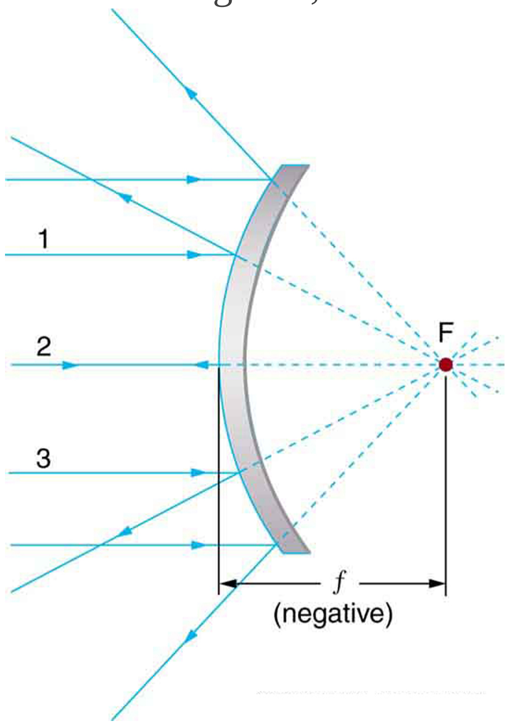
Equation:

$$f = \frac{R}{2},$$

where R is the radius of curvature of a spherical mirror. The smaller the radius of curvature, the smaller the focal length and, thus, the more powerful the mirror.

The convex mirror shown in [link] also has a focal point. Parallel rays of light reflected from the mirror seem to originate from the point F at the

focal distance f behind the mirror. The focal length and power of a convex mirror are negative, since it is a diverging mirror.



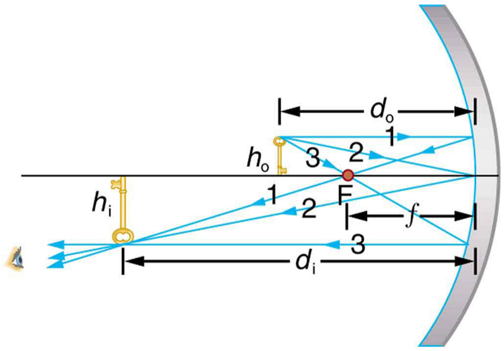
Parallel rays of light reflected from a convex spherical mirror (small in size compared with its radius of curvature) seem to originate from a well-defined focal point at the focal distance f behind the mirror. Convex mirrors diverge light rays and, thus, have a negative focal length.

Ray tracing is as useful for mirrors as for lenses. The rules for ray tracing for mirrors are based on the illustrations just discussed:

1. **Principal Ray 1**: a ray approaching a concave converging mirror parallel to its axis is reflected through the focal point F of the mirror on the same side. (See rays 1 and 3 in [link](b).); a ray approaching a convex diverging mirror parallel to its axis is reflected so that it seems to come from the focal point F behind the mirror. (See rays 1 and 3 in [link].)
2. **Principal Ray 2**: any ray striking the center of a mirror is followed by applying the law of reflection; it makes the same angle with the axis when leaving as when approaching. (See ray 2 in [link].)
3. **Principal Ray 3**: a ray approaching a concave converging mirror through its focal point is reflected parallel to its axis. (The reverse of rays 1 and 3 in [link].); a ray approaching a convex diverging mirror by heading toward its focal point on the opposite side is reflected parallel to the axis. (The reverse of rays 1 and 3 in [link].) *The third principal ray is optional and may be used to verify the accuracy of image location.*

We will use ray tracing to illustrate how images are formed by mirrors, and we can use ray tracing quantitatively to obtain numerical information. But since we assume each mirror is small compared with its radius of curvature, we can use the thin lens equations for mirrors just as we did for lenses.

Consider the situation shown in [link], concave spherical mirror reflection, in which an object is placed farther from a concave (converging) mirror than its focal length. That is, f is positive and $d_o > f$, so that we may expect an image similar to the case 1 real image formed by a converging lens. Ray tracing in [link] shows that the rays from a common point on the object all cross at a point on the same side of the mirror as the object. Thus a real image can be projected onto a screen placed at this location. The image distance is positive, and the image is inverted, so its magnification is negative. This is a *case 1 image for mirrors*. It differs from the case 1 image for lenses only in that the image is on the same side of the mirror as the object. It is otherwise identical.



A case 1 image for a mirror. An object is farther from the converging mirror than its focal length. Rays from a common point on the object are traced using the rules in the text. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the mirror, and ray 3 goes through the focal point on the way toward the mirror. All three rays cross at the same point after being reflected, locating the inverted real image. Although three rays are shown, only two of the three are needed to locate the image and determine its height.

Example:

A Concave Reflector

Electric room heaters use a concave mirror to reflect infrared (IR) radiation from hot coils. Note that IR follows the same law of reflection as visible light. Given that the mirror has a radius of curvature of 50.0 cm and produces an image of the coils 3.00 m away from the mirror, where are the coils?

Strategy and Concept

We are given that the concave mirror projects a real image of the coils at an image distance $d_{\mathrm{i}} = 3.00\ \mathrm{m}$. The coils are the object, and we are asked to find their location—that is, to find the object distance d_{o}. We are also given the radius of curvature of the mirror, so that its focal length is $f = R/2 = 25.0\ \mathrm{cm}$ (positive since the mirror is concave or converging). Assuming the mirror is small compared with its radius of curvature, we can use the thin lens equations, to solve this problem.

Solution

Since d_{i} and f are known, thin lens equation can be used to find d_{o}:

Equation:

$$\frac{1}{d_{\mathrm{o}}} + \frac{1}{d_{\mathrm{i}}} = \frac{1}{f}.$$

Rearranging to isolate d_{o} gives

Equation:

$$\frac{1}{d_{\mathrm{o}}} = \frac{1}{f} - \frac{1}{d_{\mathrm{i}}}.$$

Entering known quantities gives a value for $1/d_{\mathrm{o}}$:

Equation:

$$\frac{1}{d_{\mathrm{o}}} = \frac{1}{0.250\ \mathrm{m}} - \frac{1}{3.00\ \mathrm{m}} = \frac{3.667}{\mathrm{m}}.$$

This must be inverted to find d_{o}:

Equation:

$$d_{\mathrm{o}} = \frac{1\ \mathrm{m}}{3.667} = 27.3\ \mathrm{cm}.$$

Discussion

Note that the object (the filament) is farther from the mirror than the mirror’s focal length. This is a case 1 image ($d_o > f$ and f positive), consistent with the fact that a real image is formed. You will get the most concentrated thermal energy directly in front of the mirror and 3.00 m away from it. Generally, this is not desirable, since it could cause burns. Usually, you want the rays to emerge parallel, and this is accomplished by having the filament at the focal point of the mirror.

Note that the filament here is not much farther from the mirror than its focal length and that the image produced is considerably farther away. This is exactly analogous to a slide projector. Placing a slide only slightly farther away from the projector lens than its focal length produces an image significantly farther away. As the object gets closer to the focal distance, the image gets farther away. In fact, as the object distance approaches the focal length, the image distance approaches infinity and the rays are sent out parallel to one another.

Example:
Solar Electric Generating System

One of the solar technologies used today for generating electricity is a device (called a parabolic trough or concentrating collector) that concentrates the sunlight onto a blackened pipe that contains a fluid. This heated fluid is pumped to a heat exchanger, where its heat energy is transferred to another system that is used to generate steam—and so generate electricity through a conventional steam cycle. [link] shows such a working system in southern California. Concave mirrors are used to concentrate the sunlight onto the pipe. The mirror has the approximate shape of a section of a cylinder. For the problem, assume that the mirror is exactly one-quarter of a full cylinder.

a. If we wish to place the fluid-carrying pipe 40.0 cm from the concave mirror at the mirror’s focal point, what will be the radius of curvature of the mirror?
b. Per meter of pipe, what will be the amount of sunlight concentrated onto the pipe, assuming the insolation (incident solar radiation) is

$0.900\ \mathrm{kW/m}^2$?

c. If the fluid-carrying pipe has a 2.00-cm diameter, what will be the temperature increase of the fluid per meter of pipe over a period of one minute? Assume all the solar radiation incident on the reflector is absorbed by the pipe, and that the fluid is mineral oil.

Strategy

To solve an *Integrated Concept Problem* we must first identify the physical principles involved. Part (a) is related to the current topic. Part (b) involves a little math, primarily geometry. Part (c) requires an understanding of heat and density.

Solution to (a)

To a good approximation for a concave or semi-spherical surface, the point where the parallel rays from the sun converge will be at the focal point, so $R = 2f = 80.0\text{ cm}$.

Solution to (b)

The insolation is $900\ \mathrm{W/m}^2$. We must find the cross-sectional area A of the concave mirror, since the power delivered is $900\ \mathrm{W/m}^2 \times \mathrm{A}$. The mirror in this case is a quarter-section of a cylinder, so the area for a length L of the mirror is $\mathrm{A} = \frac{1}{4}(2\pi R)\mathrm{L}$. The area for a length of 1.00 m is then

Equation:

$$\mathrm{A} = \frac{\pi}{2}R(1.00\text{ m}) = \frac{(3.14)}{2}(0.800\text{ m})(1.00\text{ m}) = 1.26\text{ m}^2.$$

The insolation on the 1.00-m length of pipe is then

Equation:

$$\left(9.00 \times 10^2 \frac{\mathrm{W}}{\mathrm{m}^2}\right)\left(1.26\text{ m}^2\right) = 1130\text{ W}.$$

Solution to (c)

The increase in temperature is given by $Q = mc\,\Delta T$. The mass m of the mineral oil in the one-meter section of pipe is

Equation:

$$\begin{aligned} m &= \rho V = \rho\pi\left(\frac{d}{2}\right)^2(1.00\text{ m}) \\ &= \left(8.00\times10^2\text{ kg/m}^3\right)(3.14)(0.0100\text{ m})^2(1.00\text{ m}) \\ &= 0.251\text{ kg.} \end{aligned}$$

Therefore, the increase in temperature in one minute is

Equation:

$$\begin{aligned} \Delta T &= Q/mc \\ &= \frac{(1130\text{ W})(60.0\text{ s})}{(0.251\text{ kg})(1670\text{ J}\cdot\text{kg/}^\circ\text{C})} \\ &= 162^\circ\text{C.} \end{aligned}$$

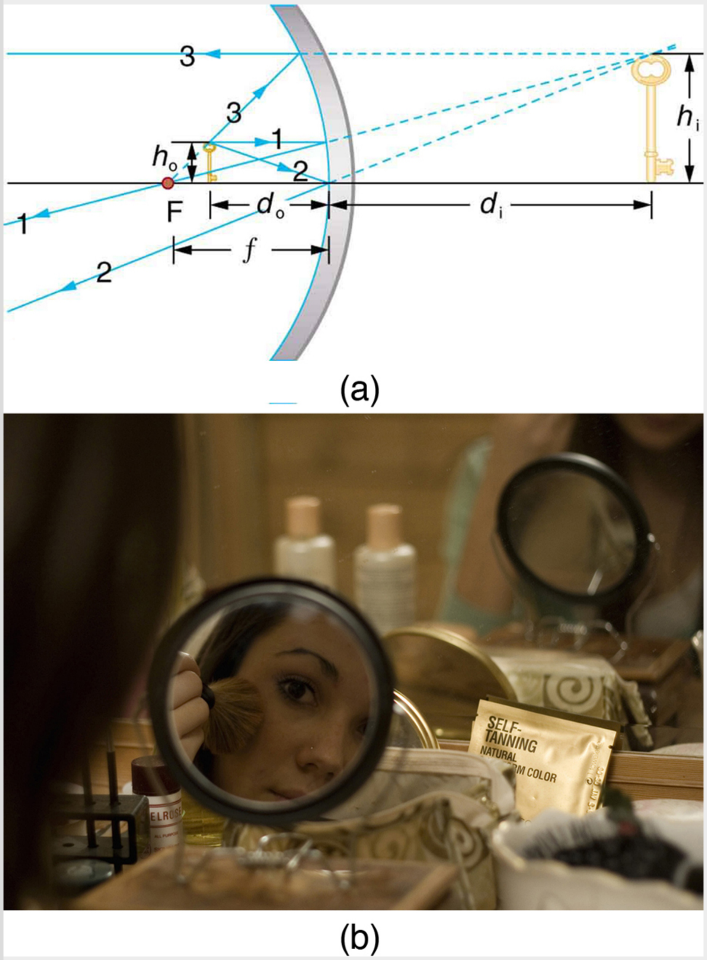
Discussion for (c)

An array of such pipes in the California desert can provide a thermal output of 250 MW on a sunny day, with fluids reaching temperatures as high as 400ºC. We are considering only one meter of pipe here, and ignoring heat losses along the pipe.

Parabolic trough collectors are used to generate electricity in southern California. (credit: kjkolb, Wikimedia Commons)

What happens if an object is closer to a concave mirror than its focal length? This is analogous to a case 2 image for lenses ($d_o < f$ and f positive), which is a magnifier. In fact, this is how makeup mirrors act as magnifiers. [link](a) uses ray tracing to locate the image of an object placed close to a concave mirror. Rays from a common point on the object

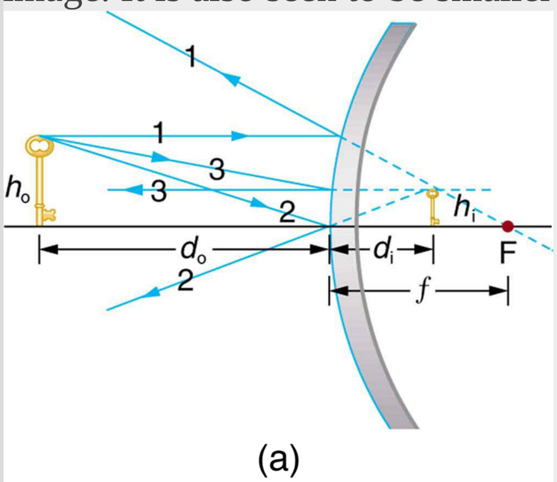
are reflected in such a manner that they appear to be coming from behind the mirror, meaning that the image is virtual and cannot be projected. As with a magnifying glass, the image is upright and larger than the object. This is a *case 2 image for mirrors* and is exactly analogous to that for lenses.



(a) Case 2 images for mirrors are formed when a converging mirror has an object closer to it than its focal length. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the mirror, and ray 3 approaches the mirror as if it came from the focal point. (b) A magnifying mirror showing the reflection. (credit: Mike Melrose, Flickr)

All three rays appear to originate from the same point after being reflected, locating the upright virtual image behind the mirror and showing it to be larger than the object. (b) Makeup mirrors are perhaps the most common use of a concave mirror to produce a larger, upright image.

A convex mirror is a diverging mirror (f is negative) and forms only one type of image. It is a *case 3* image—one that is upright and smaller than the object, just as for diverging lenses. [link](a) uses ray tracing to illustrate the location and size of the case 3 image for mirrors. Since the image is behind the mirror, it cannot be projected and is thus a virtual image. It is also seen to be smaller than the object.



Case 3 images for mirrors are formed by any convex mirror. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the

mirror, and ray 3 approaches toward the focal point. All three rays appear to originate from the same point after being reflected, locating the upright virtual image behind the mirror and showing it to be smaller than the object. (b) Security mirrors are convex, producing a smaller, upright image. Because the image is smaller, a larger area is imaged compared to what would be observed for a flat mirror (and hence security is improved). (credit: Laura D’Alessandro, Flickr)

Example:
Image in a Convex Mirror

A keratometer is a device used to measure the curvature of the cornea, particularly for fitting contact lenses. Light is reflected from the cornea, which acts like a convex mirror, and the keratometer measures the magnification of the image. The smaller the magnification, the smaller the radius of curvature of the cornea. If the light source is 12.0 cm from the cornea and the image’s magnification is 0.0320, what is the cornea’s radius of curvature?

Strategy

If we can find the focal length of the convex mirror formed by the cornea, we can find its radius of curvature (the radius of curvature is twice the focal length of a spherical mirror). We are given that the object distance is

$d_o = 12.0$ cm and that $m = 0.0320$. We first solve for the image distance d_i, and then for f.

Solution

$m = –d_i/d_o$. Solving this expression for d_i gives

Equation:

$$d_i = -md_o.$$

Entering known values yields

Equation:

$$d_i =– (0.0320)(12.0 \text{ cm}) = –0.384 \text{ cm}.$$

Equation:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Substituting known values,

Equation:

$$\frac{1}{f} = \frac{1}{12.0 \text{ cm}} + \frac{1}{-0.384 \text{ cm}} = \frac{-2.52}{\text{cm}}.$$

This must be inverted to find f:

Equation:

$$f = \frac{\text{cm}}{-2.52} = –0.400 \text{ cm}.$$

The radius of curvature is twice the focal length, so that

Equation:

$$R = 2 \mid f \mid = 0.800 \text{ cm}.$$

Discussion

Although the focal length f of a convex mirror is defined to be negative, we take the absolute value to give us a positive value for R. The radius of curvature found here is reasonable for a cornea. The distance from cornea

to retina in an adult eye is about 2.0 cm. In practice, many corneas are not spherical, complicating the job of fitting contact lenses. Note that the image distance here is negative, consistent with the fact that the image is behind the mirror, where it cannot be projected. In this section's Problems and Exercises, you will show that for a fixed object distance, the smaller the radius of curvature, the smaller the magnification.

The three types of images formed by mirrors (cases 1, 2, and 3) are exactly analogous to those formed by lenses, as summarized in the table at the end of "Image Formation by Lenses". It is easiest to concentrate on only three types of images—then remember that concave mirrors act like convex lenses, whereas convex mirrors act like concave lenses.

Note:

Take-Home Experiment: Concave Mirrors Close to Home

Find a flashlight and identify the curved mirror used in it. Find another flashlight and shine the first flashlight onto the second one, which is turned off. Estimate the focal length of the mirror. You might try shining a flashlight on the curved mirror behind the headlight of a car, keeping the headlight switched off, and determine its focal length.

Section Summary

- The characteristics of an image formed by a flat mirror are: (a) The image and object are the same distance from the mirror, (b) The image is a virtual image, and (c) The image is situated behind the mirror.
- Image length is half the radius of curvature.
  Equation:

$$f = \frac{R}{2}$$

- A convex mirror is a diverging mirror and forms only one type of image, namely a virtual image.

Conceptual Questions

Exercise:

Problem:

What are the differences between real and virtual images? How can you tell (by looking) whether an image formed by a single lens or mirror is real or virtual?

Exercise:

Problem:

Can you see a virtual image? Can you photograph one? Can one be projected onto a screen with additional lenses or mirrors? Explain your responses.

Exercise:

Problem:

Is it necessary to project a real image onto a screen for it to exist?

Exercise:

Problem:

At what distance is an image *always* located—at d_o, d_i, or f?

Exercise:

Problem:

Under what circumstances will an image be located at the focal point of a lens or mirror?

Exercise:

Problem:

What is meant by a negative magnification? What is meant by a magnification that is less than 1 in magnitude?

Exercise:

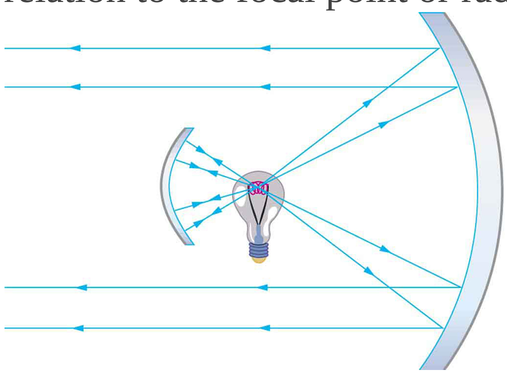
Problem:

Can a case 1 image be larger than the object even though its magnification is always negative? Explain.

Exercise:

Problem:

[link] shows a light bulb between two mirrors. One mirror produces a beam of light with parallel rays; the other keeps light from escaping without being put into the beam. Where is the filament of the light in relation to the focal point or radius of curvature of each mirror?



The two mirrors trap most of the bulb’s light and form a directional beam as in a headlight.

Exercise:

Problem:

Devise an arrangement of mirrors allowing you to see the back of your head. What is the minimum number of mirrors needed for this task?

Exercise:

Problem:

If you wish to see your entire body in a flat mirror (from head to toe), how tall should the mirror be? Does its size depend upon your distance away from the mirror? Provide a sketch.

Exercise:

Problem:

It can be argued that a flat mirror has an infinite focal length. If so, where does it form an image? That is, how are d_i and d_o related?

Exercise:

Problem:

Why are diverging mirrors often used for rear-view mirrors in vehicles? What is the main disadvantage of using such a mirror compared with a flat one?

Glossary

converging mirror
: a concave mirror in which light rays that strike it parallel to its axis converge at one or more points along the axis

diverging mirror
: a convex mirror in which light rays that strike it parallel to its axis bend away (diverge) from its axis

law of reflection
: angle of reflection equals the angle of incidence

Polarization

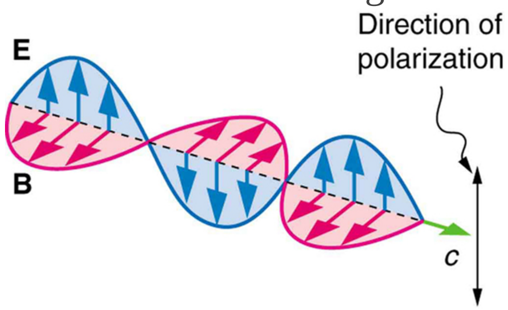
- Discuss the meaning of polarization.
- Discuss the property of optical activity of certain materials.

Polaroid sunglasses are familiar to most of us. They have a special ability to cut the glare of light reflected from water or glass (see [link]). Polaroids have this ability because of a wave characteristic of light called polarization. What is polarization? How is it produced? What are some of its uses? The answers to these questions are related to the wave character of light.

These two photographs of a river show the effect of a polarizing filter in reducing glare in light reflected from the surface of water. Part (b) of this figure was taken with a polarizing filter and part (a) was not. As a result, the reflection of clouds and sky observed in part (a) is not observed in part (b). Polarizing sunglasses are particularly useful on snow and water. (credit: Amithshs, Wikimedia Commons)

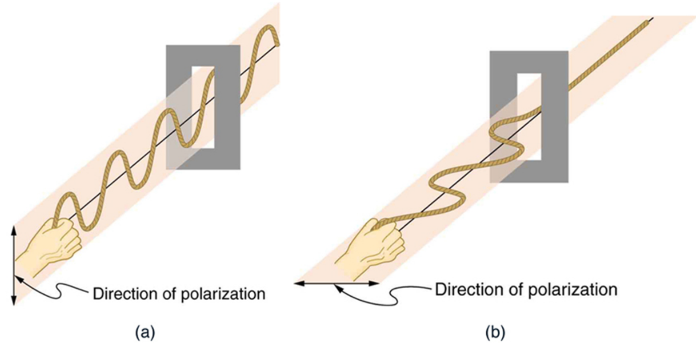
Light is one type of electromagnetic (EM) wave. As noted earlier, EM waves are *transverse waves* consisting of varying electric and magnetic fields that oscillate perpendicular to the direction of propagation (see [link]). There are specific directions for the oscillations of the electric and magnetic fields. **Polarization** is the attribute that a wave’s oscillations have

a definite direction relative to the direction of propagation of the wave. (This is not the same type of polarization as that discussed for the separation of charges.) Waves having such a direction are said to be **polarized**. For an EM wave, we define the **direction of polarization** to be the direction parallel to the electric field. Thus we can think of the electric field arrows as showing the direction of polarization, as in [link].



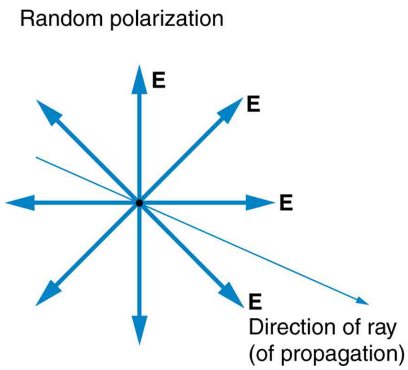
An EM wave, such as light, is a transverse wave. The electric and magnetic fields are perpendicular to the direction of propagation.

To examine this further, consider the transverse waves in the ropes shown in [link]. The oscillations in one rope are in a vertical plane and are said to be **vertically polarized**. Those in the other rope are in a horizontal plane and are **horizontally polarized**. If a vertical slit is placed on the first rope, the waves pass through. However, a vertical slit blocks the horizontally polarized waves. For EM waves, the direction of the electric field is analogous to the disturbances on the ropes.

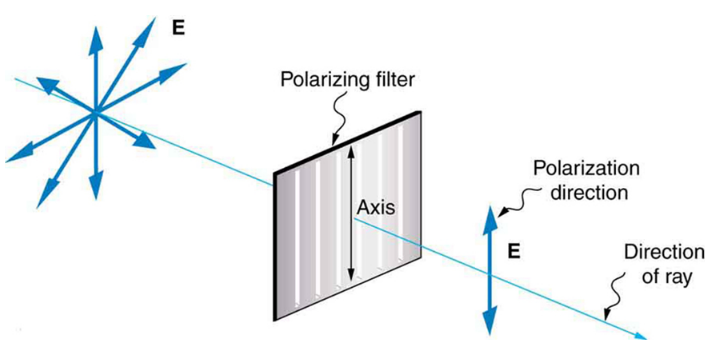


The transverse oscillations in one rope are in a vertical plane, and those in the other rope are in a horizontal plane. The first is said to be vertically polarized, and the other is said to be horizontally polarized. Vertical slits pass vertically polarized waves and block horizontally polarized waves.

The Sun and many other light sources produce waves that are randomly polarized (see [link]). Such light is said to be **unpolarized** because it is composed of many waves with all possible directions of polarization. Polaroid materials, invented by the founder of Polaroid Corporation, Edwin Land, act as a *polarizing* slit for light, allowing only polarization in one direction to pass through. Polarizing filters are composed of long molecules aligned in one direction. Thinking of the molecules as many slits, analogous to those for the oscillating ropes, we can understand why only light with a specific polarization can get through. The **axis of a polarizing filter** is the direction along which the filter passes the electric field of an EM wave (see [link]).



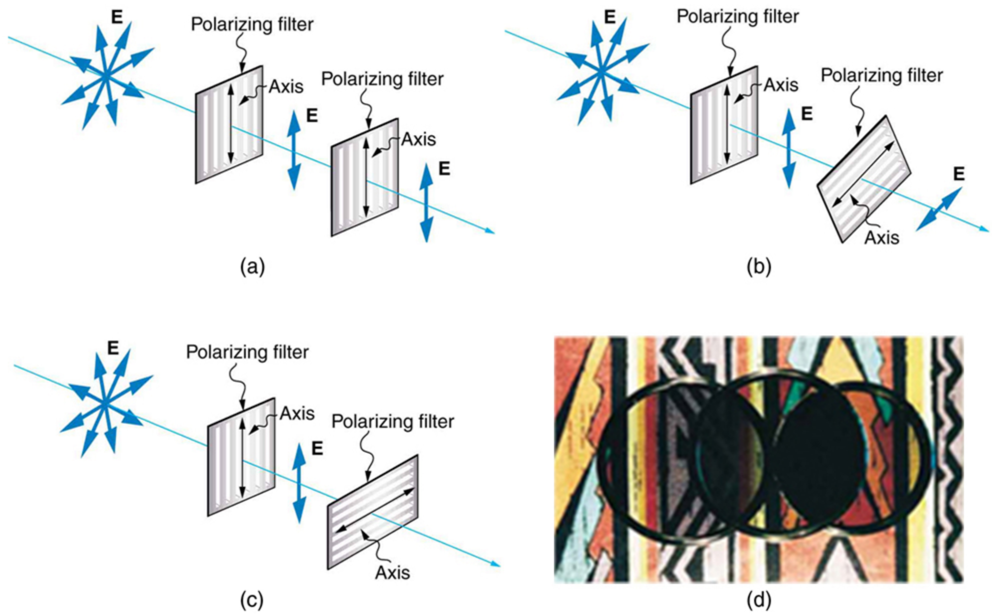
The slender arrow represents a ray of unpolarized light. The bold arrows represent the direction of polarization of the individual waves composing the ray. Since the light is unpolarized, the arrows point in all directions.



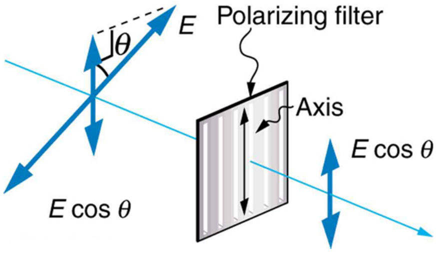
A polarizing filter has a polarization axis that acts as a slit passing through electric fields parallel to its direction. The direction of polarization of an EM

wave is defined to be the direction of its electric field.

[link] shows the effect of two polarizing filters on originally unpolarized light. The first filter polarizes the light along its axis. When the axes of the first and second filters are aligned (parallel), then all of the polarized light passed by the first filter is also passed by the second. If the second polarizing filter is rotated, only the component of the light parallel to the second filter's axis is passed. When the axes are perpendicular, no light is passed by the second.



The effect of rotating two polarizing filters, where the first polarizes the light. (a) All of the polarized light is passed by the second polarizing filter, because its axis is parallel to the first. (b) As the second is rotated, only part of the light is passed. (c) When the second is perpendicular to the first, no light is passed. (d) In this photograph, a polarizing filter is placed above two others. Its axis is perpendicular to the filter on the right (dark area) and parallel to the filter on the left (lighter area). (credit: P.P. Urone)



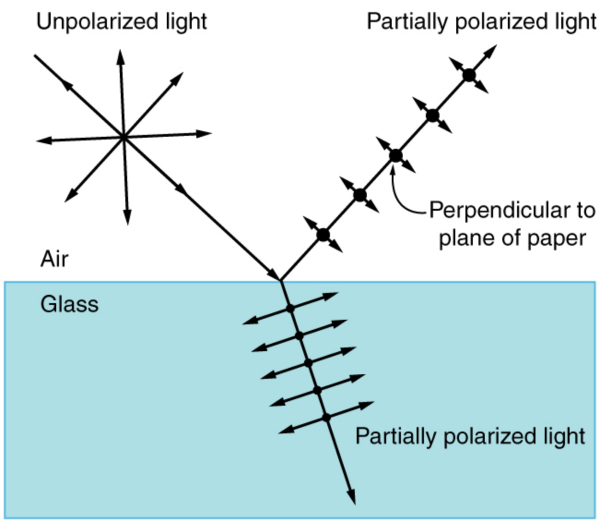
A polarizing filter transmits only the component of the wave parallel to its axis, $E\cos\theta$, reducing the intensity of any light not polarized parallel to its axis.

Polarization by Reflection

By now you can probably guess that Polaroid sunglasses cut the glare in reflected light because that light is polarized. You can check this for yourself by holding Polaroid sunglasses in front of you and rotating them while looking at light reflected from water or glass. As you rotate the sunglasses, you will notice the light gets bright and dim, but not completely black. This implies the reflected light is partially polarized and cannot be completely blocked by a polarizing filter.

[link] illustrates what happens when unpolarized light is reflected from a surface. Vertically polarized light is preferentially refracted at the surface, so that *the reflected light is left more horizontally polarized*. The reasons for this phenomenon are beyond the scope of this text, but a convenient mnemonic for remembering this is to imagine the polarization direction to be like an arrow. Vertical polarization would be like an arrow perpendicular to the surface and would be more likely to stick and not be reflected. Horizontal polarization is like an arrow bouncing on its side and would be

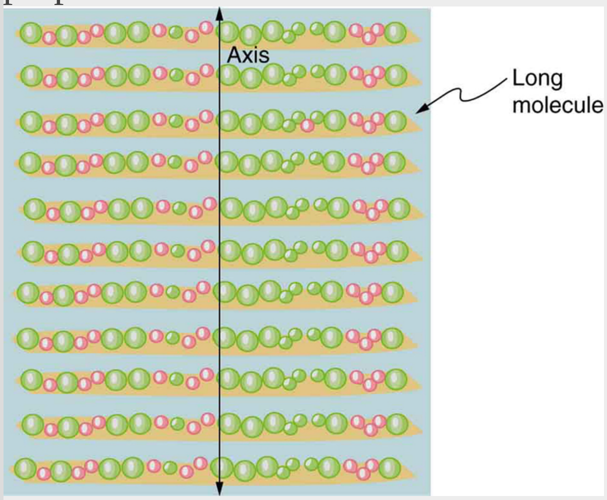
more likely to be reflected. Sunglasses with vertical axes would then block more reflected light than unpolarized light from other sources.



Polarization by reflection. Unpolarized light has equal amounts of vertical and horizontal polarization. After interaction with a surface, the vertical components are preferentially absorbed or refracted, leaving the reflected light more horizontally polarized. This is akin to arrows striking on their sides bouncing off, whereas arrows striking on their tips go into the surface.

Note:
Things Great and Small: Atomic Explanation of Polarizing Filters

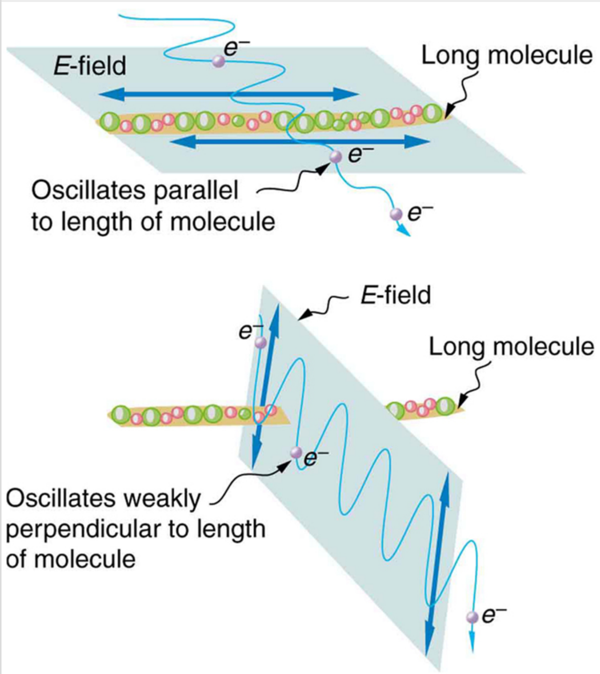
Polarizing filters have a polarization axis that acts as a slit. This slit passes electromagnetic waves (often visible light) that have an electric field parallel to the axis. This is accomplished with long molecules aligned perpendicular to the axis as shown in [link].



Long molecules are aligned perpendicular to the axis of a polarizing filter. The component of the electric field in an EM wave perpendicular to these molecules passes through the filter, while the component parallel to the molecules is absorbed.

[link] illustrates how the component of the electric field parallel to the long molecules is absorbed. An electromagnetic wave is composed of oscillating electric and magnetic fields. The electric field is strong compared with the magnetic field and is more effective in exerting force on charges in the molecules. The most affected charged particles are the electrons in the molecules, since electron masses are small. If the electron is forced to oscillate, it can absorb energy from the EM wave. This reduces the fields in the wave and, hence, reduces its intensity. In long molecules, electrons can more easily oscillate parallel to the molecule than in the perpendicular direction. The electrons are bound to the molecule and are

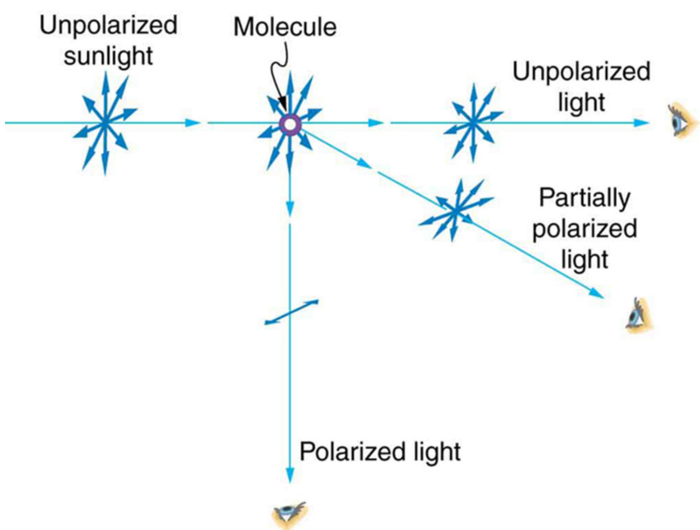
more restricted in their movement perpendicular to the molecule. Thus, the electrons can absorb EM waves that have a component of their electric field parallel to the molecule. The electrons are much less responsive to electric fields perpendicular to the molecule and will allow those fields to pass. Thus the axis of the polarizing filter is perpendicular to the length of the molecule.



Artist's conception of an electron in a long molecule oscillating parallel to the molecule. The oscillation of the electron absorbs energy and reduces the intensity of the component of the EM wave that is parallel to the molecule.

Polarization by Scattering

If you hold your Polaroid sunglasses in front of you and rotate them while looking at blue sky, you will see the sky get bright and dim. This is a clear indication that light scattered by air is partially polarized. [link] helps illustrate how this happens. Since light is a transverse EM wave, it vibrates the electrons of air molecules perpendicular to the direction it is traveling. The electrons then radiate like small antennae. Since they are oscillating perpendicular to the direction of the light ray, they produce EM radiation that is polarized perpendicular to the direction of the ray. When viewing the light along a line perpendicular to the original ray, as in [link], there can be no polarization in the scattered light parallel to the original ray, because that would require the original ray to be a longitudinal wave. Along other directions, a component of the other polarization can be projected along the line of sight, and the scattered light will only be partially polarized. Furthermore, multiple scattering can bring light to your eyes from other directions and can contain different polarizations.



Polarization by scattering. Unpolarized light scattering from air molecules shakes their electrons perpendicular to the direction of the original ray. The scattered light therefore has a polarization perpendicular to the original direction

and none parallel to the original direction.

Photographs of the sky can be darkened by polarizing filters, a trick used by many photographers to make clouds brighter by contrast. Scattering from other particles, such as smoke or dust, can also polarize light. Detecting polarization in scattered EM waves can be a useful analytical tool in determining the scattering source.

There is a range of optical effects used in sunglasses. Besides being Polaroid, other sunglasses have colored pigments embedded in them, while others use non-reflective or even reflective coatings. A recent development is photochromic lenses, which darken in the sunlight and become clear indoors. Photochromic lenses are embedded with organic microcrystalline molecules that change their properties when exposed to UV in sunlight, but become clear in artificial lighting with no UV.

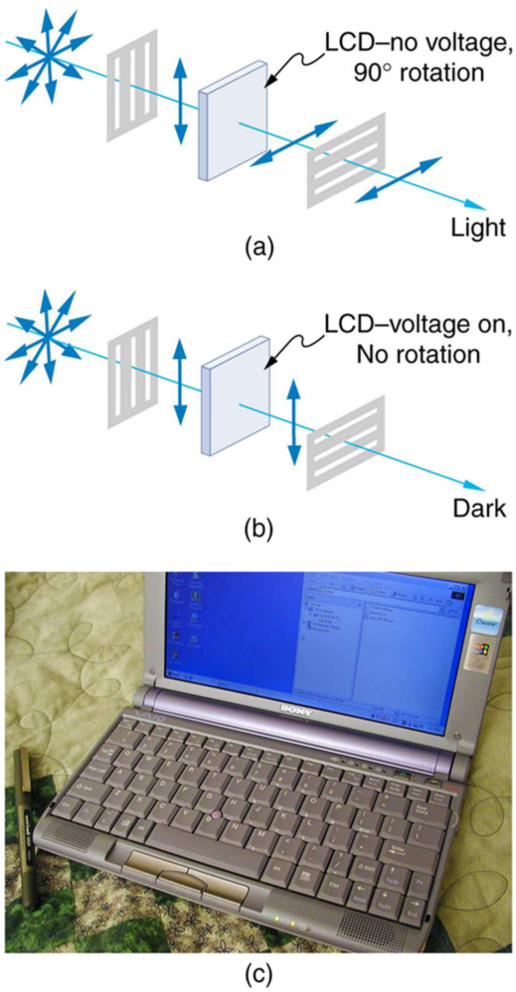
Note:
Take-Home Experiment: Polarization
Find Polaroid sunglasses and rotate one while holding the other still and look at different surfaces and objects. Explain your observations. What is the difference in angle from when you see a maximum intensity to when you see a minimum intensity? Find a reflective glass surface and do the same. At what angle does the glass need to be oriented to give minimum glare?

Liquid Crystals and Other Polarization Effects in Materials

While you are undoubtedly aware of liquid crystal displays (LCDs) found in watches, calculators, computer screens, cellphones, flat screen televisions, and other myriad places, you may not be aware that they are based on polarization. Liquid crystals are so named because their molecules

can be aligned even though they are in a liquid. Liquid crystals have the property that they can rotate the polarization of light passing through them by 90º. Furthermore, this property can be turned off by the application of a voltage, as illustrated in [link]. It is possible to manipulate this characteristic quickly and in small well-defined regions to create the contrast patterns we see in so many LCD devices.

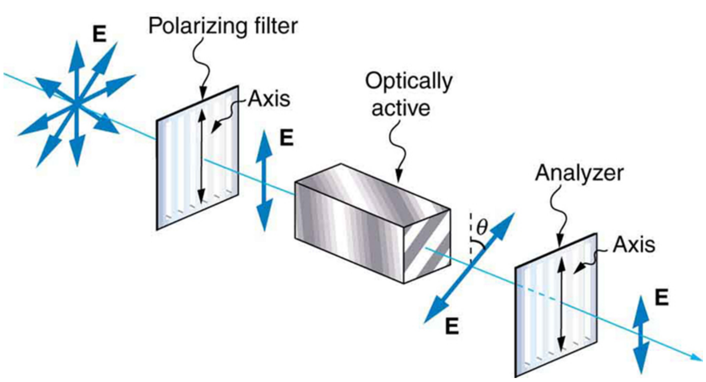
In flat screen LCD televisions, there is a large light at the back of the TV. The light travels to the front screen through millions of tiny units called pixels (picture elements). One of these is shown in [link] (a) and (b). Each unit has three cells, with red, blue, or green filters, each controlled independently. When the voltage across a liquid crystal is switched off, the liquid crystal passes the light through the particular filter. One can vary the picture contrast by varying the strength of the voltage applied to the liquid crystal.



(a) Polarized light is rotated 90º by a liquid crystal and then passed by a polarizing filter that has its axis perpendicular to the original polarization direction. (b) When a voltage is applied to the liquid crystal, the polarized light is not rotated and is blocked by the filter, making the region dark in comparison with its surroundings. (c) LCDs

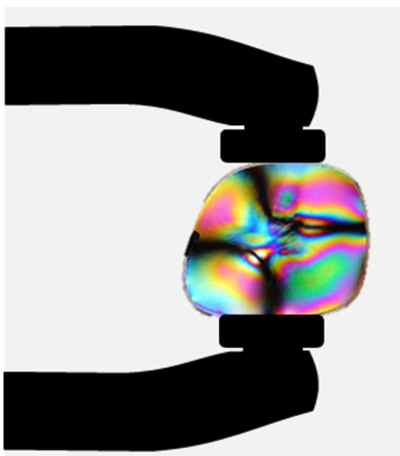
can be made color specific, small, and fast enough to use in laptop computers and TVs. (credit: Jon Sullivan)

Many crystals and solutions rotate the plane of polarization of light passing through them. Such substances are said to be **optically active**. Examples include sugar water, insulin, and collagen (see [link]). In addition to depending on the type of substance, the amount and direction of rotation depends on a number of factors. Among these is the concentration of the substance, the distance the light travels through it, and the wavelength of light. Optical activity is due to the asymmetric shape of molecules in the substance, such as being helical. Measurements of the rotation of polarized light passing through substances can thus be used to measure concentrations, a standard technique for sugars. It can also give information on the shapes of molecules, such as proteins, and factors that affect their shapes, such as temperature and pH.



Optical activity is the ability of some substances to rotate the plane of polarization of light passing through them. The rotation is detected with a polarizing filter or analyzer.

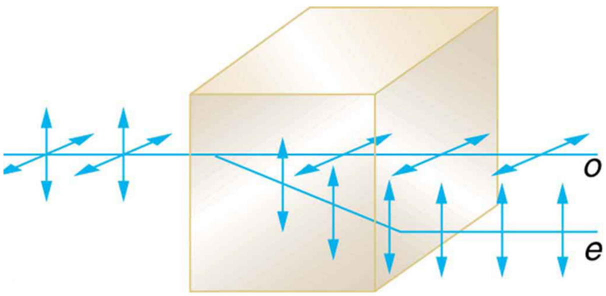
Glass and plastic become optically active when stressed; the greater the stress, the greater the effect. Optical stress analysis on complicated shapes can be performed by making plastic models of them and observing them through crossed filters, as seen in [link]. It is apparent that the effect depends on wavelength as well as stress. The wavelength dependence is sometimes also used for artistic purposes.



Optical stress analysis of a plastic lens placed between crossed polarizers. (credit: Infopro, Wikimedia Commons)

Another interesting phenomenon associated with polarized light is the ability of some crystals to split an unpolarized beam of light into two. Such crystals are said to be **birefringent** (see [link]). Each of the separated rays has a specific polarization. One behaves normally and is called the ordinary ray, whereas the other does not obey Snell's law and is called the

extraordinary ray. Birefringent crystals can be used to produce polarized beams from unpolarized light. Some birefringent materials preferentially absorb one of the polarizations. These materials are called dichroic and can produce polarization by this preferential absorption. This is fundamentally how polarizing filters and other polarizers work. The interested reader is invited to further pursue the numerous properties of materials related to polarization.



Birefringent materials, such as the common mineral calcite, split unpolarized beams of light into two. The ordinary ray behaves as expected, but the extraordinary ray does not obey Snell’s law.

Section Summary

- Polarization is the attribute that wave oscillations have a definite direction relative to the direction of propagation of the wave.
- EM waves are transverse waves that may be polarized.
- The direction of polarization is defined to be the direction parallel to the electric field of the EM wave.
- Unpolarized light is composed of many rays having random polarization directions.

- Light can be polarized by passing it through a polarizing filter or other polarizing material. The intensity of polarized light after passing through a polarizing filter depends on the direction of polarization and the axis of the filter.
- Polarization is also produced by reflection.
- Polarization can also be produced by scattering.
- There are a number of types of optically active substances that rotate the direction of polarization of light passing through them.

Conceptual Questions

Exercise:

Problem:

Under what circumstances is the phase of light changed by reflection? Is the phase related to polarization?

Exercise:

Problem: Can a sound wave in air be polarized? Explain.

Exercise:

Problem:

No light passes through two perfect polarizing filters with perpendicular axes. However, if a third polarizing filter is placed between the original two, some light can pass. Why is this? Under what circumstances does most of the light pass?

Exercise:

Problem:

Explain what happens to the energy carried by light that it is dimmed by passing it through two crossed polarizing filters.

Exercise:

Problem:

When particles scattering light are much smaller than its wavelength, the amount of scattering is proportional to $1/\lambda^4$. Does this mean there is more scattering for small λ than large λ? How does this relate to the fact that the sky is blue?

Exercise:

Problem:

Using the information given in the preceding question, explain why sunsets are red.

Glossary

axis of a polarizing filter
: the direction along which the filter passes the electric field of an EM wave

birefringent
: crystals that split an unpolarized beam of light into two beams

direction of polarization
: the direction parallel to the electric field for EM waves

horizontally polarized
: the oscillations are in a horizontal plane

optically active
: substances that rotate the plane of polarization of light passing through them

polarization
: the attribute that wave oscillations have a definite direction relative to the direction of propagation of the wave

polarized

waves having the electric and magnetic field oscillations in a definite direction

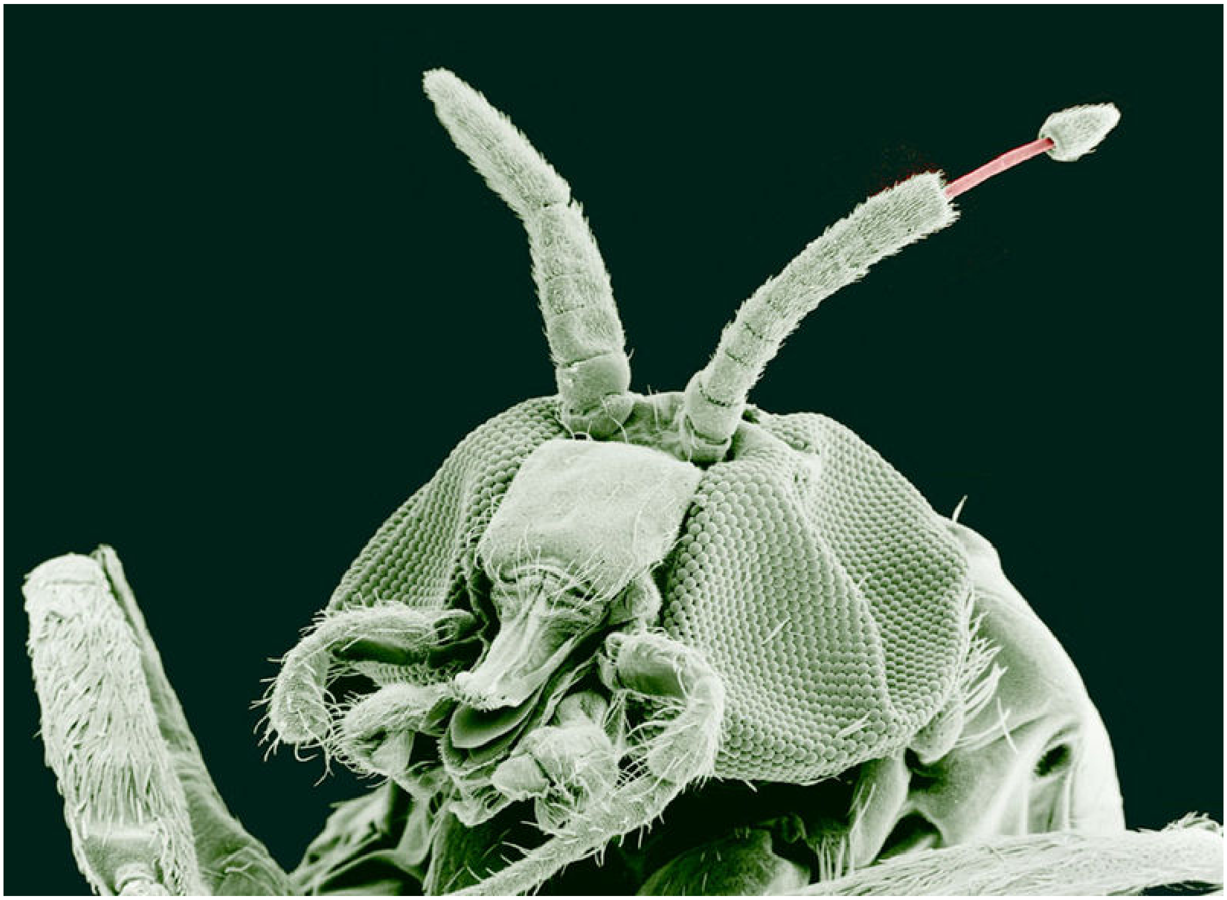
unpolarized
waves that are randomly polarized

vertically polarized
the oscillations are in a vertical plane

Introduction to Quantum Mechanics

class="introduction"

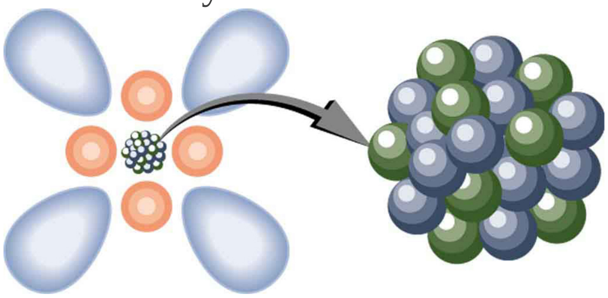
A black fly imaged by an electron microscope is as monstrous as any science-fiction creature. (credit: U.S. Department of Agriculture via Wikimedia Commons)



Quantum mechanics is the branch of physics needed to deal with submicroscopic objects. Because these objects are smaller than we can observe directly with our senses and generally must be observed with the aid of instruments, parts of quantum mechanics seem as foreign and bizarre as parts of relativity. But quantum mechanics has been shown to be valid in experiments—truth is often stranger than fiction.

Certain aspects of quantum mechanics are familiar to us. We accept as fact that matter is composed of atoms, the smallest unit of an element, and that these atoms combine to form molecules, the smallest unit of a compound. (See [link].) While we cannot see the individual water molecules in a stream, for example, we are aware that this is because molecules are so small and so numerous in that stream. When introducing atoms, we commonly say that electrons orbit atoms in discrete shells around a tiny nucleus, itself composed of smaller particles called protons and neutrons. We are also aware that electric charge comes in tiny units carried almost entirely by electrons and protons. As with water molecules in a stream, we

do not notice individual charges in the current through a lightbulb, because the charges are so small and so numerous in the macroscopic situations we sense directly.



Atoms and their substructure are familiar examples of objects that require quantum mechanics to be fully explained. Certain of their characteristics, such as the discrete electron shells, are classical physics explanations. In quantum mechanics we conceptualize discrete “electron clouds” around the nucleus.

Note:
Making Connections: Realms of Physics
Classical physics is a good approximation of modern physics under certain conditions: large, macroscopic objects moving much slower than speed of light. Quantum mechanics is valid in general, and it must be used rather than classical physics to describe small objects, such as atoms.

Atoms, molecules, and fundamental electron and proton charges are all examples of physical entities that are **quantized**—that is, they appear only in certain discrete values and do not have every conceivable value.

Quantized is the opposite of continuous. We cannot have a fraction of an atom, or part of an electron’s charge, or 14-1/3 cents, for example. Rather, everything is built of integral multiples of these substructures. Quantum physics is the branch of physics that deals with small objects and the quantization of various entities, including energy and angular momentum. Just as with classical physics, quantum physics has several subfields, such as mechanics and the study of electromagnetic forces. The **correspondence principle** states that in the classical limit (large, slow-moving objects), **quantum mechanics** becomes the same as classical physics. In this chapter, we begin the development of quantum mechanics and its description of the strange submicroscopic world. In later chapters, we will examine many areas, such as atomic and nuclear physics, in which quantum mechanics is crucial.

Glossary

quantized
: the fact that certain physical entities exist only with particular discrete values and not every conceivable value

correspondence principle
: in the classical limit (large, slow-moving objects), quantum mechanics becomes the same as classical physics

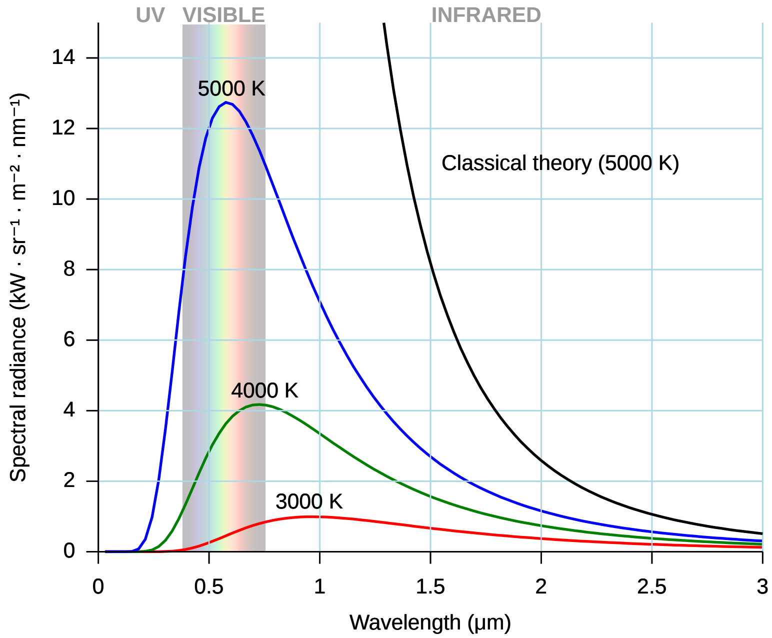
quantum mechanics
: the branch of physics that deals with small objects and with the quantization of various entities, especially energy

Blackbody Radiation

- Explain Max Planck’s contribution to the development of quantum mechanics, explaining blackbody radiation.
- Explain the significance of Planck's constant and the smallness of its value.

Ultraviolet Catastrophe

The history of quantum mechanics starts with the study of thermal physics. With the development of theory of electromagnetism and thermodynamics in the 18th century, physicists felt confident enough to attempt to develop a theory of **blackbody radiation**, which describes the EM spectrum radiated by a hot solid. It is called blackbody radiation, because an ideal radiator is jet black. It absorbs all radiation incident on it, and thus it can be at thermal equilibrium with the surrounding while emitting more radiation than a white object, which reflects radiation. So such a hypothetical object is called a **blackbody**. The first, classical attempt at theoretical description of this phenomenon was nothing short of a catastrophe. Experimentally observed spectrum showed less intensity of EM radiation at both the very short wavelength limit and the very long wavelength limit, but the classical theory prediction predicted ever-increasing intensity at shorter and shorter wavelengths. (See [link].)

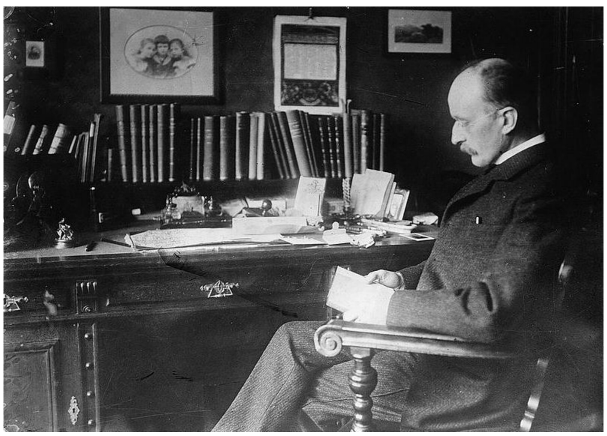


Graphs of blackbody radiation (from an ideal radiator) at three different radiator temperatures. The intensity or rate of radiation emission increases dramatically with temperature, and the peak of the spectrum shifts toward the visible and ultraviolet parts of the spectrum. The classical theory prediction is shown for the highest temperature, which shows intensity increasing without limit at short-wavelength limit. (Figure Credit: Darth Kule on Wikipedia)

This result came to be called **ultraviolet catastrophe**, due to the catastrophically large intensity of radiation predicted in the ultraviolet range of EM spectrum. This problem stumped many physicists until Max Planck came up with a novel suggestion.

Planck Law

The spectrum of blackbody radiation was extensively studied and well known. The German physicist Max Planck (1858–1947) first guessed a functional form of intensity dependence on the wavelength of EM radiation, which is now known as the **Planck Law**. The Planck Law was a good fit to the experimental results (shown in [link]), tapering off both at the long-wavelength and the short-wavelength limit, but it was only a phenomenological description that did not include a satisfying explanation of why it was so.



The German physicist Max Planck had a major influence on the early development of quantum mechanics, being the first to recognize that energy is sometimes quantized. Planck also made important contributions to special relativity and classical physics. (credit: Library of Congress, Prints and Photographs Division via Wikimedia Commons)

This led Planck to look for a reason why intensity of radiation should decrease at the short-wavelength, or high-frequency limit. Using the idea that atoms and molecules in a body act like oscillators to absorb and emit radiation, he guessed that the energy of these oscillators are quantized. That is, the energy of the oscillators could change only by a discrete amount, and this discrete amount of energy change was $\mathit{\Delta} E = hf$, where f is frequency of the oscillator and h is a fundamental constant of nature that we now call **Planck's constant**, given by

Equation:

$$h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}.$$

Without getting into the detailed mathematical derivation, we can see intuitively why this assumption would lead to the result that Planck was looking for. An object at thermal equilibrium has a certain amount of thermal energy, which is associated with kinetic energy of microscopic movements and vibrations. If this energy is randomly distributed, both very high-frequency oscillators and very low-frequency oscillators would have the same thermal energy on average. But if it was true that this thermal energy is quantized, and it has to come in integer units of hf, then at very high frequencies, there is not going to be enough thermal energy for even one unit of hf. This places an upper limit on maximum possible frequency of these oscillators, for a given amount of thermal energy per oscillator, resulting in the tapering behavior for intensity of radiation emitted by these oscillators at the higher frequency (or shorter wavelengths).

This model would also explain why the peak wavelength shifts to the shorter wavelength (higher frequency) at higher temperatures. At higher temperatures, there is more thermal energy available per oscillator, so the upper limit on maximum possible frequency will be higher, since larger unit size hf is allowed.

Note that Planck’s constant h is a very small number. So for an infrared frequency of 10^{14} Hz being emitted by a blackbody, for example, the difference between energy levels is only $\Delta E = \mathrm{hf} = (6.63 \times 10^{-34}\ \mathrm{J{\cdot}s})(10^{14}\ \mathrm{Hz}) = 6.63 \times 10^{-20}\ \mathrm{J}$, or about 0.4 eV. This 0.4 eV of energy is significant compared with typical atomic

energies, which are on the order of an electron volt, or thermal energies, which are typically fractions of an electron volt. But on a macroscopic or classical scale, energies are typically on the order of joules. Even though the macroscopic energies are quantized, the quantum steps are too small to be noticed. This is an example of the correspondence principle. For a large object, quantum mechanics produces results indistinguishable from those of classical physics.

Section Summary

- The first indication that energy is sometimes quantized came from blackbody radiation, which is the emission of EM radiation by an object that absorbs all radiation incident on it.
- The failure of classical theory to successfully explain blackbody radiation is called "ultraviolet catastrophe," after the large intensity of short-wavelength radiation incorrectly predicted.
- Planck derived the Planck Law by assuming that energy of oscillators is quantized and can change only by amount $\Delta E = hf$, where h, Planck's constant is,
  Equation:

$$h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}.$$

Conceptual Questions

Exercise:

Problem:

Give an example of a physical entity that is quantized. State specifically what the entity is and what the limits are on its values.

Exercise:

Problem:

Give an example of a physical entity that is not quantized, in that it is continuous and may have a continuous range of values.

Exercise:

Problem:

What aspect of the blackbody spectrum forced Planck to propose quantization of energy levels in the thermal oscillators?

Exercise:

Problem: Why don’t we notice quantization in everyday events?

Problems & Exercises

Exercise:

Problem:

A LiBr molecule oscillates with a frequency of $1.7 \times 10^{13}\ \text{Hz}$. (a) What is the difference in energy in eV between allowed oscillator states? (b) What is the approximate value of n for a state having an energy of 1.0 eV?

Solution:

(a) 0.070 eV

(b) 14

Exercise:

Problem:

The difference in energy between allowed oscillator states in HBr molecules is 0.330 eV. What is the oscillation frequency of this molecule?

Exercise:

Problem:

A physicist is watching a 15-kg orangutan at a zoo swing lazily in a tire at the end of a rope. He (the physicist) notices that each oscillation takes 3.00 s and hypothesizes that the energy is quantized. (a) What is the difference in energy in joules between allowed oscillator states? (b) What is the value of n for a state where the energy is 5.00 J? (c) Can the quantization be observed?

Glossary

blackbody
: an ideal radiator, which emits thermal radiation ideally and absorbs all radiation incident on it at all wavelengths

blackbody radiation
: a thermal radiation, usually modeled for a blackbody

ultraviolet catastrophe
: failure of classical theory to correctly describe blackbody radiation at short wavelengths

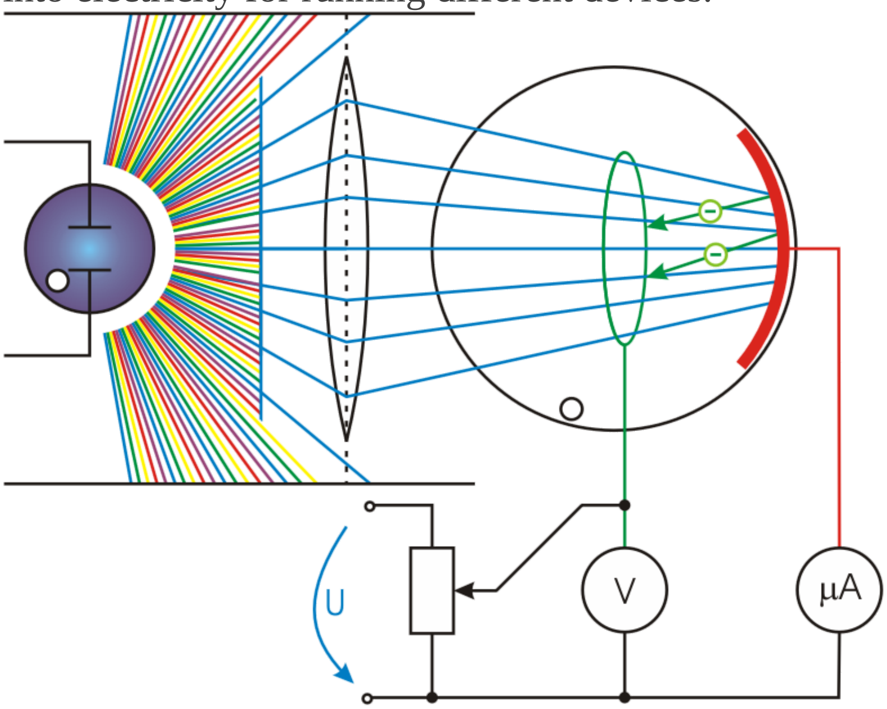
Planck’s constant
: $h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$

Planck law
: first successful theoretical description of thermal radiation which used quantization of energy hypothesis

The Photoelectric Effect

- Explain features of photoelectric effect.
- Explain why photon hypothesis is necessary to explain the observed features in photoelectric effect experiments.

When light strikes a conductor surface ("photocathode"), it can eject electrons from them. This is called the **photoelectric effect**, meaning that light (*photo*) produces electricity. One common use of the photoelectric effect is in light meters, such as those that adjust the automatic iris on various types of cameras. In a similar way, another use is in solar cells, as you probably have in your calculator or have seen on a roof top or a roadside sign. These make use of the photoelectric effect to convert light into electricity for running different devices.



The photoelectric effect can be observed by allowing light to fall on the metal plate in an evacuated tube. Electrons ejected by the light are collected on the collector wire and measured as a current. A retarding voltage between the collector wire and plate can then be adjusted so as to determine the energy of the ejected electrons.

(Figure credit: "Experiment for the Photoelectric Effect" by Stefan-Xp is licensed under CC-BY-SA.)

A photo of a photoelectric effect experiment vacuum tube is shown above. (Credit: P.P. Urone)

This effect has been known for more than a century and can be studied using a device schematically shown in [link]. This figure shows an evacuated tube with a metal plate and a collector wire that are connected by a variable voltage source, with the collector more negative than the plate. When light strikes the plate in the evacuated tube, it may eject electrons ("photoelectrons"). If the photoelectrons have kinetic energy greater than the potential energy needed to reach the negative-voltage collector wire, some electrons will be collected on the wire. Using this, the maximum electron kinetic energy can be measured by adjusting the retarding voltage between the wire and the plate. For example, if -3.00 V barely stops the electrons, then the electrons needed to overcome potential energy difference of $\Delta\text{PE} = (-e)(-3.00\text{ V}) = 3.00\text{ eV}$. This means the kinetic energy of most energetic photoelectrons was 3.00 eV. The number of photoelectrons

can also be determined by measuring the current between the wire and plate ("photocurrent"). Often, there is a direct linear relationship between the intensity of light and the number of photoelectrons and the amount of photocurrent. With a current amplifier, this can be used as a light meter.

As people studied the photoelectric effect, following properties became evident. For simplicity, these are properties shown when monochromatic (single wavelength, or frequency) EM radiation is incident on a photocathode.

1. If we vary the frequency of the EM radiation falling on a material, we find the following: For a given material, there is a threshold frequency f_0 for the EM radiation below which no electrons are ejected, no matter how intense the EM radiation.
2. Once EM radiation falls on a material, electrons are ejected without a measurable delay.
3. The number of electrons ejected per unit time is proportional to the intensity of the EM radiation, provided that the frequency of the EM radiation is high enough to eject photoelectrons.
4. If we measure the kinetic energy of ejected electrons as a function of experimental parameters, we find the following: the maximum kinetic energy of ejected electrons is dependent on the frequency of the EM radiation but not on the intensity of the EM radiation.

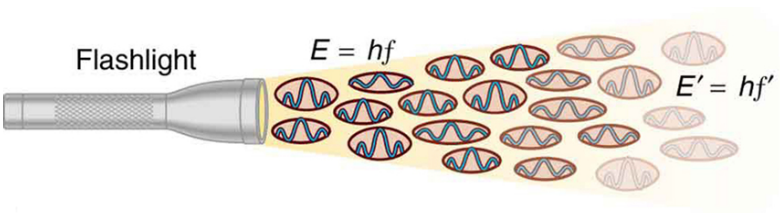
Some of these features are easy to understand from the classical understanding of EM radiation as electromagnetic wave. Larger intensity of EM radiation means larger amplitude of electromagnetic wave, which would cause more electrons to be ejected. But some of the features, especially the existence of threshold frequency, could not be explained using the classical theory of electromagnetic wave. Why is it not possible to make up for reduced frequency by increasing the intensity of EM radiation? It took the genius of Albert Einstein to realize that he can use the quantization of energy idea introduce by Max Planck to explain this. Einstein realized that these characteristics of the photoelectric effect could be fully explained if *EM radiation is itself quantized*: the apparently continuous stream of energy in an EM wave is composed of energy quanta called photons. In his explanation of the photoelectric effect, Einstein

defined a quantized unit or quantum of EM energy, which we now call a **photon**, with an energy proportional to the frequency of EM radiation. In equation form, the **photon energy** is

Equation:

$$E = \text{hf},$$

where E is the energy of a photon of frequency f and h is Planck’s constant.



An EM wave of frequency f is composed of photons, or individual quanta of EM radiation. The energy of each photon is $E = \text{hf}$, where h is Planck’s constant and f is the frequency of the EM radiation. Higher intensity means more photons per unit area. The flashlight emits large numbers of photons of many different frequencies, hence others have energy $E\prime = \text{hf}\prime$, and so on.

This is a revolutionary extension of Planck's original quantization of energy. It is an extension, because Planck never claimed EM radiation itself was quantized, only the energy levels in thermal oscillators. It is revolutionary because physicists had thought they understood EM radiation completely as an electromagnetic wave, fully described by Maxwell's equations. Physicists *had* debated intensely a century earlier on whether light was a particle (Sir Isaac Newton advanced a corpuscular theory of light) or a wave; experimental evidences convinced nearly everyone that light was a wave; and this achievement in classical physics was capped off by James

Clerk Maxwell's unified theory of electromagnetism, predicting existence of an electromagnetic wave, which turned out to be light. Should we now go back to a particle theory of light?

This is not an easy question to answer. All the experimental evidences that suggested that light is an electromagnetic wave are still true. On the other hand, the photoelectric effect cannot be explained by clinging to the classical wave theory of light. This is the beginning of quantum mechanics, and a concept we will call "wave-particle duality." We should not be too quick to answer this question. Quantum mechanics is not an easy topic to understand, not just mathematically but also conceptually. All the brilliant physicists we will talk about were wrong at one point or another about some aspect of quantum mechanics. Even Einstein himself was wrong when he said "God does not play dice," in frustration over probabilistic, non-deterministic aspects of quantum mechanics. Our goal is for you to become aware of quantum mechanical concepts and effects; it is perfectly fine not to understand everything in quantum mechanics right away. Richard Feynman (another hero in modern physics) famously said once, "If you think you understand quantum mechanics, you don't understand quantum mechanics."

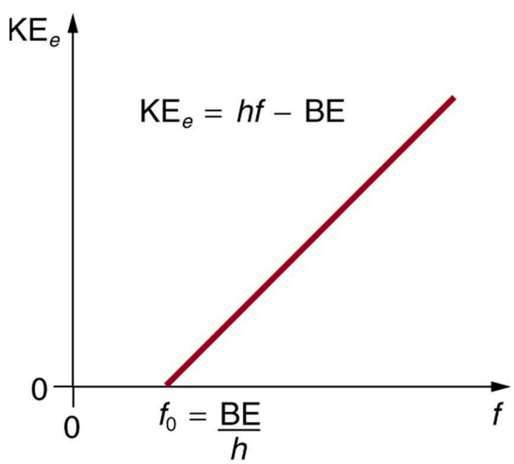
In the meantime, this is how Einstein’s idea that EM radiation is quantized explains the features of photoelectric effect (we will re-visit wave-particle duality, or particle-wave duality, later).

1. Threshold frequency is explained by individual photons interacting with individual electrons. Thus if the photon energy is too small to break a single electron away, no electrons will be ejected. (Note that if EM radiation interacts as a wave, sufficient energy could be provided by increasing the intensity of EM radiation.)
2. There are no measurable delays, because as soon as an individual photon of a sufficiently high frequency (and thus sufficiently high energy) is absorbed by a single electron, the electron is ejected. Even at a very low intensity, a single photon of enough frequency would have enough energy to eject a single electron. It is difficult to see how this could be the case, if the EM radiation interacts as a wave. For a classical wave, at some very low intensity, some measurable amount of time would be required to transfer enough energy.

3. The number of photoelectrons increasing with intensity is easy to see both under the particle and wave model of light, as mentioned before.
4. The kinetic energy (KE_e) of an ejected electron equals the photon energy minus the binding energy (BE) of the electron in the specific material, which is called the **work function**. Taking all of the photon energy (hf) to the electron, some of it is used to break the electron away from the material. The remainder goes into the ejected electron's kinetic energy. In equation form, this is expressed as:
   Equation:

   $$\text{KE}_e = \text{hf} - \text{BE}.$$

   This equation, which Einstein proposed in 1905, explains the properties of the photoelectric effect quantitatively. The binding energy can be determined in an experiment that determines the threshold frequency f_0 for the material. The binding energy is $\text{BE} = hf_0$. [link] shows a graph of maximum KE_e versus the frequency of incident EM radiation falling on a particular material, which would be used to estimate the threshold frequency.



Photoelectric effect. A graph of the kinetic energy of an ejected electron, KE_e, versus the frequency of EM radiation impinging on a certain material. There is

a threshold frequency below which no electrons are ejected, because the individual photon interacting with an individual electron has insufficient energy to break it away. Above the threshold energy, KE_e increases linearly with f, consistent with $\mathrm{KE}_e = \mathrm{hf} - \mathrm{BE}$. The slope of this line is h — the data can be used to determine Planck's constant experimentally. Einstein gave the first successful explanation of such data by proposing the idea of photons—quanta of EM radiation.

Example:
Calculating Photon Energy and the Photoelectric Effect: A Violet Light

(a) What is the energy in joules and electron volts of a photon of 420-nm violet light? (b) What is the maximum kinetic energy of electrons ejected from calcium by 420-nm violet light, given that the binding energy (or work function) of electrons for calcium metal is 2.71 eV?

Strategy

To solve part (a), note that the energy of a photon is given by $E = \mathrm{hf}$. For part (b), once the energy of the photon is calculated, it is a straightforward

application of $\mathrm{KE}_e = \mathrm{hf}–\mathrm{BE}$ to find the ejected electron’s maximum kinetic energy, since BE is given.

Solution for (a)

Photon energy is given by

Equation:

$$E = \mathrm{hf}$$

Since we are given the wavelength rather than the frequency, we solve the familiar relationship $c = f\lambda$ for the frequency, yielding

Equation:

$$f = \frac{c}{\lambda}.$$

Combining these two equations gives the useful relationship

Equation:

$$E = \frac{\mathrm{hc}}{\lambda}.$$

Now substituting known values yields

Equation:

$$E = \frac{\left(6.63 \times 10^{-34}\ \mathrm{J \cdot s}\right)\left(3.00 \times 10^{8}\ \mathrm{m/s}\right)}{420 \times 10^{-9}\ \mathrm{m}} = 4.74 \times 10^{-19}\ \mathrm{J}.$$

Converting to eV, the energy of the photon is

Equation:

$$E = \left(4.74 \times 10^{-19}\ \mathrm{J}\right)\frac{1\ \mathrm{eV}}{1.6 \times 10^{-19}\ \mathrm{J}} = 2.96\ \mathrm{eV}.$$

Solution for (b)

Finding the kinetic energy of the ejected electron is now a simple application of the equation $\mathrm{KE}_e = \mathrm{hf}–\mathrm{BE}$. Substituting the photon energy and binding energy yields

Equation:

$$\mathrm{KE}_e = \mathrm{hf} - \mathrm{BE} = 2.96\ \mathrm{eV} - 2.71\ \mathrm{eV} = 0.246\ \mathrm{eV}.$$

Discussion
The energy of this 420-nm photon of violet light is a tiny fraction of a joule, and so it is no wonder that a single photon would be difficult for us to sense directly—humans are more attuned to energies on the order of joules. But looking at the energy in electron volts, we can see that this photon has enough energy to affect atoms and molecules. A DNA molecule can be broken with about 1 eV of energy, for example, and typical atomic and molecular energies are on the order of eV, so that the UV photon in this example could have biological effects. The ejected electron (called a *photoelectron*) has a rather low energy, and it would not travel far, except in a vacuum. The electron would be stopped by a retarding potential of but 0.26 eV. This simply means that the 420-nm photons with their 2.96-eV energy are not much above the frequency threshold. You can show for yourself that the threshold wavelength is 459 nm (blue light). This means that if calcium metal is used in a light meter, the meter will be insensitive to wavelengths longer than those of blue light.

Section Summary

- The photoelectric effect is the process in which EM radiation ejects electrons from a material.
- Einstein proposed photons to be quanta of EM radiation having energy $E = \mathrm{hf}$, where f is the frequency of the radiation.
- All EM radiation is composed of photons. As Einstein explained, all characteristics of the photoelectric effect are due to the interaction of individual photons with individual electrons.
- The maximum kinetic energy KE_e of ejected electrons (photoelectrons) is given by $\mathrm{KE}_e = \mathrm{hf} - \mathrm{BE}$, where hf is the photon energy and BE is the binding energy (or work function) of the electron to the particular material.

Conceptual Questions

Exercise:

Problem:

Is visible light the only type of EM radiation that can cause the photoelectric effect?

Exercise:

Problem:

Which aspects of the photoelectric effect cannot be explained without photons? Which can be explained without photons? Are the latter inconsistent with the existence of photons?

Exercise:

Problem:

Is the photoelectric effect a direct consequence of the wave character of EM radiation or of the particle character of EM radiation? Explain briefly.

Exercise:

Problem:

Insulators (nonmetals) have a higher BE than metals, and it is more difficult for photons to eject electrons from insulators. Discuss how this relates to the free charges in metals that make them good conductors.

Exercise:

Problem:

If you pick up and shake a piece of metal that has electrons in it free to move as a current, no electrons fall out. Yet if you heat the metal, electrons can be boiled off. Explain both of these facts as they relate to the amount and distribution of energy involved with shaking the object as compared with heating it.

Problems & Exercises

Exercise:

Problem:

What is the longest-wavelength EM radiation that can eject a photoelectron from silver, given that the binding energy is 4.73 eV? Is this in the visible range?

Solution:

263 nm

Exercise:

Problem:

Find the longest-wavelength photon that can eject an electron from potassium, given that the binding energy is 2.24 eV. Is this visible EM radiation?

Exercise:

Problem:

What is the binding energy in eV of electrons in magnesium, if the longest-wavelength photon that can eject electrons is 337 nm?

Solution:

3.69 eV

Exercise:

Problem:

Calculate the binding energy in eV of electrons in aluminum, if the longest-wavelength photon that can eject them is 304 nm.

Exercise:

Problem:

What is the maximum kinetic energy in eV of electrons ejected from sodium metal by 450-nm EM radiation, given that the binding energy is 2.28 eV?

Solution:

0.483 eV

Exercise:

Problem:

UV radiation having a wavelength of 120 nm falls on gold metal, to which electrons are bound by 4.82 eV. What is the maximum kinetic energy of the ejected photoelectrons?

Exercise:

Problem:

Violet light of wavelength 400 nm ejects electrons with a maximum kinetic energy of 0.860 eV from sodium metal. What is the binding energy of electrons to sodium metal?

Solution:

2.25 eV

Exercise:

Problem:

UV radiation having a 300-nm wavelength falls on uranium metal, ejecting 0.500-eV electrons. What is the binding energy of electrons to uranium metal?

Exercise:

Problem:

What is the wavelength of EM radiation that ejects 2.00-eV electrons from calcium metal, given that the binding energy is 2.71 eV? What type of EM radiation is this?

Solution:

(a) 264 nm

(b) Ultraviolet

Exercise:

Problem:

Find the wavelength of photons that eject 0.100-eV electrons from potassium, given that the binding energy is 2.24 eV. Are these photons visible?

Exercise:

Problem:

A laser with a power output of 2.00 mW at a wavelength of 400 nm is projected onto calcium metal. (a) How many electrons per second are ejected? (b) What power is carried away by the electrons, given that the binding energy is 2.71 eV?

Solution:

(a) $4.02 \times 10^{15}\ /\text{s}$

(b) 0.256 mW

Exercise:

Problem:

(a) Calculate the number of photoelectrons per second ejected from a 1.00-mm ^{2} area of sodium metal by 500-nm EM radiation having an intensity of $1.30\ \mathrm{kW/m}^2$ (the intensity of sunlight above the Earth's atmosphere). (b) Given that the binding energy is 2.28 eV, what power is carried away by the electrons? (c) The electrons carry away less power than brought in by the photons. Where does the other power go? How can it be recovered?

Exercise:

Problem: Unreasonable Results

Red light having a wavelength of 700 nm is projected onto magnesium metal to which electrons are bound by 3.68 eV. (a) Use $\mathrm{KE}_e = \mathrm{hf} - \mathrm{BE}$ to calculate the kinetic energy of the ejected electrons. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) –1.90 eV

(b) Negative kinetic energy

(c) That the electrons would be knocked free.

Exercise:

Problem: Unreasonable Results

(a) What is the binding energy of electrons to a material from which 4.00-eV electrons are ejected by 400-nm EM radiation? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Glossary

photoelectric effect
: the phenomenon whereby some materials eject electrons when light is shined on them

photon
: a quantum, or particle, of electromagnetic radiation

photon energy
: the amount of energy a photon has; $E = \mathrm{hf}$

work function
: the amount of energy necessary to eject an electron from a material; the binding energy in photoelectric effect

The Wave Nature of Matter

- Describe de Broglie wavelength of particles.
- Explain wave-particle duality of nature.

De Broglie Wavelength

In the photoelectric effect, we have seen that light, classically understood as waves, behaves like particles sometimes. In 1923 a French physics graduate student named Prince Louis-Victor de Broglie (1892–1987) made a radical proposal based on the hope that nature is symmetric, suggesting that not only EM radiation has both particle and wave properties but that this dual-nature might be universal to all particles. De Broglie's suggestion, made as part of his doctoral thesis, was so radical that it was greeted with some skepticism. A copy of his thesis was sent to Einstein, who said it was not only probably correct, but that it might be of fundamental importance. With the support of Einstein and a few other prominent physicists, de Broglie was awarded his doctorate.

De Broglie took both relativity and quantum mechanics into account to develop the proposal that *all particles have a wavelength*, given by
Equation:

$$\lambda = \frac{h}{p} \text{ (matter and photons)},$$

where h is Planck's constant and p is momentum. This is defined to be the **de Broglie wavelength**. This is consistent with the expression for energy of a photon, $E = hf$. In relativity, energy of a massless particle like a photon is related to momentum by $E = pc$, where c is speed of light (contrast this with nonrelativistic kinetic energy of particles of mass m, $\text{KE} = p^2/2m$; check the units for yourself). From the wave relationship between wave speed, wavelength, and frequency, we have, for photons, $c = \lambda f$. Plugging in these E and f into the photon energy from the photoelectric effect, we get, $pc = hc/\lambda$, and cancelling out c from both sides gives the same expression as de Broglie wavelength.

The hallmark of a wave is interference. If matter is a wave, then it must exhibit constructive and destructive interference. Why isn't this ordinarily observed? The answer is that in order to see significant interference effects, a wave must interact with an object about the same size as its wavelength. Since h is very small, λ is also small, especially for macroscopic objects. A 3-kg bowling ball moving at 10 m/s, for example, has

Equation:

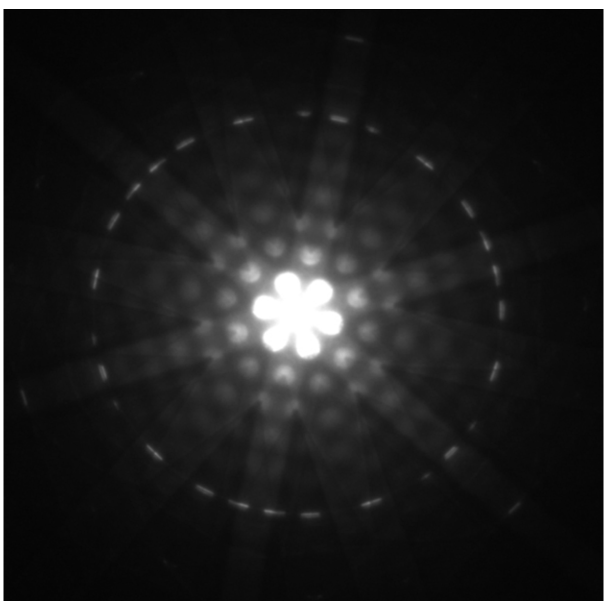
$$\lambda = h/p = (6.63 \times 10^{-34}\ \text{J·s})/[(3\ \text{kg})(10\ \text{m/s}\ )] = 2 \times 10^{-35}\ \text{m}.$$

This means that to see its wave characteristics, the bowling ball would have to interact with something about 10^{-35} m in size—far smaller than anything known. When waves interact with objects much larger than their wavelength, they show negligible interference effects and move in straight lines (such as light rays in geometric optics). To obtain longer wavelengths and make wave nature of matter readily apparent, we need smaller momentum, p, and this is easier to achieve in electrons, the lightest common matter particle we know.

American physicists Clinton J. Davisson and Lester H. Germer in 1925 and, independently, British physicist G. P. Thomson (son of J. J. Thomson, discoverer of the electron) in 1926 scattered electrons from crystals and found diffraction patterns. These patterns are exactly consistent with interference of electrons having the de Broglie wavelength and are somewhat analogous to light interacting with a diffraction grating. (See [link].)

Note:
Connections: Waves
All microscopic particles, whether massless, like photons, or having mass, like electrons, have wave properties. The relationship between momentum and wavelength is fundamental for all particles.

De Broglie’s proposal of a wave nature for all particles initiated a remarkably productive era in which the foundations for quantum mechanics were laid. In 1926, the Austrian physicist Erwin Schrödinger (1887–1961) published four papers in which the wave nature of particles was treated explicitly with wave equations. At the same time, many others began important work. Among them was German physicist Werner Heisenberg (1901–1976) who, among many other contributions to quantum mechanics, formulated a mathematical treatment of the wave nature of matter that used matrices rather than wave equations. De Broglie’s work—recognition of the universal nature of wave-particle duality, as expressed in de Broglie wavelength—was a watershed for the development of quantum mechanics. De Broglie was awarded the Nobel Prize in 1929 for his vision, as were Davisson and G. P. Thomson in 1937 for their experimental verification of de Broglie’s hypothesis.



This diffraction pattern was obtained for electrons diffracted by crystalline silicon. Bright regions are those of constructive interference, while dark regions are those of destructive

interference. (credit: Ndthe, Wikimedia Commons)

Example:
Electron Wavelength versus Velocity and Energy
For an electron having a de Broglie wavelength of 0.167 nm (appropriate for interacting with crystal lattice structures that are about this size): (a) Calculate the electron’s velocity, assuming it is nonrelativistic. (b) Calculate the electron’s kinetic energy in eV.

Strategy
For part (a), since the de Broglie wavelength is given, the electron’s velocity can be obtained from $\lambda = h/p$ by using the nonrelativistic formula for momentum, $p = \text{mv}$. For part (b), once v is obtained (and it has been verified that v is nonrelativistic), the classical kinetic energy is simply $(1/2)mv^2$.

Solution for (a)
Substituting the nonrelativistic formula for momentum ($p = \text{mv}$) into the de Broglie wavelength gives

Equation:

$$\lambda = \frac{h}{p} = \frac{h}{\text{mv}}.$$

Solving for v gives

Equation:

$$v = \frac{h}{m\lambda}.$$

Substituting known values yields

Equation:

$$v = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{(9.11 \times 10^{-31}\ \text{kg})(0.167 \times 10^{-9}\ \text{m})} = 4.36 \times 10^{6}\ \text{m/s}.$$

Solution for (b)

While fast compared with a car, this electron's speed is not highly relativistic, and so we can comfortably use the classical formula to find the electron's kinetic energy and convert it to eV as requested.

Equation:

$$\begin{aligned}
\text{KE} &= \frac{1}{2}mv^2 \\
&= \frac{1}{2}(9.11 \times 10^{-31}\text{ kg})(4.36 \times 10^{6}\text{ m/s})^2 \\
&= (8.66 \times 10^{-18}\text{ J})\left(\frac{1\text{ eV}}{1.602 \times 10^{-19}\text{ J}}\right) \\
&= 54.05\text{ eV}
\end{aligned}$$

Discussion

This low energy means that these 0.167-nm electrons could be obtained by accelerating them through a 54.0-V electrostatic potential, an easy task. The results also confirm the assumption that the electrons are nonrelativistic, since their velocity is just over 1% of the speed of light and the kinetic energy is about 0.01% of the rest energy of an electron (0.511 MeV). If the electrons had turned out to be relativistic, we would have had to use more involved calculations employing relativistic formulas.

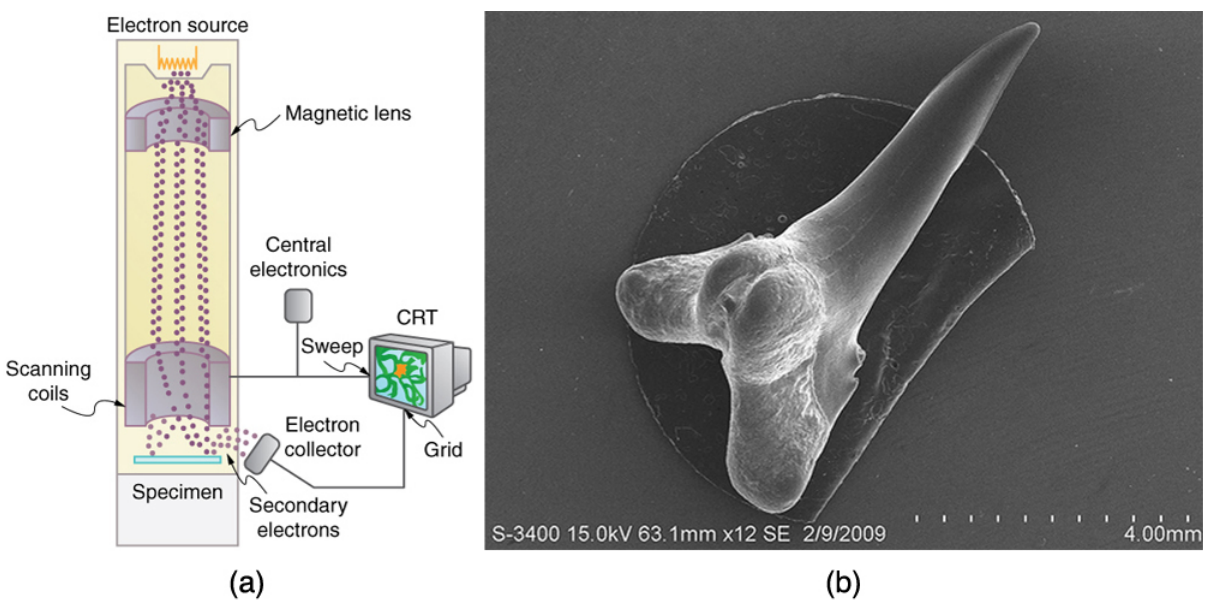
Electron Microscopes

One consequence or use of the wave nature of matter is found in the electron microscope. As we have discussed, there is a limit to the detail observed with any probe having a wavelength. Resolution, or observable detail, is limited to about one wavelength. Since a potential of only 54 V can produce electrons with sub-nanometer wavelengths, it is easy to get electrons with much smaller wavelengths than those of visible light (hundreds of nanometers). Electron microscopes can, thus, be constructed to detect much smaller details than optical microscopes. (See [link].)

There are basically two types of electron microscopes. The transmission electron microscope (TEM) accelerates electrons that are emitted from a hot filament (the cathode). The beam is broadened and then passes through the

sample. A magnetic lens focuses the beam image onto a fluorescent screen, a photographic plate, or (most probably) a CCD (light sensitive camera), from which it is transferred to a computer. The TEM is similar to the optical microscope, but it requires a thin sample examined in a vacuum. However it can resolve details as small as 0.1 nm (10^{-10} m), providing magnifications of 100 million times the size of the original object. The TEM has allowed us to see individual atoms and structure of cell nuclei.

The scanning electron microscope (SEM) provides images by using secondary electrons produced by the primary beam interacting with the surface of the sample (see [link]). The SEM also uses magnetic lenses to focus the beam onto the sample. However, it moves the beam around electrically to "scan" the sample in the *x* and *y* directions. A CCD detector is used to process the data for each electron position, producing images like the one at the beginning of this chapter. The SEM has the advantage of not requiring a thin sample and of providing a 3-D view. However, its resolution is about ten times less than a TEM.



Schematic of a scanning electron microscope (SEM) (a) used to observe small details, such as those seen in this image of a tooth of a *Himipristis*, a type of shark (b). (credit: Dallas Krentzel, Flickr)

Electrons were the first particles with mass to be directly confirmed to have the wavelength proposed by de Broglie. Subsequently, protons, helium nuclei, neutrons, and many others have been observed to exhibit interference when they interact with objects having sizes similar to their de Broglie wavelength. The de Broglie wavelength for massless particles was well established in the 1920s for photons, and it has since been observed that all massless particles have a de Broglie wavelength $\lambda = h/p.$ The wave nature of all particles is a universal characteristic of nature, and the de Broglie wavelength gives insight into two additional quantum mechanical ideas we will look at: the uncertainty principle (the idea that it is fundamentally and theoretically impossible to measure momentum and position of a particle precisely) and the Bohr's model of the hydrogen atom. The de Broglie wavelength relationship is the gateway connecting the wave nature (wavelength) of all particles with their particle nature (momentum).

Section Summary

- Particles of matter also have a wavelength, called the de Broglie wavelength, given by $\lambda = \frac{h}{p}$, where p is momentum.
- The de Broglie relationship between wavelength and momentum is universally applicable to all particles and waves.
- Matter is found to have the same *interference characteristics* as any other wave.

Conceptual Questions

Exercise:

Problem:

How does the interference of water waves differ from the interference of electrons? How are they analogous?

Exercise:

Problem: Describe one type of evidence for the wave nature of matter.

Exercise:

Problem:

Describe one type of evidence for the particle nature of EM radiation.

Problems & Exercises

Exercise:

Problem:

At what velocity will an electron have a wavelength of 1.00 m?

Solution:

7.28×10^{-4} m

Exercise:

Problem:

What is the wavelength of an electron moving at 3.00% of the speed of light?

Exercise:

Problem:

At what velocity does a proton have a 6.00-fm wavelength (about the size of a nucleus)? Assume the proton is nonrelativistic. (1 femtometer = 10^{-15} m.)

Solution:

$6.62 \times 10^{7}\ \text{m/s}$

Exercise:

Problem:

What is the velocity of a 0.400-kg billiard ball if its wavelength is 7.50 cm (large enough for it to interfere with other billiard balls)?

Exercise:

Problem:

Find the wavelength of a proton moving at 1.00% of the speed of light.

Solution:

$1.32 \times 10^{-13}\ \text{m}$

Exercise:

Problem:

Experiments are performed with ultracold neutrons having velocities as small as 1.00 m/s. (a) What is the wavelength of such a neutron? (b) What is its kinetic energy in eV?

Exercise:

Problem:

(a) Find the velocity of a neutron that has a 6.00-fm wavelength (about the size of a nucleus). Assume the neutron is nonrelativistic. (b) What is the neutron's kinetic energy in MeV?

Solution:

(a) $6.62 \times 10^{7}\ \text{m/s}$

(b) $22.9\ \text{MeV}$

Exercise:

Problem:

What is the wavelength of an electron accelerated through a 30.0-kV potential, as in a TV tube?

Exercise:

Problem:

What is the kinetic energy of an electron in a TEM having a 0.0100-nm wavelength?

Solution:
Equation:15.1 keV

Exercise:

Problem:

(a) Calculate the velocity of an electron that has a wavelength of $1.00\ \mu\text{m}$. (b) Through what voltage must the electron be accelerated to have this velocity?

Exercise:

Problem:

The velocity of a proton emerging from a Van de Graaff accelerator is 25.0% of the speed of light. (a) What is the proton's wavelength? (b) What is its kinetic energy, assuming it is nonrelativistic? (c) What was the equivalent voltage through which it was accelerated?

Solution:

(a) 5.29 fm

(b) 4.70×10^{-12} J

(c) 29.4 MV

Exercise:

Problem:

The kinetic energy of an electron accelerated in an X-ray tube is 100 keV. Assuming it is nonrelativistic, what is its wavelength?

Exercise:

Problem: **Unreasonable Results**

(a) Assuming it is nonrelativistic, calculate the velocity of an electron with a 0.100-fm wavelength (small enough to detect details of a nucleus). (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) $7.28 \times 10^{12}\ \text{m/s}$

(b) This is thousands of times the speed of light (an impossibility).

(c) The assumption that the electron is non-relativistic is unreasonable at this wavelength.

Glossary

de Broglie wavelength
: the wavelength possessed by a particle of matter, calculated by $\lambda = h/p$

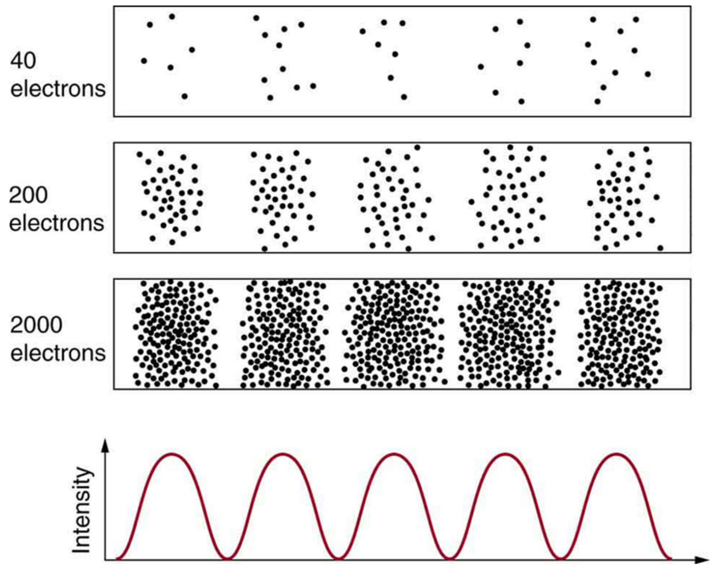
Uncertainty Principle

- Describe the position-momentum uncertainty principle.
- Explain the relationship between wave-particle dual nature and the uncertainty principle

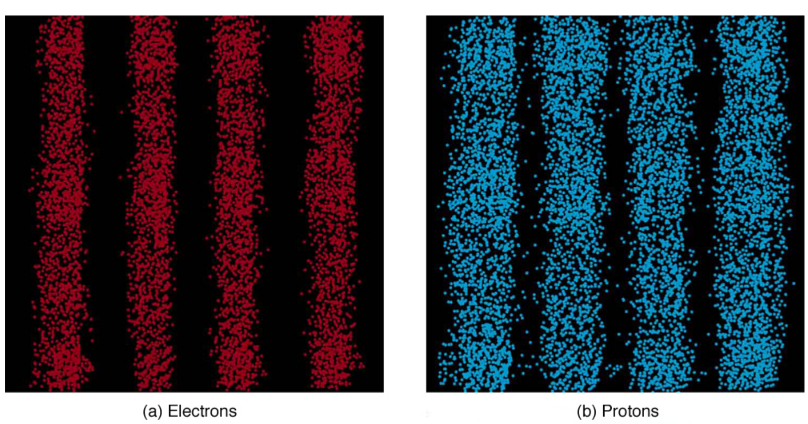
Probability Wave

The de Broglie wavelength assigns a wave nature to everything, even to the things we are used to thinking of as particles, such as an electron. So, what kind of waves are they? As far as we know, there are not smaller parts that electron can be broken into that can be oscillating. Furthermore, when we measure the position of the electron, each time, we find the electron at some definite location, as we would expect for a particle. So, what kind of a wave is electron?

[link] illustrates a result of electron interference experiment. Each dot represents the location where an electron was detected, and you see that with few electrons, dots appear spread out more or less randomly, and it is not obvious if there is any location where constructive interference or destructive interference is occurring. But as you continue to collect more data (more electrons detected), a pattern begins to emerge, where there are locations where electrons are mysteriously never detected. Those are the locations where electron waves—whatever they are—are destructively interfering.



The building up of the diffraction pattern of electrons scattered from a crystal surface. Each electron arrives at a definite location, which cannot be precisely predicted. The overall distribution shown at the bottom can be predicted as the diffraction of waves having the de Broglie wavelength of the electrons.



Double-slit interference for electrons (a) and protons (b) is identical for equal wavelengths and equal slit separations. Both patterns are probability distributions in the sense that they are built up by individual particles traversing the apparatus, the paths of which are not individually predictable.

After de Broglie proposed the wave nature of matter, many physicists, including Schrödinger and Heisenberg, explored the consequences. The Schrödinger equation is the wave equation (a type of differential equation) which describes the behavior of these matter waves. This still doesn't quite answer what kind of a wave an electron is—and we won't really answer it, in a similar way as how we can say that a photon is an electromagnetic wave—so for now, we will give this wave a name: "**probability wave**." The wave function which corresponds to the wave nature of electron describes the *probability* of detecting the particle at a particular location and time. And this wave nature becomes evident experimentally only in the statistical sense. That is, after compiling enough data, you get a distribution of the particle locations, and you can calculate a certain *probability* of finding the particle at a given location. This overall pattern is called a **probability distribution**. Those who developed quantum mechanics devised equations (of which the Schrödinger equation is one example) that predicted the probability distribution in various circumstances.

The most disquieting aspect of quantum mechanics is that the nature, at its most fundamental level, can only be described probabilistically. In [link] and [link] above, for each dot representing an electron or a photon, the experimental setup was identically. But with this identical set up, each electron (or each photon) ends up taking different paths, ending at the location represented by the dot. In classical mechanics, we would say this was because of our incomplete knowledge of the setup (we don't know the precise way an electron bounces off from the barriers on its way to the screen where it is detected). In quantum mechanics, even in theory—assuming the most complete knowledge theoretically possible—we can

only describe probability of finding the electron at one location, and the theory does not allow for prediction of individual electron paths. This aspect disturbed many physicists working on quantum mechanics, including Albert Einstein, who exclaimed, incorrectly, "God does not play dice."

Heisenberg Uncertainty Principle

If the theory does not allow prediction of individual electron path, could we simply measure the location of the electron as it travels toward the screen? When such experiment was done, the experimenters found more than what paths the electrons took to result in the interference pattern. They found that the interference pattern itself was gone, with the constructive and destructive interference fringes smeared out!

The answer here is fundamentally important—*measurement affects the system being observed*, even in theory. Even with the most sensitive of measurement devices, by the very act of measuring a physical observable (for example, particle position), you alter the state of the system. So in some cases it is impossible to measure two physical quantities to exact precision at the same time (you can measure one to exact precision and then measure the other to exact precision, but those will be two exact-precision measurements for two different particle states).

Here is an example to consider to illuminate why this is so. Suppose you want to measure the position of a moving electron. To "see" where the electron is, you need to scatter another particle (either light—photon—or another particle) off of it. But these probes will have a momentum, and by scattering from the electron, they change the momentum of the electron. So, by measuring the position of the electron, you change its momentum (and it's this change of momentum that leads to the interference pattern being smeared out). There is a limit to absolute knowledge, even in principle.

Werner Heisenberg was one of the best of those physicists who developed early quantum mechanics. Not only did his work enable a description of nature on the very small scale, it also changed our view of the availability of knowledge. Although he is universally recognized for his brilliance and the importance of his work (he received the Nobel Prize in 1932, for example), the fact that

Heisenberg remained in Germany during World War II and headed the German effort to build a nuclear bomb is sometimes held against him. (credit: Author Unknown, via Wikimedia Commons)

It was Werner Heisenberg who first stated this limit to knowledge in 1929 as a result of his work on quantum mechanics and the wave characteristics of all particles. (See [link]). Specifically, consider simultaneously measuring the position and momentum of an electron (it could be any particle). There is an **uncertainty in position** Δx that is approximately equal to the wavelength of the particle. That is,

Equation:

$$\Delta x \approx \lambda.$$

Unless the electron wave function is spread out as much as its average wavelength, the meaning of wavelength itself becomes unclear (what is the wavelength of something that exists only at one point?). To detect the position of the particle, we must interact with it, such as having it collide with a detector. In the collision, the particle will lose momentum. This change in momentum could be anywhere from close to zero to the total momentum of the particle, $p = h/\lambda$. It is not possible to tell how much momentum will be transferred to a detector, and so there is an **uncertainty in momentum** Δp, too. In fact, with these subatomic particles, the uncertainty in momentum may be as large as the momentum itself, which in equation form means (using the de Broglie relationship) that

Equation:

$$\Delta p \approx p = \frac{h}{\lambda}.$$

The uncertainty in position can be reduced by using a shorter-wavelength electron, since $\Delta x \approx \lambda$. But shortening the wavelength increases the uncertainty in momentum, since $\Delta p \approx h/\lambda$. Conversely, the uncertainty in momentum can be reduced by using a longer-wavelength electron, but this increases the uncertainty in position. Mathematically, you can express this trade-off by multiplying the uncertainties. The wavelength cancels, leaving

Equation:

$$\Delta x \Delta p \approx h.$$

So if one uncertainty is reduced, the other must increase so that their product is $\approx h$.

With the use of advanced mathematics (with uncertainties being precisely defined using statistical concepts such as standard deviation), Heisenberg showed that the best that can be done in a *simultaneous measurement of position and momentum* is

Equation:

$$\Delta x \Delta p \geq \frac{h}{4\pi}.$$

This is known as the **Heisenberg uncertainty principle**. It is impossible to measure position x and momentum p simultaneously with uncertainties Δx and Δp that multiply to be less than $h/4\pi$. Neither uncertainty can be zero. Neither uncertainty can become small without the other becoming large.

A small wavelength allows accurate position measurement, but it increases the momentum of the probe to the point that it further disturbs the momentum of a system being measured. For example, if an electron is scattered from an atom and has a wavelength small enough to detect the position of electrons in the atom, its momentum can knock the electrons

from their orbits in a manner that loses information about their original motion. It is therefore impossible to follow an electron in its orbit around an atom. If you measure the electron's position, you will find it in a definite location at that moment, but with the uncertain amount of momentum transferred, you will not know where that electron will be in the next moment.

Example:
Heisenberg Uncertainty Principle in Position and Momentum for an Atom

(a) If the position of an electron in an atom is measured to an accuracy of 0.0100 nm, what is the electron's uncertainty in velocity? (b) If the electron has this velocity, what is its kinetic energy in eV?

Strategy

The uncertainty in position is the accuracy of the measurement, or $\Delta x = 0.0100$ nm. Thus the smallest uncertainty in momentum Δp can be calculated using $\Delta x \Delta p \geq h/4\pi$. Once the uncertainty in momentum Δp is found, the uncertainty in velocity can be found from $\Delta p = m\Delta v$.

Solution for (a)

Using the equals sign in the uncertainty principle to express the minimum uncertainty, we have

Equation:

$$\Delta x \Delta p = \frac{h}{4\pi}.$$

Solving for Δp and substituting known values gives

Equation:

$$\Delta p = \frac{h}{4\pi \Delta x} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{4\pi (1.00 \times 10^{-11}\ \text{m})} = 5.28 \times 10^{-24}\ \text{kg} \cdot \text{m/s}.$$

Thus,

Equation:

$$\Delta p = 5.28 \times 10^{-24}\ \text{kg} \cdot \text{m/s} = m\Delta v.$$

Solving for Δv and substituting the mass of an electron gives

Equation:

$$\Delta v = \frac{\Delta p}{m} = \frac{5.28 \times 10^{-24}\ \text{kg} \cdot \text{m/s}}{9.11 \times 10^{-31}\ \text{kg}} = 5.79 \times 10^{6}\ \text{m/s}.$$

Solution for (b)

Although large, this velocity is not highly relativistic, and so the electron's kinetic energy is

Equation:

$$\begin{aligned} \text{KE}_e &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(9.11 \times 10^{-31}\ \text{kg})(5.79 \times 10^{6}\ \text{m/s})^2 \\ &= (1.53 \times 10^{-17}\ \text{J})\left(\frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) = 95.5\ \text{eV}. \end{aligned}$$

Discussion

Since atoms are roughly 0.1 nm in size, knowing the position of an electron to 0.0100 nm localizes it reasonably well inside the atom. This would be like being able to see details one-tenth the size of the atom. But the consequent uncertainty in velocity is large. You certainly could not follow it very well if its velocity is so uncertain. To get a further idea of how large the uncertainty in velocity is, we assumed the velocity of the electron was equal to its uncertainty and found this gave a kinetic energy of 95.5 eV. This is significantly greater than the typical energy difference between levels in atoms, so that it is impossible to get a meaningful energy for the electron if we know its position even moderately well.

Why don't we notice Heisenberg's uncertainty principle in everyday life? The answer is that Planck's constant is very small. Thus the lower limit in the uncertainty of measuring the position and momentum of large objects is negligible. We can detect sunlight reflected from Jupiter and follow the planet in its orbit around the Sun. The reflected sunlight alters the

momentum of Jupiter and creates an uncertainty in its momentum, but this is totally negligible compared with Jupiter's huge momentum. The correspondence principle tells us that the predictions of quantum mechanics become indistinguishable from classical physics for large objects, which is the case here.

Finally, note that in the discussion of particles and waves, we have stated that individual measurements produce precise or particle-like results. A definite position is determined each time we observe an electron, for example (represented as dots). But repeated measurements produce a spread in values consistent with wave characteristics, illustrating the underlying probability density. The great theoretical physicist Richard Feynman (1918–1988) commented, "What there are, are particles." When you observe enough of them, they distribute themselves as you would expect for a wave phenomenon. However, what there are as they travel we cannot tell because, when we do try to measure, we affect the traveling.

Section Summary

- Matter is found to have the same interference characteristics as any other wave.
- There is now a probability distribution for the location of a particle rather than a definite position.
- Another consequence of the wave character of all particles is the Heisenberg uncertainty principle, which limits the precision with which certain physical quantities can be known simultaneously. For position and momentum, the uncertainty principle is $\Delta x \Delta p \geq \frac{h}{4\pi}$, where Δx is the uncertainty in position and Δp is the uncertainty in momentum.
- These small limits are fundamentally important on the quantum-mechanical scale.

Conceptual Questions

Exercise:

Problem:

What is the Heisenberg uncertainty principle? Does it place limits on what can be known?

Problems & Exercises

Exercise:

Problem:

(a) If the position of an electron in a membrane is measured to an accuracy of $1.00\ \mu\text{m}$, what is the electron's minimum uncertainty in velocity? (b) If the electron has this velocity, what is its kinetic energy in eV? (c) What are the implications of this energy, comparing it to typical molecular binding energies?

Solution:

(a) 57.9 m/s

(b) 9.55×10^{-9} eV

(c) Typical molecular binding energies range from about 1eV to 10 eV, therefore the result in part (b) is approximately 9 orders of magnitude smaller than typical molecular binding energies.

Exercise:

Problem:

(a) If the position of a chlorine ion in a membrane is measured to an accuracy of $1.00\ \mu\text{m}$, what is its minimum uncertainty in velocity, given its mass is 5.86×10^{-26} kg? (b) If the ion has this velocity, what is its kinetic energy in eV, and how does this compare with typical molecular binding energies?

Exercise:

Problem:

Suppose the velocity of an electron in an atom is known to an accuracy of $2.0 \times 10^3 \text{ m/s}$ (reasonably accurate compared with orbital velocities). What is the electron's minimum uncertainty in position, and how does this compare with the approximate 0.1-nm size of the atom?

Solution:

29 nm,

290 times greater

Exercise:

Problem:

The velocity of a proton in an accelerator is known to an accuracy of 0.250% of the speed of light. (This could be small compared with its velocity.) What is the smallest possible uncertainty in its position?

Exercise:

Problem:

A relatively long-lived excited state of an atom has a lifetime of 3.00 ms. What is the minimum uncertainty in its energy?

Solution:

1.10×10^{-13} eV

Exercise:

Problem:

(a) The lifetime of a highly unstable nucleus is 10^{-20} s. What is the smallest uncertainty in its decay energy? (b) Compare this with the rest energy of an electron.

Exercise:

Problem:

The decay energy of a short-lived particle has an uncertainty of 1.0 MeV due to its short lifetime. What is the smallest lifetime it can have?

Solution:

3.3×10^{-22} s

Exercise:

Problem:

The decay energy of a short-lived nuclear excited state has an uncertainty of 2.0 eV due to its short lifetime. What is the smallest lifetime it can have?

Exercise:

Problem:

What is the approximate uncertainty in the mass of a muon, as determined from its decay lifetime?

Solution:

2.66×10^{-46} kg

Exercise:

Problem:

Derive the approximate form of Heisenberg's uncertainty principle for energy and time, $\Delta E \Delta t \approx h$, using the following arguments: Since the position of a particle is uncertain by $\Delta x \approx \lambda$, where λ is the wavelength of the photon used to examine it, there is an uncertainty in the time the photon takes to traverse Δx. Furthermore, the photon has an energy related to its wavelength, and it can transfer some or all of this energy to the object being examined. Thus the uncertainty in the energy of the object is also related to λ. Find Δt and ΔE; then multiply them to give the approximate uncertainty principle.

Glossary

Heisenberg's uncertainty principle
: a fundamental limit to the precision with which pairs of quantities (momentum and position, and energy and time) can be measured

uncertainty in momentum
: lack of precision or lack of knowledge of precise results in measurements of momentum

uncertainty in position
: lack of precision or lack of knowledge of precise results in measurements of position

probability wave
: the description of wave characteristic of matter, as revealed by probability distribution experimentally

probability distribution
: the overall spatial distribution of probabilities to find a particle at a given location

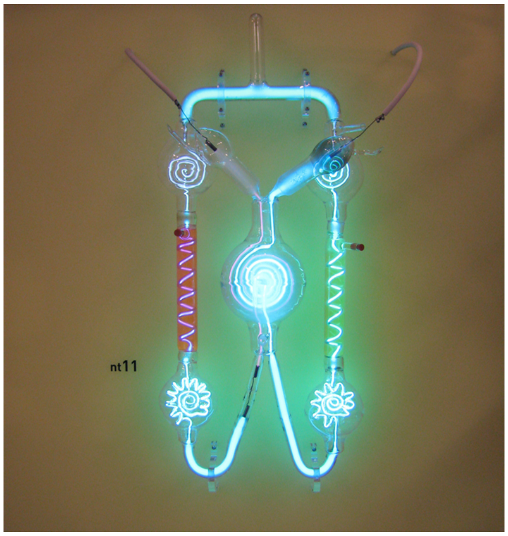
Discovery of the Atomic Nucleus

- Describe how electrons were discovered.
- Describe Rutherford’s gold foil experiment and its role in the discovery of atomic nucleus.
- Describe Rutherford’s planetary model of the atom.

Many students of physics today take for granted that atoms make up all matter and that these atoms are made up of subatomic particles, electrons, protons, and neutrons. In this section we look at the discovery of these parts, in order to look at the quantum mechanical concepts that were developed to build a self-consistent model of the atom.

The Electron

Although we have already been using the idea (and physical object) of electron to discuss electricity and some fundamental concepts of quantum mechanics, we have not yet looked at how the electrons were first discovered, well before the discovery of other subatomic particles. Electrons were discovered and their properties were measured in gas discharge tubes, similar to those shown in [link], which was also known as **cathode-ray tubes**. It consists of an evacuated glass tube containing two metal electrodes and a rarefied gas. When a high voltage is applied to the electrodes, the gas glows. They were first studied seriously by Heinrich Geissler, a German inventor and glassblower, starting in the 1860s. The English scientist William Crookes, among others, continued to study them (sometimes cathode-ray tubes are called "Crookes tubes"). The gas glows because of the interaction of the “*cathode rays*” and the rarefied gas. The cathode ray particles (we call them now "electrons") are accelerated away from the negatively charged cathode by the high voltage being applied to it. As the electrons collide with the gas atoms and molecules (most commonly argon is used, but other types of inert gas can be used), the gas atoms and molecules fluoresce and glow, making the electrons' path visible as a ray that spreads and fades as it moves away from the cathode.



A gas discharge tube glows when a high voltage is applied to it. Electrons emitted from the cathode are accelerated toward the anode; they excite atoms and molecules in the gas, which glow in response. Once called Geissler tubes and later Crookes tubes, they are now known as cathode-ray tubes (CRTs) and are found in older TVs, computer screens, and X-ray machines. When a magnetic field is applied, the beam bends in the direction expected for negative charge. (credit: Paul Downey, Flickr)

The English physicist J. J. Thomson (1856–1940) improved and expanded the scope of experiments with gas discharge tubes. (See [link] and [link].) He verified the negative charge of the cathode rays with both magnetic and electric fields. Additionally, he collected the rays in a metal cup and found an excess of negative charge. Thomson was also able to measure the ratio of the charge of the electron to its mass, q_e/m_e—an important step to finding the actual values of both q_e and m_e. [link] shows a cathode-ray tube, which produces a narrow beam of electrons that passes through charging plates connected to a high-voltage power supply. An electric field $\mathbf{E}$ is produced between the charging plates, and the cathode-ray tube is placed between the poles of a magnet so that the electric field $\mathbf{E}$ is perpendicular to the magnetic field $\mathbf{B}$ of the magnet. These fields, being perpendicular to each other, produce opposing forces on the electrons, with the two forces balancing each other out when the velocity of the charged particle is $v = E/B$. In this manner, Thomson determined the velocity of the electrons and then moved the beam up and down by adjusting the electric field.

J. J. Thomson (credit: www.firstworldwar.com, via Wikimedia Commons)

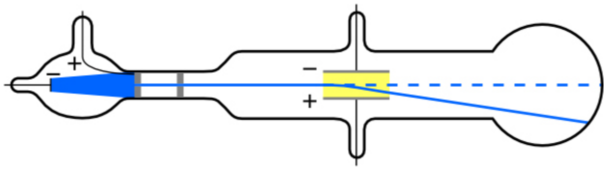
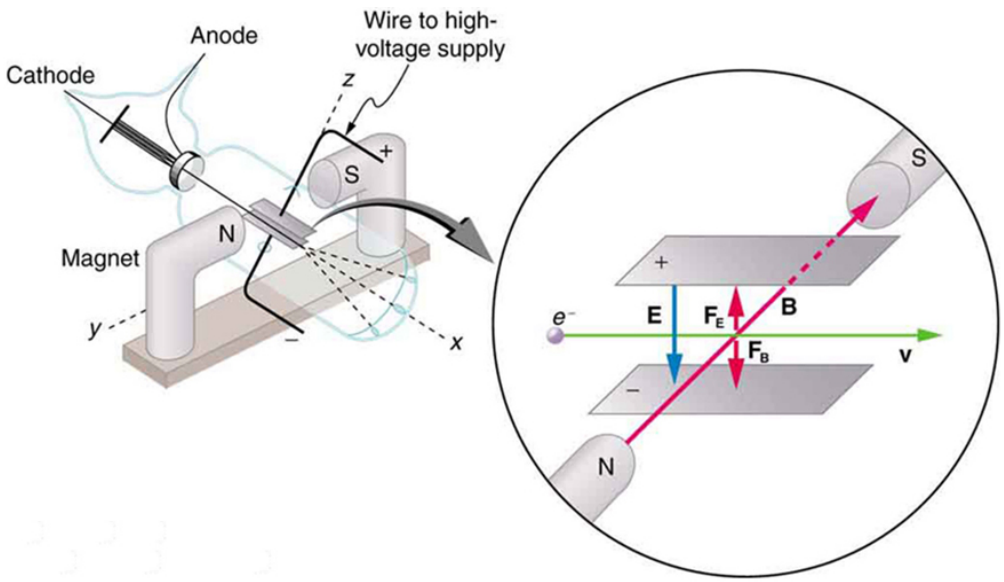


Diagram of Thomson’s CRT. (credit: Kurzon, Wikimedia Commons)



This schematic shows the electron beam in a CRT passing through crossed electric and magnetic fields and causing phosphor to glow when striking the end of the tube.

This deflection with the additional application of electric field can be used to measure and calculate the charge-to-mass ratio, q_e/m_e, by considering the electric force and the kinematics of the charged particle moving while being accelerated at a constant rate by an electric force. Another way to measure the charge-to-mass ratio is by applying a magnetic field and analyzing the circular motion that results from the centripetal magnetic force.

What is so important about q_e/m_e, the ratio of the electron's charge to its mass? The value obtained is
Equation:

$$\frac{q_e}{m_e} = -1.76 \times 10^{11}\ \mathrm{C/kg}\ (\text{electron}).$$

This is a huge number, as Thomson realized, and it implies that the electron has a very small mass. It was known from electroplating that about $10^8\ \mathrm{C/kg}$ is needed to plate a material, a factor of about 1000 less than the charge per kilogram of electrons. Thomson went on to do the same experiment for positively charged hydrogen ions (now known to be bare protons) and found a charge per kilogram about 1000 times smaller than that for the electron, implying that the proton is about 1000 times more massive than the electron. Today, we know more precisely that
Equation:

$$\frac{q_p}{m_p} = 9.58 \times 10^{7}\ \mathrm{C/kg}\ (\text{proton}),$$

where q_p is the charge of the proton and m_p is its mass. This ratio (to four significant figures) is 1836 times less charge per kilogram than for the electron. Since the charges of electrons and protons are equal in magnitude, this implies $m_p = 1836 m_e$.

Thomson performed a variety of experiments using differing gases in discharge tubes and employing other methods, such as the photoelectric effect, for freeing electrons from atoms. He always found the same

properties for the electron, proving it to be an independent particle. For his work, the important pieces of which he began to publish in 1897, Thomson was awarded the 1906 Nobel Prize in Physics. In retrospect, it is difficult to appreciate how astonishing it was to find that the atom has a substructure. Thomson himself said, “It was only when I was convinced that the experiment left no escape from it that I published my belief in the existence of bodies smaller than atoms.”

Today, numerical values of electron and proton charge (whose magnitude is also known as "elementary charge" and denoted with symbol e) and electron and proton masses are measured and known to very high precision. Their values (only up to three significant figures) are,

Equation:

$$e = 1.60 \times 10^{-19}\ \text{C (elementary charge)},$$

Equation:

$$m_e = 9.11 \times 10^{-31}\ \text{kg}\ \ \text{(electron's mass)},$$

and

Equation:

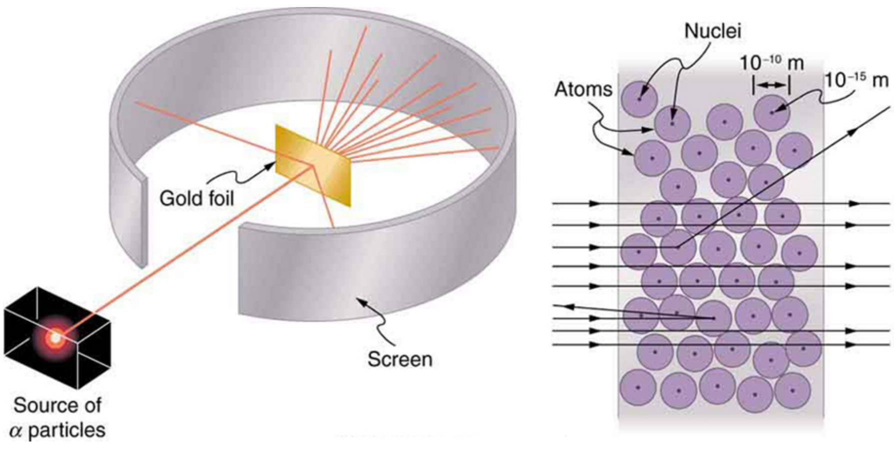
$$m_p = 1.67 \times 10^{-27}\ \text{kg}\ \ \text{(proton's mass)}.$$

Given the extremely small mass of the electron, the proton mass is nearly identical to the mass of a hydrogen atom. What Thomson (and others who measured the properties of the electron) had done was to prove the existence of one substructure of atoms, the electron, and further to show that it had only a tiny fraction of the mass of an atom. What we now know as the atomic nucleus contains most of the atom's mass, and the nature of the nucleus was completely unanticipated, as we will see.

The Nucleus

Here, we examine the first direct evidence of the size and mass of the nucleus.

Nuclear radioactivity was discovered in 1896, and it was soon the subject of intense study by a number of the best scientists in the world. Among them was New Zealander Lord Ernest Rutherford, who made numerous fundamental discoveries and earned the title of “father of nuclear physics.” Born in Nelson, Rutherford did his postgraduate studies at the Cavendish Laboratories in England before taking up a position at McGill University in Canada where he did the work that earned him a Nobel Prize in Chemistry in 1908. In the area of atomic and nuclear physics, there is much overlap between chemistry and physics, with physics providing the fundamental enabling theories. He returned to England in later years and had six future Nobel Prize winners as students. Rutherford used nuclear radiation to directly examine the size and mass of the atomic nucleus. The experiment he devised is shown in [link]. A radioactive source that emits alpha radiation was placed in a lead container with a hole in one side to produce a beam of alpha particles, which are a type of ionizing radiation ejected by the nuclei of a radioactive source. A thin gold foil was placed in the beam, and the scattering of the alpha particles was observed by the glow they caused when they struck a phosphor screen.



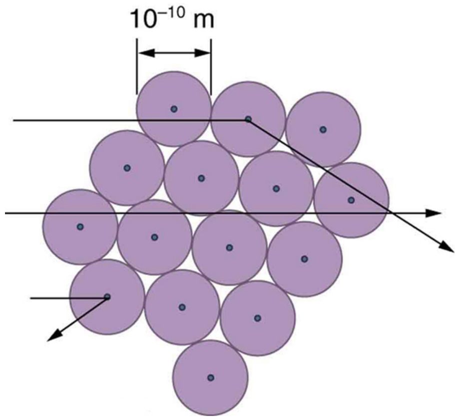
Rutherford’s experiment gave direct evidence for the size and mass of the nucleus by scattering alpha particles from a thin gold foil. Alpha

particles with energies of about 5 MeV are emitted from a radioactive source (which is a small metal container in which a specific amount of a radioactive material is sealed), are collimated into a beam, and fall upon the foil. The number of particles that penetrate the foil or scatter to various angles indicates that gold nuclei are very small and contain nearly all of the gold atom's mass. This is particularly indicated by the alpha particles that scatter to very large angles, much like a soccer ball bouncing off a goalie's head.

Alpha particles were known to be the doubly charged positive nuclei of helium atoms that had kinetic energies on the order of 5 MeV when emitted in nuclear decay, which is the disintegration of the nucleus of an unstable nuclide by the spontaneous emission of charged particles. These particles interact with matter mostly via the Coulomb force, and the manner in which they scatter from nuclei can reveal nuclear size and mass. This is analogous to observing how a bowling ball is scattered by an object you cannot see directly. Because the alpha particle's energy is so large compared with the typical energies associated with atoms (MeV versus eV), you would expect the alpha particles to simply crash through a thin foil much like a supersonic bowling ball would crash through a few dozen rows of bowling pins. Thomson had envisioned the atom to be a small sphere in which equal amounts of positive and negative charge were distributed evenly. The incident massive alpha particles would suffer only small deflections in such a model. Instead, Rutherford and his collaborators found that alpha particles occasionally were scattered to large angles, some even back in the direction from which they came! Detailed analysis using conservation of momentum and energy—particularly of the small number that came straight back—implied that there was a tightly bound, very small substructure of the gold atom that contains nearly all of the atom's mass (we now call this gold nucleus). Since the gold nucleus is several times more massive than the alpha particle, a head-on collision would scatter the alpha particle straight back toward the source.

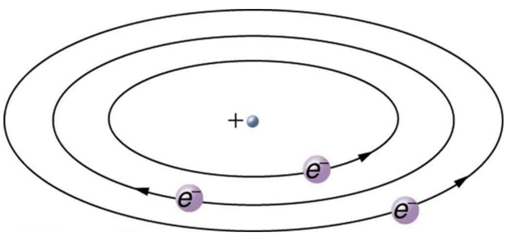
Although the results of the experiment were published by his colleagues in 1909, it took Rutherford two years to convince himself of their meaning. Like Thomson before him, Rutherford was reluctant to accept such radical results. Nature on a small scale is so unlike our classical world that even those at the forefront of discovery are sometimes surprised. Rutherford later wrote: “It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you. On consideration, I realized that this scattering backwards ... [meant] ... the greatest part of the mass of the atom was concentrated in a tiny nucleus.” In 1911, Rutherford published his analysis together with a proposed model of the atom. The size of the nucleus was determined to be about $10^{-15}\ \text{m}$, or 100,000 times smaller than the atom. This implies a huge density, on the order of $10^{15}\ \text{g/cm}^3$, vastly unlike any macroscopic matter. Also implied is the existence of previously unknown nuclear forces to counteract the huge repulsive Coulomb forces among the positive charges in the nucleus. Huge forces would also be consistent with the large energies emitted in nuclear radiation.

The small size of the nucleus also implies that the atom is mostly empty inside—today we use the "electron cloud" to describe how much of the volume of the atom is filled with the electrons, but since electrons have such small masses, energetic alpha particles pass through them mostly unaffected. There were already hints of this at the time Rutherford performed his experiments, since energetic electrons had been observed to penetrate thin foils more easily than expected. [link] shows a schematic of the atoms in a thin foil with circles representing the size of the atoms (about $10^{-10}\ \text{m}$) and dots representing the nuclei. (The dots are not to scale—if they were, you would need a microscope to see them.) Most alpha particles miss the small nuclei and are only slightly scattered by the long-range Coulomb force. Occasionally, (about once in 8000 times in Rutherford’s experiment), an alpha hits a nucleus head-on and is scattered straight backward.



An expanded view of the atoms in the gold foil in Rutherford’s experiment. Circles represent the atoms (about 10^{-10} m in diameter), while the dots represent the nuclei (about 10^{-15} m in diameter). To be visible, the dots are much larger than scale. Most alpha particles crash through but are relatively unaffected because of their high energy and the electron’s small mass. Some, however, head straight toward a nucleus and are scattered straight back. A detailed analysis gives the size and mass of the nucleus.

Based on the size and mass of the nucleus revealed by his experiment, as well as the mass of electrons, Rutherford proposed the **planetary model of the atom**. The planetary model of the atom pictures low-mass electrons orbiting a large-mass nucleus. The sizes of the electron orbits are large compared with the size of the nucleus, with mostly vacuum inside the atom. This picture is analogous to how low-mass planets in our solar system orbit the large-mass Sun at distances large compared with the size of the sun. In the atom, the attractive Coulomb force is analogous to gravitation in the planetary system. [link] is an illustration of this model, or a mental picture.



Rutherford’s planetary model of the atom incorporates the characteristics of the nucleus, electrons, and the size of the atom. This model was the first to recognize the structure of atoms, in which low-mass electrons orbit a very small, massive nucleus in orbits much larger than the nucleus. The atom is mostly empty and is analogous to our planetary system.

Rutherford’s planetary model of the atom was crucial to understanding the characteristics of atoms, and their interactions and energies. Also, it was an indication of how different nature is from the familiar classical world on the small, quantum mechanical scale. The discovery of a substructure to all matter in the form of atoms and molecules was now being taken a step further to reveal a substructure of atoms that was simpler than the 92 elements then known. We have continued to search for deeper substructures, such as those inside the nucleus, with some success. In a later chapter, we will follow this quest in the discussion of quarks and other elementary particles, and we will look at the direction the search seems now to be heading.

Section Summary

- Atoms are composed of negatively charged electrons, first proved to exist in cathode-ray-tube experiments, and a positively charged nucleus.
- All electrons are identical and have a charge-to-mass ratio of
  Equation:

  $$\frac{q_e}{m_e} = -1.76 \times 10^{11}\ \mathrm{C/kg}.$$

- The positive charge in the nuclei is carried by particles called protons, which have a charge-to-mass ratio of
  Equation:

  $$\frac{q_p}{m_p} = 9.57 \times 10^{7}\ \mathrm{C/kg}.$$

- Mass of electron,
  Equation:

  $$m_e = 9.11 \times 10^{-31}\ \mathrm{kg}.$$

- Mass of proton,
  Equation:

$$m_p = 1.67 \times 10^{-27}\text{ kg}.$$

- The planetary model of the atom pictures electrons orbiting the nucleus in the same way that planets orbit the sun.

Conceptual Questions

Exercise:

Problem:

What two pieces of evidence allowed the first calculation of m_e, the mass of the electron?

(a) The ratios q_e/m_e and q_p/m_p.

(b) The values of q_e and E_B.

(c) The ratio q_e/m_e and q_e.

Justify your response.

Exercise:

Problem:

How do the allowed orbits for electrons in atoms differ from the allowed orbits for planets around the sun? Explain how the correspondence principle applies here.

Problem Exercises

Exercise:

Problem:

Rutherford found the size of the nucleus to be about 10^{-15} m. This implied a huge density. What would this density be for gold?

Solution:

$6 \times 10^{20}\ \mathrm{kg/m^3}$

Exercise:

Problem:

In Millikan's oil-drop experiment, one looks at a small oil drop held motionless between two plates. Take the voltage between the plates to be 2033 V, and the plate separation to be 2.00 cm. The oil drop (of density $0.81\ \mathrm{g/cm^3}$) has a diameter of $4.0 \times 10^{-6}\ \mathrm{m}$. Find the charge on the drop, in terms of electron units.

Exercise:

Problem:

(a) An aspiring physicist wants to build a scale model of a hydrogen atom for her science fair project. If the atom is 1.00 m in diameter, how big should she try to make the nucleus?

(b) How easy will this be to do?

Solution:

(a) $10.0\ \mu\mathrm{m}$

(b) It isn't hard to make one of approximately this size. It would be harder to make it exactly $10.0\ \mu\mathrm{m}$.

Glossary

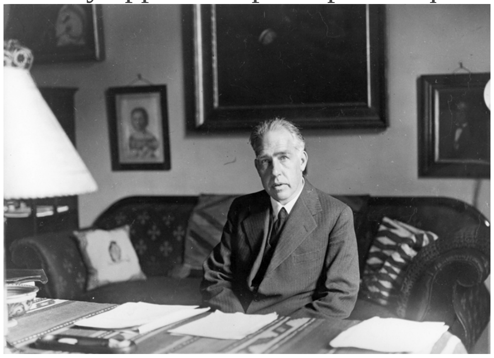
cathode-ray tube
: a vacuum tube containing a source of electrons and a screen to view images

planetary model of the atom
: the most familiar model or illustration of the structure of the atom

Bohr’s Theory of the Hydrogen Atom

- Describe early atomic models.
- Explain Bohr’s theory of the hydrogen atom.
- Distinguish between correct and incorrect features of the Bohr model, in light of modern quantum mechanics.

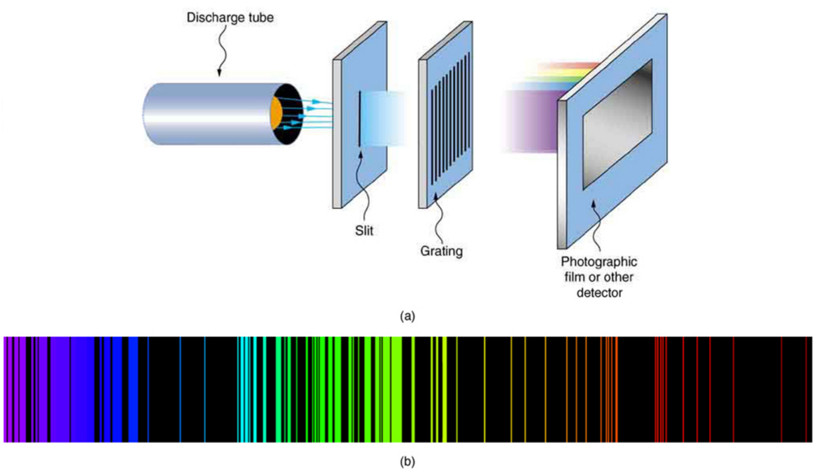
The great Danish physicist Niels Bohr (1885–1962) made immediate use of Rutherford’s planetary model of the atom. ([link]). Bohr became convinced of its validity and spent part of 1912 at Rutherford’s laboratory. In 1913, after returning to Copenhagen, he began publishing his theory of the simplest atom, hydrogen, based on the planetary model of the atom. For decades, many questions had been asked about atomic characteristics. From their sizes to their spectra, much was known about atoms, but little had been explained in terms of the laws of physics. Bohr’s theory explained the atomic spectrum of hydrogen and established new and broadly applicable principles in quantum mechanics.



Niels Bohr, Danish physicist, used the planetary model of the atom to explain the atomic spectrum and size of the hydrogen atom. His many contributions to the development of atomic physics and quantum mechanics, his personal influence on many students and colleagues, and his personal integrity, especially in the face of Nazi oppression, earned him a prominent place in history. (credit: Unknown Author, via Wikimedia Commons)

Atomic Spectra

Atomic and molecular emission and absorption spectra have been known for over a century to be discrete (or quantized). Well before they were understood from first principles, chemists have been using the emission and absorption spectra for identification of elements. [link] shows iron emission spectrum, for example. No other elements emit the exactly the same set of frequencies of light. With the discovery of substructure of the atom and the discovery of photon (or more precisely, refined understanding of the particle nature of electromagnetic waves where the particle energy is proportional to the frequency of electromagnetic waves), these resonant frequencies of light emitted by atoms could be used to infer an atomic model.



Part (a) shows, from left to right, a discharge tube, slit, and diffraction grating producing a line spectrum. Part (b) shows the emission line spectrum for iron. The discrete lines imply quantized energy states for the atoms that produce them. The line spectrum for each element is unique, providing a powerful and much used analytical tool, and many line spectra were well known for many years before they could be explained with physics. (credit for (b): Yttrium91, Wikimedia Commons)

For the hydrogen atom, the lightest element with the simplest atom, a pattern for its line spectrum was noticed by experimentalists (see [link]). All wavelengths of the line spectrum could be described by a following formula, for the suitable choice of two integers n_i and n_f:
Equation:

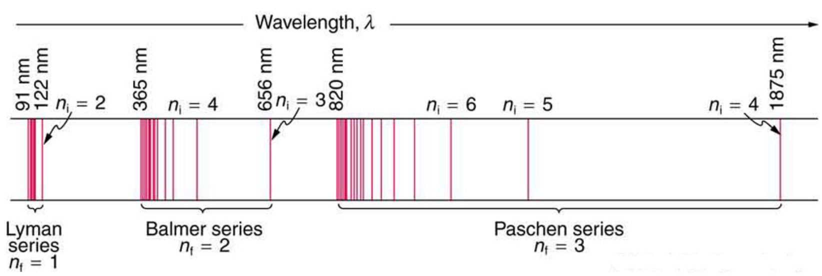
$$\frac{1}{\lambda} = R\left(\frac{1}{n_{\mathrm{f}}^2} - \frac{1}{n_{\mathrm{i}}^2}\right),$$

where λ is the wavelength of the emitted EM radiation and R is the **Rydberg constant**, determined by the experiment to be
Equation:

$$R = 1.097 \times 10^{7}/\mathrm{m}\ (\mathrm{or}\ \mathrm{m}^{-1}).$$

The n_{f} is a positive integer associated with a specific series, which are named after their discoverers. For the Lyman series, $n_{\mathrm{f}} = 1$; for the Balmer series, $n_{\mathrm{f}} = 2$; for the Paschen series, $n_{\mathrm{f}} = 3$; and so on. The Lyman series is entirely in the UV, while part of the Balmer series is visible with the remainder UV. The Paschen series and all the rest are entirely IR. There are apparently an unlimited number of series, although they lie progressively farther into the infrared and become difficult to observe as n_{f} increases. The n_{i} is a positive integer greater than n_{f}. So for example, for the Balmer series, $n_{\mathrm{f}} = 2$ and $n_{\mathrm{i}} = 3, 4, 5, 6, ...$

So, before Bohr's model of the hydrogen atom, such was the picture of atomic theory—full of suggestive (and even well-organized) data and no unifying explanation. Ernest Rutherford is quoted as saying, "All science is either physics or stamp-collecting." What he meant is, there are branches of science whose practitioners would be satisfied with a collection of interesting facts (i.e. "stamp-collecting"). But what makes physics *physics* is the search for the theoretical framework providing explanations based on fundamental principles, not idiosyncratic descriptions. Bohr's model brought the science of spectroscopy into physics.



A schematic of the hydrogen spectrum shows several series named for those who contributed most to their determination. Part of the Balmer series is in the visible spectrum, while the Lyman series is entirely in the UV, and the Paschen series and others are in the IR. Values of n_{f} and n_{i} are shown for some of the lines.

Bohr's Model for Hydrogen

The planetary model of the atom suggested by Rutherford was in trouble. While the model provided a possible picture of how the very small atomic nucleus might be arranged with the

electrons in a stable arrangement, it did not provide for the size of electron orbits (which would be related to the size of the atom), and the arrangement was not actually stable—an orbiting electron is an oscillating charge; an oscillating charge emits electromagnetic waves; electromagnetic waves carry away energy; so as the electron loses energy, it would fall into the proton. By some estimates, this would occur in as short a time as 10^{-7} s!

Bohr's starting point for his successful model was this: he proposed that *the orbits of electrons in atoms are quantized*. To fully understand this statement, we can compare the orbits of electrons in atoms to the orbits of planets in the solar system. The orbits of planets are not quantized. While laws of physics govern how planets move in the solar system (see for example, Kepler's laws, or their derivation by Newton starting with the inverse-square law of gravitation), there is no law of physics dictating how far each body in the solar system must be from the Sun. So the orbits of planets are not quantized.

So what Bohr was proposing was an entirely *new* law of physics no one had known before. In one sense, it was not completely new (Planck and Einstein already enjoyed some successes from suggesting quantization of energy in thermal oscillators and EM radiation); in another sense, it was a big break from centuries of classical mechanics. This was Bohr's quantization rule: **angular momentum of an electron in its orbit is quantized**. In mathematical form,
Equation:

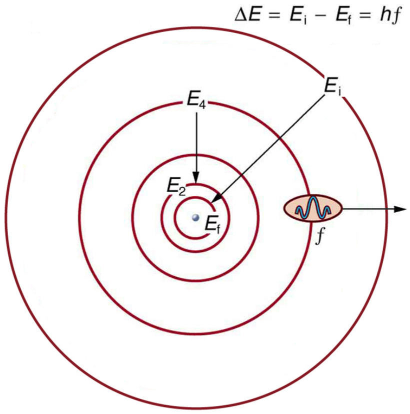
$$L = n\hbar,$$

where n could take on any positive integer value ($n = 1, 2, 3, ...$), and $\hbar$ is known as the reduced Planck constant ($\hbar = h/2\pi$). And angular momentum, L, as you might remember from earlier chapter, is given by the following for a particle in a uniform circular orbit: $L = mvr$, where m is the mass of the particle, v is the speed of the particle in orbit, and r is the radius of circular orbit. Using this as the starting point, semiclassical analysis of orbital motion yields a whole array of quantized (i.e. allowed) values of orbital distance (r_n), orbital speed (v_n), and orbital energy (E_n), among others (see: [link] for a summary).

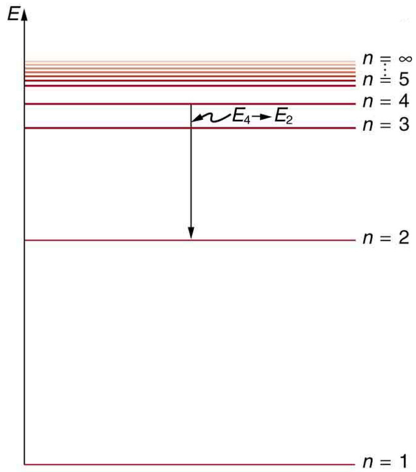
With the quantized orbital energies for the electron, we have a ready explanation for the features of atomic spectra. EM radiation is emitted when an electron transitions from a higher energy level (E_i) to a lower energy level (E_f), with the photon carrying away the energy difference,
Equation:

$$hf = \varDelta E = E_i - E_f,$$

where f is the frequency of the photon. [link] shows a schematic representation of this relationship. With only discrete values of energy E_n allowed, there are only discrete values of frequency (f) and wavelength (λ) allowed also, as shown in the line spectra.



The planetary model of the atom, as modified by Bohr, has the orbits of the electrons quantized. Only certain orbits are allowed, explaining why atomic spectra are discrete (quantized). The energy carried away from an atom by a photon comes from the electron dropping from one allowed orbit to another and is thus quantized. This is likewise true for atomic absorption of photons.



An energy-level diagram plots energy vertically and is useful in visualizing the energy states of a system and the transitions between them. This diagram is for the hydrogen-atom electrons, showing a transition between two orbits having energies E_4 and E_2.

Energy-level diagram, shown in [link], is another convenient way to illustrate these relationships. Allowed energy levels for the atom are plotted vertically with the lowest state (or **ground state**) at the bottom and with excited states above that. The energies of the lines in an atomic spectrum correspond to the *differences* in energy levels in the level diagram (figure illustrates a transition from E_4 to E_2, which would show up in the atomic spectrum as one line).

Quantized quantity	Dependence on quantum number n	Full expression
angular momentum: L_n	proportional to n	$L_n = n\hbar$

orbital radius: r_n	proportional to n^2	$r_n = \frac{n^2\hbar^2}{mke^2}$
orbital speed: v_n	proportional to $\frac{1}{n}$	$v_n = \frac{ke^2}{n\hbar}$
orbital energy: E_n	proportional to $\frac{1}{n^2}$	$E_n = -\frac{mk^2e^4}{2n^2\hbar^2} = -\frac{13.6}{n^2}\text{eV}$

Summary of quantized quantities in the Bohr model of the hydrogen atom. The full derivations take some bit of algebra, and they use: (1) centripetal force due to the Coulomb force, (2) relationship between quantized orbital radius and quantized orbital speed through quantization of angular momentum, and (3) expression for the total energy, including orbital kinetic energy and the Coulomb potential energy.

Two key results are worth highlighting. The first is the **Bohr radius**, or the smallest orbital radius a, given for $n = 1$,

Equation:

$$\begin{aligned} a &= r_1 = \hbar^2/mke^2 \\ &= 0.529 \times 10^{-10}\ \text{m}. \end{aligned}$$

This is the Bohr model's prediction for the size of the atom, made with nothing more than electric constants, mass of the electron, and the Planck's constant, and this theoretical prediction matches experimentally measured sizes of atoms fairly well.

The second is the derivation of the Rydberg formula, first given in [link]. To derive this, we start out with [link] and substitute in expressions for hydrogen energies from [link]:

Equation:

$$\begin{aligned} hf &= -\frac{mk^2e^4}{2{n_i}^2\hbar^2} - \left(-\frac{mk^2e^4}{2{n_f}^2\hbar^2}\right) \\ &= \frac{mk^2e^4}{2\hbar^2}\left(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}\right) \end{aligned}$$

Frequency f is equal to c/λ. Plugging this in and solving for $1/\lambda$ while also replacing all instances of $\hbar$ with $h/2\pi$, we get,

Equation:

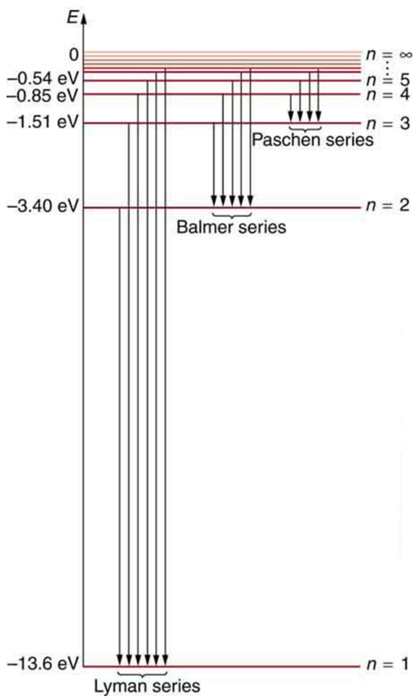
$$\frac{1}{\lambda} = \frac{2\pi^2mk^2e^4}{h^3c}\left(\frac{1}{\text{msup}} - \frac{1}{\text{msup}}\right),$$

which yields an analytical expression for the Rydberg constant,

Equation:

$$R = \frac{2\pi^2 mk^2 e^4}{h^3 c} = 1.097 \times 10^7 \text{ m}^{-1}.$$

[link] shows an energy-level diagram for hydrogen that also illustrates how the various spectral series for hydrogen are related to transitions between energy levels.



Energy-level diagram for hydrogen showing the Lyman, Balmer, and Paschen series of transitions. The orbital energies are calculated using the above equation, first derived by Bohr.

We see that Bohr’s theory of the hydrogen atom answers the question as to why this previously known formula describes the hydrogen spectrum. It is because the energy levels are proportional to $1/n^2$, where n is a non-negative integer. A downward transition releases energy, and so n_i must be greater than n_f. The various series are those where the transitions end on a certain level. For the Lyman series, $n_\mathrm{f} = 1$ — that is, all the transitions end in the ground state (see also [link]). For the Balmer series, $n_\mathrm{f} = 2$, or all the transitions end in the

first excited state; and so on. What was once a recipe is now based in physics, and something new is emerging—angular momentum is quantized.

Triumphs and Limits of the Bohr Theory

Bohr did what no one had been able to do before. Not only did he explain the spectrum of hydrogen, he correctly calculated the size of the atom from basic physics. Some of his ideas are broadly applicable. Electron orbital energies are quantized in all atoms and molecules. Angular momentum is quantized. The electrons do not spiral into the nucleus, as expected classically. These are major triumphs.

But there are limits to Bohr’s theory. It cannot be applied to multielectron atoms, even one as simple as a two-electron helium atom. Bohr’s model is a *semiclassical* model. The orbits are quantized (quantum mechanical) but are assumed to be simple circular paths (classical). As quantum mechanics was developed, it became clear that there are no well-defined orbits; rather, there are "clouds" of probability. Bohr’s theory also did not explain that some spectral lines are doublets (split into two) when examined closely. These deficiencies are addressed in later, fully-quantum-mechanical atomic models, but it should be kept in mind that Bohr did not fail. Rather, he made very important steps along the path to greater knowledge and laid the foundation.

Section Summary

- The planetary model of the atom pictures electrons orbiting the nucleus in the way that planets orbit the sun. Bohr used the planetary model to develop the first reasonable theory of hydrogen, the simplest atom. Atomic and molecular spectra are quantized, with hydrogen spectrum wavelengths given by the formula
  Equation:
  $$\frac{1}{\lambda} = R\left(\frac{1}{n_{\mathrm{f}}^2} - \frac{1}{n_{\mathrm{i}}^2}\right),$$
  where λ is the wavelength of the emitted EM radiation and R is the Rydberg constant, which has the value
  Equation:
  $$R = 1.097 \times 10^7\ \mathrm{m}^{-1}.$$
- The constants n_{i} and n_{f} are positive integers, and n_{i} must be greater than n_{f}.
- Bohr correctly proposed that the energy and radii of the orbits of electrons in atoms are quantized, with energy for transitions between orbits given by
  Equation:
  $$\Delta E = hf = E_{\mathrm{i}} - E_{\mathrm{f}},$$

where ΔE is the change in energy between the initial and final orbits and hf is the energy of an absorbed or emitted photon. It is useful to plot orbital energies on a vertical graph called an energy-level diagram.

- Bohr proposed that the allowed orbits are circular and must have quantized orbital angular momentum given by
  Equation:

  $$L = m_e v r_n = n \frac{h}{2\pi} (n = 1, 2, 3 \ldots),$$

  where L is the angular momentum, r_n is the radius of the $n\text{th}$ orbit, and h is Planck's constant.
- Additional quantized orbital quantities—orbital radius, orbital speed, and orbital energy—can be derived starting from Bohr's assumption, and they yield predictions consistent with the experimental Rydberg formula.
- While Bohr's semiclassical model of the atom does not account for all experimental facts about the atom, it is an important stepping stone to fully-quantum-mechanical models of the atom.

Conceptual Questions

Exercise:

Problem:

How do the allowed orbits for electrons in atoms differ from the allowed orbits for planets around the sun? Explain how the correspondence principle applies here.

Exercise:

Problem:

Explain how Bohr's rule for the quantization of electron orbital angular momentum differs from the actual rule.

Exercise:

Problem:

What is a hydrogen-like atom, and how are the energies and radii of its electron orbits related to those in hydrogen?

Problems & Exercises

Exercise:

Problem:

By calculating its wavelength, show that the first line in the Lyman series is UV radiation.

Solution:

$\frac{1}{\lambda} = R\left(\frac{1}{n_\mathrm{f}^2} - \frac{1}{n_\mathrm{i}^2}\right) \Rightarrow \lambda = \frac{1}{R}\left[\frac{(n_\mathrm{i} \cdot n_\mathrm{f})^2}{n_\mathrm{i}^2 - n_\mathrm{f}^2}\right]; n_\mathrm{i} = 2, n_\mathrm{f} = 1,$ so that

$\lambda = \left(\frac{\mathrm{m}}{1.097 \times 10^7}\right)\left[\frac{(2 \times 1)^2}{2^2 - 1^2}\right] = 1.22 \times 10^{-7}\ \mathrm{m} = 122\ \mathrm{nm}$, which is UV radiation.

Exercise:

Problem:

Find the wavelength of the third line in the Lyman series, and identify the type of EM radiation.

Exercise:

Problem:

Look up the values of the quantities in $a_\mathrm{B} = \frac{h^2}{4\pi^2 m_e k q_e^2}$, and verify that the Bohr radius a_B is 0.529×10^{-10} m.

Solution:

$$a_\mathrm{B} = \frac{h^2}{4\pi^2 m_e k Z q_e^2} = \frac{(6.626 \times 10^{-34}\ \mathrm{J \cdot s})^2}{4\pi^2 (9.109 \times 10^{-31}\ \mathrm{kg})(8.988 \times 10^9\ \mathrm{N \cdot m^2/C^2})(1)(1.602 \times 10^{-19}\ \mathrm{C})^2} = 0.529 \times 10^{-10}\ \mathrm{m}$$

Exercise:

Problem: Verify that the ground state energy E_0 is 13.6 eV by using $E_0 = \frac{2\pi^2 q_e^4 m_e k^2}{h^2}$.

Exercise:

Problem:

If a hydrogen atom has its electron in the $n = 4$ state, how much energy in eV is needed to ionize it?

Solution:

0.850 eV

Exercise:

Problem:

A hydrogen atom in an excited state can be ionized with less energy than when it is in its ground state. What is n for a hydrogen atom if 0.850 eV of energy can ionize it?

Exercise:

Problem:

Find the radius of a hydrogen atom in the $n = 2$ state according to Bohr's theory.

Solution:

$2.12 \times 10^{-10}\ \mathrm{m}$

Exercise:

Problem:

Show that $(13.6\ \mathrm{eV})/\mathrm{hc} = 1.097 \times 10^{7}\ \mathrm{m} = R$ (Rydberg's constant), as discussed in the text.

Exercise:

Problem:

What is the smallest-wavelength line in the Balmer series? Is it in the visible part of the spectrum?

Solution:

365 nm

It is in the ultraviolet.

Exercise:

Problem:

Show that the entire Paschen series is in the infrared part of the spectrum. To do this, you only need to calculate the shortest wavelength in the series.

Exercise:

Problem:

Do the Balmer and Lyman series overlap? To answer this, calculate the shortest-wavelength Balmer line and the longest-wavelength Lyman line.

Solution:

No overlap

365 nm

122 nm

Exercise:

Problem:

(a) Which line in the Balmer series is the first one in the UV part of the spectrum?

(b) How many Balmer series lines are in the visible part of the spectrum?

(c) How many are in the UV?

Exercise:

Problem:

A wavelength of $4.653\ \mu\text{m}$ is observed in a hydrogen spectrum for a transition that ends in the $n_\text{f} = 5$ level. What was n_i for the initial level of the electron?

Solution:

7

Exercise:

Problem:

A singly ionized helium ion has only one electron and is denoted He^+. What is the ion's radius in the ground state compared to the Bohr radius of hydrogen atom?

Exercise:

Problem:

A beryllium ion with a single electron (denoted Be^{3+}) is in an excited state with radius the same as that of the ground state of hydrogen.

(a) What is n for the Be^{3+} ion?

(b) How much energy in eV is needed to ionize the ion from this excited state?

Solution:

(a) 2

(b) 54.4 eV

Exercise:

Problem:

Atoms can be ionized by thermal collisions, such as at the high temperatures found in the solar corona. One such ion is C^{+5}, a carbon atom with only a single electron.

(a) By what factor are the energies of its hydrogen-like levels greater than those of hydrogen?

(b) What is the wavelength of the first line in this ion’s Paschen series?

(c) What type of EM radiation is this?

Exercise:

Problem:

Verify Equations $r_n = \frac{n^2}{Z} a_{\mathrm{B}}$ and $a_{\mathrm{B}} = \frac{h^2}{4\pi^2 m_e k q_e^2} = 0.529 \times 10^{-10}$ m using the approach stated in the text. That is, equate the Coulomb and centripetal forces and then insert an expression for velocity from the condition for angular momentum quantization.

Solution:

$\frac{kZq_e^2}{r_n^2} = \frac{m_e V^2}{r_n}$, so that $r_n = \frac{kZq_e^2}{m_e V^2} = \frac{kZq_e^2}{m_e} \frac{1}{V^2}$. From the equation $m_e v r_n = n \frac{h}{2\pi}$, we can substitute for the velocity, giving: $r_n = \frac{kZq_e^2}{m_e} \cdot \frac{4\pi^2 m_e^2 r_n^2}{n^2 h^2}$ so that $r_n = \frac{n^2}{Z} \frac{h^2}{4\pi^2 m_e k q_e^2} = \frac{n^2}{Z} a_{\mathrm{B}}$, where $a_{\mathrm{B}} = \frac{h^2}{4\pi^2 m_e k q_e^2}$.

Exercise:

Problem:

The wavelength of the four Balmer series lines for hydrogen are found to be 410.3, 434.2, 486.3, and 656.5 nm. What average percentage difference is found between these wavelength numbers and those predicted by $\frac{1}{\lambda} = R\left(\frac{1}{n_{\mathrm{f}}^2} - \frac{1}{n_{\mathrm{i}}^2}\right)$? It is amazing how well a simple formula (disconnected originally from theory) could duplicate this phenomenon.

Glossary

hydrogen spectrum wavelengths
: the wavelengths of visible light from hydrogen; can be calculated by $\frac{1}{\lambda} = R\left(\frac{1}{n_{\mathrm{f}}^2} - \frac{1}{n_{\mathrm{i}}^2}\right)$

Rydberg constant

a physical constant related to the atomic spectra with an established value of $1.097 \times 10^7 \text{ m}^{-1}$

double-slit interference
: an experiment in which waves or particles from a single source impinge upon two slits so that the resulting interference pattern may be observed

energy-level diagram
: a diagram used to analyze the energy level of electrons in the orbits of an atom

Bohr radius
: the mean radius of the orbit of an electron around the nucleus of a hydrogen atom in its ground state

hydrogen-like atom
: any atom with only a single electron

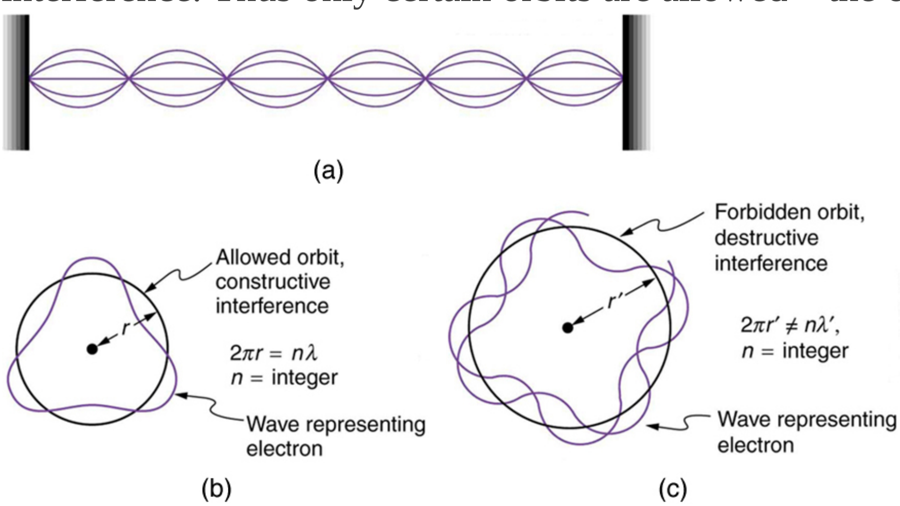
energies of hydrogen-like atoms
: Bohr formula for energies of electron states in hydrogen-like atoms: $E_n = -\frac{Z^2}{n^2} E_0 (n = 1, 2, 3, \ \ldots)$

The Wave Nature of Matter Causes Quantization

- Draw a connection between Bohr's quantization of angular momentum and electron wave.

When Bohr proposed that the orbital angular momentum of electrons in atoms was quantized, he did not have a very good reason, aside from the fact that this *ad hoc* assumption led to correct results. If we remember that all matter—including electrons in atoms—have wave property, we can give an intuitive model of *why* the orbital angular momentum would have to be quantized.

Following Bohr's initial work on the hydrogen atom, a decade was to pass before de Broglie proposed that matter has wave properties. The wave-like properties of matter were subsequently confirmed by observations of electron interference when scattered from crystals. Electrons can exist only in locations where they interfere constructively. How does this affect electrons in atomic orbits? When an electron is bound to an atom, its wavelength must fit into a small space, something like a standing wave on a string. (See [link].) Allowed orbits are those orbits in which an electron constructively interferes with itself. Not all orbits produce constructive interference. Thus only certain orbits are allowed—the orbits are quantized.



(a) Waves on a string have a wavelength related to the length of the string, allowing them to interfere constructively. (b) If we imagine the string bent into a closed circle, we get a rough idea of how

electrons in circular orbits can interfere constructively. (c) If the wavelength does not fit into the circumference, the electron interferes destructively; it cannot exist in such an orbit.

For a circular orbit, constructive interference occurs when the electron's wavelength fits neatly into the circumference, so that wave crests always align with crests and wave troughs align with troughs, as shown in [link] (b). More precisely, when an integral multiple of the electron's wavelength equals the circumference of the orbit, constructive interference is obtained. In equation form, the *condition for constructive interference and an allowed electron orbit* is

Equation:

$$n\lambda_n = 2\pi r_n (n = 1, 2, 3 \ldots),$$

where λ_n is the electron's wavelength and r_n is the radius of that circular orbit. The de Broglie wavelength is $\lambda = h/p = h/\ mv$, and so here $\lambda = h/m_e v$. Substituting this into the previous condition for constructive interference produces an interesting result:

Equation:

$$\frac{\mathrm{nh}}{m_e v} = 2\pi r_n.$$

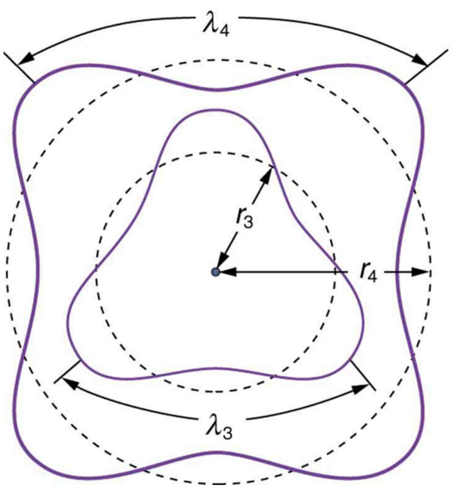
Rearranging terms, and noting that $L = \mathrm{mvr}$ for a circular orbit, we obtain the quantization of angular momentum as the condition for allowed orbits:

Equation:

$$L = m_e \mathrm{vr}_n = n\frac{h}{2\pi}(n = 1, 2, 3 \ldots).$$

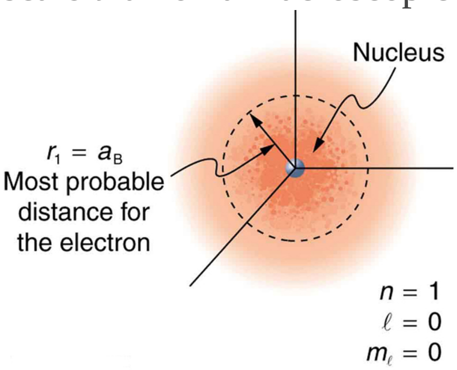
This is what Bohr was forced to hypothesize as the rule for allowed orbits, as stated earlier. We now realize that it is the condition for constructive interference of an electron in a circular orbit. [link] illustrates this for $n = 3$ and $n = 4$.

Note:
Waves and Quantization
The wave nature of matter is responsible for the quantization of energy levels in bound systems. Only those states where matter interferes constructively exist, or are “allowed.” Since there is a lowest orbit where this is possible in an atom, the electron cannot spiral into the nucleus. It cannot exist closer to or inside the nucleus. The wave nature of matter is what prevents matter from collapsing and gives atoms their sizes.



The third and fourth allowed circular orbits have three and four wavelengths, respectively, in their circumferences.

Because of the wave character of matter, the idea of well-defined orbits gives way to a model in which there is a cloud of probability, consistent with Heisenberg's uncertainty principle. [link] shows how this applies to the ground state of hydrogen. If you try to follow the electron in some well-defined orbit using a probe that has a small enough wavelength to get some details, you will instead knock the electron out of its orbit. Each measurement of the electron's position will find it to be in a definite location somewhere near the nucleus. Repeated measurements reveal a cloud of probability like that in the figure, with each speck the location determined by a single measurement. There is not a well-defined, circular-orbit type of distribution. Nature again proves to be different on a small scale than on a macroscopic scale.



The ground state of a hydrogen atom has a probability cloud describing the position of its electron. The probability of finding the electron is proportional to the darkness of the cloud. The electron can be closer or farther than the Bohr radius, but it is very unlikely to be a great distance from the nucleus.

There are many examples in which the wave nature of matter causes quantization in bound systems such as the atom. Whenever a particle is confined or bound to a small space, its allowed wavelengths are those which fit into that space. For example, the particle in a box model describes a particle free to move in a small space surrounded by impenetrable barriers. This is true in blackbody radiators (atoms and molecules) as well as in atomic and molecular spectra. Various atoms and molecules will have different sets of electron orbits, depending on the size and complexity of the system. When a system is large, such as a grain of sand, the tiny particle waves in it can fit in so many ways that it becomes impossible to see that the allowed states are discrete. Thus the correspondence principle is satisfied. As systems become large, they gradually look less grainy, and quantization becomes less evident. Unbound systems (small or not), such as an electron freed from an atom, do not have quantized energies, since their wavelengths are not constrained to fit in a certain volume.

Section Summary

- Quantization of orbital energy is caused by the wave nature of matter. Allowed orbits in atoms occur for constructive interference of electrons in the orbit, requiring an integral number of wavelengths to fit in an orbit's circumference; that is,
  Equation:
  $$n\lambda_n = 2\pi r_n (n = 1, 2, 3 \ldots),$$
  where λ_n is the electron's de Broglie wavelength.
- Owing to the wave nature of electrons and the Heisenberg uncertainty principle, there are no well-defined orbits; rather, there are clouds of probability.
- Bohr correctly proposed that the energy and radii of the orbits of electrons in atoms are quantized, with energy for transitions between orbits given by
  Equation:
  $$\Delta E = hf = E_{\text{i}} - E_{\text{f}},$$

where ΔE is the change in energy between the initial and final orbits and hf is the energy of an absorbed or emitted photon.

- It is useful to plot orbit energies on a vertical graph called an energy-level diagram.
- The allowed orbits are circular, Bohr proposed, and must have quantized orbital angular momentum given by

  Equation:

  $$L = m_e v r_n = n\frac{h}{2\pi} (n = 1, 2, 3 \ldots),$$

  where L is the angular momentum, r_n is the radius of orbit n, and h is Planck’s constant.

Conceptual Questions

Exercise:

Problem:

How is the de Broglie wavelength of electrons related to the quantization of their orbits in atoms and molecules?

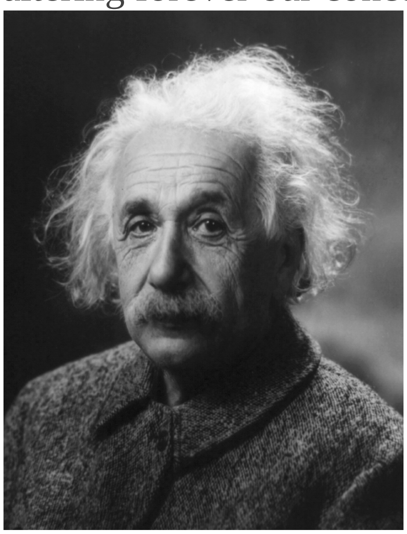
Introduction to Special Relativity
class="introduction"

Special relativity explains why traveling to other star systems, such as these in the Orion Nebula, is unreasonable using our current level of technology. (credit: s58y, Flickr)

Have you ever looked up at the night sky and dreamed of traveling to other planets in faraway star systems? Would there be other life forms? What would other worlds look like? You might imagine that such an amazing trip

would be possible if we could just travel fast enough, but you will read in this chapter why this is not true. In 1905 Albert Einstein developed the theory of special relativity. This theory explains the limit on an object's speed and describes the consequences.

Relativity. The word *relativity* might conjure an image of Einstein, but the idea did not begin with him. People have been exploring relativity for many centuries. Relativity is the study of how different observers measure the same event. Galileo and Newton developed the first correct version of classical relativity. Einstein developed the modern theory of relativity. Modern relativity is divided into two parts. *Special relativity* deals with observers who are moving at constant velocity. *General relativity* deals with observers who are undergoing acceleration. Einstein is famous because his theories of relativity made revolutionary predictions. Most importantly, his theories have been verified to great precision in a vast range of experiments, altering forever our concept of space and time.



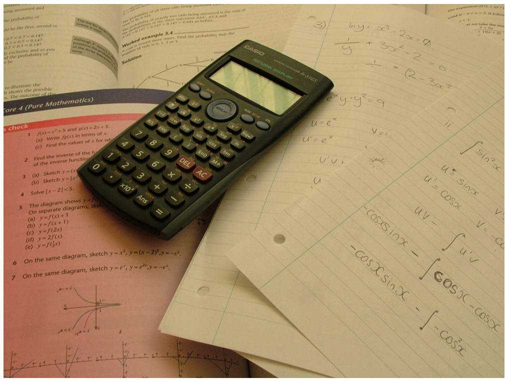
Many people think that Albert Einstein (1879–1955) was the greatest physicist of the 20th century. Not only did he develop modern relativity, thus

revolutionizing our concept of the universe, he also made fundamental contributions to the foundations of quantum mechanics. (credit: The Library of Congress)

It is important to note that although classical mechanics, in general, and classical relativity, in particular, are limited, they are extremely good approximations for large, slow-moving objects. Otherwise, we could not use classical physics to launch satellites or build bridges. In the classical limit (objects larger than submicroscopic and moving slower than about 1% of the speed of light), relativistic mechanics becomes the same as classical mechanics. This fact will be noted at appropriate places throughout this chapter.

Einstein’s Postulates

- State and explain both of Einstein’s postulates.
- Explain what an inertial frame of reference is.
- Describe one way the speed of light can be changed.



Special relativity resembles trigonometry in that both are reliable because they are based on postulates that flow one from another in a logical way. (credit: Jon Oakley, Flickr)

Have you ever used the Pythagorean Theorem and gotten a wrong answer? Probably not, unless you made a mistake in either your algebra or your arithmetic. Each time you perform the same calculation, you know that the answer will be the same. Trigonometry is reliable because of the certainty that one part always flows from another in a logical way. Each part is based on a set of postulates, and you can always connect the parts by applying those postulates. Physics is the same way with the exception that *all* parts must describe nature. If we are careful to choose the correct postulates, then our theory will follow and will be verified by experiment.

Einstein essentially did the theoretical aspect of this method for **relativity**. With two deceptively simple postulates and a careful consideration of how measurements are made, he produced the theory of **special relativity.**

Einstein’s First Postulate

The first postulate upon which Einstein based the theory of special relativity relates to reference frames. All velocities are measured relative to some frame of reference. For example, a car’s motion is measured relative to its starting point or the road it is moving over, a projectile’s motion is measured relative to the surface it was launched from, and a planet’s orbit is measured relative to the star it is orbiting around. The simplest frames of reference are those that are not accelerated and are not rotating. Newton’s first law, the law of inertia, holds exactly in such a frame.

Note:
Inertial Reference Frame
An **inertial frame of reference** is a reference frame in which a body at rest remains at rest and a body in motion moves at a constant speed in a straight line unless acted on by an outside force.

The laws of physics seem to be simplest in inertial frames. For example, when you are in a plane flying at a constant altitude and speed, physics seems to work exactly the same as if you were standing on the surface of the Earth. However, in a plane that is taking off, matters are somewhat more complicated. In these cases, the net force on an object, F, is not equal to the product of mass and acceleration, ma. Instead, F is equal to ma plus a fictitious force. This situation is not as simple as in an inertial frame. Not only are laws of physics simplest in inertial frames, but they should be the same in all inertial frames, since there is no preferred frame and no absolute motion. Einstein incorporated these ideas into his **first postulate of special relativity**.

Note:
First Postulate of Special Relativity
The laws of physics are the same and can be stated in their simplest form in all inertial frames of reference.

As with many fundamental statements, there is more to this postulate than meets the eye. The laws of physics include only those that satisfy this postulate. We shall find that the definitions of relativistic momentum and energy must be altered to fit. Another outcome of this postulate is the famous equation $E = mc^2$.

Einstein’s Second Postulate

The second postulate upon which Einstein based his theory of special relativity deals with the speed of light. Late in the 19th century, the major tenets of classical physics were well established. Two of the most important were the laws of electricity and magnetism and Newton’s laws. In particular, the laws of electricity and magnetism predict that light travels at $c = 3.00 \times 10^8 \text{ m/s}$ in a vacuum, but they do not specify the frame of reference in which light has this speed.

There was a contradiction between this prediction and Newton’s laws, in which velocities add like simple vectors. If the latter were true, then two observers moving at different speeds would see light traveling at different speeds. Imagine what a light wave would look like to a person traveling along with it at a speed c. If such a motion were possible then the wave would be stationary relative to the observer. It would have electric and magnetic fields that varied in strength at various distances from the observer but were constant in time. This is not allowed by Maxwell’s equations. So either Maxwell’s equations are wrong, or an object with mass cannot travel at speed c. Einstein concluded that the latter is true. An object with mass cannot travel at speed c. This conclusion implies that light in a vacuum must always travel at speed c relative to any observer. Maxwell’s equations are correct, and Newton’s addition of velocities is not correct for light.

Investigations such as Young's double slit experiment in the early-1800s had convincingly demonstrated that light is a wave. Many types of waves were known, and all travelled in some medium. Scientists therefore assumed that a medium carried light, even in a vacuum, and light travelled at a speed c relative to that medium. Starting in the mid-1880s, the American physicist A. A. Michelson, later aided by E. W. Morley, made a series of direct measurements of the speed of light. The results of their measurements were startling.

Note:
Michelson-Morley Experiment
The **Michelson-Morley experiment** demonstrated that the speed of light in a vacuum is independent of the motion of the Earth about the Sun.

The eventual conclusion derived from this result is that light, unlike mechanical waves such as sound, does not need a medium to carry it. Furthermore, the Michelson-Morley results implied that the speed of light c is independent of the motion of the source relative to the observer. That is, everyone observes light to move at speed c regardless of how they move relative to the source or one another. For a number of years, many scientists tried unsuccessfully to explain these results and still retain the general applicability of Newton's laws.

It was not until 1905, when Einstein published his first paper on special relativity, that the currently accepted conclusion was reached. Based mostly on his analysis that the laws of electricity and magnetism would not allow another speed for light, and only slightly aware of the Michelson-Morley experiment, Einstein detailed his **second postulate of special relativity**.

Note:
Second Postulate of Special Relativity

The speed of light c is a constant, independent of the relative motion of the source.

Deceptively simple and counterintuitive, this and the first postulate leave all else open for change. Some fundamental concepts do change. Among the changes are the loss of agreement on the elapsed time for an event, the variation of distance with speed, and the realization that matter and energy can be converted into one another. You will read about these concepts in the following sections.

Note:
Misconception Alert: Constancy of the Speed of Light
The speed of light is a constant $c = 3.00 \times 10^{8}\ \mathrm{m/s}$ *in a vacuum*. If you remember the effect of the index of refraction from The Law of Refraction, the speed of light is lower in matter.

Exercise:
Check Your Understanding

Problem: Explain how special relativity differs from general relativity.

Solution:
Answer

Special relativity applies only to unaccelerated motion, but general relativity applies to accelerated motion.

Section Summary

- Relativity is the study of how different observers measure the same event.

- Modern relativity is divided into two parts. Special relativity deals with observers who are in uniform (unaccelerated) motion, whereas general relativity includes accelerated relative motion and gravity. Modern relativity is correct in all circumstances and, in the limit of low velocity and weak gravitation, gives the same predictions as classical relativity.
- An inertial frame of reference is a reference frame in which a body at rest remains at rest and a body in motion moves at a constant speed in a straight line unless acted on by an outside force.
- Modern relativity is based on Einstein’s two postulates. The first postulate of special relativity is the idea that the laws of physics are the same and can be stated in their simplest form in all inertial frames of reference. The second postulate of special relativity is the idea that the speed of light c is a constant, independent of the relative motion of the source.
- The Michelson-Morley experiment demonstrated that the speed of light in a vacuum is independent of the motion of the Earth about the Sun.

Conceptual Questions

Exercise:

Problem:

Which of Einstein’s postulates of special relativity includes a concept that does not fit with the ideas of classical physics? Explain.

Exercise:

Problem:

Is Earth an inertial frame of reference? Is the Sun? Justify your response.

Exercise:

Problem:

When you are flying in a commercial jet, it may appear to you that the airplane is stationary and the Earth is moving beneath you. Is this point of view valid? Discuss briefly.

Glossary

relativity
: the study of how different observers measure the same event

special relativity
: the theory that, in an inertial frame of reference, the motion of an object is relative to the frame from which it is viewed or measured

inertial frame of reference
: a reference frame in which a body at rest remains at rest and a body in motion moves at a constant speed in a straight line unless acted on by an outside force

first postulate of special relativity
: the idea that the laws of physics are the same and can be stated in their simplest form in all inertial frames of reference

second postulate of special relativity
: the idea that the speed of light c is a constant, independent of the source

Michelson-Morley experiment
: an investigation performed in 1887 that proved that the speed of light in a vacuum is the same in all frames of reference from which it is viewed

Simultaneity And Time Dilation

- Describe simultaneity.
- Describe time dilation.
- Calculate γ.
- Compare proper time and the observer’s measured time.
- Explain why the twin paradox is a false paradox.

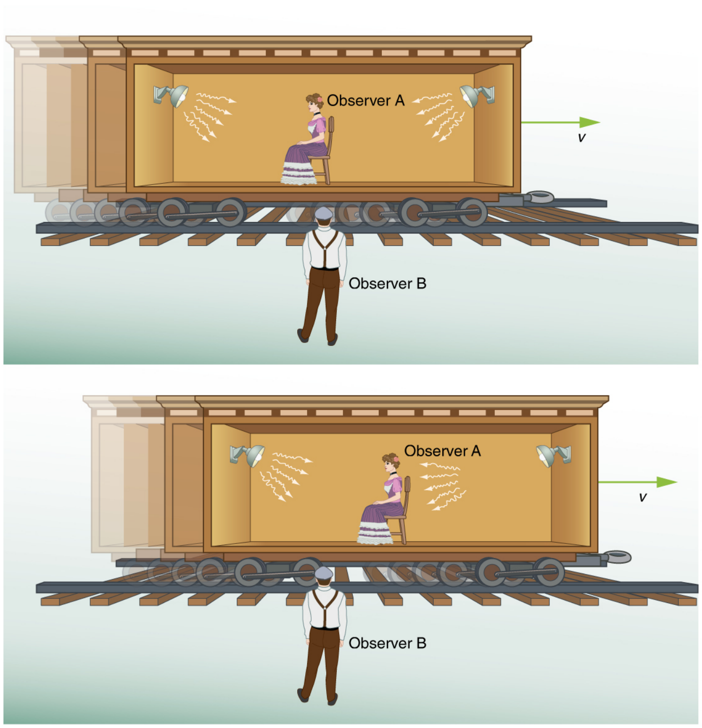
Elapsed time for a foot race is the same for all observers, but at relativistic speeds, elapsed time depends on the relative motion of the observer and the event that is observed. (credit: Jason Edward Scott Bain, Flickr)

Do time intervals depend on who observes them? Intuitively, we expect the time for a process, such as the elapsed time for a foot race, to be the same for all observers. Our experience has been that disagreements over elapsed time have to do with the accuracy of measuring time. When we carefully consider just how time is measured, however, we will find that elapsed time depends on the relative motion of an observer with respect to the process being measured.

Simultaneity

Consider how we measure elapsed time. If we use a stopwatch, for example, how do we know when to start and stop the watch? One method is to use the arrival of light from the event, such as observing a light turning green to start a drag race. The timing will be more accurate if some sort of electronic detection is used, avoiding human reaction times and other complications.

Now suppose we use this method to measure the time interval between two flashes of light produced by flash lamps. (See [link].) Two flash lamps with observer A midway between them are on a rail car that moves to the right relative to observer B. Observer B arranges for the light flashes to be emitted just as A passes B, so that both A and B are equidistant from the lamps when the light is emitted. Observer B measures the time interval between the arrival of the light flashes. According to postulate 2, the speed of light is not affected by the motion of the lamps relative to B. Therefore, light travels equal distances to him at equal speeds. Thus observer B measures the flashes to be simultaneous.



Observer B measures the elapsed time between the arrival of light flashes as described in the text. Observer A moves with the lamps on a rail car. Observer B views the light flashes occurring simultaneously. Observer B views the light on the right reaching observer A before the light on the left does.

Now consider what observer B sees happen to observer A. Observer B views light from the right reaching observer A before light from the left, because she has moved toward that flash lamp, lessening the distance the light must travel and reducing the time it takes to get to her. Light travels at speed c relative to both observers, but observer B remains equidistant between the points where the flashes were emitted, while A gets closer to the emission point on the right. From observer B’s point of view, then, there is a time interval between the arrival of the flashes to observer A. Observer B measures the flashes to arrive simultaneously relative to him but not relative to A.

Now consider what observer A sees happening. She sees the light from the right at the same time that she sees the light from the left. Since both lamps are the same distance from her in her reference frame, from her perspective, the flashes occurred at the same time. Here a relative velocity between observers affects whether two events are observed to be simultaneous. *Simultaneity is not absolute*

This illustrates the power of clear thinking. We might have guessed incorrectly that if light is emitted simultaneously, then two observers halfway between the sources would see the flashes simultaneously. But careful analysis shows this not to be the case. Einstein was brilliant at this type of *thought experiment* (in German, “Gedankenexperiment”). He very carefully considered how an observation is made and disregarded what might seem obvious. The validity of thought experiments, of course, is

determined by actual observation. The genius of Einstein is evidenced by the fact that experiments have repeatedly confirmed his theory of relativity.

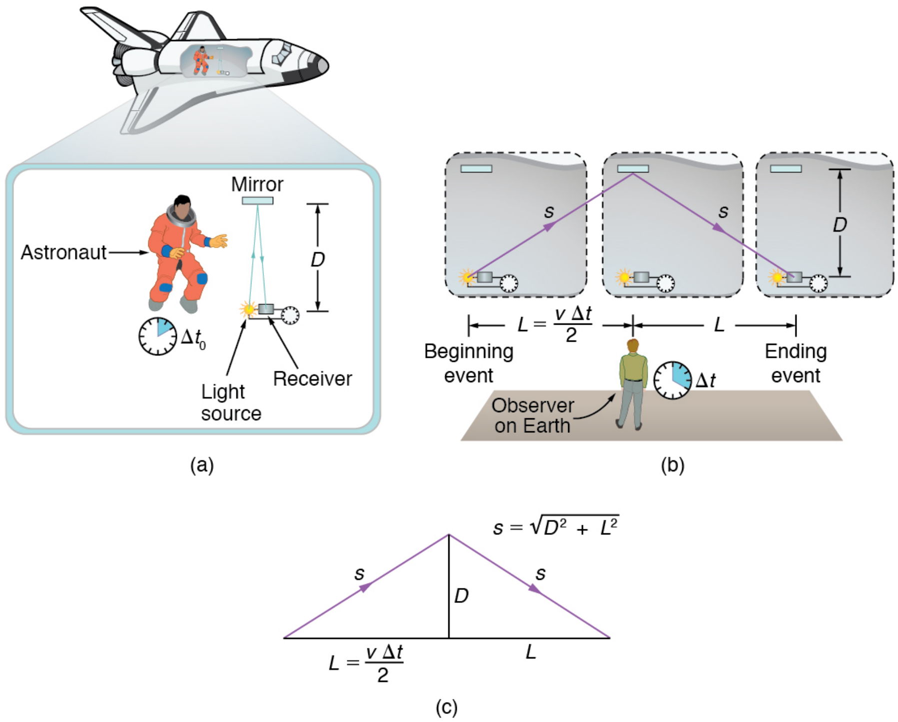
In summary: Two events are defined to be simultaneous if an observer measures them as occurring at the same time (such as by receiving light from the events). Two events are not necessarily simultaneous to all observers.

Time Dilation

The consideration of the measurement of elapsed time and simultaneity leads to an important relativistic effect.

Note:
Time dilation
Time dilation is the phenomenon of time passing slower for an observer who is moving relative to another observer.

Suppose, for example, an astronaut measures the time it takes for light to cross her ship, bounce off a mirror, and return. (See [link].) How does the elapsed time the astronaut measures compare with the elapsed time measured for the same event by a person on the Earth? Asking this question (another thought experiment) produces a profound result. We find that the elapsed time for a process depends on who is measuring it. In this case, the time measured by the astronaut is smaller than the time measured by the Earth-bound observer. The passage of time is different for the observers because the distance the light travels in the astronaut's frame is smaller than in the Earth-bound frame. Light travels at the same speed in each frame, and so it will take longer to travel the greater distance in the Earth-bound frame.



(a) An astronaut measures the time Δt_0 for light to cross her ship using an electronic timer. Light travels a distance $2D$ in the astronaut's frame. (b) A person on the Earth sees the light follow the longer path $2s$ and take a longer time Δt. (c) These triangles are used to find the relationship between the two distances $2D$ and $2s$.

To quantitatively verify that time depends on the observer, consider the paths followed by light as seen by each observer. (See [link](c).) The astronaut sees the light travel straight across and back for a total distance of $2D$, twice the width of her ship. The Earth-bound observer sees the light travel a total distance $2s$. Since the ship is moving at speed v to the right relative to the Earth, light moving to the right hits the mirror in this frame. Light travels at a speed c in both frames, and because time is the distance divided by speed, the time measured by the astronaut is

Equation:

$$\Delta t_0 = \frac{2D}{c}.$$

This time has a separate name to distinguish it from the time measured by the Earth-bound observer.

> **Note:**
> **Proper Time**
> **Proper time** Δt_0 is the time measured by an observer at rest relative to the event being observed.

In the case of the astronaut observe the reflecting light, the astronaut measures proper time. The time measured by the Earth-bound observer is
Equation:

$$\Delta t = \frac{2s}{c}.$$

To find the relationship between Δt_0 and Δt, consider the triangles formed by D and s. (See [link](c).) The third side of these similar triangles is L, the distance the astronaut moves as the light goes across her ship. In the frame of the Earth-bound observer,
Equation:

$$L = \frac{v\Delta t}{2}.$$

Using the Pythagorean Theorem, the distance s is found to be
Equation:

$$s = \sqrt{D^2 + \left(\frac{v\Delta t}{2}\right)^2}.$$

Substituting s into the expression for the time interval Δt gives
Equation:

$$\Delta t = \frac{2s}{c} = \frac{2\sqrt{D^2 + \left(\frac{v\Delta t}{2}\right)^2}}{c}.$$

We square this equation, which yields
Equation:

$$(\Delta t)^2 = \frac{4\left(D^2 + \frac{v^2(\Delta t)^2}{4}\right)}{c^2} = \frac{4D^2}{c^2} + \frac{v^2}{c^2}(\Delta t)^2.$$

Note that if we square the first expression we had for Δt_0, we get $(\Delta t_0)^2 = \frac{4D^2}{c^2}$. This term appears in the preceding equation, giving us a means to relate the two time intervals. Thus,
Equation:

$$(\Delta t)^2 = (\Delta t_0)^2 + \frac{v^2}{c^2}(\Delta t)^2.$$

Gathering terms, we solve for Δt:
Equation:

$$(\Delta t)^2\left(1 - \frac{v^2}{c^2}\right) = (\Delta t_0)^2.$$

Thus,
Equation:

$$(\Delta t)^2 = \frac{(\Delta t_0)^2}{1 - \frac{v^2}{c^2}}.$$

Taking the square root yields an important relationship between elapsed times:

Equation:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}} = \gamma \Delta t_0,$$

where

Equation:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

This equation for Δt is truly remarkable. First, as contended, elapsed time is not the same for different observers moving relative to one another, even though both are in inertial frames. Proper time Δt_0 measured by an observer, like the astronaut moving with the apparatus, is smaller than time measured by other observers. Since those other observers measure a longer time Δt, the effect is called time dilation. The Earth-bound observer sees time dilate (get longer) for a system moving relative to the Earth. Alternatively, according to the Earth-bound observer, time slows in the moving frame, since less time passes there. All clocks moving relative to an observer, including biological clocks such as aging, are observed to run slow compared with a clock stationary relative to the observer.

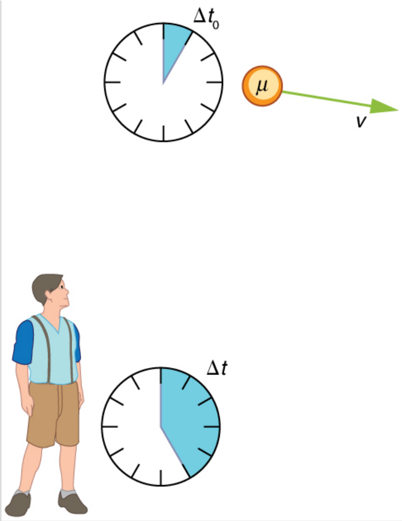
Note that if the relative velocity is much less than the speed of light ($v << c$), then $\frac{v^2}{c^2}$ is extremely small, and the elapsed times Δt and Δt_0 are nearly equal. At low velocities, modern relativity approaches classical physics—our everyday experiences have very small relativistic effects.

The equation $\Delta t = \gamma \Delta t_0$ also implies that relative velocity cannot exceed the speed of light. As v approaches c, Δt approaches infinity. This would imply that time in the astronaut's frame stops at the speed of light. If v exceeded c, then we would be taking the square root of a negative number, producing an imaginary value for Δt.

There is considerable experimental evidence that the equation $\Delta t = \gamma \Delta t_0$ is correct. One example is found in cosmic ray particles that continuously rain down on the Earth from deep space. Some collisions of these particles with nuclei in the upper atmosphere result in short-lived particles called muons. The half-life (amount of time for half of a material to decay) of a muon is $1.52\ \mu$s when it is at rest relative to the observer who measures the half-life. This is the proper time Δt_0. Muons produced by cosmic ray particles have a range of velocities, with some moving near the speed of light. It has been found that the muon's half-life as measured by an Earth-bound observer (Δt) varies with velocity exactly as predicted by the equation $\Delta t = \gamma \Delta t_0$. The faster the muon moves, the longer it lives. We on the Earth see the muon's half-life time dilated—as viewed from our frame, the muon decays more slowly than it does when at rest relative to us.

Example:
Calculating Δt for a Relativistic Event: How Long Does a Speedy Muon Live?

Suppose a cosmic ray colliding with a nucleus in the Earth's upper atmosphere produces a muon that has a velocity $v = 0.950c$. The muon then travels at constant velocity and lives $1.52\ \mu$s as measured in the muon's frame of reference. (You can imagine this as the muon's internal clock.) How long does the muon live as measured by an Earth-bound observer? (See [link].)



A muon in the Earth’s atmosphere lives longer as measured by an Earth-bound observer than measured by the muon’s internal clock.

Strategy

A clock moving with the system being measured observes the proper time, so the time we are given is $\Delta t_0 = 1.52\ \mu$s. The Earth-bound observer measures Δt as given by the equation $\Delta t = \gamma \Delta t_0$. Since we know the velocity, the calculation is straightforward.

Solution

1) Identify the knowns. $v = 0.950c$, $\Delta t_0 = 1.52\ \mu$s

2) Identify the unknown. Δt

3) Choose the appropriate equation.

Use,

Equation:

$$\Delta t = \gamma \Delta t_0,$$

where
Equation:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

4) Plug the knowns into the equation.
First find γ.
Equation:

$$\begin{aligned} \gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \\ &= \frac{1}{\sqrt{1-\frac{(0.950c)^2}{c^2}}} \\ &= \frac{1}{\sqrt{1-(0.950)^2}} \\ &= 3.20. \end{aligned}$$

Use the calculated value of γ to determine Δt.
Equation:

$$\begin{aligned} \Delta t &= \gamma \Delta t_0 \\ &= (3.20)(1.52\ \mu\text{s}) \\ &= 4.87\ \mu\text{s} \end{aligned}$$

Discussion
One implication of this example is that since $\gamma = 3.20$ at 95.0% of the speed of light ($v = 0.950c$), the relativistic effects are significant. The two time intervals differ by this factor of 3.20, where classically they would be the same. Something moving at $0.950c$ is said to be highly relativistic.

Another implication of the preceding example is that everything an astronaut does when moving at 95.0% of the speed of light relative to the Earth takes 3.20 times longer when observed from the Earth. Does the

astronaut sense this? Only if she looks outside her spaceship. All methods of measuring time in her frame will be affected by the same factor of 3.20. This includes her wristwatch, heart rate, cell metabolism rate, nerve impulse rate, and so on. She will have no way of telling, since all of her clocks will agree with one another because their relative velocities are zero. Motion is relative, not absolute. But what if she does look out the window?

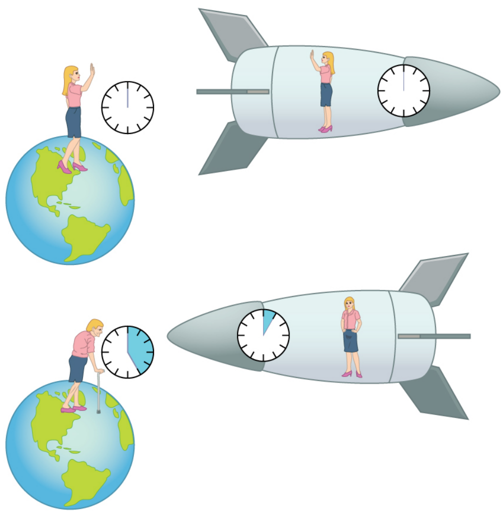
Note:
Real-World Connections
It may seem that special relativity has little effect on your life, but it is probably more important than you realize. One of the most common effects is through the Global Positioning System (GPS). Emergency vehicles, package delivery services, electronic maps, and communications devices are just a few of the common uses of GPS, and the GPS system could not work without taking into account relativistic effects. GPS satellites rely on precise time measurements to communicate. The signals travel at relativistic speeds. Without corrections for time dilation, the satellites could not communicate, and the GPS system would fail within minutes.

The Twin Paradox

An intriguing consequence of time dilation is that a space traveler moving at a high velocity relative to the Earth would age less than her Earth-bound twin. Imagine the astronaut moving at such a velocity that $\gamma = 30.0$, as in [link]. A trip that takes 2.00 years in her frame would take 60.0 years in her Earth-bound twin's frame. Suppose the astronaut traveled 1.00 year to another star system. She briefly explored the area, and then traveled 1.00 year back. If the astronaut was 40 years old when she left, she would be 42 upon her return. Everything on the Earth, however, would have aged 60.0 years. Her twin, if still alive, would be 100 years old.

The situation would seem different to the astronaut. Because motion is relative, the spaceship would seem to be stationary and the Earth would appear to move. (This is the sensation you have when flying in a jet.) If the

astronaut looks out the window of the spaceship, she will see time slow down on the Earth by a factor of $\gamma = 30.0$. To her, the Earth-bound sister will have aged only 2/30 (1/15) of a year, while she aged 2.00 years. The two sisters cannot both be correct.



The twin paradox asks why the traveling twin ages less than the Earth-bound twin. That is the prediction we obtain if we consider the Earth-bound twin's frame. In the astronaut's frame, however, the Earth is moving and time runs slower there. Who is correct?

As with all paradoxes, the premise is faulty and leads to contradictory conclusions. In fact, the astronaut's motion is significantly different from that of the Earth-bound twin. The astronaut accelerates to a high velocity

and then decelerates to view the star system. To return to the Earth, she again accelerates and decelerates. The Earth-bound twin does not experience these accelerations. So the situation is not symmetric, and it is not correct to claim that the astronaut will observe the same effects as her Earth-bound twin. If you use special relativity to examine the twin paradox, you must keep in mind that the theory is expressly based on inertial frames, which by definition are not accelerated or rotating. Einstein developed general relativity to deal with accelerated frames and with gravity, a prime source of acceleration. You can also use general relativity to address the twin paradox and, according to general relativity, the astronaut will age less. Some important conceptual aspects of general relativity are discussed in General Relativity and Quantum Gravity of this course.

In 1971, American physicists Joseph Hafele and Richard Keating verified time dilation at low relative velocities by flying extremely accurate atomic clocks around the Earth on commercial aircraft. They measured elapsed time to an accuracy of a few nanoseconds and compared it with the time measured by clocks left behind. Hafele and Keating's results were within experimental uncertainties of the predictions of relativity. Both special and general relativity had to be taken into account, since gravity and accelerations were involved as well as relative motion.

Exercise:

Check Your Understanding

Problem:1. What is γ if $v = 0.650c$?

Solution

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{1}{\sqrt{1-\frac{(0.650c)^2}{c^2}}} = 1.32$$

2. A particle travels at 1.90×10^8 m/s and lives 2.10×10^{-8} s when at rest relative to an observer. How long does the particle live as viewed in the laboratory?

Solution:

$$\Delta t = \frac{\Delta_t^0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{2.10\times10^{-8}\text{ s}}{\sqrt{1-\frac{(1.90\times10^8\text{ m/s})^2}{(3.00\times10^8\text{ m/s})^2}}} = 2.71\times10^{-8}\text{ s}$$

Section Summary

- Two events are defined to be simultaneous if an observer measures them as occurring at the same time. They are not necessarily simultaneous to all observers—simultaneity is not absolute.
- Time dilation is the phenomenon of time passing slower for an observer who is moving relative to another observer.
- Observers moving at a relative velocity v do not measure the same elapsed time for an event. Proper time Δt_0 is the time measured by an observer at rest relative to the event being observed. Proper time is related to the time Δt measured by an Earth-bound observer by the equation
  Equation:

  $$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma\Delta t_0,$$

  where
  Equation:

  $$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.$$

- The equation relating proper time and time measured by an Earth-bound observer implies that relative velocity cannot exceed the speed of light.
- The twin paradox asks why a twin traveling at a relativistic speed away and then back towards the Earth ages less than the Earth-bound twin. The premise to the paradox is faulty because the traveling twin is accelerating. Special relativity does not apply to accelerating frames of reference.

- Time dilation is usually negligible at low relative velocities, but it does occur, and it has been verified by experiment.

Conceptual Questions

Exercise:

Problem:

Does motion affect the rate of a clock as measured by an observer moving with it? Does motion affect how an observer moving relative to a clock measures its rate?

Exercise:

Problem:

To whom does the elapsed time for a process seem to be longer, an observer moving relative to the process or an observer moving with the process? Which observer measures proper time?

Exercise:

Problem:

How could you travel far into the future without aging significantly? Could this method also allow you to travel into the past?

Problems & Exercises

Exercise:

Problem: (a) What is γ if $v = 0.250c$? (b) If $v = 0.500c$?

Solution:

(a) 1.0328

(b) 1.15

Exercise:

Problem: (a) What is γ if $v = 0.100c$? (b) If $v = 0.900c$?

Exercise:

Problem:

Particles called π-mesons are produced by accelerator beams. If these particles travel at 2.70×10^8 m/s and live 2.60×10^{-8} s when at rest relative to an observer, how long do they live as viewed in the laboratory?

Solution:

5.96×10^{-8} s

Exercise:

Problem:

Suppose a particle called a kaon is created by cosmic radiation striking the atmosphere. It moves by you at $0.980c$, and it lives 1.24×10^{-8} s when at rest relative to an observer. How long does it live as you observe it?

Exercise:

Problem:

A neutral π-meson is a particle that can be created by accelerator beams. If one such particle lives 1.40×10^{-16} s as measured in the laboratory, and 0.840×10^{-16} s when at rest relative to an observer, what is its velocity relative to the laboratory?

Solution:

$0.800c$

Exercise:

Problem:

A neutron lives 900 s when at rest relative to an observer. How fast is the neutron moving relative to an observer who measures its life span to be 2065 s?

Exercise:

Problem:

If relativistic effects are to be less than 1%, then γ must be less than 1.01. At what relative velocity is $\gamma = 1.01$?

Solution:

$0.140c$

Exercise:

Problem:

If relativistic effects are to be less than 3%, then γ must be less than 1.03. At what relative velocity is $\gamma = 1.03$?

Exercise:

Problem:

(a) At what relative velocity is $\gamma = 1.50$? (b) At what relative velocity is $\gamma = 100$?

Solution:

(a) $0.745c$

(b) $0.99995c$ (to five digits to show effect)

Exercise:

Problem:

(a) At what relative velocity is $\gamma = 2.00$? (b) At what relative velocity is $\gamma = 10.0$?

Exercise:

Problem: **Unreasonable Results**

(a) Find the value of γ for the following situation. An Earth-bound observer measures 23.9 h to have passed while signals from a high-velocity space probe indicate that $24.0\ \mathrm{h}$ have passed on board. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) 0.996

(b) γ cannot be less than 1.

(c) Assumption that time is longer in moving ship is unreasonable.

Glossary

time dilation
: the phenomenon of time passing slower to an observer who is moving relative to another observer

proper time
: Δt_0. the time measured by an observer at rest relative to the event being observed: $\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \gamma \Delta t_0$, where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

twin paradox
: this asks why a twin traveling at a relativistic speed away and then back towards the Earth ages less than the Earth-bound twin. The

premise to the paradox is faulty because the traveling twin is accelerating, and special relativity does not apply to accelerating frames of reference

Length Contraction

- Describe proper length.
- Calculate length contraction.
- Explain why we don't notice these effects at everyday scales.

People might describe distances differently, but at relativistic speeds, the distances really are different. (credit: Corey Leopold, Flickr)

Have you ever driven on a road that seems like it goes on forever? If you look ahead, you might say you have about 10 km left to go. Another traveler might say the road ahead looks like it's about 15 km long. If you both measured the road, however, you would agree. Traveling at everyday speeds, the distance you both measure would be the same. You will read in this section, however, that this is not true at relativistic speeds. Close to the speed of light, distances measured are not the same when measured by different observers.

Proper Length

One thing all observers agree upon is relative speed. Even though clocks measure different elapsed times for the same process, they still agree that relative speed, which is distance divided by elapsed time, is the same. This

implies that distance, too, depends on the observer’s relative motion. If two observers see different times, then they must also see different distances for relative speed to be the same to each of them.

The muon discussed in [link] illustrates this concept. To an observer on the Earth, the muon travels at for from the time it is produced until it decays. Thus it travels a distance

Equation:

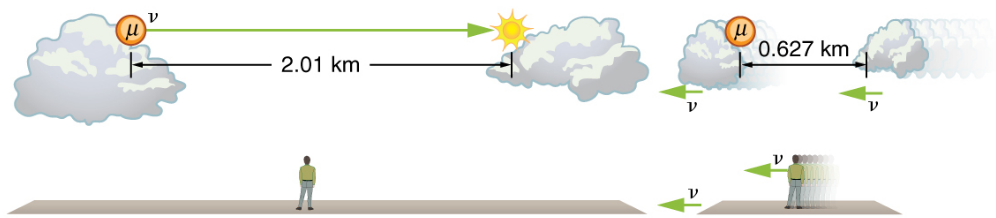
relative to the Earth. In the muon’s frame of reference, its lifetime is only . It has enough time to travel only

Equation:

The distance between the same two events (production and decay of a muon) depends on who measures it and how they are moving relative to it.

> **Note:**
> Proper Length
> **Proper length** is the distance between two points measured by an observer who is at rest relative to both of the points.

The Earth-bound observer measures the proper length , because the points at which the muon is produced and decays are stationary relative to the Earth. To the muon, the Earth, air, and clouds are moving, and so the distance it sees is not the proper length.



(a) The Earth-bound observer sees the muon travel 2.01 km between clouds. (b) The muon sees itself travel the same path, but only a distance of 0.627 km. The Earth, air, and clouds are moving relative to the muon in its frame, and all appear to have smaller lengths along the direction of travel.

Length Contraction

To develop an equation relating distances measured by different observers, we note that the velocity relative to the Earth-bound observer in our muon example is given by

Equation:

The time relative to the Earth-bound observer is , since the object being timed is moving relative to this observer. The velocity relative to the moving observer is given by

Equation:

The moving observer travels with the muon and therefore observes the proper time . The two velocities are identical; thus,

Equation:

We know that . Substituting this equation into the relationship above gives

Equation:

Substituting for gives an equation relating the distances measured by different observers.

Note:

Length Contraction

Length contraction is the shortening of the measured length of an object moving relative to the observer's frame.

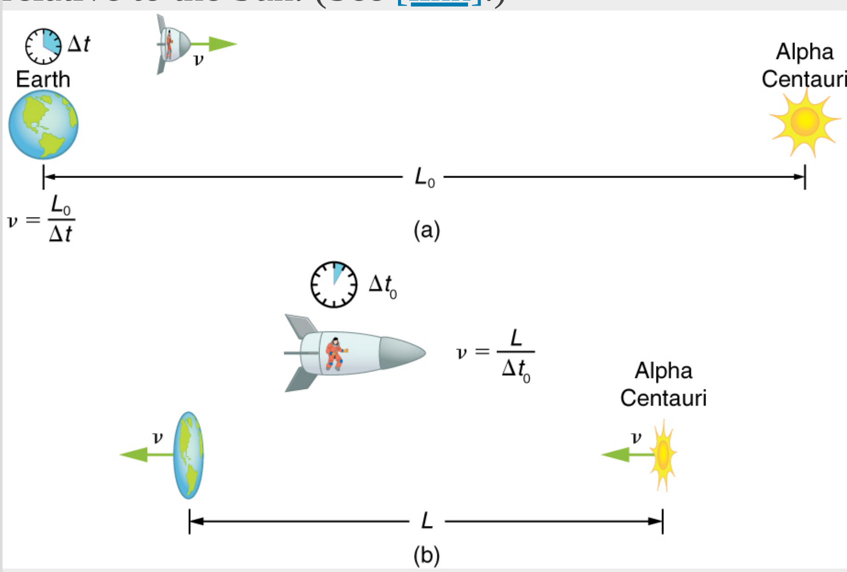
Equation:

If we measure the length of anything moving relative to our frame, we find its length to be smaller than the proper length that would be measured if the object were stationary. For example, in the muon's reference frame, the distance between the points where it was produced and where it decayed is shorter. Those points are fixed relative to the Earth but moving relative to the muon. Clouds and other objects are also contracted along the direction of motion in the muon's reference frame.

Example:

Calculating Length Contraction: The Distance between Stars Contracts when You Travel at High Velocity

Suppose an astronaut, such as the twin discussed in [Simultaneity and Time Dilation](), travels so fast that . (a) She travels from the Earth to the nearest star system, Alpha Centauri, 4.300 light years (ly) away as measured by an Earth-bound observer. How far apart are the Earth and Alpha Centauri as measured by the astronaut? (b) In terms of , what is her velocity relative to the Earth? You may neglect the motion of the Earth relative to the Sun. (See [link].)



(a) The Earth-bound observer measures the proper distance between the Earth and the Alpha Centauri. (b) The astronaut observes a length contraction, since the Earth and the Alpha Centauri move relative to her ship. She can travel this shorter distance in a smaller time (her proper time) without exceeding the speed of light.

Strategy

First note that a light year (ly) is a convenient unit of distance on an astronomical scale—it is the distance light travels in a year. For part (a), note that the 4.300 ly distance between the Alpha Centauri and the Earth is

the proper distance , because it is measured by an Earth-bound observer to whom both stars are (approximately) stationary. To the astronaut, the Earth and the Alpha Centauri are moving by at the same velocity, and so the distance between them is the contracted length . In part (b), we are given , and so we can find by rearranging the definition of to express in terms of .

Solution for (a)

1. Identify the knowns. ;
2. Identify the unknown.
3. Choose the appropriate equation. —
4. Rearrange the equation to solve for the unknown.
   Equation:

—

———

Solution for (b)

1. Identify the known.
2. Identify the unknown. in terms of
3. Choose the appropriate equation.
4. Rearrange the equation to solve for the unknown.
   Equation:

   Squaring both sides of the equation and rearranging terms gives
   Equation:

so that
Equation:

and
Equation:

Taking the square root, we find
Equation:

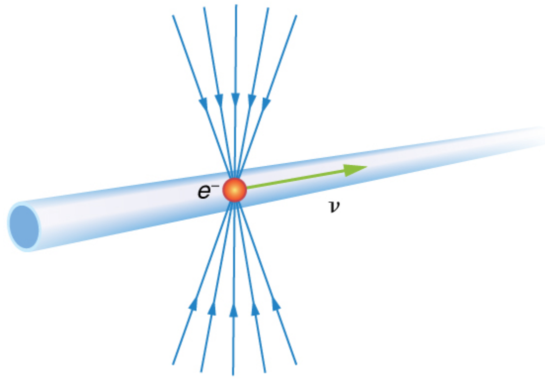
which is rearranged to produce a value for the velocity
Equation:

Discussion
First, remember that you should not round off calculations until the final result is obtained, or you could get erroneous results. This is especially true for special relativity calculations, where the differences might only be revealed after several decimal places. The relativistic effect is large here (γ), and we see that is approaching (not equaling) the speed of light. Since the distance as measured by the astronaut is so much smaller, the astronaut can travel it in much less time in her frame.

People could be sent very large distances (thousands or even millions of light years) and age only a few years on the way if they traveled at extremely high velocities. But, like emigrants of centuries past, they would leave the Earth they know forever. Even if they returned, thousands to millions of years would have passed on the Earth, obliterating most of what now exists. There is also a more serious practical obstacle to traveling at such velocities; immensely greater energies than classical physics predicts would be needed to achieve such high velocities. This will be discussed in Relatavistic Energy.

Why don’t we notice length contraction in everyday life? The distance to the grocery shop does not seem to depend on whether we are moving or not.

Examining the equation —, we see that at low velocities ( ) the lengths are nearly equal, the classical expectation. But length contraction is real, if not commonly experienced. For example, a charged particle, like an electron, traveling at relativistic velocity has electric field lines that are compressed along the direction of motion as seen by a stationary observer. (See [link].) As the electron passes a detector, such as a coil of wire, its field interacts much more briefly, an effect observed at particle accelerators such as the 3 km long Stanford Linear Accelerator (SLAC). In fact, to an electron traveling down the beam pipe at SLAC, the accelerator and the Earth are all moving by and are length contracted. The relativistic effect is so great than the accelerator is only 0.5 m long to the electron. It is actually easier to get the electron beam down the pipe, since the beam does not have to be as precisely aimed to get down a short pipe as it would down one 3 km long. This, again, is an experimental verification of the Special Theory of Relativity.



The electric field lines of a high-velocity charged particle are compressed along the direction of motion by length contraction. This produces a different signal when the particle goes through a coil, an experimentally verified effect of length contraction.

Exercise:
Check Your Understanding

Problem:

A particle is traveling through the Earth's atmosphere at a speed of . To an Earth-bound observer, the distance it travels is 2.50 km. How far does the particle travel in the particle's frame of reference?

Solution:
Answer
Equation:

Summary

- All observers agree upon relative speed.
- Distance depends on an observer’s motion. Proper length is the distance between two points measured by an observer who is at rest relative to both of the points. Earth-bound observers measure proper length when measuring the distance between two points that are stationary relative to the Earth.
- Length contraction is the shortening of the measured length of an object moving relative to the observer’s frame:

  Equation:

Conceptual Questions

Exercise:

Problem:

To whom does an object seem greater in length, an observer moving with the object or an observer moving relative to the object? Which observer measures the object’s proper length?

Exercise:

Problem:

Relativistic effects such as time dilation and length contraction are present for cars and airplanes. Why do these effects seem strange to us?

Exercise:

Problem:

Suppose an astronaut is moving relative to the Earth at a significant fraction of the speed of light. (a) Does he observe the rate of his clocks to have slowed? (b) What change in the rate of Earth-bound clocks does he see? (c) Does his ship seem to him to shorten? (d) What about the distance between stars that lie on lines parallel to his motion? (e) Do he and an Earth-bound observer agree on his velocity relative to the Earth?

Problems & Exercises

Exercise:

Problem:

A spaceship, 200 m long as seen on board, moves by the Earth at . What is its length as measured by an Earth-bound observer?

Solution:

48.6 m

Exercise:

Problem:

How fast would a 6.0 m-long sports car have to be going past you in order for it to appear only 5.5 m long?

Exercise:

Problem:

(a) How far does the muon in [link] travel according to the Earth-bound observer? (b) How far does it travel as viewed by an observer moving with it? Base your calculation on its velocity relative to the Earth and the time it lives (proper time). (c) Verify that these two distances are related through length contraction γ .

Solution:

(a) 1.387 km = 1.39 km

(b) 0.433 km

(c)

Thus, the distances in parts (a) and (b) are related when .

Exercise:

Problem:

(a) How long would the muon in [link] have lived as observed on the Earth if its velocity was ? (b) How far would it have traveled as observed on the Earth? (c) What distance is this in the muon's frame?

Exercise:

Problem:

(a) How long does it take the astronaut in [link] to travel 4.30 ly at (as measured by the Earth-bound observer)? (b) How long does it take according to the astronaut? (c) Verify that these two times are related through time dilation with γ as given.

Solution:

(a) 4.303 y (to four digits to show any effect)

(b) 0.1434 y

(c) γ

Thus, the two times are related when .

Exercise:

Problem:

(a) How fast would an athlete need to be running for a 100-m race to look 100 yd long? (b) Is the answer consistent with the fact that relativistic effects are difficult to observe in ordinary circumstances? Explain.

Exercise:

Problem: **Unreasonable Results**

(a) Find the value of for the following situation. An astronaut measures the length of her spaceship to be 25.0 m, while an Earth-bound observer measures it to be 100 m. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) 0.250

(b) must be ≥1

(c) The Earth-bound observer must measure a shorter length, so it is unreasonable to assume a longer length.

Exercise:

Problem: **Unreasonable Results**

A spaceship is heading directly toward the Earth at a velocity of . The astronaut on board claims that he can send a canister toward the Earth at relative to the Earth. (a) Calculate the velocity the canister must have relative to the spaceship. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Glossary

proper length

; the distance between two points measured by an observer who is at rest relative to both of the points; Earth-bound observers measure proper length when measuring the distance between two points that are stationary relative to the Earth

length contraction

, the shortening of the measured length of an object moving relative to the observer's frame: — —

Relativistic Addition of Velocities

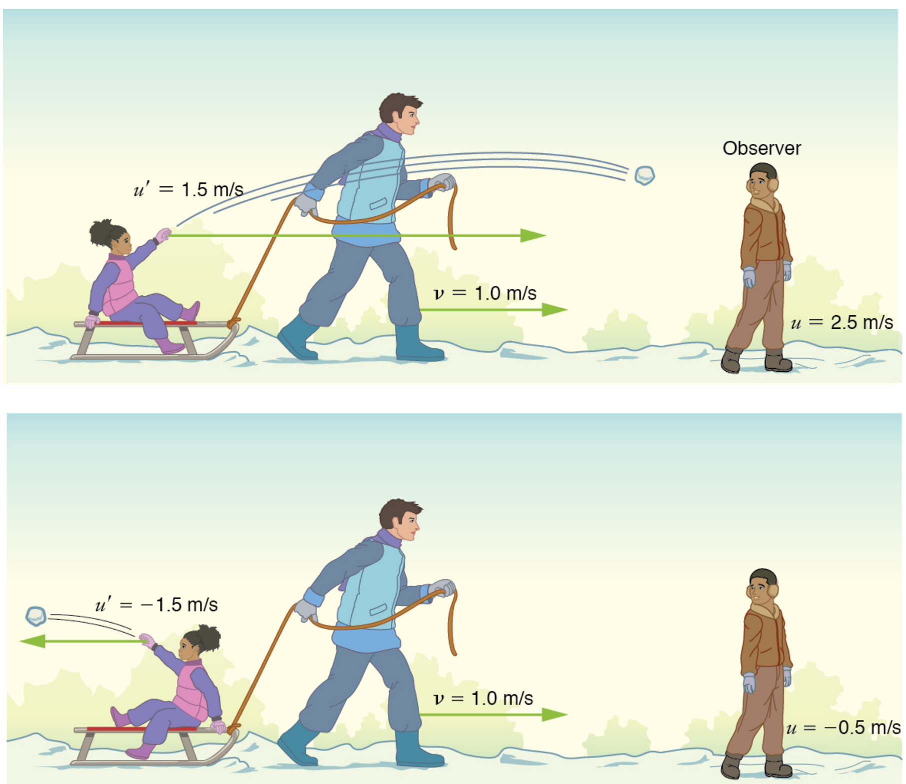
- Calculate relativistic velocity addition.
- Explain when relativistic velocity addition should be used instead of classical addition of velocities.
- Calculate relativistic Doppler shift.

The total velocity of a kayak, like this one on the Deerfield River in Massachusetts, is its velocity relative to the water as well as the water’s velocity relative to the riverbank. (credit: abkfenris, Flickr)

If you’ve ever seen a kayak move down a fast-moving river, you know that remaining in the same place would be hard. The river current pulls the kayak along. Pushing the oars back against the water can move the kayak forward in the water, but that only accounts for part of the velocity. The kayak’s motion is an example of classical addition of velocities. In classical physics, velocities add as vectors. The kayak’s velocity is the vector sum of its velocity relative to the water and the water’s velocity relative to the riverbank.

Classical Velocity Addition

For simplicity, we restrict our consideration of velocity addition to one-dimensional motion. Classically, velocities add like regular numbers in one-dimensional motion. (See [link].) Suppose, for example, a girl is riding in a sled at a speed 1.0 m/s relative to an observer. She throws a snowball first forward, then backward at a speed of 1.5 m/s relative to the sled. We denote direction with plus and minus signs in one dimension; in this example, forward is positive. Let v be the velocity of the sled relative to the Earth, u the velocity of the snowball relative to the Earth-bound observer, and $u\prime$ the velocity of the snowball relative to the sled.



Classically, velocities add like ordinary numbers in one-dimensional motion. Here the girl throws a snowball forward and then backward from a sled. The velocity of the sled relative to the Earth is $v=1.0\text{ m/s}$. The velocity of the snowball relative

to the sled is $u\prime$, while its velocity relative to the Earth is u. Classically, $\text{u=v+u}\prime$.

Note:
Classical Velocity Addition
Equation:

$$\text{u=v+u}\prime$$

Thus, when the girl throws the snowball forward, $u=1.0\text{ m/s}+1.5\text{ m/s}=2.5\text{ m/s}$. It makes good intuitive sense that the snowball will head towards the Earth-bound observer faster, because it is thrown forward from a moving vehicle. When the girl throws the snowball backward, $u=1.0\text{ m/s}+(-1.5\text{ m/s})=-0.5\text{ m/s}$. The minus sign means the snowball moves away from the Earth-bound observer.

Relativistic Velocity Addition

The second postulate of relativity (verified by extensive experimental observation) says that classical velocity addition does not apply to light. Imagine a car traveling at night along a straight road, as in [link]. If classical velocity addition applied to light, then the light from the car's headlights would approach the observer on the sidewalk at a speed u=v+c. But we know that light will move away from the car at speed c relative to the driver of the car, and light will move towards the observer on the sidewalk at speed c, too.

According to experiment and the second postulate of relativity, light from the car's headlights moves away from the car at speed c and towards the observer on the sidewalk at speed c. Classical velocity addition is not valid.

Note:

Relativistic Velocity Addition

Either light is an exception, or the classical velocity addition formula only works at low velocities. The latter is the case. The correct formula for one-dimensional **relativistic velocity addition** is

Equation:

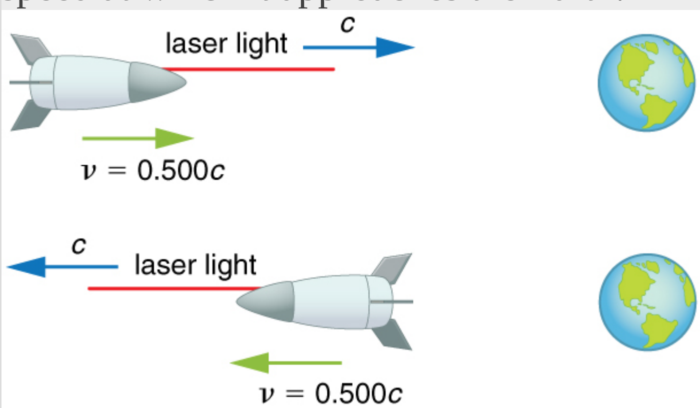
$$u = \frac{v + u\prime}{1 + \frac{vu\prime}{c^2}},$$

where v is the relative velocity between two observers, u is the velocity of an object relative to one observer, and $u\prime$ is the velocity relative to the other observer. (For ease of visualization, we often choose to measure u in our reference frame, while someone moving at v relative to us measures $u\prime$.) Note that the term $\frac{vu\prime}{c^2}$ becomes very small at low velocities, and $u = \frac{v+u\prime}{1+\frac{vu\prime}{c^2}}$ gives a result very close to classical velocity addition. As

before, we see that classical velocity addition is an excellent approximation to the correct relativistic formula for small velocities. No wonder that it seems correct in our experience.

Example:
Showing that the Speed of Light towards an Observer is Constant (in a Vacuum): The Speed of Light is the Speed of Light

Suppose a spaceship heading directly towards the Earth at half the speed of light sends a signal to us on a laser-produced beam of light. Given that the light leaves the ship at speed c as observed from the ship, calculate the speed at which it approaches the Earth.



Strategy

Because the light and the spaceship are moving at relativistic speeds, we cannot use simple velocity addition. Instead, we can determine the speed at which the light approaches the Earth using relativistic velocity addition.

Solution

1. Identify the knowns. v$=0.500c$; $u\prime= c$
2. Identify the unknown. u
3. Choose the appropriate equation. $u = \frac{v+u\prime}{1+\frac{vu\prime}{c^2}}$
4. Plug the knowns into the equation.
   Equation:

$$\begin{aligned} u &= \frac{v+u\prime}{1+\frac{vu\prime}{c^2}} \\ &= \frac{0.500c+c}{1+\frac{(0.500c)(c)}{c^2}} \\ &= \frac{(0.500+1)c}{1+\frac{0.500c^2}{c^2}} \\ &= \frac{1.500c}{1+0.500} \\ &= \frac{1.500c}{1.500} \\ &= c \end{aligned}$$

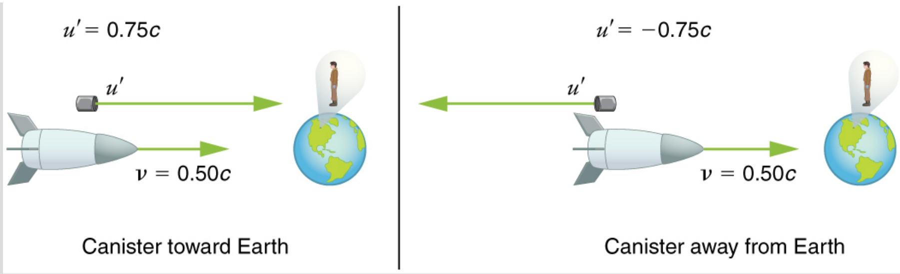
Discussion

Relativistic velocity addition gives the correct result. Light leaves the ship at speed c and approaches the Earth at speed c. The speed of light is independent of the relative motion of source and observer, whether the observer is on the ship or Earth-bound.

Velocities cannot add to greater than the speed of light, provided that v is less than c and $u\prime$ does not exceed c. The following example illustrates that relativistic velocity addition is not as symmetric as classical velocity addition.

Example:
Comparing the Speed of Light towards and away from an Observer: Relativistic Package Delivery

Suppose the spaceship in the previous example is approaching the Earth at half the speed of light and shoots a canister at a speed of $0.750c$. (a) At what velocity will an Earth-bound observer see the canister if it is shot directly towards the Earth? (b) If it is shot directly away from the Earth? (See [link].)



Strategy

Because the canister and the spaceship are moving at relativistic speeds, we must determine the speed of the canister by an Earth-bound observer using relativistic velocity addition instead of simple velocity addition.

Solution for (a)

1. Identify the knowns. v=0.500c; $u\prime = 0.750c$
2. Identify the unknown. u
3. Choose the appropriate equation. u=$\frac{v+u\prime}{1+\frac{vu\prime}{c^2}}$
4. Plug the knowns into the equation.
   Equation:

$$\begin{aligned} u &= \frac{v+u\prime}{1+\frac{vu\prime}{c^2}} \\ &= \frac{0.500\text{c} + 0.750c}{1+\frac{(0.500c)(0.750c)}{c^2}} \\ &= \frac{1.250c}{1+0.375} \\ &= 0.909c \end{aligned}$$

Solution for (b)

1. Identify the knowns. $v = 0.500c$; $u\prime = -0.750c$
2. Identify the unknown. u
3. Choose the appropriate equation. $u = \frac{v+u\prime}{1+\frac{vu\prime}{c^2}}$
4. Plug the knowns into the equation.
   Equation:

$$\begin{aligned} u &= \frac{v+u\prime}{1+\frac{vu\prime}{c^2}} \\ &= \frac{0.500c + (-0.750c)}{1+\frac{(0.500c)(-0.750c)}{c^2}} \\ &= \frac{-0.250c}{1-0.375} \\ &= -0.400c \end{aligned}$$

Discussion
The minus sign indicates velocity away from the Earth (in the opposite direction from v), which means the canister is heading towards the Earth in part (a) and away in part (b), as expected. But relativistic velocities do not add as simply as they do classically. In part (a), the canister does approach the Earth faster, but not at the simple sum of $1.250c$. The total velocity is less than you would get classically. And in part (b), the canister moves away from the Earth at a velocity of $-0.400c$, which is *faster* than the $-0.250c$ you would expect classically. The velocities are not even symmetric. In part (a) the canister moves $0.409c$ faster than the ship relative to the Earth, whereas in part (b) it moves $0.900c$ slower than the ship.

Doppler Shift

Although the speed of light does not change with relative velocity, the frequencies and wavelengths of light do. First discussed for sound waves, a Doppler shift occurs in any wave when there is relative motion between source and observer.

Note:
Relativistic Doppler Effects
The observed wavelength of electromagnetic radiation is longer (called a red shift) than that emitted by the source when the source moves away

from the observer and shorter (called a blue shift) when the source moves towards the observer.

Equation:

$$=\lambda_{\text{obs}}=\lambda_s\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}.$$

In the Doppler equation, λ_{obs} is the observed wavelength, λ_s is the source wavelength, and u is the relative velocity of the source to the observer. The velocity u is positive for motion away from an observer and negative for motion toward an observer. In terms of source frequency and observed frequency, this equation can be written

Equation:

$$f_{\text{obs}}=\text{f}_s\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}}.$$

Notice that the – and + signs are different than in the wavelength equation.

Note:

Career Connection: Astronomer

If you are interested in a career that requires a knowledge of special relativity, there’s probably no better connection than astronomy. Astronomers must take into account relativistic effects when they calculate distances, times, and speeds of black holes, galaxies, quasars, and all other astronomical objects. To have a career in astronomy, you need at least an undergraduate degree in either physics or astronomy, but a Master’s or doctoral degree is often required. You also need a good background in high-level mathematics.

Example:
Calculating a Doppler Shift: Radio Waves from a Receding Galaxy
Suppose a galaxy is moving away from the Earth at a speed $0.825c$. It emits radio waves with a wavelength of 0.525 m. What wavelength would we detect on the Earth?
Strategy
Because the galaxy is moving at a relativistic speed, we must determine the Doppler shift of the radio waves using the relativistic Doppler shift instead of the classical Doppler shift.
Solution

1. Identify the knowns. u=$0.825c$; $\lambda_s = 0.525\ m$
2. Identify the unknown. λ_{obs}
3. Choose the appropriate equation. $\lambda_{\text{obs}}=\lambda_s\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}$
4. Plug the knowns into the equation.
   Equation:

$$\begin{aligned}\lambda_{\text{obs}} &= \lambda_s\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}\\ &= (0.525\text{ m})\sqrt{\frac{1+\frac{0.825c}{c}}{1-\frac{0.825c}{c}}}\\ &= 1.70\text{ m.}\end{aligned}$$

Discussion
Because the galaxy is moving away from the Earth, we expect the wavelengths of radiation it emits to be redshifted. The wavelength we calculated is 1.70 m, which is redshifted from the original wavelength of 0.525 m.

The relativistic Doppler shift is easy to observe. This equation has everyday applications ranging from Doppler-shifted radar velocity measurements of transportation to Doppler-radar storm monitoring. In astronomical

observations, the relativistic Doppler shift provides velocity information such as the motion and distance of stars.

Exercise:

Check Your Understanding

Problem:

Suppose a space probe moves away from the Earth at a speed $0.350c$. It sends a radio wave message back to the Earth at a frequency of 1.50 GHz. At what frequency is the message received on the Earth?

Solution:

Answer

Equation:

$$f_{\text{obs}} = \mathrm{f}_s \sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}} = (1.50\text{ GHz})\sqrt{\frac{1-\frac{0.350\mathrm{c}}{c}}{1+\frac{0.350\mathrm{c}}{c}}} = 1.04\text{ GHz}$$

Section Summary

- With classical velocity addition, velocities add like regular numbers in one-dimensional motion: $u=v+u\prime$, where v is the velocity between two observers, u is the velocity of an object relative to one observer, and $u\prime$ is the velocity relative to the other observer.
- Velocities cannot add to be greater than the speed of light. Relativistic velocity addition describes the velocities of an object moving at a relativistic speed:
  Equation:

$$\mathrm{u} = \frac{v+u\prime}{1+\frac{vu\prime}{c^2}}$$

- An observer of electromagnetic radiation sees **relativistic Doppler effects** if the source of the radiation is moving relative to the observer. The wavelength of the radiation is longer (called a red shift) than that

emitted by the source when the source moves away from the observer and shorter (called a blue shift) when the source moves toward the observer. The shifted wavelength is described by the equation
Equation:

$$\lambda_{\text{obs}}=\lambda_s\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}$$

λ_{obs} is the observed wavelength, λ_s is the source wavelength, and u is the relative velocity of the source to the observer.

Conceptual Questions

Exercise:

Problem:

Explain the meaning of the terms “red shift” and “blue shift” as they relate to the relativistic Doppler effect.

Exercise:

Problem:

What happens to the relativistic Doppler effect when relative velocity is zero? Is this the expected result?

Exercise:

Problem:

Is the relativistic Doppler effect consistent with the classical Doppler effect in the respect that λ_{obs} is larger for motion away?

Exercise:

Problem:

All galaxies farther away than about 50×10^6 ly exhibit a red shift in their emitted light that is proportional to distance, with those farther and farther away having progressively greater red shifts. What does this imply, assuming that the only source of red shift is relative motion? (Hint: At these large distances, it is space itself that is expanding, but the effect on light is the same.)

Problems & Exercises

Exercise:

Problem:

Suppose a spaceship heading straight towards the Earth at $0.750c$ can shoot a canister at $0.500c$ relative to the ship. (a) What is the velocity of the canister relative to the Earth, if it is shot directly at the Earth? (b) If it is shot directly away from the Earth?

Solution:

(a) $0.909c$

(b) $0.400c$

Exercise:

Problem:

Repeat the previous problem with the ship heading directly away from the Earth.

Exercise:

Problem:

If a spaceship is approaching the Earth at $0.100c$ and a message capsule is sent toward it at $0.100c$ relative to the Earth, what is the speed of the capsule relative to the ship?

Solution:

$0.198c$

Exercise:

Problem:

(a) Suppose the speed of light were only 3000 m/s. A jet fighter moving toward a target on the ground at 800 m/s shoots bullets, each having a muzzle velocity of 1000 m/s. What are the bullets' velocity relative to the target? (b) If the speed of light was this small, would you observe relativistic effects in everyday life? Discuss.

Exercise:

Problem:

If a galaxy moving away from the Earth has a speed of 1000 km/s and emits 656 nm light characteristic of hydrogen (the most common element in the universe). (a) What wavelength would we observe on the Earth? (b) What type of electromagnetic radiation is this? (c) Why is the speed of the Earth in its orbit negligible here?

Solution:

a) 658 nm

b) red

c) $v/\text{c} = 9.92 \times 10^{-5}$ (negligible)

Exercise:

Problem:

A space probe speeding towards the nearest star moves at $0.250c$ and sends radio information at a broadcast frequency of 1.00 GHz. What frequency is received on the Earth?

Exercise:

Problem:

If two spaceships are heading directly towards each other at $0.800c$, at what speed must a canister be shot from the first ship to approach the other at $0.999c$ as seen by the second ship?

Solution:

$0.991c$

Exercise:

Problem:

Two planets are on a collision course, heading directly towards each other at $0.250c$. A spaceship sent from one planet approaches the second at $0.750c$ as seen by the second planet. What is the velocity of the ship relative to the first planet?

Exercise:

Problem:

When a missile is shot from one spaceship towards another, it leaves the first at $0.950c$ and approaches the other at $0.750c$. What is the relative velocity of the two ships?

Solution:

$-0.696c$

Exercise:

Problem:

What is the relative velocity of two spaceships if one fires a missile at the other at $0.750c$ and the other observes it to approach at $0.950c$?

Exercise:

Problem:

Near the center of our galaxy, hydrogen gas is moving directly away from us in its orbit about a black hole. We receive 1900 nm electromagnetic radiation and know that it was 1875 nm when emitted by the hydrogen gas. What is the speed of the gas?

Solution:

$0.01324c$

Exercise:

Problem:

A highway patrol officer uses a device that measures the speed of vehicles by bouncing radar off them and measuring the Doppler shift. The outgoing radar has a frequency of 100 GHz and the returning echo has a frequency 15.0 kHz higher. What is the velocity of the vehicle? Note that there are two Doppler shifts in echoes. Be certain not to round off until the end of the problem, because the effect is small.

Exercise:

Problem:

Prove that for any relative velocity v between two observers, a beam of light sent from one to the other will approach at speed c (provided that v is less than c, of course).

Solution:

$u\prime = c$, so

$$\begin{aligned} u &= \frac{\mathrm{v+u\prime}}{1+(\mathrm{vu\prime}/c^2)} = \frac{\mathrm{v+c}}{1+(\mathrm{vc}/c^2)} = \frac{\mathrm{v+c}}{1+(v/c)} \\ &= \frac{c(\mathrm{v+c})}{\mathrm{c+v}} = c \end{aligned}$$

Exercise:

Problem:

Show that for any relative velocity v between two observers, a beam of light projected by one directly away from the other will move away at the speed of light (provided that v is less than c, of course).

Exercise:

Problem:

(a) All but the closest galaxies are receding from our own Milky Way Galaxy. If a galaxy 12.0×10^9 ly ly away is receding from us at 0. $0.900c$, at what velocity relative to us must we send an exploratory probe to approach the other galaxy at $0.990c$, as measured from that galaxy? (b) How long will it take the probe to reach the other galaxy as measured from the Earth? You may assume that the velocity of the other galaxy remains constant. (c) How long will it then take for a radio signal to be beamed back? (All of this is possible in principle, but not practical.)

Solution:

a) $0.99947c$

b) 1.2064×10^{11} y

c) 1.2058×10^{11} y (all to sufficient digits to show effects)

Glossary

classical velocity addition
: the method of adding velocities when $v << c$; velocities add like regular numbers in one-dimensional motion: $u = v + u\prime$, where v is the

velocity between two observers, u is the velocity of an object relative to one observer, and $u\prime$ is the velocity relative to the other observer

relativistic velocity addition

the method of adding velocities of an object moving at a relativistic speed: $\text{u}=\frac{v+u\prime}{1+\frac{vu\prime}{c^2}}$, where v is the relative velocity between two observers, u is the velocity of an object relative to one observer, and $u\prime$ is the velocity relative to the other observer

relativistic Doppler effects

a change in wavelength of radiation that is moving relative to the observer; the wavelength of the radiation is longer (called a red shift) than that emitted by the source when the source moves away from the observer and shorter (called a blue shift) when the source moves toward the observer; the shifted wavelength is described by the equation

Equation:

$$\lambda_{\text{obs}}=\lambda_s\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}$$

where λ_{obs} is the observed wavelength, λ_s is the source wavelength, and u is the velocity of the source to the observer

Relativistic Momentum

- Calculate relativistic momentum.
- Explain why the only mass it makes sense to talk about is rest mass.

Momentum is an important concept for these football players from the University of California at Berkeley and the University of California at Davis. Players with more mass often have a larger impact because their momentum is larger. For objects moving at relativistic speeds, the effect is even greater. (credit: John Martinez Pavliga)

In classical physics, momentum is a simple product of mass and velocity. However, we saw in the last section that when special relativity is taken into account, massive objects have a speed limit. What effect do you think mass and velocity have on the momentum of objects moving at relativistic speeds?

Momentum is one of the most important concepts in physics. The broadest form of Newton's second law is stated in terms of momentum. Momentum is conserved whenever the net external force on a system is zero. This makes momentum conservation a fundamental tool for analyzing collisions. All of Work, Energy, and Energy Resources is devoted to momentum, and momentum has been important for many other topics as well, particularly where collisions were involved. We will see that momentum has the same importance in modern physics. Relativistic momentum is conserved, and much of what we know about subatomic structure comes from the analysis of collisions of accelerator-produced relativistic particles.

The first postulate of relativity states that the laws of physics are the same in all inertial frames. Does the law of conservation of momentum survive this requirement at high velocities? The answer is yes, provided that the momentum is defined as follows.

Note:

Relativistic Momentum

Relativistic momentum p is classical momentum multiplied by the relativistic factor γ.

Equation:

$$p = \gamma m u,$$

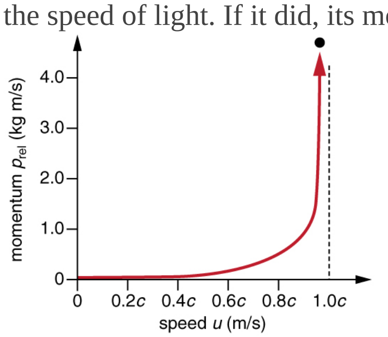
where m is the **rest mass** of the object, u is its velocity relative to an observer, and the relativistic factor

Equation:

$$\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}.$$

Note that we use u for velocity here to distinguish it from relative velocity v between observers. Only one observer is being considered here. With p defined in this way, total momentum p_{tot} is conserved whenever the net external force is zero, just as in classical physics. Again we see that the relativistic quantity becomes virtually the same as the classical at low velocities. That is, relativistic momentum $\gamma m u$ becomes the classical $m u$ at low velocities, because γ is very nearly equal to 1 at low velocities.

Relativistic momentum has the same intuitive feel as classical momentum. It is greatest for large masses moving at high velocities, but, because of the factor γ, relativistic momentum approaches infinity as u approaches c. (See [link].) This is another indication that an object with mass cannot reach the speed of light. If it did, its momentum would become infinite, an unreasonable value.



Relativistic momentum approaches infinity as the velocity of an object approaches the speed of light.

Note:

Misconception Alert: Relativistic Mass and Momentum

The relativistically correct definition of momentum as $p = \gamma m u$ is sometimes taken to imply that mass varies with velocity: $m_{\text{var}} = \gamma m$, particularly in older textbooks. However, note that m is the mass of

the object as measured by a person at rest relative to the object. Thus, m is defined to be the rest mass, which could be measured at rest, perhaps using gravity. When a mass is moving relative to an observer, the only way that its mass can be determined is through collisions or other means in which momentum is involved. Since the mass of a moving object cannot be determined independently of momentum, the only meaningful mass is rest mass. Thus, when we use the term mass, assume it to be identical to rest mass.

Relativistic momentum is defined in such a way that the conservation of momentum will hold in all inertial frames. Whenever the net external force on a system is zero, relativistic momentum is conserved, just as is the case for classical momentum. This has been verified in numerous experiments.

In Relativistic Energy, the relationship of relativistic momentum to energy is explored. That subject will produce our first inkling that objects without mass may also have momentum.
Exercise:
Check Your Understanding

Problem:

What is the momentum of an electron traveling at a speed $0.985c$? The rest mass of the electron is 9.11×10^{-31} kg.

Solution:
Answer
Equation:

$$p = \gamma \text{mu} = \frac{\text{mu}}{\sqrt{1 - \frac{u^2}{c^2}}} = \frac{(9.11 \times 10^{-31}\ \text{kg})(0.985)(3.00 \times 10^8\ \text{m/s})}{\sqrt{1 - \frac{(0.985c)^2}{c^2}}} = 1.56 \times 10^{-21}\ \text{kg} \cdot \text{m/s}$$

Section Summary

- The law of conservation of momentum is valid whenever the net external force is zero and for relativistic momentum. Relativistic momentum p is classical momentum multiplied by the relativistic factor γ.
- $p = \gamma \text{mu}$, where m is the rest mass of the object, u is its velocity relative to an observer, and the relativistic factor $\gamma = \frac{1}{\sqrt{1 - \frac{u^2}{c^2}}}$.
- At low velocities, relativistic momentum is equivalent to classical momentum.
- Relativistic momentum approaches infinity as u approaches c. This implies that an object with mass cannot reach the speed of light.
- Relativistic momentum is conserved, just as classical momentum is conserved.

Conceptual Questions

Exercise:

Problem: How does modern relativity modify the law of conservation of momentum?

Exercise:

Problem:

Is it possible for an external force to be acting on a system and relativistic momentum to be conserved? Explain.

Problem Exercises

Exercise:

Problem:

Find the momentum of a helium nucleus having a mass of 6.68×10^{-27} kg that is moving at $0.200c$.

Solution:

$4.09 \times 10^{-19}\ \text{kg} \cdot \text{m/s}$

Exercise:

Problem: What is the momentum of an electron traveling at $0.980c$?

Exercise:

Problem:

(a) Find the momentum of a 1.00×10^{9} kg asteroid heading towards the Earth at $30.0\ \text{km/s}$. (b) Find the ratio of this momentum to the classical momentum. (Hint: Use the approximation that $\gamma = 1 + (1/2)v^2/c^2$ at low velocities.)

Solution:

(a) $3.000000015 \times 10^{13}\ \text{kg} \cdot \text{m/s}$.

(b) Ratio of relativistic to classical momenta equals 1.000000005 (extra digits to show small effects)

Exercise:

Problem:

(a) What is the momentum of a 2000 kg satellite orbiting at 4.00 km/s? (b) Find the ratio of this momentum to the classical momentum. (Hint: Use the approximation that $\gamma = 1 + (1/2)v^2/c^2$ at low velocities.)

Exercise:

Problem:

What is the velocity of an electron that has a momentum of 3.04×10^{-21} kg·m/s? Note that you must calculate the velocity to at least four digits to see the difference from c.

Solution:

$2.9957 \times 10^{8}\ \mathrm{m/s}$

Exercise:

Problem: Find the velocity of a proton that has a momentum of $4.48 \times -10^{-19}\ \mathrm{kg{\cdot}m/s}$.

Exercise:

Problem:

(a) Calculate the speed of a 1.00-μg particle of dust that has the same momentum as a proton moving at $0.999c$. (b) What does the small speed tell us about the mass of a proton compared to even a tiny amount of macroscopic matter?

Solution:

(a) $1.121 \times 10^{-8}\ \mathrm{m/s}$

(b) The small speed tells us that the mass of a proton is substantially smaller than that of even a tiny amount of macroscopic matter!

Exercise:

Problem:

(a) Calculate γ for a proton that has a momentum of $1.00\ \mathrm{kg{\cdot}m/s}$. (b) What is its speed? Such protons form a rare component of cosmic radiation with uncertain origins.

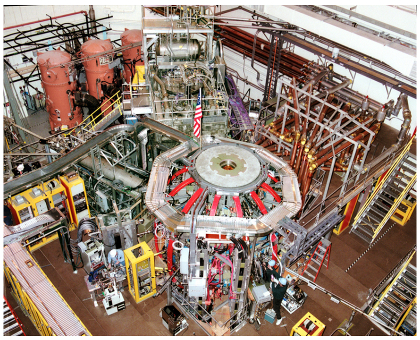
Glossary

relativistic momentum
: p, the momentum of an object moving at relativistic velocity; $p = \gamma m u$, where m is the rest mass of the object, u is its velocity relative to an observer, and the relativistic factor $\gamma = \frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$

rest mass
: the mass of an object as measured by a person at rest relative to the object

Relativistic Energy

- Compute total energy of a relativistic object.
- Compute the kinetic energy of a relativistic object.
- Describe rest energy, and explain how it can be converted to other forms.
- Explain why massive particles cannot reach C.



The National Spherical Torus Experiment (NSTX) has a fusion reactor in which hydrogen isotopes undergo fusion to produce helium. In this process, a relatively small mass of fuel is converted into a large amount of energy. (credit: Princeton Plasma Physics Laboratory)

A tokamak is a form of experimental fusion reactor, which can change mass to energy. Accomplishing this requires an understanding of relativistic energy. Nuclear reactors are proof of the conservation of relativistic energy.

Conservation of energy is one of the most important laws in physics. Not only does energy have many important forms, but each form can be converted to any other. We know that classically the total amount of energy in a system remains constant. Relativistically, energy is still conserved, provided its definition is altered to include the possibility of mass changing to energy, as in the reactions that occur within a nuclear reactor. Relativistic energy is intentionally defined so that it will be conserved in all inertial frames, just as is the case for relativistic momentum. As a consequence, we learn that several fundamental quantities are related in ways not known in classical physics. All of these relationships are verified by experiment and have fundamental consequences. The altered definition of energy

contains some of the most fundamental and spectacular new insights into nature found in recent history.

Total Energy and Rest Energy

The first postulate of relativity states that the laws of physics are the same in all inertial frames. Einstein showed that the law of conservation of energy is valid relativistically, if we define energy to include a relativistic factor.

Note:
Total Energy
Total energy E is defined to be
Equation:

$$E = \gamma mc^2,$$

where m is mass, c is the speed of light, $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, and v is the velocity of the mass relative to an observer. There are many aspects of the total energy E that we will discuss—among them are how kinetic and potential energies are included in E, and how E is related to relativistic momentum. But first, note that at rest, total energy is not zero. Rather, when $v = 0$, we have $\gamma = 1$, and an object has rest energy.

Note:
Rest Energy
Rest energy is
Equation:

$$E_0 = mc^2.$$

This is the correct form of Einstein's most famous equation, which for the first time showed that energy is related to the mass of an object at rest. For example, if energy is stored in the object, its rest mass increases. This also implies that mass can be destroyed to release energy. The implications of these first two equations regarding relativistic energy are so broad that they were not completely recognized for some years after Einstein published them in 1907, nor was the experimental proof that they are correct widely recognized at first. Einstein, it should be noted, did understand and describe the meanings and implications of his theory.

Example:
Calculating Rest Energy: Rest Energy is Very Large
Calculate the rest energy of a 1.00-g mass.
Strategy
One gram is a small mass—less than half the mass of a penny. We can multiply this mass, in SI units, by the speed of light squared to find the equivalent rest energy.
Solution

1. Identify the knowns. $m = 1.00 \times 10^{-3}\ \text{kg}$; $c = 3.00 \times 10^{8}\ \text{m/s}$
2. Identify the unknown. E_0
3. Choose the appropriate equation. $E_0 = mc^2$
4. Plug the knowns into the equation.
   Equation:

$$\begin{aligned} E_0 &= mc^2 = (1.00 \times 10^{-3}\ \text{kg})(3.00 \times 10^{8}\ \text{m/s})^2 \\ &= 9.00 \times 10^{13}\ \text{kg} \cdot \text{m}^2/\text{s}^2 \end{aligned}$$

5. Convert units.

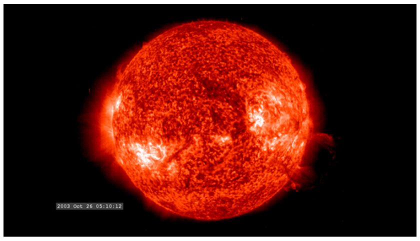
   Noting that $1\ \text{kg} \cdot \text{m}^2/\text{s}^2 = 1\ \text{J}$, we see the rest mass energy is
   Equation:

$$E_0 = 9.00 \times 10^{13}\ \text{J}.$$

Discussion
This is an enormous amount of energy for a 1.00-g mass. We do not notice this energy, because it is generally not available. Rest energy is large because the speed of light c is a large number and c^2 is a very large number, so that mc^2 is huge for any macroscopic mass. The 9.00×10^{13} J rest mass energy for 1.00 g is about twice the energy released by the Hiroshima atomic bomb and about 10,000 times the kinetic energy of a large aircraft carrier. If a way can be found to convert rest mass energy into some other form (and all forms of energy can be converted into one another), then huge amounts of energy can be obtained from the destruction of mass.

Today, the practical applications of *the conversion of mass into another form of energy*, such as in nuclear weapons and nuclear power plants, are well known. But examples also existed when Einstein first proposed the correct form of relativistic energy, and he did describe some of them. Nuclear radiation had been discovered in the previous decade, and it had been a mystery as to where its energy originated. The explanation was that, in certain nuclear processes, a small amount of mass is destroyed and energy is released and carried by nuclear radiation. But the amount of mass destroyed is so small that it is difficult to detect that any is missing. Although Einstein proposed this as the source of energy in the radioactive salts

then being studied, it was many years before there was broad recognition that mass could be and, in fact, commonly is converted to energy. (See [link].)


(a)


(b)

The Sun (a) and the Susquehanna Steam Electric Station (b) both convert mass into energy—the Sun via nuclear fusion, the electric station via nuclear fission. (credits: (a) NASA/Goddard Space Flight Center, Scientific Visualization Studio; (b) U.S. government)

Because of the relationship of rest energy to mass, we now consider mass to be a form of energy rather than something separate. There had not even been a hint of this prior to Einstein's work. Such conversion is now known to be the source of the Sun's energy, the energy of nuclear decay, and even the source of energy keeping Earth's interior hot.

Stored Energy and Potential Energy

What happens to energy stored in an object at rest, such as the energy put into a battery by charging it, or the energy stored in a toy gun's compressed spring? The energy input becomes part of the total energy of the object and, thus, increases its rest mass. All stored and potential energy becomes mass in a system. Why is it we don't ordinarily notice this? In fact, conservation of mass (meaning total mass is constant) was one of the great laws verified by 19th-century science. Why was it not noticed to be incorrect? The following example helps answer these questions.

Example:
Calculating Rest Mass: A Small Mass Increase due to Energy Input
A car battery is rated to be able to move 600 ampere-hours $(\mathrm{A{\cdot}h})$ of charge at 12.0 V. (a) Calculate the increase in rest mass of such a battery when it is taken from being fully depleted to being fully charged. (b) What percent increase is this, given the battery's mass is 20.0 kg?

Strategy
In part (a), we first must find the energy stored in the battery, which equals what the battery can supply in the form of electrical potential energy. Since $\mathrm{PE_{elec}} = \mathrm{qV}$, we have to calculate the charge q in $600\ \mathrm{A{\cdot}h}$, which is the product of the current I and the time t. We then multiply the result by 12.0 V. We can then calculate the battery's increase in mass using $\Delta E = \mathrm{PE_{elec}} = (\Delta m)c^2$. Part (b) is a simple ratio converted to a percentage.

Solution for (a)

1. Identify the knowns. $I \cdot t = 600\ \mathrm{A \cdot h}$; $V = 12.0\ \mathrm{V}$; $c = 3.00 \times 10^8\ \mathrm{m/s}$
2. Identify the unknown. Δm
3. Choose the appropriate equation. $\mathrm{PE_{elec}} = (\Delta m)c^2$
4. Rearrange the equation to solve for the unknown. $\Delta m = \frac{\mathrm{PE_{elec}}}{c^2}$
5. Plug the knowns into the equation.
   Equation:

$$\begin{aligned} \Delta m &= \frac{\mathrm{PE_{elec}}}{c^2} \\ &= \frac{\mathrm{qV}}{c^2} \\ &= \frac{(It)V}{c^2} \\ &= \frac{(600\ \mathrm{A{\cdot}h})(12.0\ \mathrm{V})}{(3.00 \times 10^8)^2}. \end{aligned}$$

   Write amperes A as coulombs per second (C/s), and convert hours to seconds.
   Equation:

$$
\begin{aligned}
\Delta m &= \frac{(600\ \mathrm{C/s{\cdot}h}\left(\frac{3600\ \mathrm{s}}{1\ \mathrm{h}}\right)(12.0\ \mathrm{J/C})}{(3.00\times10^{8}\ \mathrm{m/s})^2} \\
&= \frac{(2.16\times10^{6}\ \mathrm{C})(12.0\ \mathrm{J/C})}{(3.00\times10^{8}\ \mathrm{m/s})^2}
\end{aligned}
$$

Using the conversion $1\ \mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}^2 = 1\ \mathrm{J}$, we can write the mass as

$\Delta m = 2.88\times10^{-10}\ \mathrm{kg}.$

Solution for (b)

1. Identify the knowns. $\Delta m = 2.88\times10^{-10}\ \mathrm{kg}$; $m = 20.0\ \mathrm{kg}$
2. Identify the unknown. % change
3. Choose the appropriate equation. $\%\ \mathrm{increase} = \frac{\Delta m}{m}\times 100\%$
4. Plug the knowns into the equation.
   Equation:

$$
\begin{aligned}
\%\ \mathrm{increase} &= \frac{\Delta m}{m}\times 100\% \\
&= \frac{2.88\times10^{-10}\ \mathrm{kg}}{20.0\ \mathrm{kg}}\times 100\% \\
&= 1.44\times10^{-9}\%.
\end{aligned}
$$

Discussion
Both the actual increase in mass and the percent increase are very small, since energy is divided by c^2, a very large number. We would have to be able to measure the mass of the battery to a precision of a billionth of a percent, or 1 part in 10^{11}, to notice this increase. It is no wonder that the mass variation is not readily observed. In fact, this change in mass is so small that we may question how you could verify it is real. The answer is found in nuclear processes in which the percentage of mass destroyed is large enough to be measured. The mass of the fuel of a nuclear reactor, for example, is measurably smaller when its energy has been used. In that case, stored energy has been released (converted mostly to heat and electricity) and the rest mass has decreased. This is also the case when you use the energy stored in a battery, except that the stored energy is much greater in nuclear processes, making the change in mass measurable in practice as well as in theory.

Kinetic Energy and the Ultimate Speed Limit

Kinetic energy is energy of motion. Classically, kinetic energy has the familiar expression $\frac{1}{2}mv^2$. The relativistic expression for kinetic energy is obtained from the work-energy theorem. This theorem states that the net work on a system goes into kinetic energy. If our system starts from rest, then the work-energy theorem is
Equation:

$$W_{\text{net}} = \text{KE}.$$

Relativistically, at rest we have rest energy $E_0 = mc^2$. The work increases this to the total energy $E = \gamma mc^2$. Thus,
Equation:

$$W_{\text{net}} = E - E_0 = \gamma mc^2 - mc^2 = (\gamma - 1)\, mc^2.$$

Relativistically, we have $W_{\text{net}} = \text{KE}_{\text{rel}}$.

Note:
Relativistic Kinetic Energy
Relativistic kinetic energy is
Equation:

$$\text{KE}_{\text{rel}} = (\gamma - 1)\, mc^2.$$

When motionless, we have $v = 0$ and
Equation:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = 1,$$

so that $\text{KE}_{\text{rel}} = 0$ at rest, as expected. But the expression for relativistic kinetic energy (such as total energy and rest energy) does not look much like the classical $\frac{1}{2}mv^2$. To show that the classical expression for kinetic energy is obtained at low velocities, we note that the binomial expansion for γ at low velocities gives
Equation:

$$\gamma = 1 + \frac{1}{2}\frac{v^2}{c^2}.$$

A binomial expansion is a way of expressing an algebraic quantity as a sum of an infinite series of terms. In some cases, as in the limit of small velocity here, most terms are very small. Thus the expression derived for γ here is not exact, but it is a very accurate approximation. Thus, at low velocities,
Equation:

$$\gamma - 1 = \frac{1}{2}\frac{v^2}{c^2}.$$

Entering this into the expression for relativistic kinetic energy gives
Equation:

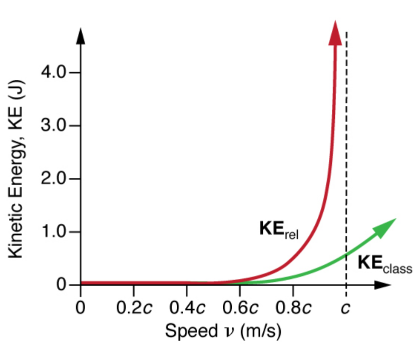
$$\text{KE}_{\text{rel}} = \left[\frac{1}{2}\frac{v^2}{c^2}\right]mc^2 = \frac{1}{2}mv^2 = \text{KE}_{\text{class}}.$$

So, in fact, relativistic kinetic energy does become the same as classical kinetic energy when $v << c$.

It is even more interesting to investigate what happens to kinetic energy when the velocity of an object approaches the speed of light. We know that γ becomes infinite as v approaches c, so that KE_{rel} also becomes infinite as the velocity approaches the speed of light. (See [link].) An infinite amount of work (and, hence, an infinite amount of energy input) is required to accelerate a mass to the speed of light.

Note:
The Speed of Light
No object with mass can attain the speed of light.

So the speed of light is the ultimate speed limit for any particle having mass. All of this is consistent with the fact that velocities less than c always add to less than c. Both the relativistic form for kinetic energy and the ultimate speed limit being c have been confirmed in detail in numerous experiments. No matter how much energy is put into accelerating a mass, its velocity can only approach—not reach—the speed of light.



This graph of KE_{rel}

versus velocity shows how kinetic energy approaches infinity as velocity approaches the speed of light. It is thus not possible for an object having mass to reach the speed of light. Also shown is $\mathrm{KE}_{\mathrm{class}}$, the classical kinetic energy, which is similar to relativistic kinetic energy at low velocities. Note that much more energy is required to reach high velocities than predicted classically.

Example:
Comparing Kinetic Energy: Relativistic Energy Versus Classical Kinetic Energy
An electron has a velocity $v = 0.990c$. (a) Calculate the kinetic energy in MeV of the electron. (b) Compare this with the classical value for kinetic energy at this velocity. (The mass of an electron is 9.11×10^{-31} kg.)

Strategy

The expression for relativistic kinetic energy is always correct, but for (a) it must be used since the velocity is highly relativistic (close to c). First, we will calculate the relativistic factor γ, and then use it to determine the relativistic kinetic energy. For (b), we will calculate the classical kinetic energy (which would be close to the relativistic value if v were less than a few percent of c) and see that it is not the same.

Solution for (a)

1. Identify the knowns. $v = 0.990c$; $m = 9.11 \times 10^{-31}$ kg
2. Identify the unknown. $\mathrm{KE}_{\mathrm{rel}}$
3. Choose the appropriate equation. $\mathrm{KE}_{\mathrm{rel}} = (\gamma - 1)\, mc^2$
4. Plug the knowns into the equation.

   First calculate γ. We will carry extra digits because this is an intermediate calculation.
   Equation:

$$\begin{aligned}\gamma &= \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \\ &= \frac{1}{\sqrt{1-\frac{(0.990\mathrm{c})^2}{c^2}}} \\ &= \frac{1}{\sqrt{1-(0.990)^2}} \\ &= 7.0888\end{aligned}$$

Next, we use this value to calculate the kinetic energy.
Equation:

$$\begin{aligned}\mathrm{KE}_{\mathrm{rel}} &= (\gamma-1)\,mc^2 \\ &= (7.0888-1)(9.11\times10^{-31}\ \mathrm{kg})(3.00\times10^{8}\ \mathrm{m/s})^2 \\ &= 4.99\times10^{-13}\ \mathrm{J}\end{aligned}$$

5. Convert units.
   Equation:

$$\begin{aligned}\mathrm{KE}_{\mathrm{rel}} &= (4.99\times10^{-13}\ \mathrm{J})\left(\frac{1\ \mathrm{MeV}}{1.60\times10^{-13}\ \mathrm{J}}\right) \\ &= 3.12\ \mathrm{MeV}\end{aligned}$$

Solution for (b)

1. List the knowns. $v = 0.990c$; $m = 9.11\times10^{-31}\ \mathrm{kg}$
2. List the unknown. $\mathrm{KE}_{\mathrm{class}}$
3. Choose the appropriate equation. $\mathrm{KE}_{\mathrm{class}} = \frac{1}{2}mv^2$
4. Plug the knowns into the equation.
   Equation:

$$\begin{aligned}\mathrm{KE}_{\mathrm{class}} &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(9.00\times10^{-31}\ \mathrm{kg})(0.990)^2(3.00\times10^{8}\ \mathrm{m/s})^2 \\ &= 4.02\times10^{-14}\ \mathrm{J}\end{aligned}$$

5. Convert units.
   Equation:

$$\begin{aligned}\mathrm{KE}_{\mathrm{class}} &= 4.02\times10^{-14}\ \mathrm{J}\left(\frac{1\ \mathrm{MeV}}{1.60\times10^{-13}\ \mathrm{J}}\right) \\ &= 0.251\ \mathrm{MeV}\end{aligned}$$

Discussion

As might be expected, since the velocity is 99.0% of the speed of light, the classical kinetic energy is significantly off from the correct relativistic value. Note also that the classical value is much smaller than the relativistic value. In fact, $\mathrm{KE}_{\mathrm{rel}}/\mathrm{KE}_{\mathrm{class}} = 12.4$ here. This is some indication of how difficult it is to get a mass moving close to the speed of light. Much more energy is required than predicted classically. Some people interpret this extra energy as going into increasing the mass of the system, but, as discussed in Relativistic Momentum, this cannot be verified unambiguously. What is certain is that ever-increasing amounts of energy are needed to get the velocity of a mass a little closer to that of light. An energy of 3 MeV is a very small amount for an electron, and it can be achieved with present-day particle accelerators. SLAC, for example, can accelerate electrons to over $50 \times 10^{9}\ \mathrm{eV} = 50{,}000\ \mathrm{MeV}$.

Is there any point in getting v a little closer to c than 99.0% or 99.9%? The answer is yes. We learn a great deal by doing this. The energy that goes into a high-velocity mass can be converted to any other form, including into entirely new masses. (See [link].) Most of what we know about the substructure of matter and the collection of exotic short-lived particles in nature has been learned this way. Particles are accelerated to extremely relativistic energies and made to collide with other particles, producing totally new species of particles. Patterns in the characteristics of these previously unknown particles hint at a basic substructure for all matter. These particles and some of their characteristics will be covered in Particle Physics.

The Fermi National Accelerator Laboratory, near Batavia, Illinois, was a subatomic particle collider that accelerated protons and antiprotons to attain energies up to 1 Tev (a trillion electronvolts). The circular ponds near the rings were built to dissipate waste heat. This accelerator was shut down in September 2011. (credit: Fermilab, Reidar Hahn)

Relativistic Energy and Momentum

We know classically that kinetic energy and momentum are related to each other, since
Equation:

$$\mathrm{KE_{class}} = \frac{p^2}{2m} = \frac{(mv)^2}{2m} = \frac{1}{2}mv^2.$$

Relativistically, we can obtain a relationship between energy and momentum by algebraically manipulating their definitions. This produces
Equation:

$$E^2 = (\mathrm{pc})^2 + (\mathrm{mc}^2)^2,$$

where E is the relativistic total energy and p is the relativistic momentum. This relationship between relativistic energy and relativistic momentum is more complicated than the classical, but we can gain some interesting new insights by examining it. First, total energy is related to momentum and rest mass. At rest, momentum is zero, and the equation gives the total energy to be the rest energy mc^2 (so this equation is consistent with the discussion of rest energy above). However, as the mass is accelerated, its momentum p increases, thus increasing the total energy. At sufficiently high velocities, the rest energy term $(mc^2)^2$ becomes negligible compared with the momentum term $(pc)^2$; thus, $E = pc$ at extremely relativistic velocities.

If we consider momentum p to be distinct from mass, we can determine the implications of the equation $E^2 = (\mathrm{pc})^2 + (\mathrm{mc}^2)^2$, for a particle that has no mass. If we take m to be zero in this equation, then $E = \mathrm{pc}$, or $p = E/c$. Massless particles have this momentum. There are several massless particles found in nature, including photons (these are quanta of electromagnetic radiation). Another implication is that a massless particle must travel at speed c and only at speed c. While it is beyond the scope of this text to examine the relationship in the equation $E^2 = (\mathrm{pc})^2 + (\mathrm{mc}^2)^2$, in detail, we can see that the relationship has important implications in special relativity.

Note:
Problem-Solving Strategies for Relativity

Examine the situation to	. Relativistic	$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$, the quantitative	γ is very close to 1, then relativistic effects are small and differ very

determine that it is necessary to use relativity	effects are related to relativistic factor. If little from the usually easier classical calculations.
Identify exactly what needs to be determined in the problem (identify the unknowns).	
Make a list of what is given or can be inferred from the problem as stated (identify the knowns).	Look in particular for information on relative velocity v.
Make certain you understand the conceptual aspects of the problem before making any calculations.	Decide, for example, which observer sees time dilated or length contracted before plugging into equations. If you have thought about who sees what, who is moving with the event being observed, who sees proper time, and so on, you will find it much easier to determine if your calculation is reasonable.
Determine the primary type of calculation to be done to find the unknowns identified above.	You will find the section summary helpful in determining whether a length contraction, relativistic kinetic energy, or some other concept is involved.
Do not round off during the calculation.	As noted in the text, you must often perform your calculations to many digits to see the desired effect. You may round off at the very end of the problem, but do not use a rounded number in a subsequent calculation.
Check the answer to see if it is reasonable: Does it make sense?	This may be more difficult for relativity, since we do not encounter it directly. But you can look for velocities greater than c or relativistic effects that are in the wrong direction (such as a time contraction where a dilation was expected).

Exercise:
Check Your Understanding

Problem:

A photon decays into an electron-positron pair. What is the kinetic energy of the electron if its speed is $0.992c$?

Solution:
Answer
Equation:

$$\begin{aligned} \mathrm{KE}_{\mathrm{rel}} &= (\gamma - 1)\,mc^2 = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right) mc^2 \\ &= \left(\frac{1}{\sqrt{1-\frac{(0.992c)^2}{c^2}}} - 1\right)(9.11 \times 10^{-31}\ \mathrm{kg})(3.00 \times 10^{8}\ \mathrm{m/s})^2 = 5.67 \times 10^{-13}\ \mathrm{J} \end{aligned}$$

Section Summary

- Relativistic energy is conserved as long as we define it to include the possibility of mass changing to energy.
- Total Energy is defined as: $E = \gamma mc^2$, where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$.
- Rest energy is $E_0 = mc^2$, meaning that mass is a form of energy. If energy is stored in an object, its mass increases. Mass can be destroyed to release energy.
- We do not ordinarily notice the increase or decrease in mass of an object because the change in mass is so small for a large increase in energy.
- The relativistic work-energy theorem is $W_{\text{net}} = E - E_0 = \gamma mc^2 - mc^2 = (\gamma - 1) mc^2$.
- Relativistically, $W_{\text{net}} = \text{KE}_{\text{rel}}$, where KE_{rel} is the relativistic kinetic energy.
- Relativistic kinetic energy is $\text{KE}_{\text{rel}} = (\gamma - 1) mc^2$, where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$. At low velocities, relativistic kinetic energy reduces to classical kinetic energy.
- **No object with mass can attain the speed of light** because an infinite amount of work and an infinite amount of energy input is required to accelerate a mass to the speed of light.
- The equation $E^2 = (pc)^2 + (mc^2)^2$ relates the relativistic total energy E and the relativistic momentum p. At extremely high velocities, the rest energy mc^2 becomes negligible, and $E = pc$.

Conceptual Questions

Exercise:

Problem:

How are the classical laws of conservation of energy and conservation of mass modified by modern relativity?

Exercise:

Problem:

What happens to the mass of water in a pot when it cools, assuming no molecules escape or are added? Is this observable in practice? Explain.

Exercise:

Problem:

Consider a thought experiment. You place an expanded balloon of air on weighing scales outside in the early morning. The balloon stays on the scales and you are able to measure changes in its mass. Does the mass of the balloon change as the day progresses? Discuss the difficulties in carrying out this experiment.

Exercise:

Problem:

The mass of the fuel in a nuclear reactor decreases by an observable amount as it puts out energy. Is the same true for the coal and oxygen combined in a conventional power plant? If so, is this observable in practice for the coal and oxygen? Explain.

Exercise:

Problem:

We know that the velocity of an object with mass has an upper limit of c. Is there an upper limit on its momentum? Its energy? Explain.

Exercise:

Problem: Given the fact that light travels at c, can it have mass? Explain.

Exercise:

Problem:

If you use an Earth-based telescope to project a laser beam onto the Moon, you can move the spot across the Moon's surface at a velocity greater than the speed of light. Does this violate modern relativity? (Note that light is being sent from the Earth to the Moon, not across the surface of the Moon.)

Problems & Exercises

Exercise:

Problem:

What is the rest energy of an electron, given its mass is 9.11×10^{-31} kg? Give your answer in joules and MeV.

Solution:

8.20×10^{-14} J

0.512 MeV

Exercise:

Problem:

Find the rest energy in joules and MeV of a proton, given its mass is 1.67×10^{-27} kg.

Exercise:

Problem:

If the rest energies of a proton and a neutron (the two constituents of nuclei) are 938.3 and 939.6 MeV respectively, what is the difference in their masses in kilograms?

Solution:

$2.3 \times 10^{-30}\ \text{kg}$

Exercise:

Problem:

The Big Bang that began the universe is estimated to have released $10^{68}\ \text{J}$ of energy. How many stars could half this energy create, assuming the average star's mass is $4.00 \times 10^{30}\ \text{kg}$?

Exercise:

Problem:

A supernova explosion of a $2.00 \times 10^{31}\ \text{kg}$ star produces $1.00 \times 10^{44}\ \text{J}$ of energy. (a) How many kilograms of mass are converted to energy in the explosion? (b) What is the ratio $\Delta m/m$ of mass destroyed to the original mass of the star?

Solution:

(a) $1.11 \times 10^{27}\ \text{kg}$

(b) 5.56×10^{-5}

Exercise:

Problem:

(a) Using data from [link], calculate the mass converted to energy by the fission of 1.00 kg of uranium. (b) What is the ratio of mass destroyed to the original mass, $\Delta m/m$?

Exercise:

Problem:

(a) Using data from [link], calculate the amount of mass converted to energy by the fusion of 1.00 kg of hydrogen. (b) What is the ratio of mass destroyed to the original mass, $\Delta m/m$? (c) How does this compare with $\Delta m/m$ for the fission of 1.00 kg of uranium?

Solution:

$7.1 \times 10^{-3}\ \text{kg}$

7.1×10^{-3}

The ratio is greater for hydrogen.

Exercise:

Problem:

There is approximately 10^{34} J of energy available from fusion of hydrogen in the world's oceans. (a) If 10^{33} J of this energy were utilized, what would be the decrease in mass of the oceans? Assume that 0.08% of the mass of a water molecule is converted to energy during the fusion of hydrogen. (b) How great a volume of water does this correspond to? (c) Comment on whether this is a significant fraction of the total mass of the oceans.

Exercise:

Problem:

A muon has a rest mass energy of 105.7 MeV, and it decays into an electron and a massless particle. (a) If all the lost mass is converted into the electron's kinetic energy, find γ for the electron. (b) What is the electron's velocity?

Solution:

208

$0.999988c$

Exercise:

Problem:

A π-meson is a particle that decays into a muon and a massless particle. The π-meson has a rest mass energy of 139.6 MeV, and the muon has a rest mass energy of 105.7 MeV. Suppose the π-meson is at rest and all of the missing mass goes into the muon's kinetic energy. How fast will the muon move?

Exercise:

Problem:

(a) Calculate the relativistic kinetic energy of a 1000-kg car moving at 30.0 m/s if the speed of light were only 45.0 m/s. (b) Find the ratio of the relativistic kinetic energy to classical.

Solution:

6.92×10^{5} J

1.54

Exercise:

Problem:

Alpha decay is nuclear decay in which a helium nucleus is emitted. If the helium nucleus has a mass of 6.80×10^{-27} kg and is given 5.00 MeV of kinetic energy, what is its velocity?

Exercise:

Problem:

(a) Beta decay is nuclear decay in which an electron is emitted. If the electron is given 0.750 MeV of kinetic energy, what is its velocity? (b) Comment on how the high velocity is consistent with the kinetic energy as it compares to the rest mass energy of the electron.

Solution:

(a) $0.914c$

(b) The rest mass energy of an electron is 0.511 MeV, so the kinetic energy is approximately 150% of the rest mass energy. The electron should be traveling close to the speed of light.

Exercise:

Problem:

A positron is an antimatter version of the electron, having exactly the same mass. When a positron and an electron meet, they annihilate, converting all of their mass into energy. (a) Find the energy released, assuming negligible kinetic energy before the annihilation. (b) If this energy is given to a proton in the form of kinetic energy, what is its velocity? (c) If this energy is given to another electron in the form of kinetic energy, what is its velocity?

Exercise:

Problem:

What is the kinetic energy in MeV of a π-meson that lives 1.40×10^{-16} s as measured in the laboratory, and 0.840×10^{-16} s when at rest relative to an observer, given that its rest energy is 135 MeV?

Solution:

90.0 MeV

Exercise:

Problem:

Find the kinetic energy in MeV of a neutron with a measured life span of 2065 s, given its rest energy is 939.6 MeV, and rest life span is 900s.

Exercise:

Problem:

(a) Show that $(pc)^2/(mc^2)^2 = \gamma^2 - 1$. This means that at large velocities $pc >> mc^2$. (b) Is $E \approx$ pc when $\gamma = 30.0$, as for the astronaut discussed in the twin paradox?

Solution:

$$E^2 = p^2c^2 + m^2c^4 = \gamma^2 m^2 c^4\text{, so that}$$

(a) $$p^2c^2 = \left(\gamma^2 - 1\right)m^2c^4 \text{ , and therefore}$$

$$\frac{(pc)^2}{\left(mc^2\right)^2} = \gamma^2 - 1$$

(b) yes

Exercise:

Problem:

One cosmic ray neutron has a velocity of $0.250c$ relative to the Earth. (a) What is the neutron's total energy in MeV? (b) Find its momentum. (c) Is $E \approx$ pc in this situation? Discuss in terms of the equation given in part (a) of the previous problem.

Exercise:

Problem:

What is γ for a proton having a mass energy of 938.3 MeV accelerated through an effective potential of 1.0 TV (teravolt) at Fermilab outside Chicago?

Solution:

1.07×10^3

Exercise:

Problem:

(a) What is the effective accelerating potential for electrons at the Stanford Linear Accelerator, if $\gamma = 1.00 \times 10^5$ for them? (b) What is their total energy (nearly the same as kinetic in this case) in GeV?

Exercise:

Problem:

(a) Using data from [link], find the mass destroyed when the energy in a barrel of crude oil is released. (b) Given these barrels contain 200 liters and assuming the density of crude oil is $750\ \text{kg/m}^3$, what is the ratio of mass destroyed to original mass, $\Delta m/m$?

Solution:

$6.56\times10^{-8}\ \text{kg}$

4.37×10^{-10}

Exercise:

Problem:

(a) Calculate the energy released by the destruction of 1.00 kg of mass. (b) How many kilograms could be lifted to a 10.0 km height by this amount of energy?

Exercise:

Problem:

A Van de Graaff accelerator utilizes a 50.0 MV potential difference to accelerate charged particles such as protons. (a) What is the velocity of a proton accelerated by such a potential? (b) An electron?

Solution:

$0.314c$

$0.99995c$

Exercise:

Problem:

Suppose you use an average of $500\ \text{kW}\cdot\text{h}$ of electric energy per month in your home. (a) How long would 1.00 g of mass converted to electric energy with an efficiency of 38.0% last you? (b) How many homes could be supplied at the $500\ \text{kW}\cdot\text{h}$ per month rate for one year by the energy from the described mass conversion?

Exercise:

Problem:

(a) A nuclear power plant converts energy from nuclear fission into electricity with an efficiency of 35.0%. How much mass is destroyed in one year to produce a continuous 1000 MW of electric power? (b) Do you think it would be possible to observe this mass loss if the total mass of the fuel is 10^4 kg?

Solution:

(a) 1.00 kg

(b) This much mass would be measurable, but probably not observable just by looking because it is 0.01% of the total mass.

Exercise:

Problem:

Nuclear-powered rockets were researched for some years before safety concerns became paramount. (a) What fraction of a rocket's mass would have to be destroyed to get it into a low Earth orbit, neglecting the decrease in gravity? (Assume an orbital altitude of 250 km, and calculate both the kinetic energy (classical) and the gravitational potential energy needed.) (b) If the ship has a mass of 1.00×10^5 kg (100 tons), what total yield nuclear explosion in tons of TNT is needed?

Exercise:

Problem:

The Sun produces energy at a rate of 4.00×10^{26} W by the fusion of hydrogen. (a) How many kilograms of hydrogen undergo fusion each second? (b) If the Sun is 90.0% hydrogen and half of this can undergo fusion before the Sun changes character, how long could it produce energy at its current rate? (c) How many kilograms of mass is the Sun losing per second? (d) What fraction of its mass will it have lost in the time found in part (b)?

Solution:

(a) 6.3×10^{11} kg/s

(b) 4.5×10^{10} y

(c) 4.44×10^9 kg

(d) 0.32%

Exercise:

Problem: **Unreasonable Results**

A proton has a mass of 1.67×10^{-27} kg. A physicist measures the proton's total energy to be 50.0 MeV. (a) What is the proton's kinetic energy? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Exercise:

Problem: Construct Your Own Problem

Consider a highly relativistic particle. Discuss what is meant by the term “highly relativistic.” (Note that, in part, it means that the particle cannot be massless.) Construct a problem in which you calculate the wavelength of such a particle and show that it is very nearly the same as the wavelength of a massless particle, such as a photon, with the same energy. Among the things to be considered are the rest energy of the particle (it should be a known particle) and its total energy, which should be large compared to its rest energy.

Exercise:

Problem: Construct Your Own Problem

Consider an astronaut traveling to another star at a relativistic velocity. Construct a problem in which you calculate the time for the trip as observed on the Earth and as observed by the astronaut. Also calculate the amount of mass that must be converted to energy to get the astronaut and ship to the velocity travelled. Among the things to be considered are the distance to the star, the velocity, and the mass of the astronaut and ship. Unless your instructor directs you otherwise, do not include any energy given to other masses, such as rocket propellants.

Glossary

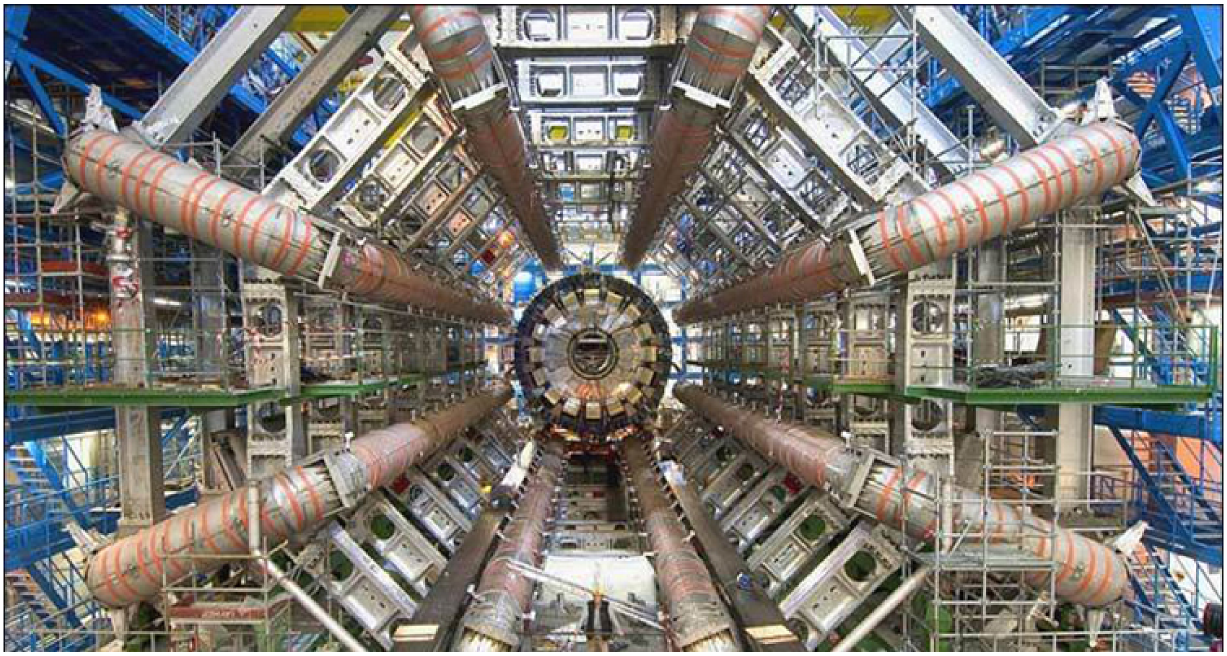
total energy
: defined as $E = \gamma mc^2$, where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

rest energy
: the energy stored in an object at rest: $E_0 = mc^2$

relativistic kinetic energy
: the kinetic energy of an object moving at relativistic speeds: $\text{KE}_{\text{rel}} = (\gamma - 1)\, mc^2$, where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

Introduction to Nuclear and Particle Physics
class="introduction"

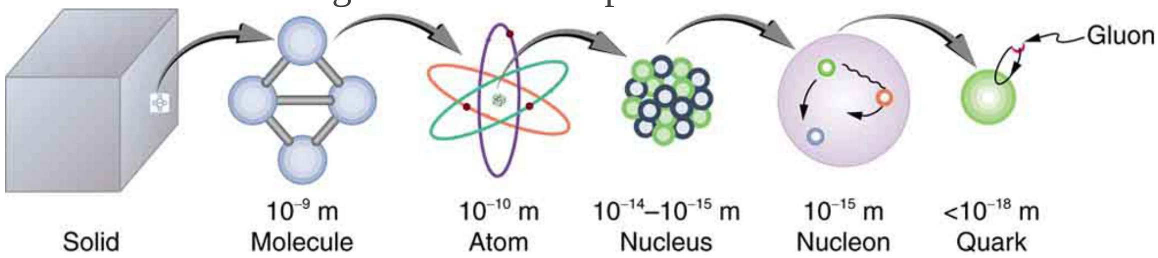
Part of the Large Hadron Collider at CERN, on the border of Switzerland and France. The LHC is a particle accelerator, designed to study fundamental particles. (credit: Image Editor, Flickr)



There is an ongoing quest to find substructures of matter. At one time, it was thought that atoms would be the ultimate substructure, but just when the first direct evidence of atoms was obtained, it became clear that they have a substructure and a *nucleus*. The nucleus itself has spectacular characteristics. For example, certain nuclei are unstable, and their decay emits radiations with energies millions of times greater than atomic energies. Some of the mysteries of nature, such as why the core of the earth remains molten and how the sun produces its energy, are explained by nuclear phenomena. The exploration of *radioactivity* and the nucleus revealed fundamental and previously unknown particles, forces, and conservation laws. That exploration has evolved into a search for further underlying structures, such as quarks. In this chapter, we will start with the fundamentals of nuclear radioactivity and the nucleus, moving on to some of the important applications of nuclear physics. We will end the chapter by exploring the basics of what we know about quarks and other substructures smaller than nuclei.

[link] shows the structures and length scales involved in study of nucleus and elementary particles. In atomic and molecular physics, we were already dealing with structures at nanometer scale, about 500 times shorter than the wavelength of green light. The nuclear scale goes 100,000 times smaller; the size of largest stable nuclei is about 10 femtometers, or 10^{-14} m. All

other objects we are studying in nuclear and particle physics are smaller than this, with the most elementary particles, quarks and leptons, having no known size and being considered as "point masses."

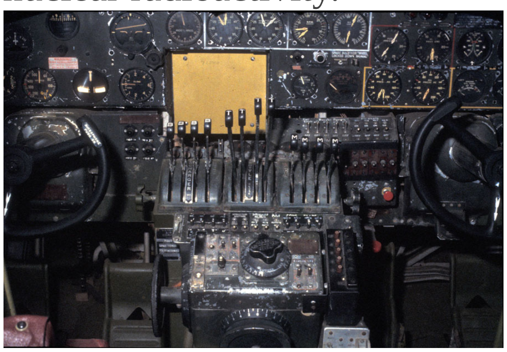


The properties of matter are based on substructures called molecules and atoms. Atoms have the substructure of a nucleus with orbiting electrons, the interactions of which explain atomic properties. Protons and neutrons, the interactions of which explain the stability and abundance of elements, form the substructure of nuclei. Protons and neutrons are not fundamental—they are composed of quarks. Like electrons and a few other particles, quarks may be the fundamental building blocks of all there is, lacking any further substructure. But the story is not complete, because quarks and electrons may have substructure smaller than details that are presently observable.

Nuclear Radioactivity

- Explain nuclear radiation.
- Explain the types of radiation—alpha rays, beta rays, and gamma rays.
- Explain the ionization of an atom by radiation.
- Define the range of radiation.

The discovery and study of nuclear radioactivity quickly revealed evidence of revolutionary new physics. In addition, uses for nuclear radiation also emerged quickly—for example, people such as Ernest Rutherford used it to determine the size of the nucleus and devices were painted with radon-doped paint to make them glow in the dark (see [link]). We therefore begin our study of nuclear physics with the discovery and basic features of nuclear radioactivity.



The dials of this World War II aircraft will glow in the dark, because they are painted with radium-doped phosphorescent paint. It is a poignant reminder of the dual nature of radiation. Although radium paint dials are conveniently visible day and night, they emit radon, a hazardous radioactive gas that is odorless, colorless,

and tasteless. (credit: U.S. Air Force Photo)

Discovery of Nuclear Radioactivity

In 1896, the French physicist Antoine Henri Becquerel (1852–1908) accidentally found that a uranium-rich mineral called pitchblende emits invisible, penetrating rays that can darken a photographic plate enclosed in an opaque envelope. The rays therefore carry energy; but amazingly, the pitchblende emits them continuously without any energy input. This is an apparent violation of the law of conservation of energy, one that we now understand is due to the conversion of a small amount of mass into energy, as related in Einstein's famous equation $E = mc^2$. It was soon evident that Becquerel's rays originate in the nuclei of the atoms and have other unique characteristics. The emission of these rays is called **nuclear radioactivity** or simply **radioactivity**. The rays themselves are called **nuclear radiation**. A nucleus that spontaneously destroys part of its mass to emit radiation is said to **decay** (a term also used to describe the emission of radiation by atoms in excited states). A substance or object that emits nuclear radiation is said to be **radioactive**.

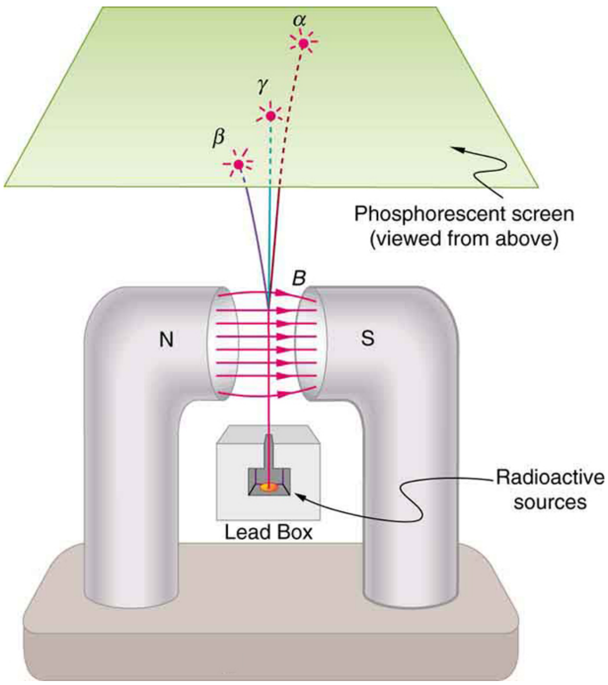
Two types of experimental evidence imply that Becquerel's rays originate not from the atomic structure, but something smaller inside. First, the radiation is found to be associated with certain elements, such as uranium. Radiation does not vary with chemical state—that is, uranium is radioactive whether it is in the form of an element or compound. In addition, radiation does not vary with temperature, pressure, or ionization state of the uranium atom. Since all of these factors affect electrons in an atom, the radiation cannot come from electron transitions, as atomic spectra do. The huge energy emitted during each event is the second piece of evidence that the radiation cannot be atomic in origin. Nuclear radiation has energies of the order of 10^6 eV per event, which is much greater than the typical atomic energies (a few eV), such as that observed in spectra and chemical reactions, and more than ten times as high as the most energetic characteristic X-rays. Becquerel did not vigorously pursue his discovery for very long. In 1898, Marie Curie (1867–1934) began her doctoral study of

Becquerel’s rays. She and her husband—French physicist Pierre Curie (1859–1906)—soon discovered two new radioactive elements, which she named *polonium* (after her native land) and *radium* (because it radiates). These two new elements filled holes in the periodic table and, further, displayed much higher levels of radioactivity per gram of material than uranium. Over a period of four years, working under poor conditions and spending their own funds, the Curies processed more than a ton of uranium ore to isolate a gram of radium salt. Radium became highly sought after, because it was about two million times as radioactive as uranium. Curie’s radium salt glowed visibly from the radiation that took its toll on them and other unaware researchers. Shortly after completing her Ph.D., both Curies and Becquerel shared the 1903 Nobel Prize in physics for their work on radioactivity. Pierre was killed in a horse cart accident in 1906, but Marie continued her study of radioactivity for nearly 30 more years. Awarded the 1911 Nobel Prize in chemistry for her discovery of two new elements, she remains the only person to win Nobel Prizes in physics and chemistry. Marie’s radioactive fingerprints on some pages of her notebooks can still expose film, and she suffered from radiation-induced lesions. She died of leukemia likely caused by long-term exposure to radiation, but she was active in research almost until her death in 1934.

Alpha, Beta, and Gamma

Research begun by people such as New Zealander Ernest Rutherford soon after the discovery of nuclear radiation indicated that different types of rays are emitted. Eventually, three types were distinguished and named **alpha** (α), **beta**(β), and **gamma**(γ), because, like X-rays, their identities were initially unknown. [link] shows what happens if the rays are passed through a magnetic field. The γs are unaffected, while the αs and βs are deflected in opposite directions, indicating that the αs are positive, the βs negative, and the γs uncharged. Rutherford used both magnetic and electric fields to show that αs have a positive charge twice the magnitude of an electron, or $+2\mid q_e\mid$. In the process, he found the αs charge to mass ratio to be several thousand times smaller than the electron’s. Later on, Rutherford collected α s from a radioactive source and passed an electric discharge through them, obtaining the spectrum of recently discovered helium gas. Among many important discoveries made by Rutherford and his collaborators was the

proof that α *radiation is the emission of a helium nucleus*. Rutherford won the Nobel Prize in chemistry in 1908 for his early work. He continued to make important contributions until his death in 1934.



Alpha, beta, and gamma rays are passed through a magnetic field on the way to a phosphorescent screen. The αs and βs bend in opposite directions, while the γs are unaffected, indicating a positive charge for αs, negative for βs, and neutral for γs. Consistent results are obtained with electric fields. Collection of the radiation offers further confirmation from the direct measurement of excess charge.

Other researchers had already proved that βs are negative and have the same mass and same charge-to-mass ratio as the recently discovered electron. By 1902, it was recognized that *β radiation is the emission of an electron*. Although βs are electrons, they do not exist in the nucleus before it decays and are not ejected atomic electrons—the electron is created in the nucleus at the instant of decay.

Since γs remain unaffected by electric and magnetic fields, it is natural to think they might be photons. Evidence for this grew, but it was not until 1914 that this was proved by Rutherford and collaborators. By scattering γ radiation from a crystal and observing interference, they demonstrated that *γ radiation is the emission of a high-energy photon by a nucleus*. In fact, γ radiation comes from the de-excitation of a nucleus, just as an X-ray comes from the de-excitation of an atom. The names "γ ray" and "X-ray" identify the source of the radiation. At the same energy, γ rays and X-rays are otherwise identical.

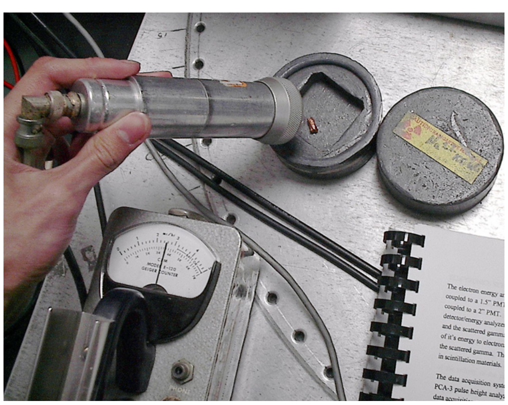
Type of Radiation	**Range**
α	A sheet of paper, a few cm of air, fractions of a mm of tissue
β	A thin aluminum plate, or tens of cm of tissue
γ	Several cm of lead or meters of concrete

Properties of Nuclear Radiation

Ionization and Range

Two of the most important characteristics of α, β, and γ rays were recognized very early. All three types of nuclear radiation produce *ionization* in materials, but they penetrate different distances in materials—that is, they have different *ranges*. Let us examine why they have these characteristics and what are some of the consequences.

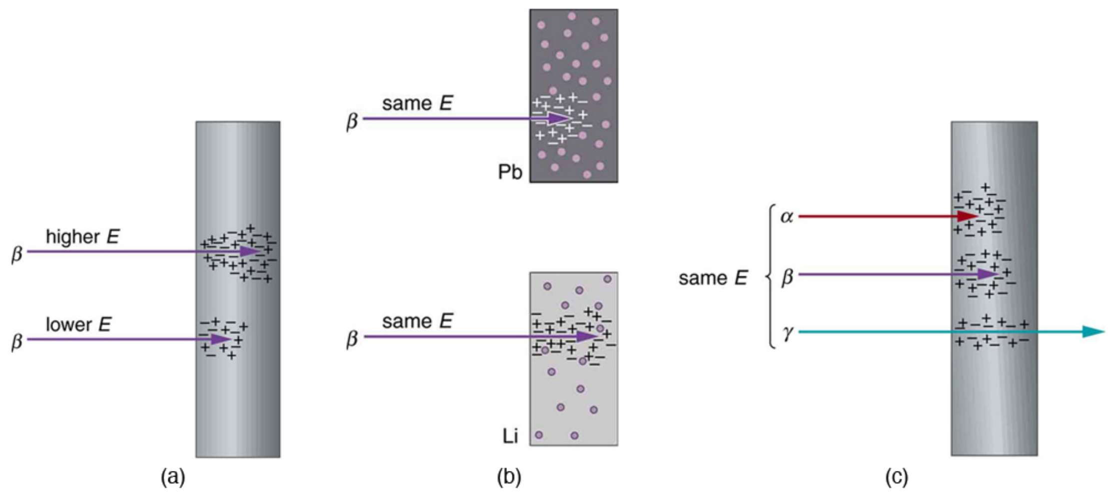
Like X-rays, nuclear radiation in the form of αs, βs, and γs has enough energy per event to ionize atoms and molecules in any material. The energy emitted in various nuclear decays ranges from a few keV to more than 10 MeV, while only a few eV are needed to produce ionization. The effects of X-rays and nuclear radiation on biological tissues and other materials, such as solid state electronics, are directly related to the ionization they produce. All of them, for example, can damage electronics or kill cancer cells. In addition, methods for detecting X-rays and nuclear radiation are based on ionization, directly or indirectly. All of them can ionize the air between the plates of a capacitor, for example, causing it to discharge. This is the basis of inexpensive personal radiation monitors, such as pictured in [link]. Apart from α, β, and γ, there are other forms of nuclear radiation as well, and these also produce ionization with similar effects. We define **ionizing radiation** as any form of radiation that produces ionization whether nuclear in origin or not, since the effects and detection of the radiation are related to ionization.



These dosimeters

(literally, dose meters) are personal radiation monitors that detect the amount of radiation by the discharge of a rechargeable internal capacitor. The amount of discharge is related to the amount of ionizing radiation encountered, a measurement of dose. One dosimeter is shown in the charger. Its scale is read through an eyepiece on the top. (credit: L. Chang, Wikimedia Commons)

The **range of radiation** is defined to be the distance it can travel through a material. Range is related to several factors, including the energy of the radiation, the material encountered, and the type of radiation (see [link]). The higher the *energy*, the greater the range, all other factors being the same. This makes good sense, since radiation loses its energy in materials primarily by producing ionization in them, and each ionization of an atom or a molecule requires energy that is removed from the radiation. The amount of ionization is, thus, directly proportional to the energy of the particle of radiation, as is its range.



The penetration or range of radiation depends on its energy, the material it encounters, and the type of radiation. (a) Greater energy means greater range. (b) Radiation has a smaller range in materials with high electron density. (c) Alphas have the smallest range, betas have a greater range, and gammas penetrate the farthest.

Radiation can be absorbed or shielded by materials, such as the lead aprons dentists drape on us when taking X-rays. Lead is a particularly effective shield compared with other materials, such as plastic or air. How does the range of radiation depend on *material*? Ionizing radiation interacts best with charged particles in a material. Since electrons have small masses, they most readily absorb the energy of the radiation in collisions. The greater the density of a material and, in particular, the greater the density of electrons within a material, the smaller the range of radiation.

Different *types* of radiation have different ranges when compared at the same energy and in the same material. Alphas have the shortest range, betas penetrate farther, and gammas have the greatest range. This is directly related to charge and speed of the particle or type of radiation. At a given energy, each α, β, or γ will produce the same number of ionizations in a material (each ionization requires a certain amount of energy on average). The more readily the particle produces ionization, the more quickly it will lose its energy. The effect of *charge* is as follows: The α has a charge of

$+2q_e$, the β has a charge of $-q_e$, and the γ is uncharged. The electromagnetic force exerted by the α is thus twice as strong as that exerted by the β and it is more likely to produce ionization. Although chargeless, the γ does interact weakly because it is an electromagnetic wave, but it is less likely to produce ionization in any encounter.

The *speed* at which they travel is the other major factor affecting the range of αs, βs, and γs. The faster they move, the less time they spend in the vicinity of an atom or a molecule, and the less likely they are to interact. Since αs and βs are particles with mass (helium nuclei and electrons, respectively), their energy is kinetic. The mass of the β particle is thousands of times less than that of the αs, so that βs must travel much faster than αs to have the same energy. Since βs move faster, they have less time to interact than αs. Gamma rays are photons, which must travel at the speed of light. They are even less likely to interact than a β, since they spend even less time near a given atom (and they have no charge). The range of γs is thus greater than the range of βs.

Alpha radiation from radioactive sources has a range much less than a millimeter of biological tissues, usually not enough to even penetrate the dead layers of our skin. On the other hand, the same α radiation can penetrate a few centimeters of air, so mere distance from a source prevents α radiation from reaching us. This makes α radiation relatively safe for our body compared to β and γ radiation. Typical β radiation can penetrate a few millimeters of tissue or about a meter of air. Beta radiation is thus hazardous even when not ingested. The range of βs in lead is about a millimeter, and so it is easy to store β sources in lead radiation-proof containers. Gamma rays have a much greater range than either αs or βs. In fact, if a given thickness of material, like a lead brick, absorbs 90% of the γ s, then a second lead brick will only absorb 90% of what got through the first. Thus, γs do not have a well-defined range; we can only cut down the amount that gets through. Typically, γs can penetrate many meters of air, go right through our bodies, and are effectively shielded (that is, reduced in intensity to acceptable levels) by many centimeters of lead. One benefit of γ s is that they can be used as radioactive tracers (see [link]).

This image of the concentration of a radioactive tracer in a patient's body reveals where the most active bone cells are, an indication of bone cancer. A short-lived radioactive substance that locates itself selectively is given to the patient, and the radiation is measured with an external detector. The emitted γ radiation has a sufficient range to leave the body—the range of αs and βs is too small for them to be observed outside the patient. (credit: Kieran Maher, Wikimedia Commons)

Section Summary

- Some nuclei are radioactive—they spontaneously decay destroying some part of their mass and emitting energetic rays, a process called nuclear radioactivity.
- Nuclear radiation, like X-rays, is ionizing radiation, because energy sufficient to ionize matter is emitted in each decay.
- The range (or distance traveled in a material) of ionizing radiation is affected by the charge of the emitted particle and its energy. Greater electric charge and lower energy lead to shorter ranges; less electric charge and higher energy lead to longer ranges.
- Radiation detectors are based directly or indirectly upon the ionization created by radiation, as are the effects of radiation on living and inert materials.

Conceptual Questions

Exercise:

Problem:

Suppose the range for $5.0\text{ MeV}\alpha$ ray is known to be 2.0 mm in a certain material. Does this mean that every $5.0\text{ MeV}\alpha$ a ray that strikes this material travels 2.0 mm, or does the range have an average value with some statistical fluctuations in the distances traveled? Explain.

Exercise:

Problem:

What is the difference between γ rays and characteristic X-rays? Is either necessarily more energetic than the other? Which can be the most energetic?

Exercise:

Problem:

Ionizing radiation interacts with matter by scattering from electrons and nuclei in the substance. Based on the law of conservation of momentum and energy, explain why electrons tend to absorb more energy than nuclei in these interactions.

Exercise:

Problem:

What characteristics of radioactivity show it to be nuclear in origin and not atomic?

Exercise:

Problem:

What is the source of the energy emitted in radioactive decay? Identify an earlier conservation law, and describe how it was modified to take such processes into account.

Exercise:

Problem:

Consider [link]. If an electric field is substituted for the magnetic field with positive charge instead of the north pole and negative charge instead of the south pole, in which directions will the α, β , and γ rays bend?

Exercise:

Problem:

Explain how an α particle can have a larger range in air than a β particle with the same energy in lead.

Exercise:

Problem:

Arrange the following according to their ability to act as radiation shields, with the best first and worst last. Explain your ordering in terms of how radiation loses its energy in matter.

(a) A solid material with low density composed of low-mass atoms.

(b) A gas composed of high-mass atoms.

(c) A gas composed of low-mass atoms.

(d) A solid with high density composed of high-mass atoms.

Exercise:

Problem:

Often, when people have to work around radioactive materials spills, we see them wearing white coveralls (usually a plastic material). What types of radiation (if any) do you think these suits protect the worker from, and how?

Glossary

alpha ray
: a component of nuclear radioactivity; consists of two protons and two neutrons

beta ray
: a component of nuclear radioactivity; consists of an electron

gamma ray
: a component of nuclear radioactivity; consists of a high-energy photon

ionizing radiation
: a radiation that produces ionization; examples include alpha rays, beta rays, gamma rays, and X-rays

nuclear radiation

radiation that originate in the nuclei of atoms; alpha, beta, and gamma rays are among most common forms; also see "radioactivity"

radioactivity

the emission of rays from the nuclei of atoms

radioactive

a substance or object that emits nuclear radiation

range of radiation

the distance that the radiation can travel through a material

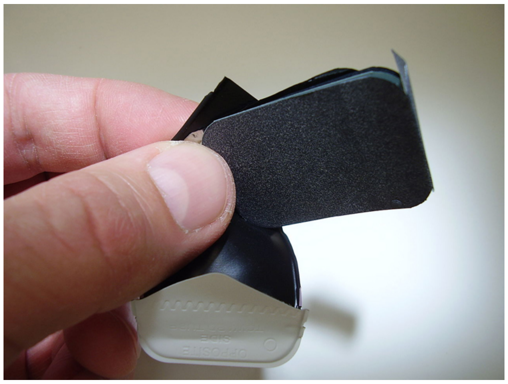
Radiation Detection and Detectors

- Explain the working principle of a Geiger tube.
- Define and discuss radiation detectors.

It is well known that ionizing radiation affects us but does not trigger nerve impulses. Newspapers carry stories about unsuspecting victims of radiation poisoning who fall ill with radiation sickness, such as burns and blood count changes, but who never felt the radiation directly. This makes the detection of radiation by instruments more than an important research tool. This section is a brief overview of radiation detection and some of its applications.

Human Application

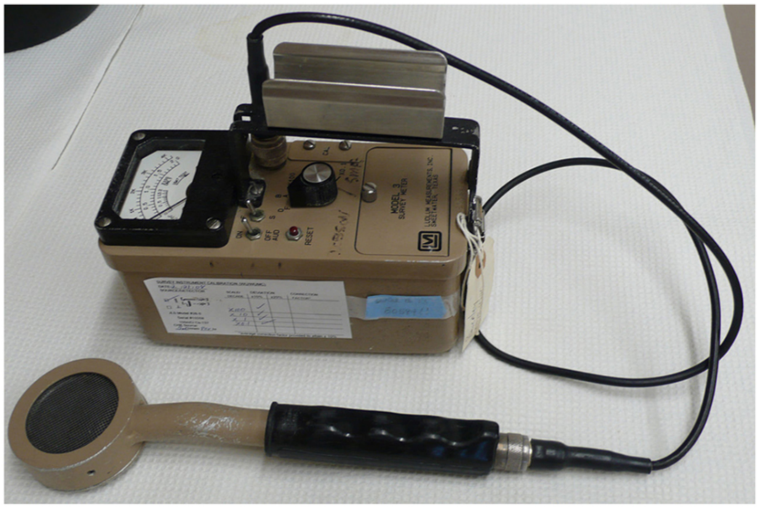
The first direct detection of radiation was Becquerel's fogged photographic plate. Photographic film is still the most common detector of ionizing radiation, being used routinely in medical and dental x rays. Nuclear radiation is also captured on film, such as seen in [link]. The mechanism for film exposure by ionizing radiation is similar to that by photons. A quantum of energy interacts with the emulsion and alters it chemically, thus exposing the film. The quantum come from an α-particle, β-particle, or photon, provided it has more than the few eV of energy needed to induce the chemical change (as does all ionizing radiation). The process is not 100% efficient, since not all incident radiation interacts and not all interactions produce the chemical change. The amount of film darkening is related to exposure, but the darkening also depends on the type of radiation, so that absorbers and other devices must be used to obtain energy, charge, and particle-identification information.



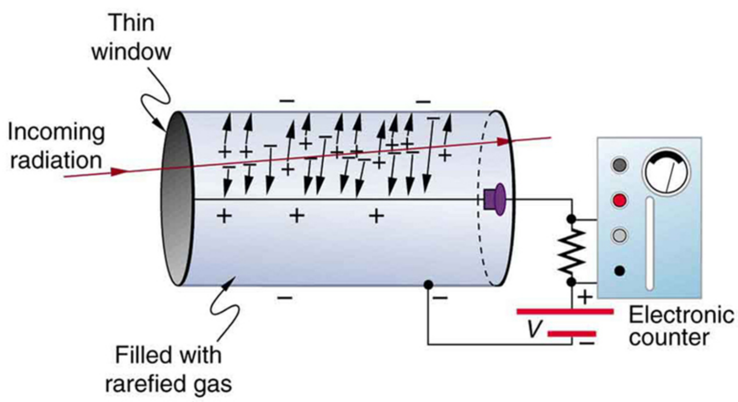
Film badges contain film similar to that used in this dental x-ray film and is sandwiched between various absorbers to determine the penetrating ability of the radiation as well as the amount. (credit: Werneuchen, Wikimedia Commons)

Another very common **radiation detector** is the **Geiger tube**. The clicking and buzzing sound we hear in dramatizations and documentaries, as well as in our own physics labs, is usually an audio output of events detected by a Geiger counter. These relatively inexpensive radiation detectors are based on the simple and sturdy Geiger tube, shown schematically in [link](b). A conducting cylinder with a wire along its axis is filled with an insulating gas so that a voltage applied between the cylinder and wire produces almost no current. Ionizing radiation passing through the tube produces free ion pairs that are attracted to the wire and cylinder, forming a current that is detected as a count. The word count implies that there is no information on energy, charge, or type of radiation with a simple Geiger counter. They do not detect every particle, since some radiation can pass through without producing enough ionization to be detected. However, Geiger counters are very useful in producing a prompt output that reveals the existence and relative intensity of ionizing radiation.

(a)



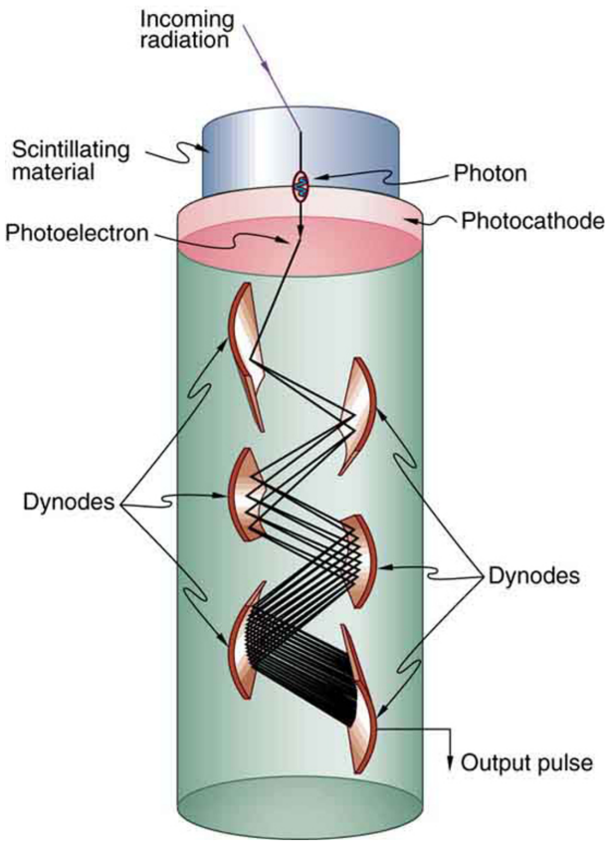
(b)



(a) Geiger counters such as this one are used for prompt monitoring of radiation levels, generally giving only relative intensity and not identifying the type or energy of the radiation. (credit: TimVickers, Wikimedia Commons) (b) Voltage applied between the cylinder and wire in a Geiger tube causes ions and electrons produced by radiation passing through the gas-filled cylinder to move towards them. The resulting current is detected and registered as a count.

Another radiation detection method records light produced when radiation interacts with materials. The energy of the radiation is sufficient to excite atoms in a material that may fluoresce, such as the phosphor used by Rutherford’s group. Materials called **scintillators** use a more complex collaborative process to convert radiation energy into light. Scintillators may be liquid or solid, and they can be very efficient. Their light output can provide information about the energy, charge, and type of radiation. Scintillator light flashes are very brief in duration, enabling the detection of a huge number of particles in short periods of time. Scintillator detectors are used in a variety of research and diagnostic applications. Among these are the detection by satellite-mounted equipment of the radiation from distant galaxies, the analysis of radiation from a person indicating body burdens, and the detection of exotic particles in accelerator laboratories.

Light from a scintillator is converted into electrical signals by devices such as the **photomultiplier** tube shown schematically in [link]. These tubes are based on the photoelectric effect, which is multiplied in stages into a cascade of electrons, hence the name photomultiplier. Light entering the photomultiplier strikes a metal plate, ejecting an electron that is attracted by a positive potential difference to the next plate, giving it enough energy to eject two or more electrons, and so on. The final output current can be made proportional to the energy of the light entering the tube, which is in turn proportional to the energy deposited in the scintillator. Very sophisticated information can be obtained with scintillators, including energy, charge, particle identification, direction of motion, and so on.



Photomultipliers use the photoelectric effect on the photocathode to convert the light output of a scintillator into an electrical signal. Each successive dynode has a more-positive potential than the last and attracts the ejected electrons, giving them more energy. The number of electrons is thus multiplied at each dynode, resulting in an easily detected output current.

Solid-state radiation detectors convert ionization produced in a semiconductor (like those found in computer chips) directly into an

electrical signal. Semiconductors can be constructed that do not conduct current in one particular direction. When a voltage is applied in that direction, current flows only when ionization is produced by radiation, similar to what happens in a Geiger tube. Further, the amount of current in a solid-state detector is closely related to the energy deposited and, since the detector is solid, it can have a high efficiency (since ionizing radiation is stopped in a shorter distance in solids fewer particles escape detection). As with scintillators, very sophisticated information can be obtained from solid-state detectors.

Note:
PhET Explorations: Radioactive Dating Game
Learn about different types of radiometric dating, such as carbon dating. Understand how decay and half life work to enable radiometric dating to work. Play a game that tests your ability to match the percentage of the dating element that remains to the age of the object.
https://archive.cnx.org/specials/d709a8b0-068c-11e6-bcfb-f38266817c66/radioactive-dating-game/#sim-half-life

Section Summary

- Radiation detectors are based directly or indirectly upon the ionization created by radiation, as are the effects of radiation on living and inert materials.

Conceptual Questions

Exercise:

Problem:

Is it possible for light emitted by a scintillator to be too low in frequency to be used in a photomultiplier tube? Explain.

Problems & Exercises

Exercise:

Problem:

The energy of 30.0 eV is required to ionize a molecule of the gas inside a Geiger tube, thereby producing an ion pair. Suppose a particle of ionizing radiation deposits 0.500 MeV of energy in this Geiger tube. What maximum number of ion pairs can it create?

Solution:

1.67×10^4

Exercise:

Problem:

A particle of ionizing radiation creates 4000 ion pairs in the gas inside a Geiger tube as it passes through. What minimum energy was deposited, if 30.0 eV is required to create each ion pair?

Exercise:

Problem:

(a) Repeat [link], and convert the energy to joules or calories. (b) If all of this energy is converted to thermal energy in the gas, what is its temperature increase, assuming $50.0\ \text{cm}^3$ of ideal gas at 0.250-atm pressure? (The small answer is consistent with the fact that the energy is large on a quantum mechanical scale but small on a macroscopic scale.)

Exercise:

Problem:

Suppose a particle of ionizing radiation deposits 1.0 MeV in the gas of a Geiger tube, all of which goes to creating ion pairs. Each ion pair requires 30.0 eV of energy. (a) The applied voltage sweeps the ions out of the gas in 1.00 μs. What is the current? (b) This current is smaller than the actual current since the applied voltage in the Geiger tube accelerates the separated ions, which then create other ion pairs in subsequent collisions. What is the current if this last effect multiplies the number of ion pairs by 900?

Glossary

Geiger tube
: a very common radiation detector that usually gives an audio output

photomultiplier
: a device that converts light into electrical signals

radiation detector
: a device that is used to detect and track the radiation from a radioactive reaction

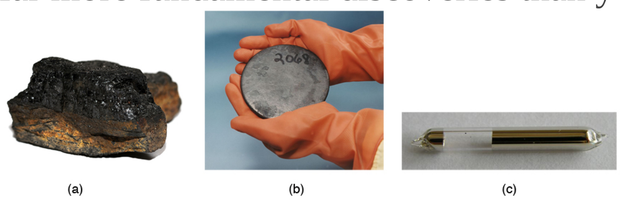
scintillators
: a radiation detection method that records light produced when radiation interacts with materials

solid-state radiation detectors
: semiconductors fabricated to directly convert incident radiation into electrical current

Substructure of the Nucleus

- Define and discuss the nucleus in an atom.
- Define atomic number.
- Define and discuss isotopes.
- Calculate the density of the nucleus.
- Explain nuclear force.

What is inside the nucleus? Why are some nuclei stable while others decay? (See [link].) Why are there different types of decay (α, β and γ)? Why are nuclear decay energies so large? Pursuing natural questions like these has led to far more fundamental discoveries than you might imagine.



Why is most of the carbon in this coal stable (a), while the uranium in the disk (b) slowly decays over billions of years? Why is cesium in this ampule (c) even less stable than the uranium, decaying in far less than 1/1,000,000 the time? What is the reason uranium and cesium undergo different types of decay (α and β, respectively)? (credits: (a) Bresson Thomas, Wikimedia Commons; (b) U.S. Department of Energy; (c) Tomihahndorf, Wikimedia Commons)

We have already identified **protons** as the particles that carry positive charge in the nuclei. However, there are actually *two* types of particles in the nuclei—the *proton* and the *neutron*, referred to collectively as **nucleons**, the constituents of nuclei. As its name implies, the **neutron** is a neutral particle ($q = 0$) that has

nearly the same mass and intrinsic spin as the proton. [link] compares the masses of protons, neutrons, and electrons. Note how close the proton and neutron masses are, but the neutron is slightly more massive once you look past the third digit. Both nucleons are much more massive than an electron. In fact, $m_p = 1836 m_e$ and $m_n = 1839 m_e$.

[link] also gives masses in terms of mass units that are more convenient than kilograms on the atomic and nuclear scale. The first of these is the *unified* ***atomic mass*** *unit* (u), defined as

Equation:

$$1\text{ u} = 1.6605 \times 10^{-27}\text{ kg}.$$

This unit is defined so that a neutral carbon ^{12}C atom has a mass of exactly 12 u. Masses are also expressed in units of MeV/c^2. These units are very convenient when considering the conversion of mass into energy (and vice versa), as is so prominent in nuclear processes. Using $E = mc^2$ and units of m in MeV/c^2, we find that c^2 cancels and E comes out conveniently in MeV. For example, if the rest mass of a proton is converted entirely into energy, then

Equation:

$$E = mc^2 = (938.27\text{ MeV}/c^2)c^2 = 938.27\text{ MeV}.$$

It is useful to note that 1 u of mass converted to energy produces 931.5 MeV, or

Equation:

$$1\text{ u} = 931.5\text{ MeV}/c^2.$$

All properties of a nucleus are determined by the number of protons and neutrons it has. A specific combination of protons and neutrons is called a **nuclide** and is a unique nucleus. The following notation is used to represent a particular nuclide:

Equation:

$$^{A}_{Z}\text{X}_{N},$$

where the symbols A, X, Z , and N are defined as follows: The *number of protons in a nucleus* is the **atomic number** Z. X is the *symbol for the element*, such as Ca for calcium. However, once Z is known, the element is known; hence, Z and X are redundant. For example, $Z = 20$ is always calcium, and calcium always has $Z = 20$. N is the *number of neutrons* in a nucleus. In the notation for a nuclide, the subscript N is usually omitted. The symbol A is defined as the number of nucleons or the *total number of protons and neutrons*,
Equation:

$$A = N + Z,$$

where A is also called the **mass number**. This name for A is logical; the mass of an atom is nearly equal to the mass of its nucleus, since electrons have so little mass. The mass of the nucleus turns out to be nearly equal to the sum of the masses of the protons and neutrons in it, which is proportional to A. In this context, it is particularly convenient to express masses in units of u. Both protons and neutrons have masses close to 1 u, and so the mass of an atom is close to A u. For example, in an oxygen nucleus with eight protons and eight neutrons, $A = 16$, and its mass is 16 u. As noticed, the unified atomic mass unit is defined so that a neutral carbon atom (actually a $^{12}\mathrm{C}$ atom) has a mass of *exactly* 12 u. Carbon was chosen as the standard, partly because of its importance in organic chemistry.

Particle	Symbol	kg	u	MeVc^2
Proton	p	1.67262×10^{-27}	1.007276	938.27
Neutron	n	1.67493×10^{-27}	1.008665	939.57

Particle	Symbol	kg	u	MeVc^2
Electron	*e*	9.1094×10^{-31}	0.00054858	0.511

Masses of the Proton, Neutron, and Electron

Let us look at a few examples of nuclides expressed in the ${}_{Z}^{A}\mathrm{X}_{N}$ notation. The nucleus of the simplest atom, hydrogen, is a single proton, or ${}_{1}^{1}\mathrm{H}$ (the zero for no neutrons is often omitted). To check this symbol, refer to the periodic table—you see that the atomic number Z of hydrogen is 1. Since you are given that there are no neutrons, the mass number A is also 1. Suppose you are told that the helium nucleus or α particle has two protons and two neutrons. You can then see that it is written ${}_{2}^{4}\mathrm{He}_{2}$. There is a scarce form of hydrogen found in nature called deuterium; its nucleus has one proton and one neutron and, hence, twice the mass of common hydrogen. The symbol for deuterium is, thus, ${}_{1}^{2}\mathrm{H}_{1}$ (sometimes D is used, as for deuterated water $\mathrm{D_2O}$). An even rarer—and radioactive—form of hydrogen is called tritium, since it has a single proton and two neutrons, and it is written ${}_{1}^{3}\mathrm{H}_{2}$. These three varieties of hydrogen have nearly identical chemistries, but the nuclei differ greatly in mass, stability, and other characteristics. Nuclei (such as those of hydrogen) having the same Z and different N s are defined to be **isotopes** of the same element.

There is some redundancy in the symbols A, X, Z, and N . If the element X is known, then Z can be found in a periodic table and is always the same for a given element. If both A and X are known, then N can also be determined (first find Z; then, $N = A - Z$). Thus the simpler notation for nuclides is
Equation:

$$^{A}\mathrm{X},$$

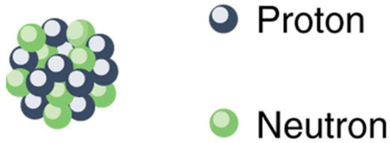
which is sufficient and is most commonly used. For example, in this simpler notation, the three isotopes of hydrogen are $^{1}\mathrm{H}$, $^{2}\mathrm{H}$, and $^{3}\mathrm{H}$, while the α particle is $^{4}\mathrm{He}$. We read this backward, saying helium-4 for $^{4}\mathrm{He}$, or uranium-238 for $^{238}\mathrm{U}$. So for $^{238}\mathrm{U}$, should we need to know, we can determine that $Z = 92$ for uranium from the periodic table, and, thus, $N = 238 - 92 = 146$.

A variety of experiments indicate that a nucleus behaves something like a tightly packed ball of nucleons, as illustrated in [link]. These nucleons have large kinetic energies and, thus, move rapidly in very close contact. Nucleons can be separated by a large force, such as in a collision with another nucleus, but resist strongly being pushed closer together. The most compelling evidence that nucleons are closely packed in a nucleus is that the **radius of a nucleus**, r, is found to be given approximately by

Equation:

$$r = r_0 A^{1/3},$$

where $r_0 = 1.2\ \text{fm}$ and A is the mass number of the nucleus. Note that $r^3 \propto A$. Since many nuclei are spherical, and the volume of a sphere is $V = (4/3)\pi r^3$, we see that $V \propto A$ —that is, the volume of a nucleus is proportional to the number of nucleons in it. This is what would happen if you pack nucleons so closely that there is no empty space between them.



A model of the nucleus.

Nucleons are held together by nuclear forces and resist both being pulled apart and pushed inside one another. The volume of the nucleus is the sum of the volumes of the nucleons in it, here shown in different colors to represent protons and neutrons.

Example:
How Small and Dense Is a Nucleus?
(a) Find the radius of an iron-56 nucleus. (b) Find its approximate density in kg/m^3, approximating the mass of ^{56}Fe to be 56 u.
Strategy and Concept

(a) Finding the radius of ^{56}Fe is a straightforward application of $r = r_0 A^{1/3}$, given $A = 56$. (b) To find the approximate density, we assume the nucleus is spherical (this one actually is), calculate its volume using the radius found in part (a), and then find its density from $\rho = m/V$. Finally, we will need to convert density from units of u/fm^3 to kg/m^3.

Solution

(a) The radius of a nucleus is given by

Equation:

$$r = r_0 A^{1/3}.$$

Substituting the values for r_0 and A yields

Equation:

$$\begin{aligned} r &= (1.2\ \text{fm})(56)^{1/3} = (1.2\ \text{fm})(3.83) \\ &= 4.6\ \text{fm}. \end{aligned}$$

(b) Density is defined to be $\rho = m/V$, which for a sphere of radius r is

Equation:

$$\rho = \frac{m}{V} = \frac{m}{(4/3)\pi r^3}.$$

Substituting known values gives

Equation:

$$\begin{aligned} \rho &= \frac{56\ \text{u}}{(1.33)(3.14)(4.6\ \text{fm})^3} \\ &= 0.138\ \text{u/fm}^3. \end{aligned}$$

Converting to units of kg/m^3, we find

Equation:

$$\begin{aligned} \rho &= (0.138\ \text{u/fm}^3)(1.66 \times 10^{-27}\ \text{kg/u})\left(\frac{1\ \text{fm}}{10^{-15}\ \text{m}}\right) \\ &= 2.3 \times 10^{17}\ \text{kg/m}^3. \end{aligned}$$

Discussion

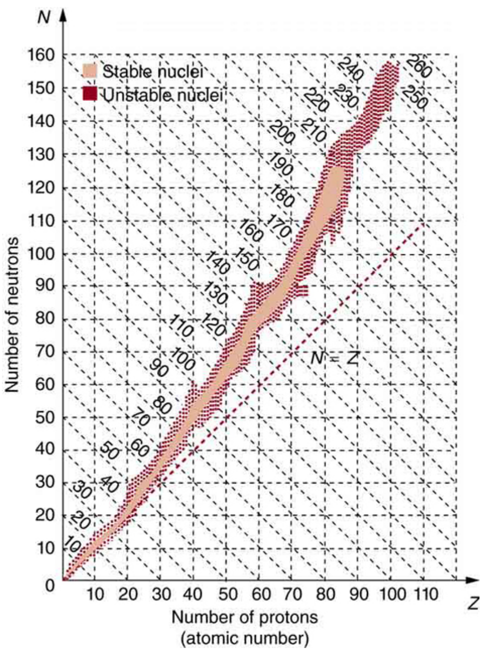
(a) The radius of this medium-sized nucleus is found to be approximately 4.6 fm, and so its diameter is about 10 fm, or 10^{-14} m. In our discussion of Rutherford's discovery of the nucleus, we noticed that it is about 10^{-15} m in diameter (which is for lighter nuclei), consistent with this result to an order of magnitude. The nucleus is much smaller in diameter than the typical atom, which has a diameter of the order of 10^{-10} m.
(b) The density found here is so large as to cause disbelief. It is consistent with earlier discussions we have had about the nucleus being very small and containing nearly all of the mass of the atom. Nuclear densities, such as found here, are about 2×10^{14} times greater than that of water, which has a density of "only" $10^{3}\ \mathrm{kg/m^3}$. One cubic meter of nuclear matter, such as found in a neutron star, has the same mass as a cube of water 61 km on a side.

Nuclear Forces and Stability

What forces hold a nucleus together? The nucleus is very small and its protons, being positive, exert tremendous repulsive forces on one another. (The Coulomb force increases as charges get closer, since it is proportional to $1/r^2$, even at the tiny distances found in nuclei.) The answer is that two previously unknown forces hold the nucleus together and make it into a tightly packed ball of nucleons. These forces are called the *weak and strong nuclear forces*. Nuclear forces are so short ranged that they fall to zero strength when nucleons are separated by only a few fm. However, like glue, they are strongly attracted when the nucleons get close to one another. The strong nuclear force is about 100 times more attractive than the repulsive EM force, easily holding the nucleons together. Nuclear forces become extremely repulsive if the nucleons get too close, making nucleons strongly resist being pushed inside one another, something like ball bearings.

The fact that nuclear forces are very strong is responsible for the very large energies emitted in nuclear decay. During decay, the forces do work, and since work is force times the distance, a large force can result in a large emitted energy. In fact, we know that there are *two* distinct nuclear forces because of the different types of nuclear decay—the strong nuclear force is responsible for α decay, while the weak nuclear force is responsible for β decay.

The many stable and unstable nuclei we have explored, and the hundreds we have not discussed, can be arranged in a table called the **chart of the nuclides**, a simplified version of which is shown in [link]. Nuclides are located on a plot of N versus Z. Examination of a detailed chart of the nuclides reveals patterns in the characteristics of nuclei, such as stability, abundance, and types of decay, analogous to but more complex than the systematics in the periodic table of the elements.



Simplified chart of the nuclides, a graph of N versus Z for known nuclides. The patterns of stable and unstable nuclides reveal characteristics of the nuclear forces. The dashed line is for $N = Z$. Numbers along diagonals are mass numbers A.

In principle, a nucleus can have any combination of protons and neutrons, but [link] shows a definite pattern for those that are stable. For low-mass nuclei, there is a strong tendency for N and Z to be nearly equal. This means that the nuclear force is more attractive when $N = Z$. More detailed examination reveals greater stability when N and Z are even numbers—nuclear forces are more attractive when neutrons and protons are in pairs. For increasingly higher masses, there are progressively more neutrons than protons in stable nuclei. This is due to the ever-growing repulsion between protons. Since nuclear forces are short ranged, and the Coulomb force is long ranged, an excess of neutrons keeps the protons a little farther apart, reducing Coulomb repulsion. Decay modes of nuclides out of the region of stability consistently produce nuclides closer to the region of stability. There are more stable nuclei having certain numbers of protons and neutrons, called **magic numbers**. Magic numbers indicate a shell structure for the nucleus in which closed shells are more stable. Nuclear shell theory has been very successful in explaining nuclear energy levels, nuclear decay, and the greater stability of nuclei with closed shells. We have been producing ever-heavier transuranic elements since the early 1940s, and we have now produced the element with $Z = 118$. There are theoretical predictions of an island of relative stability for nuclei with such high Z s.

The German-born American physicist Maria Goeppert Mayer (1906–1972)

shared the 1963 Nobel Prize in physics with J. Jensen for the creation of the nuclear shell model. This successful nuclear model has nucleons filling shells analogous to electron shells in atoms. It was inspired by patterns observed in nuclear properties. (credit: Nobel Foundation via Wikimedia Commons)

Section Summary

- Two particles, both called nucleons, are found inside nuclei. The two types of nucleons are protons and neutrons; they are very similar, except that the proton is positively charged while the neutron is neutral. Some of their characteristics are given in [link] and compared with those of the electron. A mass unit convenient to atomic and nuclear processes is the unified atomic mass unit (u), defined to be
  Equation:

  $$1\ \text{u} = 1.6605 \times 10^{-27}\ \text{kg} = 931.46\ \text{MeV}/c^2.$$

- A nuclide is a specific combination of protons and neutrons, denoted by
  Equation:

$$ {}^{A}_{Z}\mathrm{X}_{N} \text{ or simply} {}^{A}\mathrm{X}, $$

Z is the number of protons or atomic number, X is the symbol for the element, N is the number of neutrons, and A is the mass number or the total number of protons and neutrons,
Equation:

$$ A = N + Z. $$

- Nuclides having the same Z but different N are isotopes of the same element.
- The radius of a nucleus, r, is approximately
  Equation:

  $$ r = r_0 A^{1/3}, $$

  where $r_0 = 1.2 \text{ fm}$. Nuclear volumes are proportional to A. There are two nuclear forces, the weak and the strong. Systematics in nuclear stability seen on the chart of the nuclides indicate that there are shell closures in nuclei for values of Z and N equal to the magic numbers, which correspond to highly stable nuclei.

Conceptual Questions

Exercise:

Problem:

The weak and strong nuclear forces are basic to the structure of matter. Why we do not experience them directly?

Exercise:

Problem:

Define and make clear distinctions between the terms neutron, nucleon, nucleus, nuclide, and neutrino.

Exercise:

Problem:

What are isotopes? Why do different isotopes of the same element have similar chemistries?

Problems & Exercises

Exercise:

Problem:

Verify that a 2.3×10^{17} kg mass of water at normal density would make a cube 60 km on a side, as claimed in [link]. (This mass at nuclear density would make a cube 1.0 m on a side.)

Solution:
Equation:

$$\begin{aligned} m = \rho V = \rho d^3 \quad \Rightarrow \quad a &= \left(\frac{m}{\rho}\right)^{1/3} = \left(\frac{2.3 \times 10^{17}\ \text{kg}}{1000\ \text{kg/m}^3}\right)^{\frac{1}{3}} \\ &= 61 \times 10^3\ \text{m} = 61\ \text{km} \end{aligned}$$

Exercise:

Problem:

Find the length of a side of a cube having a mass of 1.0 kg and the density of nuclear matter, taking this to be $2.3 \times 10^{17}\ \text{kg/m}^3$.

Exercise:

Problem: What is the radius of an α particle?

Solution:

1.9 fm

Exercise:

Problem:

Find the radius of a ^{238}Pu nucleus. ^{238}Pu is a manufactured nuclide that is used as a power source on some space probes.

Exercise:

Problem:

(a) Calculate the radius of ^{58}Ni, one of the most tightly bound stable nuclei.

(b) What is the ratio of the radius of ^{58}Ni to that of ^{258}Ha, one of the largest nuclei ever made? Note that the radius of the largest nucleus is still much smaller than the size of an atom.

Solution:

(a) 4.6 fm

(b) $0.61\text{ to }1$

Exercise:

Problem:

The unified atomic mass unit is defined to be $1\text{ u}=1.6605\times10^{-27}\text{ kg}$. Verify that this amount of mass converted to energy yields 931.5 MeV. Note that you must use four-digit or better values for c and $\mid q_e\mid$.

Exercise:

Problem:

What is the ratio of the velocity of a β particle to that of an α particle, if they have the same nonrelativistic kinetic energy?

Solution:

$85.4\text{ to }1$

Exercise:

Problem:

If a 1.50-cm-thick piece of lead can absorb 90.0% of the γ rays from a radioactive source, how many centimeters of lead are needed to absorb all but 0.100% of the γ rays?

Exercise:

Problem:

The detail observable using a probe is limited by its wavelength. Calculate the energy of a γ-ray photon that has a wavelength of 1×10^{-16} m, small enough to detect details about one-tenth the size of a nucleon. Note that a photon having this energy is difficult to produce and interacts poorly with the nucleus, limiting the practicability of this probe.

Solution:

12.4 GeV

Exercise:

Problem:

(a) Show that if you assume the average nucleus is spherical with a radius $r = r_0 A^{1/3}$, and with a mass of A u, then its density is independent of A.

(b) Calculate that density in u/fm^3 and kg/m^3, and compare your results with those found in [link] for ^{56}Fe.

Exercise:

Problem:

What is the ratio of the velocity of a 5.00-MeV β ray to that of an α particle with the same kinetic energy? This should confirm that βs travel much faster than αs even when relativity is taken into consideration. (See also [link].)

Solution:

19.3 to 1

Glossary

atomic mass
: the total mass of the protons, neutrons, and electrons in a single atom

atomic number
: number of protons in a nucleus

chart of the nuclides
: a table comprising stable and unstable nuclei

isotopes
: nuclei having the same Z and different Ns

magic numbers
: a number that indicates a shell structure for the nucleus in which closed shells are more stable

mass number
: number of nucleons in a nucleus

neutron
: a neutral particle that is found in a nucleus

nucleons
: the particles found inside nuclei

nucleus
: a region consisting of protons and neutrons at the center of an atom

nuclide
: a type of atom whose nucleus has specific numbers of protons and neutrons

protons
: the positively charged nucleons found in a nucleus

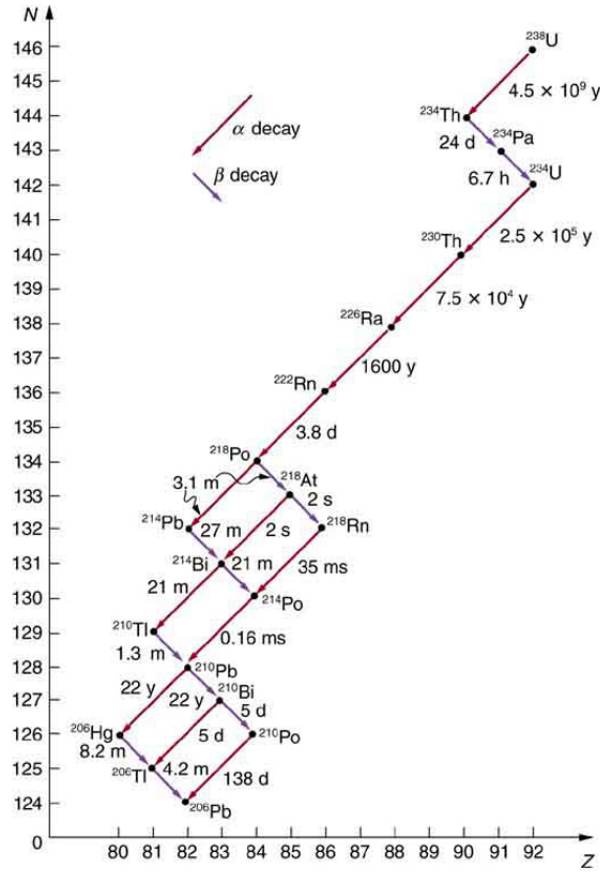
radius of a nucleus
: the radius of a nucleus is $r = r_0 A^{1/3}$

Nuclear Decay and Conservation Laws

- Define and discuss nuclear decay.
- State the conservation laws.
- Explain parent and daughter nucleus.
- Calculate the energy emitted during nuclear decay.

Nuclear **decay** has provided an amazing window into the realm of the very small. Nuclear decay gave the first indication of the connection between mass and energy, and it revealed the existence of two of the four basic forces in nature. In this section, we explore the major modes of nuclear decay; and, like those who first explored them, we will discover evidence of previously unknown particles and conservation laws.

Some nuclides are stable, apparently living forever. Unstable nuclides decay (that is, they are radioactive), eventually producing a stable nuclide after many decays. We call the original nuclide the **parent** and its decay products the **daughters**. Some radioactive nuclides decay in a single step to a stable nucleus. For example, ${}^{60}\text{Co}$ is unstable and decays directly to ${}^{60}\text{Ni}$, which is stable. Others, such as ${}^{238}\text{U}$, decay to another unstable nuclide, resulting in a **decay series** in which each subsequent nuclide decays until a stable nuclide is finally produced. The decay series that starts from ${}^{238}\text{U}$ is of particular interest, since it produces the radioactive isotopes ${}^{226}\text{Ra}$ and ${}^{210}\text{Po}$, which the Curies first discovered (see [link]). Radon gas is also produced (${}^{222}\text{Rn}$ in the series), an increasingly recognized naturally occurring hazard. Since radon is a noble gas, it emanates from materials, such as soil, containing even trace amounts of ${}^{238}\text{U}$ and can be inhaled. The decay of radon and its daughters produces internal damage. The ${}^{238}\text{U}$ decay series ends with ${}^{206}\text{Pb}$, a stable isotope of lead.



The decay series produced by ^{238}U, the most common uranium isotope. Nuclides are graphed in the same manner as in the chart of nuclides. The type of decay for each member of the series is shown, as well as the half-lives. Note that some nuclides decay by more than one mode. You can see why radium and polonium are found in uranium ore. A stable isotope of lead is the end product of the series.

Note that the daughters of α decay shown in [link] always have two fewer protons and two fewer neutrons than the parent. This seems reasonable, since we know that α decay is the emission of a ^{4}He nucleus, which has two protons and two neutrons. The daughters of β decay have one less neutron and one more proton than their parent. Beta decay is a little more subtle, as we shall see. No γ decays are shown in the figure, because they do not produce a daughter that differs from the parent.

Alpha Decay

In **alpha decay**, a ^{4}He nucleus simply breaks away from the parent nucleus, leaving a daughter with two fewer protons and two fewer neutrons than the parent (see [link]). One example of α decay is shown in [link] for ^{238}U. Another nuclide that undergoes α decay is ^{239}Pu. The decay equations for these two nuclides are

Equation:

$$^{238}\text{U} \rightarrow {}^{234}\text{Th}_{92}^{234} + {}^{4}\text{He}$$

and

Equation:

$$^{239}\text{Pu} \rightarrow {}^{235}\text{U} + {}^{4}\text{He}.$$

Alpha decay is the separation of a ^{4}He nucleus from the parent. The daughter nucleus has two fewer protons and two fewer neutrons than the parent. Alpha decay occurs spontaneously only if the daughter and ^{4}He nucleus have less total mass than the parent.

If you examine the periodic table of the elements, you will find that Th has $Z = 90$, two fewer than U, which has $Z = 92$. Similarly, in the second **decay equation**, we see that U has two fewer protons than Pu, which has $Z = 94$. The general rule for α decay is best written in the format $^{A}_{Z}\text{X}_{N}$. If a certain nuclide is known to α decay (generally this information must be looked up in a table of isotopes, such as in Appendix B), its α **decay equation** is

Equation:

$$ {}^{A}_{Z}\mathrm{X}_{N} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y}_{N-2} + {}^{4}_{2}\mathrm{He}_{2} \quad (\alpha \text{ decay}) $$

where Y is the nuclide that has two fewer protons than X, such as Th having two fewer than U. So if you were told that ${}^{239}\mathrm{Pu}$ α decays and were asked to write the complete decay equation, you would first look up which element has two fewer protons (an atomic number two lower) and find that this is uranium. Then since four nucleons have broken away from the original 239, its atomic mass would be 235.

It is instructive to examine conservation laws related to α decay. You can see from the equation ${}^{A}_{Z}\mathrm{X}_{N} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y}_{N-2} + {}^{4}_{2}\mathrm{He}_{2}$ that total charge is conserved. Linear and angular momentum are conserved, too. Although conserved angular momentum is not of great consequence in this type of decay, conservation of linear momentum has interesting consequences. If the nucleus is at rest when it decays, its momentum is zero. In that case, the fragments must fly in opposite directions with equal-magnitude momenta so that total momentum remains zero. This results in the α particle carrying away most of the energy, as a bullet from a heavy rifle carries away most of the energy of the powder burned to shoot it. Total mass–energy is also conserved: the energy produced in the decay comes from conversion of a fraction of the original mass. The general relationship is
Equation:

$$ E = (\Delta m)c^2. $$

Here, E is the **nuclear reaction energy** (the reaction can be nuclear decay or any other reaction), and Δm is the difference in mass between initial and final products. When the final products have less total mass, Δm is positive, and the reaction releases energy (is exothermic). When the products have greater total mass, the reaction is endothermic (Δm is negative) and must be induced with an energy input. For α decay to be spontaneous, the decay products must have smaller mass than the parent.

Example:
Alpha Decay Energy Found from Nuclear Masses
Find the energy emitted in the α decay of ${}^{239}\mathrm{Pu}$.
Strategy
Nuclear reaction energy, such as released in α decay, can be found using the equation $E = (\Delta m)c^2$. We must first find Δm, the difference in mass between the parent nucleus and the products of the decay. This is easily done using masses given in Appendix A.
Solution
The decay equation was given earlier for ${}^{239}\mathrm{Pu}$; it is
Equation:

$$^{239}\text{Pu} \rightarrow {}^{235}\text{U} + {}^{4}\text{He}.$$

Thus the pertinent masses are those of ^{239}Pu, ^{235}U, and the α particle or ^{4}He, all of which are listed in Appendix A. The initial mass was $m(^{239}\text{Pu}) = 239.052157 \text{ u}$. The final mass is the sum $m(^{235}\text{U})+m(^{4}\text{He})= 235.043924 \text{ u} + 4.002602 \text{ u} = 239.046526 \text{ u}$. Thus,
Equation:

$$\begin{aligned}\Delta m &= m(^{239}\text{Pu}) - [m(^{235}\text{U}) + m(^{4}\text{He})] \\ &= 239.052157 \text{ u} - 239.046526 \text{ u} \\ &= 0.005631 \text{ u}.\end{aligned}$$

Now we can find E by entering Δm into the equation:
Equation:

$$E = (\Delta m)c^2 = (0.005631 \text{ u})c^2.$$

We know $1 \text{ u} = 931.5 \text{ MeV}/c^2$, and so
Equation:

$$E = (0.005631)(931.5 \text{ MeV}/c^2)(c^2) = 5.25 \text{ MeV}.$$

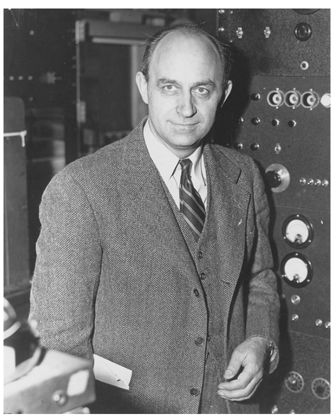
Discussion
The energy released in this α decay is in the MeV range, about 10^6 times as great as typical chemical reaction energies, consistent with many previous discussions. Most of this energy becomes kinetic energy of the α particle (or ^{4}He nucleus), which moves away at high speed. The energy carried away by the recoil of the ^{235}U nucleus is much smaller in order to conserve momentum. The ^{235}U nucleus can be left in an excited state to later emit photons (γ rays). This decay is spontaneous and releases energy, because the products have less mass than the parent nucleus. The question of why the products have less mass will be discussed in "Binding Energy." Note that the masses given in Appendix A are atomic masses of neutral atoms, including their electrons. The mass of the electrons is the same before and after α decay, and so their masses subtract out when finding Δm. In this case, there are 94 electrons before and after the decay.

Beta Decay

There are actually *three* types of **beta decay**. The first discovered was “ordinary” beta decay and is called β^- decay or electron emission. The symbol β^- represents *an electron emitted in nuclear beta decay*. Cobalt-60 is a nuclide that β^- decays in the following manner:
Equation:

$$^{60}\text{Co} \rightarrow {}^{60}\text{Ni} + \beta^- + \text{neutrino}.$$

The **neutrino** is a particle emitted in beta decay that was unanticipated and is of fundamental importance. The neutrino was not even proposed in theory until more than 20 years after beta decay was known to involve electron emissions. Neutrinos are so difficult to detect that the first direct evidence of them was not obtained until 1953. Neutrinos are nearly massless, have no charge, and do not interact with nucleons via the strong nuclear force. Traveling approximately at the speed of light, they have little time to affect any nucleus they encounter. This is, owing to the fact that they have no charge (and they are not EM waves), they do not interact through the EM force. They do interact via the relatively weak and very short range weak nuclear force. Consequently, neutrinos escape almost any detector and penetrate almost any shielding. However, neutrinos do carry energy, angular momentum (they are fermions with half-integral spin), and linear momentum away from a beta decay. When accurate measurements of beta decay were made, it became apparent that energy, angular momentum, and linear momentum were not accounted for by the daughter nucleus and electron alone. Either a previously unsuspected particle was carrying them away, or three conservation laws were being violated. Wolfgang Pauli made a formal proposal for the existence of neutrinos in 1930. The Italian-born American physicist Enrico Fermi (1901–1954) gave neutrinos their name, meaning little neutral ones, when he developed a sophisticated theory of beta decay (see [link]). Part of Fermi's theory was the identification of the weak nuclear force as being distinct from the strong nuclear force and in fact responsible for beta decay.



Enrico Fermi was nearly unique among 20th-century physicists—he made significant contributions both as an experimentalist and a theorist. His many contributions to theoretical

physics included the identification of the weak nuclear force. The fermi (fm) is named after him, as are an entire class of subatomic particles (fermions), an element (Fermium), and a major research laboratory (Fermilab). His experimental work included studies of radioactivity, for which he won the 1938 Nobel Prize in physics, and creation of the first nuclear chain reaction. (credit: United States Department of Energy, Office of Public Affairs)

The neutrino also reveals a new conservation law. There are various families of particles, one of which is the electron family. We propose that the number of members of the electron family is constant in any process or any closed system. In our example of beta decay, there are no members of the electron family present before the decay, but after, there is an electron and a neutrino. So electrons are given an electron family number of $+1$. The neutrino in β^- decay is an **electron's antineutrino**, given the symbol ν_e, where ν is the Greek letter nu, and the subscript e means this neutrino is related to the electron. The bar indicates this is a particle of **antimatter**. (All particles have antimatter counterparts that are nearly identical except that they have the opposite charge. Antimatter is almost entirely absent on Earth, but it is found in nuclear decay and other nuclear and particle reactions as well as in outer space.) The electron's antineutrino ν_e, being antimatter, has an electron family number of –1. The total is zero, before and after the decay. The new conservation law, obeyed in all circumstances, states that the *total electron family number is constant*. An electron cannot be created without also creating an antimatter family member. This law is analogous to the conservation of charge in a situation where total charge is originally zero, and equal amounts of positive and negative charge must be created in a reaction to keep the total zero.

If a nuclide ${}_{Z}^{A}\text{X}_{N}$ is known to β^- decay, then its β^- decay equation is

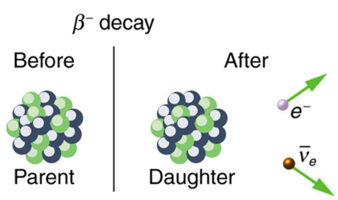
Equation:

$$ {}_{Z}^{A}\text{X}_{N} \rightarrow {}_{Z+1}^{A}\text{Y}_{N-1} + \beta^- + \bar{\nu}_e \; (\beta^- \text{ decay}), $$

where Y is the nuclide having one more proton than X (see [link]). So if you know that a certain nuclide β^- decays, you can find the daughter nucleus by first looking up Z for the parent and then determining which element has atomic number $Z+1$. In the example of the β^- decay of ${}^{60}\text{Co}$ given earlier, we see that $Z=27$ for Co and $Z=28$ is Ni. It is as if one of the neutrons in the parent nucleus decays into a proton, electron, and neutrino. In fact, neutrons outside of nuclei do just that—they live only an average of a few minutes and β^- decay in the following manner:

Equation:

$$ \text{n} \rightarrow \text{p} + \beta^- + \bar{\nu}_e. $$



In β^- decay, the parent nucleus emits an electron and an antineutrino. The daughter nucleus has one more proton and one less neutron than its parent. Neutrinos interact so weakly that they are almost never directly observed, but they play a fundamental role in particle physics.

We see that charge is conserved in β^- decay, since the total charge is Z before and after the decay. For example, in ^{60}Co decay, total charge is 27 before decay, since cobalt has $Z = 27$. After decay, the daughter nucleus is Ni, which has $Z = 28$, and there is an electron, so that the total charge is also $28 + (-1)$ or 27. Angular momentum is conserved, but not obviously (you have to examine the spins and angular momenta of the final products in detail to verify this). Linear momentum is also conserved, again imparting most of the decay energy to the electron and the antineutrino, since they are of low and zero mass, respectively. Another new conservation law is obeyed here and elsewhere in nature. *The total number of nucleons A is conserved*. In ^{60}Co decay, for example, there are 60 nucleons before and after the decay. Note that total A is also conserved in α decay. Also note that the total number of protons changes, as does the total number of neutrons, so that total Z and total N are *not* conserved in β^- decay, as they are in α decay. Energy released in β^- decay can be calculated given the masses of the parent and products.

Example:

β^- Decay Energy from Masses

Find the energy emitted in the β^- decay of ^{60}Co.

Strategy and Concept

As in the preceding example, we must first find Δm, the difference in mass between the parent nucleus and the products of the decay, using masses given in Appendix A. Then the emitted energy is calculated as before, using $E = (\Delta m)c^2$. The initial mass is just that of the parent nucleus, and the final mass is that of the daughter nucleus and the electron created in the decay. The neutrino is massless, or nearly so. However, since the masses given in Appendix A are for neutral atoms, the daughter nucleus has one more electron than the parent, and so the extra electron mass that corresponds to the β^- is included in the atomic mass of Ni. Thus,

Equation:

$$\Delta m = m(^{60}\text{Co}) - m(^{60}\text{Ni }).$$

Solution

The β^- decay equation for ^{60}Co is

Equation:

$$^{60}_{27}\text{Co}_{33} \rightarrow {}^{60}_{28}\text{Ni}_{32} + \beta^- + \nu_e.$$

As noticed,

Equation:

$$\Delta m = m(^{60}\text{Co}) - m(^{60}\text{Ni }).$$

Entering the masses found in Appendix A gives

Equation:

$$\Delta m = 59.933820 \text{ u} - 59.930789 \text{ u} = 0.003031 \text{ u}.$$

Thus,
Equation:

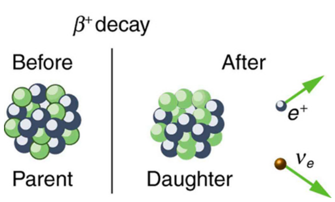
$$E = (\Delta m)c^2 = (0.003031 \text{ u})c^2.$$

Using $1 \text{ u} = 931.5 \text{ MeV}/c^2$, we obtain
Equation:

$$E = (0.003031)(931.5 \text{ MeV}/c^2)(c^2) = 2.82 \text{ MeV}.$$

Discussion and Implications
Perhaps the most difficult thing about this example is convincing yourself that the β^- mass is included in the atomic mass of ^{60}Ni. Beyond that are other implications. Again the decay energy is in the MeV range. This energy is shared by all of the products of the decay. In many ^{60}Co decays, the daughter nucleus ^{60}Ni is left in an excited state and emits photons (γ rays). Most of the remaining energy goes to the electron and neutrino, since the recoil kinetic energy of the daughter nucleus is small. One final note: the electron emitted in β^- decay is created in the nucleus at the time of decay.

The second type of beta decay is less common than the first. It is β^+decay. Certain nuclides decay by the emission of a *positive* electron. This is **antielectron** or **positron decay** (see [link]).



β^+ decay is the emission of a positron that eventually finds an electron to annihilate, characteristically producing gammas in opposite directions.

The antielectron is often represented by the symbol e^+, but in beta decay it is written as β^+ to indicate the antielectron was emitted in a nuclear decay. Antielectrons are the antimatter counterpart to electrons, being nearly identical, having the same mass, spin, and so on, but having a positive charge and an electron family number of –1. When a **positron** encounters an electron, there is a mutual annihilation in which all the mass of the antielectron-electron pair is converted into pure photon energy. (The reaction, $e^+ + e^- \rightarrow \gamma + \gamma$, conserves electron family number as well as all other conserved quantities.) If a nuclide ${}^A_Z\mathrm{X}_N$ is known to β^+ decay, then its β^+**decay equation** is

Equation:

$${}^A_Z\mathrm{X}_N \rightarrow {}_{Z-1}^{A}\mathrm{Y}_{N+1} + \beta^+ + \nu_e \;(\beta^+ \text{ decay}),$$

where Y is the nuclide having one less proton than X (to conserve charge) and ν_e is the symbol for the **electron's neutrino**, which has an electron family number of $+1$. Since an antimatter member of the electron family (the β^+) is created in the decay, a matter member of the family (here the ν_e) must also be created. Given, for example, that ${}^{22}\mathrm{Na}$ β^+ decays, you can write its full decay equation by first finding that $Z = 11$ for ${}^{22}\mathrm{Na}$, so that the daughter nuclide will have $Z = 10$, the atomic number for neon. Thus the β^+ decay equation for ${}^{22}\mathrm{Na}$ is

Equation:

$${}^{22}_{11}\mathrm{Na}_{11} \rightarrow {}^{22}_{10}\mathrm{Ne}_{12} + \beta^+ + \nu_e.$$

In β^+ decay, it is as if one of the protons in the parent nucleus decays into a neutron, a positron, and a neutrino. Protons do not do this outside of the nucleus, and so the decay is due to the complexities of the nuclear force. Note again that the total number of nucleons is constant in this and any other reaction. To find the energy emitted in β^+ decay, you must again count the number of electrons in the neutral atoms, since atomic masses are used. The daughter has one less electron than the parent, and one electron mass is created in the decay. Thus, in β^+ decay,

Equation:

$$\Delta m = m(\text{parent}) - [m(\text{daughter}) + 2m_e],$$

since we use the masses of neutral atoms.

Electron capture is the third type of beta decay. Here, a nucleus captures an inner-shell electron and undergoes a nuclear reaction that has the same effect as β^+ decay. Electron capture is sometimes denoted by the letters EC. We know that electrons cannot reside in the nucleus, but this is a nuclear reaction that consumes the electron and occurs spontaneously only when the products have less mass than the parent plus the electron. If a nuclide ${}^A_Z\mathrm{X}_N$ is known to undergo electron capture, then its **electron capture equation** is

Equation:

$$ {}^{A}_{Z}\mathrm{X}_{N} + e^{-} \rightarrow {}_{Z-1}^{A}\mathrm{Y}_{N+1} + \nu_{e}(\text{electron capture, or EC}). $$

Any nuclide that can β^{+} decay can also undergo electron capture (and often does both). The same conservation laws are obeyed for EC as for β^{+} decay. It is good practice to confirm these for yourself.

All forms of beta decay occur because the parent nuclide is unstable and lies outside the region of stability in the chart of nuclides. Those nuclides that have relatively more neutrons than those in the region of stability will β^{-} decay to produce a daughter with fewer neutrons, producing a daughter nearer the region of stability. Similarly, those nuclides having relatively more protons than those in the region of stability will β^{-} decay or undergo electron capture to produce a daughter with fewer protons, nearer the region of stability.

Gamma Decay

Gamma decay is the simplest form of nuclear decay—it is the emission of energetic photons by nuclei left in an excited state by some earlier process. Protons and neutrons in an excited nucleus are in higher orbitals, and they fall to lower levels by photon emission (analogous to electrons in excited atoms). Nuclear excited states have lifetimes typically of only about 10^{-14} s, an indication of the great strength of the forces pulling the nucleons to lower states. The γ decay equation is simply

Equation:

$$ {}^{A}_{Z}\mathrm{X}^{*}_{N} \rightarrow {}^{A}_{Z}\mathrm{X}_{N} + \gamma_{1} + \gamma_{2} + \cdots \; (\gamma \text{ decay}) $$

where the asterisk indicates the nucleus is in an excited state. There may be one or more γ s emitted, depending on how the nuclide de-excites. In radioactive decay, γ emission is common and is preceded by γ or β decay. For example, when $^{60}\mathrm{Co}$ β^{-} decays, it most often leaves the daughter nucleus in an excited state, written ^{60}Ni*. Then the nickel nucleus quickly γ decays by the emission of two penetrating γ s:

Equation:

$$ {}^{60}\mathrm{Ni}^{*} \rightarrow {}^{60}\mathrm{Ni} + \gamma_{1} + \gamma_{2}. $$

These are called cobalt γ rays, although they come from nickel—they are used for cancer therapy, for example. It is again constructive to verify the conservation laws for gamma decay. Finally, since γ decay does not change the nuclide to another species, it is not prominently featured in charts of decay series, such as that in [link].

There are other types of nuclear decay, but they occur less commonly than α, β, and γ decay. Spontaneous fission is the most important of the other forms of nuclear decay because of its applications in nuclear power and weapons. It is covered in the next chapter.

Section Summary

- When a parent nucleus decays, it produces a daughter nucleus following rules and conservation laws. There are three major types of nuclear decay, called alpha (α), beta (β), and gamma (γ). The α decay equation is
  Equation:

  $$ {}^{A}_{Z}\mathrm{X}_{N} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y}_{N-2} + {}^{4}_{2}\mathrm{He}_{2}. $$

- Nuclear decay releases an amount of energy E related to the mass destroyed Δm by
  Equation:

  $$ E = (\Delta m)c^2. $$

- There are three forms of beta decay. The β^-decay equation is
  Equation:

  $$ {}^{A}_{Z}\mathrm{X}_{N} \rightarrow {}^{A}_{Z+1}\mathrm{Y}_{N-1} + \beta^- + \nu_e. $$

- The β^+ decay equation is
  Equation:

  $$ {}^{A}_{Z}\mathrm{X}_{N} \rightarrow {}^{A}_{Z-1}\mathrm{Y}_{N+1} + \beta^+ + \nu_e. $$

- The electron capture equation is
  Equation:

  $$ {}^{A}_{Z}\mathrm{X}_{N} + e^- \rightarrow {}^{A}_{Z-1}\mathrm{Y}_{N+1} + \nu_e. $$

- β^- is an electron, β^+ is an antielectron or positron, ν_e represents an electron's neutrino, and ν_e is an electron's antineutrino. In addition to all previously known conservation laws, two new ones arise— conservation of electron family number and conservation of the total number of nucleons. The γ decay equation is
  Equation:

  $$ {}^{A}_{Z}\mathrm{X}^{*}_{N} \rightarrow {}^{A}_{Z}\mathrm{X}_{N} + \gamma_1 + \gamma_2 + \cdots $$

  γ is a high-energy photon originating in a nucleus.

Conceptual Questions

Exercise:

Problem:

Star Trek fans have often heard the term “antimatter drive.” Describe how you could use a magnetic field to trap antimatter, such as produced by nuclear decay, and later combine it with matter to produce energy. Be specific about the type of antimatter, the need for vacuum storage, and the fraction of matter converted into energy.

Exercise:

Problem:

What conservation law requires an electron’s neutrino to be produced in electron capture? Note that the electron no longer exists after it is captured by the nucleus.

Exercise:

Problem:

Neutrinos are experimentally determined to have an extremely small mass. Huge numbers of neutrinos are created in a supernova at the same time as massive amounts of light are first produced. When the 1987A supernova occurred in the Large Magellanic Cloud, visible primarily in the Southern Hemisphere and some 100,000 light-years away from Earth, neutrinos from the explosion were observed at about the same time as the light from the blast. How could the relative arrival times of neutrinos and light be used to place limits on the mass of neutrinos?

Exercise:

Problem:

What do the three types of beta decay have in common that is distinctly different from alpha decay?

Problems & Exercises

In the following eight problems, write the complete decay equation for the given nuclide in the complete ${}^{A}_{Z}\mathrm{X}_{N}$ notation. Refer to the periodic table for values of Z.

Exercise:

Problem:

β^- decay of ${}^{3}\mathrm{H}$ (tritium), a manufactured isotope of hydrogen used in some digital watch displays, and manufactured primarily for use in hydrogen bombs.

Solution:
Equation:

$${}^{3}_{1}\mathrm{H}_{2} \rightarrow {}^{3}_{2}\mathrm{He}_{1} + \beta^{-} + \nu_{e}$$

Exercise:

Problem:

β^- decay of ${}^{40}\text{K}$, a naturally occurring rare isotope of potassium responsible for some of our exposure to background radiation.

Exercise:

Problem: β^+ decay of ${}^{50}\text{Mn}$.

Solution:
Equation:

$${}^{50}_{25}M_{25} \rightarrow {}^{50}_{24}\text{Cr}_{26} + \beta^+ + \nu_e$$

Exercise:

Problem: β^+ decay of ${}^{52}\text{Fe}$.

Exercise:

Problem: Electron capture by ${}^{7}\text{Be}$.

Solution:
Equation:

$${}^{7}_{4}\text{Be}_3 + e^- \rightarrow {}^{7}_{3}\text{Li}_4 + \nu_e$$

Exercise:

Problem: Electron capture by ${}^{106}\text{In}$.

Exercise:

Problem:

α decay of ${}^{210}\text{Po}$, the isotope of polonium in the decay series of ${}^{238}\text{U}$ that was discovered by the Curies. A favorite isotope in physics labs, since it has a short half-life and decays to a stable nuclide.

Solution:
Equation:

$${}^{210}_{84}\text{Po}_{126} \rightarrow {}^{206}_{82}\text{Pb}_{124} + {}^{4}_{2}\text{He}_2$$

Exercise:

Problem:

α decay of ^{226}Ra, another isotope in the decay series of ^{238}U, first recognized as a new element by the Curies. Poses special problems because its daughter is a radioactive noble gas.

In the following four problems, identify the parent nuclide and write the complete decay equation in the $^{A}_{Z}\text{X}_{N}$ notation. Refer to the periodic table for values of Z.

Exercise:

Problem:

β^{-} decay producing ^{137}Ba. The parent nuclide is a major waste product of reactors and has chemistry similar to potassium and sodium, resulting in its concentration in your cells if ingested.

Solution:
Equation:

$$^{137}_{55}\text{Cs}_{82} \rightarrow {}^{137}_{56}\text{Ba}_{81} + \beta^{-} + \nu_{e}$$

Exercise:

Problem:

β^{-} decay producing ^{90}Y. The parent nuclide is a major waste product of reactors and has chemistry similar to calcium, so that it is concentrated in bones if ingested (^{90}Y is also radioactive.)

Exercise:

Problem:

α decay producing ^{228}Ra. The parent nuclide is nearly 100% of the natural element and is found in gas lantern mantles and in metal alloys used in jets (^{228}Ra is also radioactive).

Solution:
Equation:

$$^{232}_{90}\text{Th}_{142} \rightarrow {}^{228}_{88}\text{Ra}_{140} + {}^{4}_{2}\text{He}_{2}$$

Exercise:

Problem:

α decay producing ^{208}Pb. The parent nuclide is in the decay series produced by ^{232}Th, the only naturally occurring isotope of thorium.

Exercise:

Problem:

When an electron and positron annihilate, both their masses are destroyed, creating two equal energy photons to preserve momentum. (a) Confirm that the annihilation equation $e^+ + e^- \rightarrow \gamma + \gamma$ conserves charge, electron family number, and total number of nucleons. To do this, identify the values of each before and after the annihilation. (b) Find the energy of each γ ray, assuming the electron and positron are initially nearly at rest. (c) Explain why the two γ rays travel in exactly opposite directions if the center of mass of the electron-positron system is initially at rest.

Solution:

(a)
$\text{charge:}(+1) + (-1) = 0;\ \ \text{electron family number: } (+1) + (-1) = 0;\ \ A\text{: } 0 + 0 = 0$

(b) 0.511 MeV

(c) The two γ rays must travel in exactly opposite directions in order to conserve momentum, since initially there is zero momentum if the center of mass is initially at rest.

Exercise:

Problem:

Confirm that charge, electron family number, and the total number of nucleons are all conserved by the rule for α decay given in the equation ${}^{A}_{Z}\text{X}_N \rightarrow {}^{A-4}_{Z-2}\text{Y}_{N-2} + {}^{4}_{2}\text{He}_2$. To do this, identify the values of each before and after the decay.

Exercise:

Problem:

Confirm that charge, electron family number, and the total number of nucleons are all conserved by the rule for β^- decay given in the equation ${}^{A}_{Z}\text{X}_N \rightarrow {}^{A}_{Z+1}\text{Y}_{N-1} + \beta^- + \nu_e$. To do this, identify the values of each before and after the decay.

Solution:
Equation:

$$Z = (Z+1) - 1;\quad A = A;\quad \text{efn} : 0 = (+1) + (-1)$$

Exercise:

Problem:

Confirm that charge, electron family number, and the total number of nucleons are all conserved by the rule for β^- decay given in the equation ${}^{A}_{Z}\text{X}_N \rightarrow {}^{A}_{Z-1}\text{Y}_{N-1} + \beta^- + \nu_e$. To do this, identify the values of each before and after the decay.

Exercise:

Problem:

Confirm that charge, electron family number, and the total number of nucleons are all conserved by the rule for electron capture given in the equation ${}^{A}_{Z}\text{X}_N + e^- \rightarrow {}^{A}_{Z-1}\text{Y}_{N+1} + \nu_e$. To do this, identify the values of each before and after the capture.

Solution:
Equation:

$$Z - 1 = Z - 1; \quad A = A; \quad \text{efn :}(+1) = (+1)$$

Exercise:

Problem:

A rare decay mode has been observed in which ${}^{222}\text{Ra}$ emits a ${}^{14}\text{C}$ nucleus. (a) The decay equation is ${}^{222}\text{Ra} \rightarrow {}^{A}\text{X} + {}^{14}\text{C}$. Identify the nuclide ${}^{A}\text{X}$. (b) Find the energy emitted in the decay. The mass of ${}^{222}\text{Ra}$ is 222.015353 u.

Exercise:

Problem: (a) Write the complete α decay equation for ${}^{226}\text{Ra}$.

(b) Find the energy released in the decay.

Solution:

(a) ${}^{226}_{88}\text{Ra}_{138} \rightarrow {}^{222}_{86}\text{Rn}_{136} + {}^{4}_{2}\text{He}_2$

(b) 4.87 MeV

Exercise:

Problem: (a) Write the complete α decay equation for ${}^{249}\text{Cf}$.

(b) Find the energy released in the decay.

Exercise:

Problem:

(a) Write the complete β^- decay equation for the neutron. (b) Find the energy released in the decay.

Solution:

(a) $\mathrm{n} \to \mathrm{p} + \beta^- + \nu_e$

(b)) 0.783 MeV

Exercise:

Problem:

(a) Write the complete β^- decay equation for $^{90}\mathrm{Sr}$, a major waste product of nuclear reactors. (b) Find the energy released in the decay.

Exercise:

Problem:

Calculate the energy released in the β^+ decay of $^{22}\mathrm{Na}$, the equation for which is given in the text. The masses of $^{22}\mathrm{Na}$ and $^{22}\mathrm{Ne}$ are 21.994434 and 21.991383 u, respectively.

Solution:

1.82 MeV

Exercise:

Problem: (a) Write the complete β^+ decay equation for $^{11}\mathrm{C}$.

(b) Calculate the energy released in the decay. The masses of $^{11}\mathrm{C}$ and $^{11}\mathrm{B}$ are 11.011433 and 11.009305 u, respectively.

Exercise:

Problem: (a) Calculate the energy released in the α decay of $^{238}\mathrm{U}$.

(b) What fraction of the mass of a single $^{238}\mathrm{U}$ is destroyed in the decay? The mass of $^{234}\mathrm{Th}$ is 234.043593 u.

(c) Although the fractional mass loss is large for a single nucleus, it is difficult to observe for an entire macroscopic sample of uranium. Why is this?

Solution:

(a) 4.274 MeV

(b) 1.927×10^{-5}

(c) Since U-238 is a slowly decaying substance, only a very small number of nuclei decay on human timescales; therefore, although those nuclei that decay lose a noticeable fraction of their mass, the change in the total mass of the sample is not detectable for a macroscopic sample.

Exercise:

Problem: (a) Write the complete reaction equation for electron capture by ^{7}Be.

(b) Calculate the energy released.

Exercise:

Problem: (a) Write the complete reaction equation for electron capture by ^{15}O.

(b) Calculate the energy released.

Solution:

(a) $^{15}_{8}\text{O}_7 + e^- \rightarrow {}^{15}_{7}\text{N}_8 + \nu_e$

(b) 2.754 MeV

Glossary

parent
: the original state of nucleus before decay

daughter
: the nucleus obtained when parent nucleus decays and produces another nucleus following the rules and the conservation laws

positron
: the particle that results from positive beta decay; also known as an antielectron

decay
: the process by which an atomic nucleus of an unstable atom loses mass and energy by emitting ionizing particles

alpha decay
: type of radioactive decay in which an atomic nucleus emits an alpha particle

beta decay
type of radioactive decay in which an atomic nucleus emits a beta particle

gamma decay
type of radioactive decay in which an atomic nucleus emits a gamma particle

decay equation
the equation to find out how much of a radioactive material is left after a given period of time

nuclear reaction energy
the energy created in a nuclear reaction

neutrino
an electrically neutral, weakly interacting elementary subatomic particle

electron’s antineutrino
antiparticle of electron’s neutrino

positron decay
type of beta decay in which a proton is converted to a neutron, releasing a positron and a neutrino

antielectron
another term for positron

decay series
process whereby subsequent nuclides decay until a stable nuclide is produced

electron’s neutrino
a subatomic elementary particle which has no net electric charge

antimatter
composed of antiparticles

electron capture
the process in which a proton-rich nuclide absorbs an inner atomic electron and simultaneously emits a neutrino

electron capture equation
equation representing the electron capture

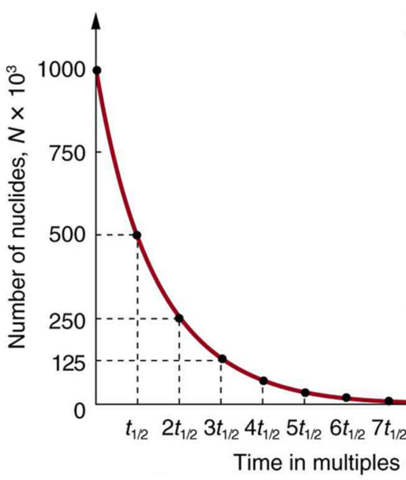
Half-Life and Activity

- Define half-life.
- Define dating.
- Calculate age of old objects by radioactive dating.

Unstable nuclei decay. However, some nuclides decay faster than others. For example, radium and polonium, discovered by the Curies, decay faster than uranium. This means they have shorter lifetimes, producing a greater rate of decay. In this section we explore half-life and activity, the quantitative terms for lifetime and rate of decay.

Half-Life

Why use a term like half-life rather than lifetime? The answer can be found by examining [link], which shows how the number of radioactive nuclei in a sample decreases with time. The *time in which half of the original number of nuclei decay* is defined as the **half-life**, $t_{1/2}$. Half of the remaining nuclei decay in the next half-life. Further, half of that amount decays in the following half-life. Therefore, the number of radioactive nuclei decreases from N to $N/2$ in one half-life, then to $N/4$ in the next, and to $N/8$ in the next, and so on. If N is a large number, then *many* half-lives (not just two) pass before all of the nuclei decay. Nuclear decay is an example of a purely statistical process. A more precise definition of half-life is that *each nucleus has a 50% chance of living for a time equal to one half-life* $t_{1/2}$. Thus, if N is reasonably large, half of the original nuclei decay in a time of one half-life. If an individual nucleus makes it through that time, it still has a 50% chance of surviving through another half-life. Even if it happens to make it through hundreds of half-lives, it still has a 50% chance of surviving through one more. The probability of decay is the same no matter when you start counting. This is like random coin flipping. The chance of heads is 50%, no matter what has happened before.



Time	N
0	1,000,000
$t_{1/2}$	500,000
$2t_{1/2}$	250,000
$3t_{1/2}$	125,000
$4t_{1/2}$	62,500
$5t_{1/2}$	31,250
$6t_{1/2}$	15,625
$7t_{1/2}$	7,813
$8t_{1/2}$	3,906
$9t_{1/2}$	1,953
$10t_{1/2}$	977

Radioactive decay reduces the number of radioactive nuclei over time. In one half-life $t_{1/2}$, the number decreases to half of its original value. Half of what remains decay in the next half-life, and half of those in the next, and so on. This is an exponential decay, as seen in the graph of the number of nuclei present as a function of time.

There is a tremendous range in the half-lives of various nuclides, from as short as 10^{-23} s for the most unstable, to more than 10^{16} y for the least unstable, or about 46 orders of magnitude. Nuclides with the shortest half-lives are those for which the nuclear forces are least attractive, an indication of the extent to which the nuclear force can depend on the particular combination of neutrons and protons. The concept of half-life is applicable to other subatomic particles. It is also applicable to the decay of excited states in atoms and nuclei.

For simple multiples of half-life $(t = t_{1/2}, 2\,t_{1/2}, 3\,t_{1/2}, \ldots, n\,t_{1/2})$, the following intuitive expression relates original N_0 and future N amounts.
Equation:

$$N = N_0 \left(\frac{1}{2}\right)^n$$

This expression shows that the amount is reduced by half during each half-life. For example, using the data in Figure 1, after three half-lives we see that one-eighth of the original number remains.

$$t = 3\,t_{1/2},\ N_0 = 1,000,000$$

$$N = N_0 \left(\frac{1}{2}\right)^n = (1,000,000)\left(\frac{1}{2}\right)^3 = (1,000,000)\left(\frac{1}{8}\right) = 125,000$$

And after ten half-lives 1/1024 of the original remains.

$$t = 10\,t_{1/2},\ N_0 = 1,000,000$$

$$N = N_0 \left(\frac{1}{2}\right)^n = (1,000,000)\left(\frac{1}{2}\right)^{10} = (1,000,000)\left(\frac{1}{1024}\right) = 977$$

For an arbitrary time, not just a multiple of the half-life, the exponential relationship shown below is generally used. The following equation gives the quantitative relationship between the original number of nuclei present at time zero N_0 and the number N at a later time t:
Equation:

$$N = N_0 e^{-\lambda t},$$

where $e = 2.71828...$ is the base of the natural logarithm, and λ is the **decay constant** for the nuclide. The shorter the half-life, the larger is the value of λ, and the faster the exponential $e^{-\lambda t}$ decreases with time. The relationship between the decay constant λ and the half-life $t_{1/2}$ is

Equation:

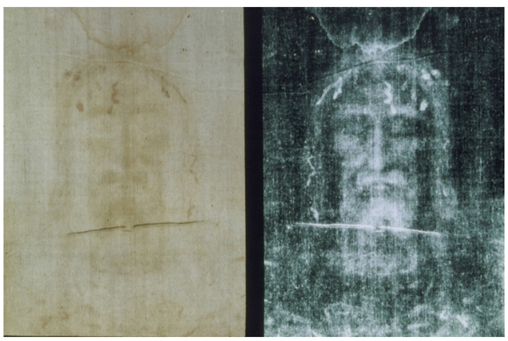
$$\lambda = \frac{\ln(2)}{t_{1/2}} \approx \frac{0.693}{t_{1/2}}.$$

To see how the number of nuclei declines to half its original value in one half-life, let $t = t_{1/2}$ in the exponential in the equation $N = N_0 e^{-\lambda t}$. This gives $N = N_0 e^{-\lambda t} = N_0 e^{-0.693} = 0.500 N_0$.

Radioactive dating is a clever use of naturally occurring radioactivity. Its most famous application is **carbon-14 dating**. Carbon-14 has a half-life of 5730 years and is produced in a nuclear reaction induced when solar neutrinos strike ^{14}N in the atmosphere. Radioactive carbon has the same chemistry as stable carbon, and so it mixes into the ecosphere, where it is consumed and becomes part of every living organism. Carbon-14 has an abundance of 1.3 parts per trillion of normal carbon. Thus, if you know the number of carbon nuclei in an object (perhaps determined by mass and Avogadro’s number), you multiply that number by 1.3×10^{-12} to find the number of ^{14}C nuclei in the object. When an organism dies, carbon exchange with the environment ceases, and ^{14}C is not replenished as it decays. By comparing the abundance of ^{14}C in an artifact, such as mummy wrappings, with the normal abundance in living tissue, it is possible to determine the artifact’s age (or time since death). Carbon-14 dating can be used for biological tissues as old as 50 or 60 thousand years, but is most accurate for younger samples, since the abundance of ^{14}C nuclei in them is greater. Very old biological materials contain no ^{14}C at all. There are instances in which the date of an artifact can be determined by other means, such as historical knowledge or tree-ring counting. These cross-references have confirmed the validity of carbon-14 dating and permitted us to calibrate the technique as well. Carbon-14 dating revolutionized parts of archaeology and is of such importance that it earned the 1960 Nobel Prize in chemistry for its developer, the American chemist Willard Libby (1908–1980).

One of the most famous cases of carbon-14 dating involves the Shroud of Turin, a long piece of fabric purported to be the burial shroud of Jesus (see

[link]). This relic was first displayed in Turin in 1354 and was denounced as a fraud at that time by a French bishop. Its remarkable negative imprint of an apparently crucified body resembles the then-accepted image of Jesus, and so the shroud was never disregarded completely and remained controversial over the centuries. Carbon-14 dating was not performed on the shroud until 1988, when the process had been refined to the point where only a small amount of material needed to be destroyed. Samples were tested at three independent laboratories, each being given four pieces of cloth, with only one unidentified piece from the shroud, to avoid prejudice. All three laboratories found samples of the shroud contain 92% of the ^{14}C found in living tissues, allowing the shroud to be dated (see [link]).



Part of the Shroud of Turin, which shows a remarkable negative imprint likeness of Jesus complete with evidence of crucifixion wounds. The shroud first surfaced in the 14th century and was only recently carbon-14 dated. It has not been determined how the image was placed on the material. (credit: Butko, Wikimedia Commons)

Example:

How Old Is the Shroud of Turin?

Calculate the age of the Shroud of Turin given that the amount of ^{14}C found in it is 92% of that in living tissue.

Strategy

Knowing that 92% of the ^{14}C remains means that $N/N_0 = 0.92$. Therefore, the equation $N = N_0 e^{-\lambda t}$ can be used to find λt. We also know that the half-life of ^{14}C is 5730 y, and so once λt is known, we can use the equation $\lambda = \frac{0.693}{t_{1/2}}$ to find λ and then find t as requested. Here, we postulate that the decrease in ^{14}C is solely due to nuclear decay.

Solution

Solving the equation $N = N_0 e^{-\lambda t}$ for N/N_0 gives

Equation:

$$\frac{N}{N_0} = e^{-\lambda t}.$$

Thus,

Equation:

$$0.92 = e^{-\lambda t}.$$

Taking the natural logarithm of both sides of the equation yields

Equation:

$$\ln 0.92 = -\lambda t$$

so that

Equation:

$$-0.0834 = -\lambda t.$$

Rearranging to isolate t gives

Equation:

$$t = \frac{0.0834}{\lambda}.$$

Now, the equation $\lambda = \frac{0.693}{t_{1/2}}$ can be used to find λ for ^{14}C. Solving for λ and substituting the known half-life gives

Equation:

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{5730\ \text{y}}.$$

We enter this value into the previous equation to find t:

Equation:

$$t = \frac{0.0834}{\frac{0.693}{5730\ \text{y}}} = 690\ \text{y}.$$

Discussion

This dates the material in the shroud to 1988–690 = a.d. 1300. Our calculation is only accurate to two digits, so that the year is rounded to 1300. The values obtained at the three independent laboratories gave a weighted average date of a.d. 1320 ± 60. The uncertainty is typical of carbon-14 dating and is due to the small amount of ^{14}C in living tissues, the amount of material available, and experimental uncertainties (reduced by having three independent measurements). It is meaningful that the date of the shroud is consistent with the first record of its existence and inconsistent with the period in which Jesus lived.

There are other forms of radioactive dating. Rocks, for example, can sometimes be dated based on the decay of ^{238}U. The decay series for ^{238}U ends with ^{206}Pb, so that the ratio of these nuclides in a rock is an indication of how long it has been since the rock solidified. The original composition of the rock, such as the absence of lead, must be known with some confidence. However, as with carbon-14 dating, the technique can be verified by a consistent body of knowledge. Since ^{238}U has a half-life of 4.5×10^9 y, it is useful for dating only very old materials, showing, for

example, that the oldest rocks on Earth solidified about 3.5×10^9 years ago.

Activity, the Rate of Decay

What do we mean when we say a source is highly radioactive? Generally, this means the number of decays per unit time is very high. We define **activity** R to be the **rate of decay** expressed in decays per unit time. In equation form, this is
Equation:

$$R = \frac{\Delta N}{\Delta t}$$

where ΔN is the number of decays that occur in time Δt. The SI unit for activity is one decay per second and is given the name **becquerel** (Bq) in honor of the discoverer of radioactivity. That is,
Equation:

$$1 \text{ Bq} = 1 \text{ decay/s}.$$

Activity R is often expressed in other units, such as decays per minute or decays per year. One of the most common units for activity is the **curie** (Ci), defined to be the activity of 1 g of ^{226}Ra, in honor of Marie Curie's work with radium. The definition of curie is
Equation:

$$1 \text{ Ci} = 3.70 \times 10^{10} \text{ Bq},$$

or 3.70×10^{10} decays per second. A curie is a large unit of activity, while a becquerel is a relatively small unit. $1 \text{ MBq} = 100 \text{ microcuries } (\mu\text{Ci})$. In countries like Australia and New Zealand that adhere more to SI units, most radioactive sources, such as those used in medical diagnostics or in physics laboratories, are labeled in Bq or megabecquerel (MBq).

Intuitively, you would expect the activity of a source to depend on two things: the amount of the radioactive substance present, and its half-life. The greater the number of radioactive nuclei present in the sample, the more will decay per unit of time. The shorter the half-life, the more decays per unit time, for a given number of nuclei. So activity R should be proportional to the number of radioactive nuclei, N, and inversely proportional to their half-life, $t_{1/2}$. In fact, your intuition is correct. It can be shown that the activity of a source is

Equation:

$$R = \frac{0.693N}{t_{1/2}}$$

where N is the number of radioactive nuclei present, having half-life $t_{1/2}$. This relationship is useful in a variety of calculations, as the next two examples illustrate.

Example:

How Great Is the ^{14}C Activity in Living Tissue?

Calculate the activity due to ^{14}C in 1.00 kg of carbon found in a living organism. Express the activity in units of Bq and Ci.

Strategy

To find the activity R using the equation $R = \frac{0.693N}{t_{1/2}}$, we must know N and $t_{1/2}$. The half-life of ^{14}C can be found in Appendix B, and was stated above as 5730 y. To find N, we first find the number of ^{12}C nuclei in 1.00 kg of carbon using the concept of a mole. As indicated, we then multiply by 1.3×10^{-12} (the abundance of ^{14}C in a carbon sample from a living organism) to get the number of ^{14}C nuclei in a living organism.

Solution

One mole of carbon has a mass of 12.0 g, since it is nearly pure ^{12}C. (A mole has a mass in grams equal in magnitude to A found in the periodic table.) Thus the number of carbon nuclei in a kilogram is

Equation:

$$N(^{12}\text{C}) = \frac{6.02 \times 10^{23}\ \text{mol}^{-1}}{12.0\ \text{g/mol}} \times (1000\ \text{g}) = 5.02 \times 10^{25}.$$

So the number of ^{14}C nuclei in 1 kg of carbon is
Equation:

$$N(^{14}\text{C}) = (5.02 \times 10^{25})(1.3 \times 10^{-12}) = 6.52 \times 10^{13}.$$

Now the activity R is found using the equation $R = \frac{0.693N}{t_{1/2}}$.
Entering known values gives
Equation:

$$R = \frac{0.693(6.52 \times 10^{13})}{5730\ \text{y}} = 7.89 \times 10^{9}\ \text{y}^{-1},$$

or 7.89×10^{9} decays per year. To convert this to the unit Bq, we simply convert years to seconds. Thus,
Equation:

$$R = (7.89 \times 10^{9}\ \text{y}^{-1})\frac{1.00\ \text{y}}{3.16 \times 10^{7}\ \text{s}} = 250\ \text{Bq},$$

or 250 decays per second. To express R in curies, we use the definition of a curie,
Equation:

$$R = \frac{250\ \text{Bq}}{3.7 \times 10^{10}\ \text{Bq/Ci}} = 6.76 \times 10^{-9}\ \text{Ci}.$$

Thus,
Equation:

$$R = 6.76\ \text{nCi}.$$

Discussion
Our own bodies contain kilograms of carbon, and it is intriguing to think there are hundreds of ^{14}C decays per second taking place in us. Carbon-14

and other naturally occurring radioactive substances in our bodies contribute to the background radiation we receive. The small number of decays per second found for a kilogram of carbon in this example gives you some idea of how difficult it is to detect ^{14}C in a small sample of material. If there are 250 decays per second in a kilogram, then there are 0.25 decays per second in a gram of carbon in living tissue. To observe this, you must be able to distinguish decays from other forms of radiation, in order to reduce background noise. This becomes more difficult with an old tissue sample, since it contains less ^{14}C, and for samples more than 50 thousand years old, it is impossible.

Human-made (or artificial) radioactivity has been produced for decades and has many uses. Some of these include medical therapy for cancer, medical imaging and diagnostics, and food preservation by irradiation. It is clear that radiation is hazardous. A number of tragic examples of this exist, one of the most disastrous being the meltdown and fire at the Chernobyl reactor complex in the Ukraine (see [link]). Several radioactive isotopes were released in huge quantities, contaminating many thousands of square kilometers and directly affecting hundreds of thousands of people. The most significant releases were of ^{131}I, ^{90}Sr, ^{137}Cs, ^{239}Pu, ^{238}U, and ^{235}U. Estimates are that the total amount of radiation released was about 100 million curies.

Human and Medical Applications

The Chernobyl reactor. More than 100 people died soon after its meltdown, and there will be thousands of deaths from radiation-induced cancer in the future. While the accident was due to a series of human errors, the cleanup efforts were heroic. Most of the immediate fatalities were firefighters and reactor personnel. (credit: Elena Filatova)

Example:

What Mass of ^{137}Cs Escaped Chernobyl?

It is estimated that the Chernobyl disaster released 6.0 MCi of ^{137}Cs into the environment. Calculate the mass of ^{137}Cs released.

Strategy

We can calculate the mass released using Avogadro's number and the concept of a mole if we can first find the number of nuclei N released. Since the activity R is given, and the half-life of ^{137}Cs is found in Appendix B to be 30.2 y, we can use the equation $R = \frac{0.693N}{t_{1/2}}$ to find N.

Solution

Solving the equation $R = \frac{0.693N}{t_{1/2}}$ for N gives

Equation:

$$N = \frac{Rt_{1/2}}{0.693}.$$

Entering the given values yields
Equation:

$$N = \frac{(6.0\text{ MCi})(30.2\text{ y})}{0.693}.$$

Converting curies to becquerels and years to seconds, we get
Equation:

$$\begin{aligned} N &= \frac{(6.0\times10^{6}\text{ Ci})(3.7\times10^{10}\text{ Bq/Ci})(30.2\text{ y})(3.16\times10^{7}\text{ s/y})}{0.693} \\ &= 3.1\times10^{26}. \end{aligned}$$

One mole of a nuclide ${}^{A}X$ has a mass of A grams, so that one mole of ${}^{137}\text{Cs}$ has a mass of 137 g. A mole has 6.02×10^{23} nuclei. Thus the mass of ${}^{137}\text{Cs}$ released was
Equation:

$$\begin{aligned} m &= \left(\frac{137\text{ g}}{6.02\times10^{23}}\right)(3.1\times10^{26}) = 70\times10^{3}\text{ g} \\ &= 70\text{ kg}. \end{aligned}$$

Discussion
While 70 kg of material may not be a very large mass compared to the amount of fuel in a power plant, it is extremely radioactive, since it only has a 30-year half-life. Six megacuries (6.0 MCi) is an extraordinary amount of activity but is only a fraction of what is produced in nuclear reactors. Similar amounts of the other isotopes were also released at Chernobyl. Although the chances of such a disaster may have seemed small, the consequences were extremely severe, requiring greater caution than was used. More will be said about safe reactor design in the next chapter, but it should be noted that Western reactors have a fundamentally safer design.

Activity R decreases in time, going to half its original value in one half-life, then to one-fourth its original value in the next half-life, and so on. Since

$R = \frac{0.693N}{t_{1/2}}$, the activity decreases as the number of radioactive nuclei decreases. The equation for R as a function of time is found by combining the equations $N = N_0 e^{-\lambda t}$ and $R = \frac{0.693N}{t_{1/2}}$, yielding

Equation:

$$R = R_0 e^{-\lambda t},$$

where R_0 is the activity at $t = 0$. This equation shows exponential decay of radioactive nuclei. For example, if a source originally has a 1.00-mCi activity, it declines to 0.500 mCi in one half-life, to 0.250 mCi in two half-lives, to 0.125 mCi in three half-lives, and so on. For times other than whole half-lives, the equation $R = R_0 e^{-\lambda t}$ must be used to find R.

Section Summary

- Half-life $t_{1/2}$ is the time in which there is a 50% chance that a nucleus will decay. The number of nuclei N as a function of time is
  Equation:

  $$N = N_0 e^{-\lambda t},$$

  where N_0 is the number present at $t = 0$, and λ is the decay constant, related to the half-life by
  Equation:

  $$\lambda = \frac{0.693}{t_{1/2}}.$$

- One of the applications of radioactive decay is radioactive dating, in which the age of a material is determined by the amount of radioactive decay that occurs. The rate of decay is called the activity R:
  Equation:

  $$R = \frac{\Delta N}{\Delta t}.$$

- The SI unit for R is the becquerel (Bq), defined by
  Equation:

$$1\ \text{Bq} = 1\ \text{decay/s}.$$

- R is also expressed in terms of curies (Ci), where
  Equation:

$$1\ \text{Ci} = 3.70 \times 10^{10}\ \text{Bq}.$$

- The activity R of a source is related to N and $t_{1/2}$ by
  Equation:

$$R = \frac{0.693N}{t_{1/2}}.$$

- Since N has an exponential behavior as in the equation $N = N_0 e^{-\lambda t}$, the activity also has an exponential behavior, given by
  Equation:

$$R = R_0 e^{-\lambda t},$$

  where R_0 is the activity at $t = 0$.

Conceptual Questions

Exercise:

Problem:

In a 3×10^9-year-old rock that originally contained some ^{238}U, which has a half-life of 4.5×10^9 years, we expect to find some ^{238}U remaining in it. Why are ^{226}Ra, ^{222}Rn, and ^{210}Po also found in such a rock, even though they have much shorter half-lives (1600 years, 3.8 days, and 138 days, respectively)?

Exercise:

Problem:

Does the number of radioactive nuclei in a sample decrease to *exactly* half its original value in one half-life? Explain in terms of the statistical nature of radioactive decay.

Exercise:

Problem:

Radioactivity depends on the nucleus and not the atom or its chemical state. Why, then, is one kilogram of uranium more radioactive than one kilogram of uranium hexafluoride?

Exercise:

Problem:

Explain how a bound system can have less mass than its components. Why is this not observed classically, say for a building made of bricks?

Exercise:

Problem:

Spontaneous radioactive decay occurs only when the decay products have less mass than the parent, and it tends to produce a daughter that is more stable than the parent. Explain how this is related to the fact that more tightly bound nuclei are more stable. (Consider the binding energy per nucleon.)

Exercise:

Problem:

To obtain the most precise value of BE from the equation $\text{BE}=\left[ZM\left({}^{1}\text{H}\right)+Nm_{n}\right]c^{2}-m\left({}^{A}\text{X}\right)c^{2}$, we should take into account the binding energy of the electrons in the neutral atoms. Will doing this produce a larger or smaller value for BE? Why is this effect usually negligible?

Exercise:

Problem:

How does the finite range of the nuclear force relate to the fact that BE/A is greatest for A near 60?

Problems & Exercises

Data from the appendices and the periodic table may be needed for these problems.

Exercise:

Problem:

An old campfire is uncovered during an archaeological dig. Its charcoal is found to contain less than 1/1000 the normal amount of ^{14}C. Estimate the minimum age of the charcoal, noting that $2^{10} = 1024$.

Solution:

57,300 y

Exercise:

Problem:

A ^{60}Co source is labeled 4.00 mCi, but its present activity is found to be 1.85×10^7 Bq. (a) What is the present activity in mCi? (b) How long ago did it actually have a 4.00-mCi activity?

Exercise:

Problem:

(a) Calculate the activity R in curies of 1.00 g of ^{226}Ra. (b) Discuss why your answer is not exactly 1.00 Ci, given that the curie was originally supposed to be exactly the activity of a gram of radium.

Solution:

(a) 0.988 Ci

(b) The half-life of $^{226}\mathrm{Ra}$ is now better known.

Exercise:

Problem:

Show that the activity of the $^{14}\mathrm{C}$ in 1.00 g of $^{12}\mathrm{C}$ found in living tissue is 0.250 Bq.

Exercise:

Problem:

Mantles for gas lanterns contain thorium, because it forms an oxide that can survive being heated to incandescence for long periods of time. Natural thorium is almost 100% $^{232}\mathrm{Th}$, with a half-life of 1.405×10^{10} y. If an average lantern mantle contains 300 mg of thorium, what is its activity?

Solution:

1.22×10^{3} Bq

Exercise:

Problem:

Cow’s milk produced near nuclear reactors can be tested for as little as 1.00 pCi of $^{131}\mathrm{I}$ per liter, to check for possible reactor leakage. What mass of $^{131}\mathrm{I}$ has this activity?

Exercise:

Problem:

(a) Natural potassium contains $^{40}\mathrm{K}$, which has a half-life of 1.277×10^{9} y. What mass of $^{40}\mathrm{K}$ in a person would have a decay rate of 4140 Bq? (b) What is the fraction of $^{40}\mathrm{K}$ in natural potassium, given that the person has 140 g in his body? (These numbers are typical for a 70-kg adult.)

Solution:

(a) 16.0 mg

(b) 0.0114%

Exercise:

Problem:

There is more than one isotope of natural uranium. If a researcher isolates 1.00 mg of the relatively scarce ^{235}U and finds this mass to have an activity of 80.0 Bq, what is its half-life in years?

Exercise:

Problem:

^{50}V has one of the longest known radioactive half-lives. In a difficult experiment, a researcher found that the activity of 1.00 kg of ^{50}V is 1.75 Bq. What is the half-life in years?

Solution:

1.48×10^{17} y

Exercise:

Problem:

You can sometimes find deep red crystal vases in antique stores, called uranium glass because their color was produced by doping the glass with uranium. Look up the natural isotopes of uranium and their half-lives, and calculate the activity of such a vase assuming it has 2.00 g of uranium in it. Neglect the activity of any daughter nuclides.

Exercise:

Problem:

A tree falls in a forest. How many years must pass before the ^{14}C activity in 1.00 g of the tree’s carbon drops to 1.00 decay per hour?

Solution:

5.6×10^4 y

Exercise:

Problem:

What fraction of the ^{40}K that was on Earth when it formed 4.5×10^9 years ago is left today?

Exercise:

Problem:

A 5000-Ci ^{60}Co source used for cancer therapy is considered too weak to be useful when its activity falls to 3500 Ci. How long after its manufacture does this happen?

Solution:

2.71 y

Exercise:

Problem:

Natural uranium is 0.7200% ^{235}U and 99.27% ^{238}U. What were the percentages of ^{235}U and ^{238}U in natural uranium when Earth formed 4.5×10^9 years ago?

Exercise:

Problem:

The β^- particles emitted in the decay of ^{3}H (tritium) interact with matter to create light in a glow-in-the-dark exit sign. At the time of manufacture, such a sign contains 15.0 Ci of ^{3}H. (a) What is the mass of the tritium? (b) What is its activity 5.00 y after manufacture?

Solution:

(a) 1.56 mg

(b) 11.3 Ci

Exercise:

Problem:

World War II aircraft had instruments with glowing radium-painted dials. The activity of one such instrument was 1.0×10^5 Bq when new. (a) What mass of ${}^{226}\mathrm{Ra}$ was present? (b) After some years, the phosphors on the dials deteriorated chemically, but the radium did not escape. What is the activity of this instrument 57.0 years after it was made?

Exercise:

Problem:

(a) The ${}^{210}\mathrm{Po}$ source used in a physics laboratory is labeled as having an activity of $1.0\ \mu\mathrm{Ci}$ on the date it was prepared. A student measures the radioactivity of this source with a Geiger counter and observes 1500 counts per minute. She notices that the source was prepared 120 days before her lab. What fraction of the decays is she observing with her apparatus? (b) Identify some of the reasons that only a fraction of the α s emitted are observed by the detector.

Solution:

(a) 1.23×10^{-3}

(b) Only part of the emitted radiation goes in the direction of the detector. Only a fraction of that causes a response in the detector. Some of the emitted radiation (mostly α particles) is observed within the source. Some is absorbed within the source, some is absorbed by the detector, and some does not penetrate the detector.

Exercise:

Problem:

Armor-piercing shells with depleted uranium cores are fired by aircraft at tanks. (The high density of the uranium makes them effective.) The uranium is called depleted because it has had its ^{235}U removed for reactor use and is nearly pure ^{238}U. Depleted uranium has been erroneously called non-radioactive. To demonstrate that this is wrong: (a) Calculate the activity of 60.0 g of pure ^{238}U. (b) Calculate the activity of 60.0 g of natural uranium, neglecting the ^{234}U and all daughter nuclides.

Exercise:

Problem:

The ceramic glaze on a red-orange Fiestaware plate is U_2O_3 and contains 50.0 grams of ^{238}U , but very little ^{235}U. (a) What is the activity of the plate? (b) Calculate the total energy that will be released by the ^{238}U decay. (c) If energy is worth 12.0 cents per $kW \cdot h$, what is the monetary value of the energy emitted? (These plates went out of production some 30 years ago, but are still available as collectibles.)

Solution:

(a) 1.68×10^{-5} Ci

(b) 8.65×10^{10} J

(c) $\$2.9 \times 10^{3}$

Exercise:

Problem:

Large amounts of depleted uranium (^{238}U) are available as a by-product of uranium processing for reactor fuel and weapons. Uranium is very dense and makes good counter weights for aircraft. Suppose you have a 4000-kg block of ^{238}U. (a) Find its activity. (b) How many calories per day are generated by thermalization of the decay energy? (c) Do you think you could detect this as heat? Explain.

Exercise:

Problem:

The *Galileo* space probe was launched on its long journey past several planets in 1989, with an ultimate goal of Jupiter. Its power source is 11.0 kg of $^{238}\mathrm{Pu}$, a by-product of nuclear weapons plutonium production. Electrical energy is generated thermoelectrically from the heat produced when the 5.59-MeV α particles emitted in each decay crash to a halt inside the plutonium and its shielding. The half-life of $^{238}\mathrm{Pu}$ is 87.7 years. (a) What was the original activity of the $^{238}\mathrm{Pu}$ in becquerel? (b) What power was emitted in kilowatts? (c) What power was emitted 12.0 y after launch? You may neglect any extra energy from daughter nuclides and any losses from escaping γ rays.

Solution:

(a) $6.97 \times 10^{15}\ \mathrm{Bq}$

(b) 6.24 kW

(c) 5.67 kW

Glossary

becquerel
: SI unit for rate of decay of a radioactive material

half-life

the time in which there is a 50% chance that a nucleus will decay

radioactive dating
: an application of radioactive decay in which the age of a material is determined by the amount of radioactivity of a particular type that occurs

decay constant
: quantity that is inversely proportional to the half-life and that is used in equation for number of nuclei as a function of time

carbon-14 dating
: a radioactive dating technique based on the radioactivity of carbon-14

activity
: the rate of decay for radioactive nuclides

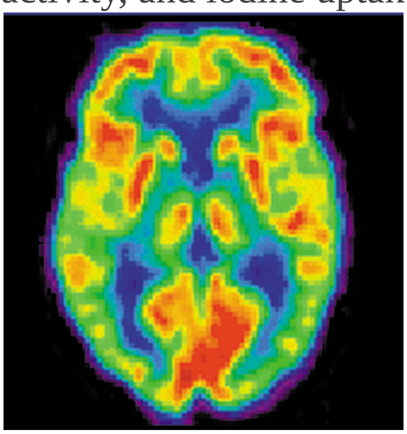
rate of decay
: the number of radioactive events per unit time

curie
: the activity of 1g of $^{226}\mathrm{Ra}$, equal to $3.70 \times 10^{10}\ \mathrm{Bq}$

Medical Imaging and Diagnostics

- Explain the working principle behind an anger camera.
- Describe the SPECT and PET imaging techniques.

A host of medical imaging techniques employ nuclear radiation. What makes nuclear radiation so useful? First, γ radiation can easily penetrate tissue; hence, it is a useful probe to monitor conditions inside the body. Second, nuclear radiation depends on the nuclide and not on the chemical compound it is in, so that a radioactive nuclide can be put into a compound designed for specific purposes. The compound is said to be **tagged**. A tagged compound used for medical purposes is called a **radiopharmaceutical**. Radiation detectors external to the body can determine the location and concentration of a radiopharmaceutical to yield medically useful information. For example, certain drugs are concentrated in inflamed regions of the body, and this information can aid diagnosis and treatment as seen in [link]. Another application utilizes a radiopharmaceutical which the body sends to bone cells, particularly those that are most active, to detect cancerous tumors or healing points. Images can then be produced of such bone scans. Radioisotopes are also used to determine the functioning of body organs, such as blood flow, heart muscle activity, and iodine uptake in the thyroid gland.



A radiopharmaceutical is used to produce this brain image of a patient with

Alzheimer's disease. Certain features are computer enhanced. (credit: National Institutes of Health)

Medical Application

[link] lists certain medical diagnostic uses of radiopharmaceuticals, including isotopes and activities that are typically administered. Many organs can be imaged with a variety of nuclear isotopes replacing a stable element by a radioactive isotope. One common diagnostic employs iodine to image the thyroid, since iodine is concentrated in that organ. The most active thyroid cells, including cancerous cells, concentrate the most iodine and, therefore, emit the most radiation. Conversely, hypothyroidism is indicated by lack of iodine uptake. Note that there is more than one isotope that can be used for several types of scans. Another common nuclear diagnostic is the thallium scan for the cardiovascular system, particularly used to evaluate blockages in the coronary arteries and examine heart activity. The salt TlCl can be used, because it acts like NaCl and follows the blood. Gallium-67 accumulates where there is rapid cell growth, such as in tumors and sites of infection. Hence, it is useful in cancer imaging. Usually, the patient receives the injection one day and has a whole body scan 3 or 4 days later because it can take several days for the gallium to build up.

Procedure, isotope	Typical activity (mCi), where $1\text{ mCi} = 3.7 \times 10^7\text{ Bq}$

Procedure, isotope	**Typical activity (mCi), where** $1\ \text{mCi} = 3.7 \times 10^7\ \text{Bq}$
Brain scan	
$^{99\text{m}}\text{Tc}$	7.5
$^{113\text{m}}\text{In}$	7.5
^{11}C (PET)	20
^{13}N (PET)	20
^{15}O (PET)	50
^{18}F (PET)	10
Lung scan	
$^{99\text{m}}\text{Tc}$	2

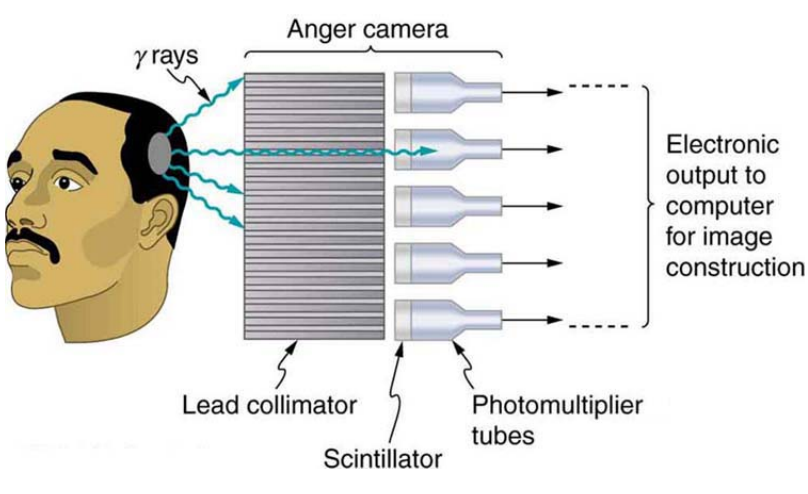
Procedure, isotope	**Typical activity (mCi), where** $1\ \mathrm{mCi} = 3.7 \times 10^7\ \mathrm{Bq}$
$^{133}\mathrm{Xe}$	7.5
Cardiovascular blood pool	
$^{131}\mathrm{I}$	0.2
$^{99m}\mathrm{Tc}$	2
Cardiovascular arterial flow	
$^{201}\mathrm{Tl}$	3
$^{24}\mathrm{Na}$	7.5
Thyroid scan	
$^{131}\mathrm{I}$	0.05
$^{123}\mathrm{I}$	0.07

Procedure, isotope	Typical activity (mCi), where $1\text{ mCi} = 3.7 \times 10^7\text{ Bq}$
Liver scan	
^{198}Au (colloid)	0.1
$^{99\text{m}}\text{Tc}$ (colloid)	2
Bone scan	
^{85}Sr	0.1
$^{99\text{m}}\text{Tc}$	10
Kidney scan	
^{197}Hg	0.1
$^{99\text{m}}\text{Tc}$	1.5

Diagnostic Uses of Radiopharmaceuticals

Note that [link] lists many diagnostic uses for ^{99m}Tc, where "m" stands for a metastable state of the technetium nucleus. Perhaps 80 percent of all radiopharmaceutical procedures employ ^{99m}Tc because of its many advantages. One is that the decay of its metastable state produces a single, easily identified 0.142-MeV γ ray. Additionally, the radiation dose to the patient is limited by the short 6.0-h half-life of ^{99m}Tc. And, although its half-life is short, it is easily and continuously produced on site. The basic process for production is neutron activation of molybdenum, which quickly β decays into ^{99m}Tc. Technetium-99m can be attached to many compounds to allow the imaging of the skeleton, heart, lungs, kidneys, etc.

[link] shows one of the simpler methods of imaging the concentration of nuclear activity, employing a device called an **Anger camera** or **gamma camera**. A piece of lead with holes bored through it collimates γ rays emerging from the patient, allowing detectors to receive γ rays from specific directions only. The computer analysis of detector signals produces an image. One of the disadvantages of this detection method is that there is no depth information (i.e., it provides a two-dimensional view of the tumor as opposed to a three-dimensional view), because radiation from any location under that detector produces a signal.



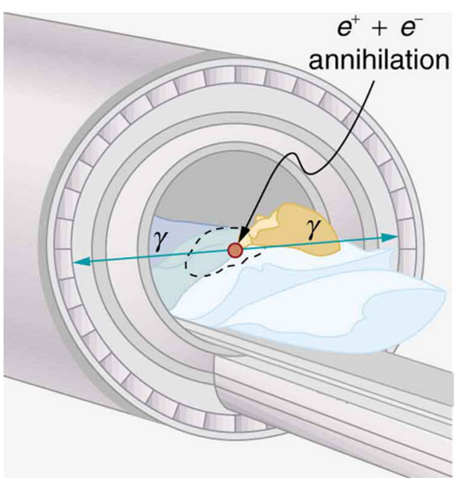
An Anger or gamma camera consists of a lead collimator and an array of detectors. Gamma rays produce light flashes in the

scintillators. The light output is converted to an electrical signal by the photomultipliers. A computer constructs an image from the detector output.

Imaging techniques much like those in x-ray computed tomography (CT) scans use nuclear activity in patients to form three-dimensional images. [link] shows a patient in a circular array of detectors that may be stationary or rotated, with detector output used by a computer to construct a detailed image. This technique is called **single-photon-emission computed tomography(SPECT)** or sometimes simply SPET. The spatial resolution of this technique is poor, about 1 cm, but the contrast (i.e. the difference in visual properties that makes an object distinguishable from other objects and the background) is good.

SPECT uses a geometry similar to a CT scanner to form an image of the concentration of a radiopharmaceutical compound. (credit: Woldo, Wikimedia Commons)

Images produced by β^+ emitters have become important in recent years. When the emitted positron (β^+) encounters an electron, mutual annihilation occurs, producing two γ rays. These γ rays have identical 0.511-MeV energies (the energy comes from the destruction of an electron or positron mass) and they move directly away from one another, allowing detectors to determine their point of origin accurately, as shown in [link]. The system is called **positron emission tomography (PET)**. It requires detectors on opposite sides to simultaneously (i.e., at the same time) detect photons of 0.511-MeV energy and utilizes computer imaging techniques similar to those in SPECT and CT scans. Examples of β^+ -emitting isotopes used in PET are ^{11}C, ^{13}N, ^{15}O, and ^{18}F, as seen in [link]. This list includes C, N, and O, and so they have the advantage of being able to function as tags for natural body compounds. Its resolution of 0.5 cm is better than that of SPECT; the accuracy and sensitivity of PET scans make them useful for examining the brain's anatomy and function. The brain's use of oxygen and water can be monitored with ^{15}O. PET is used extensively for diagnosing brain disorders. It can note decreased metabolism in certain regions prior to a confirmation of Alzheimer's disease. PET can locate regions in the brain that become active when a person carries out specific activities, such as speaking, closing their eyes, and so on.



A PET system takes

advantage of the two identical γ-ray photons produced by positron-electron annihilation. These γ rays are emitted in opposite directions, so that the line along which each pair is emitted is determined. Various events detected by several pairs of detectors are then analyzed by the computer to form an accurate image.

Note:
PhET Explorations: Simplified MRI
Is it a tumor? Magnetic Resonance Imaging (MRI) can tell. Your head is full of tiny radio transmitters (the nuclear spins of the hydrogen nuclei of your water molecules). In an MRI unit, these little radios can be made to broadcast their positions, giving a detailed picture of the inside of your head. Click to open media in new browser.

Section Summary

- Radiopharmaceuticals are compounds that are used for medical imaging and therapeutics.
- The process of attaching a radioactive substance is called tagging.

- [link] lists certain diagnostic uses of radiopharmaceuticals including the isotope and activity typically used in diagnostics.
- One common imaging device is the Anger camera, which consists of a lead collimator, radiation detectors, and an analysis computer.
- Tomography performed with γ-emitting radiopharmaceuticals is called SPECT and has the advantages of x-ray CT scans coupled with organ- and function-specific drugs.
- PET is a similar technique that uses β^+ emitters and detects the two annihilation γ rays, which aid to localize the source.

Conceptual Questions

Exercise:

Problem:

In terms of radiation dose, what is the major difference between medical diagnostic uses of radiation and medical therapeutic uses?

Exercise:

Problem:

One of the methods used to limit radiation dose to the patient in medical imaging is to employ isotopes with short half-lives. How would this limit the dose?

Problems & Exercises

Exercise:

Problem:

A neutron generator uses an α source, such as radium, to bombard beryllium, inducing the reaction $^4\text{He} + {}^9\text{Be} \rightarrow {}^{12}\text{C} + n$. Such neutron sources are called RaBe sources, or PuBe sources if they use plutonium to get the α s. Calculate the energy output of the reaction in MeV.

Solution:

5.701 MeV

Exercise:

Problem:

Neutrons from a source (perhaps the one discussed in the preceding problem) bombard natural molybdenum, which is 24 percent $^{98}\mathrm{Mo}$. What is the energy output of the reaction $^{98}\mathrm{Mo} + n \rightarrow {}^{99}\mathrm{Mo} + \gamma$? The mass of $^{98}\mathrm{Mo}$ is given in Appendix A: Atomic Masses, and that of $^{99}\mathrm{Mo}$ is 98.907711 u.

Exercise:

Problem:

The purpose of producing $^{99}\mathrm{Mo}$ (usually by neutron activation of natural molybdenum, as in the preceding problem) is to produce $^{99\mathrm{m}}\mathrm{Tc}$. Using the rules, verify that the β^- decay of $^{99}\mathrm{Mo}$ produces $^{99\mathrm{m}}\mathrm{Tc}$. (Most $^{99\mathrm{m}}\mathrm{Tc}$ nuclei produced in this decay are left in a metastable excited state denoted $^{99\mathrm{m}}\mathrm{Tc}$.)

Solution:

$${}^{99}_{42}\mathrm{Mo}_{57} \rightarrow {}^{99}_{43}\mathrm{Tc}_{56} + \beta^- + v_e$$

Exercise:

Problem:

(a) Two annihilation γ rays in a PET scan originate at the same point and travel to detectors on either side of the patient. If the point of origin is 9.00 cm closer to one of the detectors, what is the difference in arrival times of the photons? (This could be used to give position information, but the time difference is small enough to make it difficult.)

(b) How accurately would you need to be able to measure arrival time differences to get a position resolution of 1.00 mm?

Exercise:

Problem:

[link] indicates that 7.50 mCi of ^{99m}Tc is used in a brain scan. What is the mass of technetium?

Solution:

1.43×10^{-9} g

Exercise:

Problem:

The activities of ^{131}I and ^{123}I used in thyroid scans are given in [link] to be 50 and $70\ \mu\text{Ci}$, respectively. Find and compare the masses of ^{131}I and ^{123}I in such scans, given their respective half-lives are 8.04 d and 13.2 h. The masses are so small that the radioiodine is usually mixed with stable iodine as a carrier to ensure normal chemistry and distribution in the body.

Exercise:

Problem:

(a) Neutron activation of sodium, which is 100% ^{23}Na, produces ^{24}Na, which is used in some heart scans, as seen in [link]. The equation for the reaction is $^{23}\text{Na} + n \rightarrow {}^{24}\text{Na} + \gamma$. Find its energy output, given the mass of ^{24}Na is 23.990962 u.

(b) What mass of ^{24}Na produces the needed 5.0-mCi activity, given its half-life is 15.0 h?

Solution:

(a) 6.958 MeV

(b) 5.7×10^{-10} g

Glossary

Anger camera
: a common medical imaging device that uses a scintillator connected to a series of photomultipliers

gamma camera
: another name for an Anger camera

positron emission tomography (PET)
: tomography technique that uses β^+ emitters and detects the two annihilation γ rays, aiding in source localization

radiopharmaceutical
: compound used for medical imaging

single-photon-emission computed tomography (SPECT)
: tomography performed with γ-emitting radiopharmaceuticals

tagged
: process of attaching a radioactive substance to a chemical compound

Biological Effects of Ionizing Radiation

- Define various units of radiation.
- Describe effects of ionizing radiation on living organisms.

We hear many seemingly contradictory things about the biological effects of ionizing radiation. It can cause cancer, burns, and hair loss, yet it is used to treat and even cure cancer. How do we understand these effects? Once again, there is an underlying simplicity in nature, even in complicated biological organisms. All the effects of ionizing radiation on biological tissue can be understood by knowing that **ionizing radiation affects molecules within cells, particularly DNA molecules.**

Let us take a brief look at molecules within cells and how cells operate. Cells have long, double-helical DNA molecules containing chemical codes called genetic codes that govern the function and processes undertaken by the cell. It is for unraveling the double-helical structure of DNA that James Watson, Francis Crick, and Maurice Wilkins received the Nobel Prize. Damage to DNA consists of breaks in chemical bonds or other changes in the structural features of the DNA chain, leading to changes in the genetic code. In human cells, we can have as many as a million individual instances of damage to DNA per cell per day. It is remarkable that DNA contains codes that check whether the DNA is damaged or can repair itself. It is like an auto check and repair mechanism. This repair ability of DNA is vital for maintaining the integrity of the genetic code and for the normal functioning of the entire organism. It should be constantly active and needs to respond rapidly. The rate of DNA repair depends on various factors such as the cell type and age of the cell. A cell with a damaged ability to repair DNA, which could have been induced by ionizing radiation, can do one of the following:

- The cell can go into an irreversible state of dormancy, known as senescence.
- The cell can commit suicide, known as programmed cell death.
- The cell can go into unregulated cell division leading to tumors and cancers.

Since ionizing radiation damages the DNA, which is critical in cell reproduction, it has its greatest effect on cells that rapidly reproduce, including most types of cancer. Thus, cancer cells are more sensitive to radiation than normal cells and can be killed by it easily. Cancer is characterized by a malfunction of cell reproduction, and can also be caused by ionizing radiation. Without contradiction, ionizing radiation can be both a cure and a cause.

To discuss quantitatively the biological effects of ionizing radiation, we need a radiation dose unit that is directly related to those effects. All effects of radiation are assumed to be directly proportional to the amount of ionization produced in the biological organism. The amount of ionization is in turn proportional to the amount of deposited energy. Therefore, we define a **radiation dose unit** called the **rad**, as of a joule of ionizing energy deposited per kilogram of tissue, which is

Equation:

For example, if a 50.0-kg person is exposed to ionizing radiation over her entire body and she absorbs 1.00 J, then her whole-body radiation dose is

Equation:

If the same 1.00 J of ionizing energy were absorbed in her 2.00-kg forearm alone, then the dose to the forearm would be

Equation:

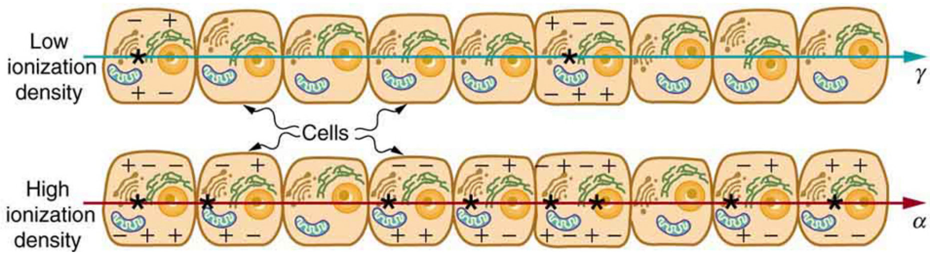
and the unaffected tissue would have a zero rad dose. While calculating radiation doses, you divide the energy absorbed by the mass of affected tissue. You must specify the affected region, such as the whole body or forearm in addition to giving the numerical dose in rads. The SI unit for radiation dose is the **gray (Gy)**, which is defined to be

Equation:

However, the rad is still commonly used. Although the energy per kilogram in 1 rad is small, it has significant effects since the energy causes ionization. The energy needed for a single ionization is a few eV, or less than . Thus, 0.01 J of ionizing energy can create a huge number of ion pairs and have an effect at the cellular level.

The effects of ionizing radiation may be directly proportional to the dose in rads, but they also depend on the type of radiation and the type of tissue. That is, for a given dose in rads, the effects depend on whether the radiation is X-ray, or some other type of ionizing radiation. In the earlier discussion of the range of ionizing radiation, it was noted that energy is deposited in a series of ionizations and not in a single interaction. Each ion pair or ionization requires a certain amount of energy, so that the number of ion pairs is directly proportional to the amount of the deposited ionizing energy. But, if the range of the radiation is small, as it is for s, then the ionization and the damage created is more concentrated and harder for the organism to repair, as seen in [link]. Concentrated damage is more difficult for biological organisms to repair than damage that is spread out, so short-range particles have greater biological effects. The **relative biological effectiveness** (RBE) or **quality factor** (QF) is given in [link] for several types of ionizing radiation—the effect of the radiation is directly proportional to the RBE. A dose unit more closely related to effects in biological tissue is called the **roentgen equivalent man** or rem and is defined to be the dose in rads multiplied by the relative biological effectiveness.

Equation:



The image shows ionization created in cells by and radiation. Because of its shorter range, the ionization and damage created by is more concentrated and harder for the organism to repair. Thus, the RBE for s is greater than the RBE for s, even though they create the same amount of ionization at the same energy.

So, if a person had a whole-body dose of 2.00 rad of radiation, the dose in rem would be . If the person had a whole-body dose of 2.00 rad of radiation, then the dose in rem would be . The s would have 20 times the effect on the person than the s for the same deposited energy. The SI equivalent of the rem is the **sievert** (Sv), defined to be , so that

Equation:

The RBEs given in [link] are approximate, but they yield certain insights. For example, the eyes are more sensitive to radiation, because the cells of the lens do not repair themselves. Neutrons cause more damage than rays, although both are neutral and have large ranges, because neutrons often cause secondary radiation when they are captured. Note that the RBEs are 1 for higher-energy s, s, and X-rays, three of the most common types of radiation. For those types of radiation, the numerical values of the dose in rem and rad are identical. For example, 1 rad of radiation is also 1 rem. For that reason, rads are still widely quoted rather than rem. [link] summarizes the units that are used for radiation.

Type and energy of radiation	RBE[footnote] Values approximate, difficult to determine.
X-rays	1
rays	1
rays greater than 32 keV	1
rays less than 32 keV	1.7
Neutrons, thermal to slow (<20 keV)	2–5
Neutrons, fast (1–10 MeV)	10 (body), 32 (eyes)
Protons (1–10 MeV)	10 (body), 32 (eyes)
rays from radioactive decay	10–20
Heavy ions from accelerators	10–20

Relative Biological Effectiveness

Quantity	SI unit name	Definition	Former unit	Conversion
Activity	Becquerel (bq)	decay/sec	Curie (Ci)	
Absorbed dose	Gray (Gy)	1 J/kg	rad	
Dose Equivalent	Sievert (Sv)	1 J/kg × RBE	rem	

Units for Radiation

Note:
Misconception Alert: Activity vs. Dose
“Activity” refers to the radioactive source while “dose” refers to the amount of energy from the radiation that is deposited in a person or object. A high level of activity doesn’t mean much danger if a person is far away from the source, or if substantial shielding is placed between the source and the person. A radioactive source can have a high activity in two different ways—the type of radioactive isotope may have a short half-life, resulting in more number of decays per second for a given amount of material, or there may be more material, resulting in more total number of decays per second.
For a given radioactive source with given activity, the dose received by nearby personnel can be reduced by radiation safety practices of establishing minimum distance perimeters (intensity of radiation follows inverse-square law) and designing and placing shielding materials around the source.

The large-scale effects of radiation on humans can be divided into two categories: immediate effects and long-term effects. [link] gives the immediate effects of whole-body exposures received in less than one day. If the radiation exposure is spread out over more time, greater doses are needed to cause the effects listed. This is due to the body’s ability to partially repair the damage. Any dose less than 100 mSv (10 rem) is called a **low dose**, 0.1 Sv to 1 Sv (10 to 100 rem) is called a **moderate dose**, and anything greater than 1 Sv (100 rem) is called a **high dose**. There is no known way to determine after the fact if a person has been exposed to less than 10 mSv.

Dose in Sv [footnote] Multiply by 100 to obtain dose in rem.	**Effect**
0–0.10	No observable effect.
0.1 – 1	Slight to moderate decrease in white blood cell counts.
0.5	Temporary sterility; 0.35 for women, 0.50 for men.
1 – 2	Significant reduction in blood cell counts, brief nausea and vomiting. Rarely fatal.

Dose in Sv [footnote] Multiply by 100 to obtain dose in rem.	**Effect**
2 – 5	Nausea, vomiting, hair loss, severe blood damage, hemorrhage, fatalities.
4.5	LD50/32. Lethal to 50% of the population within 32 days after exposure if not treated.
5 – 20	Worst effects due to malfunction of small intestine and blood systems. Limited survival.
>20	Fatal within hours due to collapse of central nervous system.

Immediate Effects of Radiation (Adults, Whole Body, Single Exposure)

Immediate effects are explained by the effects of radiation on cells and the sensitivity of rapidly reproducing cells to radiation. The first clue that a person has been exposed to radiation is a change in blood count, which is not surprising since blood cells are the most rapidly reproducing cells in the body. At higher doses, nausea and hair loss are observed, which may be due to interference with cell reproduction. Cells in the lining of the digestive system also rapidly reproduce, and their destruction causes nausea. When the growth of hair cells slows, the hair follicles become thin and break off. High doses cause significant cell death in all systems, but the lowest doses that cause fatalities do so by weakening the immune system through the loss of white blood cells.

The two known long-term effects of radiation are cancer and genetic defects. Both are directly attributable to the interference of radiation with cell reproduction. For high doses of radiation, the risk of cancer is reasonably well known from studies of exposed groups. Hiroshima and Nagasaki survivors and a smaller number of people exposed by their occupation, such as radium dial painters, have been fully documented. Chernobyl victims will be studied for many decades, with some data already available. For example, a significant increase in childhood thyroid cancer has been observed. The risk of a radiation-induced cancer for low and moderate doses is generally *assumed* to be proportional to the risk known for high doses. Under this assumption, any dose of radiation, no matter how small, involves a risk to human health. This is called the **linear hypothesis** and it may be prudent, but it *is* controversial. There is some evidence that, unlike the immediate effects of radiation, the long-term effects are cumulative and there is little self-repair. This is analogous to the risk of skin cancer from UV exposure, which is known to be cumulative.

There is a latency period for the onset of radiation-induced cancer of about 2 years for leukemia and 15 years for most other forms. The person is at risk for at least 30 years after the latency period. Omitting many details, the overall risk of a radiation-induced cancer death per year per rem of exposure is about 10 in a million, which can be written as

.

If a person receives a dose of 1 rem, his risk each year of dying from radiation-induced cancer is 10 in a million and that risk continues for about 30 years. The lifetime risk is thus 300 in a million, or 0.03 percent. Since about 20 percent of all worldwide deaths are from cancer, the increase due to a 1 rem exposure is impossible to detect demographically. But 100 rem (1 Sv), which was the dose received by the average Hiroshima and Nagasaki survivor, causes an additional 3 percent risk, which can be observed with statistical significance above the 20 percent natural incidence rate.

The incidence of genetic defects induced by radiation is about one-third that of cancer deaths, but is much more poorly known. The lifetime risk of a genetic defect due to a 1 rem exposure is about 100 in a million or , but the normal incidence is 60,000 in a million. Evidence of such a small increase, tragic as it is, is nearly impossible to obtain. For example, there is no evidence of increased genetic defects among the offspring of Hiroshima and Nagasaki survivors. Animal studies do not seem to correlate well with effects on humans and are not very helpful. For both cancer and genetic defects, the approach to safety has been to use the linear hypothesis, which is likely to be an overestimate of the risks of low doses. Certain researchers even claim that low doses are *beneficial*. **Hormesis** is a term used to describe generally favorable biological responses to low exposures of toxins or radiation. Such low levels may help certain repair mechanisms to develop or enable cells to adapt to the effects of the low exposures. Positive effects may occur at low doses that could be a problem at high doses.

Even the linear hypothesis estimates of the risks are relatively small, and the average person is not exposed to large amounts of radiation. [link] lists average annual background radiation doses from natural and artificial sources for Australia, the United States, Germany, and world-wide averages. Cosmic rays are partially shielded by the atmosphere, and the dose depends upon altitude and latitude, but the average is about 0.40 mSv/yr. A good example of the variation of cosmic radiation dose with altitude comes from the airline industry. Monitored personnel show an average of 2 mSv/yr. A 12-hour flight might give you an exposure of 0.02 to 0.03 mSv.

Doses from the Earth itself are mainly due to the isotopes of uranium, thorium, and potassium, and vary greatly by location. Some places have great natural concentrations of uranium and thorium, yielding doses ten times as high as the average value. Internal doses come from foods and liquids that we ingest. Fertilizers containing phosphates have potassium and uranium. So we are all a little radioactive. Carbon-14 has about 66 Bq/kg radioactivity whereas fertilizers may have more than 3000 Bq/kg radioactivity. Medical and dental diagnostic exposures are mostly from X-rays. It should be noted that X-ray

doses tend to be localized and are becoming much smaller with improved techniques. [link] shows typical doses received during various diagnostic X-ray examinations. Note the large dose from a CT scan. While CT scans only account for less than 20 percent of the X-ray procedures done today, they account for about 50 percent of the annual dose received.

Radon is usually more pronounced underground and in buildings with low air exchange with the outside world. Almost all soil contains some and , but radon is lower in mainly sedimentary soils and higher in granite soils. Thus, the exposure to the public can vary greatly, even within short distances. Radon can diffuse from the soil into homes, especially basements. The estimated exposure for is controversial. Recent studies indicate there is more radon in homes than had been realized, and it is speculated that radon may be responsible for 20 percent of lung cancers, being particularly hazardous to those who also smoke. Many countries have introduced limits on allowable radon concentrations in indoor air, often requiring the measurement of radon concentrations in a house prior to its sale. Ironically, it could be argued that the higher levels of radon exposure and their geographic variability, taken with the lack of demographic evidence of any effects, means that low-level radiation is *less* dangerous than previously thought.

Radiation Protection

Laws regulate radiation doses to which people can be exposed. The greatest occupational whole-body dose that is allowed depends upon the country and is about 20 to 50 mSv/yr and is rarely reached by medical and nuclear power workers. Higher doses are allowed for the hands. Much lower doses are permitted for the reproductive organs and the fetuses of pregnant women. Inadvertent doses to the public are limited to of occupational doses, except for those caused by nuclear power, which cannot legally expose the public to more than of the occupational limit or 0.05 mSv/yr (5 mrem/yr). This has been exceeded in the United States only at the time of the Three Mile Island (TMI) accident in 1979. Chernobyl is another story. Extensive monitoring with a variety of radiation detectors is performed to assure radiation safety. Increased ventilation in uranium mines has lowered the dose there to about 1 mSv/yr.

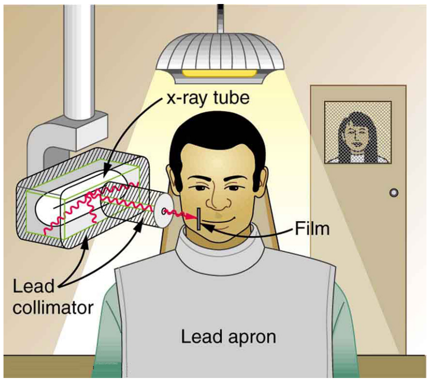
Source	**Dose (mSv/yr)[footnote] Multiply by 100 to obtain dose in mrem/yr.**			
Source	Australia	Germany	United States	World

Source	**Dose (mSv/yr)[footnote]** **Multiply by 100 to obtain dose in mrem/yr.**			
Natural Radiation - external				
Cosmic Rays	0.30	0.28	0.30	0.39
Soil, building materials	0.40	0.40	0.30	0.48
Radon gas	0.90	1.1	2.0	1.2
Natural Radiation - internal				
	0.24	0.28	0.40	0.29
Medical & Dental	0.80	0.90	0.53	0.40
TOTAL	2.6	3.0	3.5	2.8

Background Radiation Sources and Average Doses

To physically limit radiation doses, we use **shielding**, increase the **distance** from a source, and limit the **time of exposure**.

[link] illustrates how these are used to protect both the patient and the dental technician when an X-ray is taken. Shielding absorbs radiation, and the appropriate type of shielding depends on the type of radiation. The greater the distance from the source, the more the radiation spreads out. The less time a person is exposed to a given source, the smaller is the dose received by the person. Doses from most medical diagnostics have decreased in recent years due to faster films that require less exposure time.



A lead apron is placed over the dental patient and shielding surrounds the X-ray tube to limit exposure to tissue other than the tissue that is being imaged. Fast films limit the time needed to obtain images, reducing exposure to the imaged tissue. The technician stands a few meters away behind a lead-lined door with a lead glass window, reducing her occupational exposure.

Procedure	Effective dose (mSv)
Chest	0.02
Dental	0.01
Skull	0.07
Leg	0.02

Procedure	Effective dose (mSv)
Mammogram	0.40
Barium enema	7.0
Upper GI	3.0
CT head	2.0
CT abdomen	10.0

Typical Doses Received During Diagnostic X-ray Exams

Risk versus Benefit

Medical doses of radiation are also limited. Diagnostic doses are generally low and have further lowered with improved techniques and faster films. With the possible exception of routine dental X-rays, radiation is used diagnostically only when needed so that the low risk is justified by the benefit of the diagnosis. Chest X-rays give the lowest doses—about 0.1 mSv to the tissue affected, with less than 5 percent scattering into tissues that are not directly imaged. Other X-ray procedures range upward to about 10 mSv in a CT scan, and about 5 mSv (0.5 rem) per dental X-ray, again both only affecting the tissue imaged. Medical images with radiopharmaceuticals give doses ranging from 1 to 5 mSv, usually localized. One exception is the thyroid scan using . Because of its relatively long half-life, it exposes the thyroid to about 0.75 Sv. The isotope is more difficult to produce, but its short half-life limits thyroid exposure to about 15 mSv.

Section Summary

- The biological effects of ionizing radiation are due to two effects it has on cells: interference with cell reproduction, and destruction of cell function.
- A radiation dose unit called the rad is defined in terms of the ionizing energy deposited per kilogram of tissue:
  Equation:
- The SI unit for radiation dose is the gray (Gy), which is defined to be
- To account for the effect of the type of particle creating the ionization, we use the relative biological effectiveness (RBE) or quality factor (QF) given in [link] and define a unit called the roentgen equivalent man (rem) as

Equation:

- Particles that have short ranges or create large ionization densities have RBEs greater than 1. The SI equivalent of the rem is the sievert (Sv), defined to be
Equation:

- Whole-body, single-exposure doses of 0.1 Sv or less are low doses while those of 0.1 to 1 Sv are moderate, and those over 1 Sv are high doses. Some immediate radiation effects are given in [link]. Effects due to low doses are not observed, but their risk is assumed to be directly proportional to those of high doses, an assumption known as the linear hypothesis. Long-term effects are cancer deaths at the rate of and genetic defects at roughly one-third this rate. Background radiation doses and sources are given in [link]. World-wide average radiation exposure from natural sources, including radon, is about 3 mSv, or 300 mrem. Radiation protection utilizes shielding, distance, and time to limit exposure.

Conceptual Questions

Exercise:

Problem:

Isotopes that emit radiation are relatively safe outside the body and exceptionally hazardous inside. Yet those that emit radiation are hazardous outside and inside. Explain why.

Exercise:

Problem:

Why is radon more closely associated with inducing lung cancer than other types of cancer?

Exercise:

Problem:

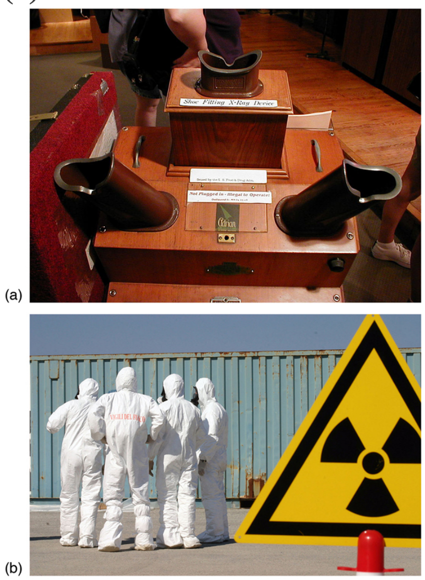
The RBE for low-energy s is 1.7, whereas that for higher-energy s is only 1. Explain why, considering how the range of radiation depends on its energy.

Exercise:

Problem:

Which methods of radiation protection were used in the device shown in the first photo in [link]? Which were used in the situation shown in the second photo?

(a)



(a) This X-ray fluorescence machine is one of the thousands used in shoe stores to produce images of feet as a check on the fit of shoes. They are unshielded and remain on as long as the feet are in them, producing doses much greater than medical images. Children were fascinated with them. These machines were used in shoe stores until laws preventing such unwarranted radiation exposure were enacted in the 1950s. (credit: Andrew Kuchling) (b) Now that we know the effects of exposure to

radioactive material, safety is a priority. (credit: U.S. Navy)

Exercise:

Problem:

What radioisotope could be a problem in homes built of cinder blocks made from uranium mine tailings? (This is true of homes and schools in certain regions near uranium mines.)

Exercise:

Problem:

Are some types of cancer more sensitive to radiation than others? If so, what makes them more sensitive?

Exercise:

Problem:

Suppose a person swallows some radioactive material by accident. What information is needed to be able to assess possible damage?

Problems & Exercises

Exercise:

Problem:

What is the dose in mSv for: (a) a 0.1 Gy X-ray? (b) 2.5 mGy of neutron exposure to the eye? (c) 1.5 mGy of exposure?

Solution:

(a) 100 mSv

(b) 80 mSv

(c) ~30 mSv

Exercise:

Problem:

Find the radiation dose in Gy for: (a) A 10-mSv fluoroscopic X-ray series. (b) 50 mSv of skin exposure by an emitter. (c) 160 mSv of β and rays from the in your body.

Exercise:

Problem:

How many Gy of exposure is needed to give a cancerous tumor a dose of 40 Sv if it is exposed to activity?

Solution:

~2 Gy

Exercise:

Problem:

What is the dose in Sv in a cancer treatment that exposes the patient to 200 Gy of rays?

Exercise:

Problem:

One half the rays from are absorbed by a 0.170-mm-thick lead shielding. Half of the rays that pass through the first layer of lead are absorbed in a second layer of equal thickness. What thickness of lead will absorb all but one in 1000 of these rays?

Solution:

1.69 mm

Exercise:

Problem:

A plumber at a nuclear power plant receives a whole-body dose of 30 mSv in 15 minutes while repairing a crucial valve. Find the radiation-induced yearly risk of death from cancer and the chance of genetic defect from this maximum allowable exposure.

Exercise:

Problem:

In the 1980s, the term picowave was used to describe food irradiation in order to overcome public resistance by playing on the well-known safety of microwave radiation. Find the energy in MeV of a photon having a wavelength of a picometer.

Solution:

1.24 MeV

Exercise:

Problem: Find the mass of that has an activity of μ .

Glossary

gray (Gy)
: the SI unit for radiation dose which is defined to be

linear hypothesis
: assumption that risk is directly proportional to risk from high doses

rad
: the ionizing energy deposited per kilogram of tissue

sievert
: the SI equivalent of the rem

relative biological effectiveness (RBE)
: a number that expresses the relative amount of damage that a fixed amount of ionizing radiation of a given type can inflict on biological tissues

quality factor
: same as relative biological effectiveness

roentgen equivalent man (rem)
: a dose unit more closely related to effects in biological tissue

low dose
: a dose less than 100 mSv (10 rem)

moderate dose
: a dose from 0.1 Sv to 1 Sv (10 to 100 rem)

high dose
: a dose greater than 1 Sv (100 rem)

hormesis
: a term used to describe generally favorable biological responses to low exposures of toxins or radiation

shielding
: a technique to limit radiation exposure

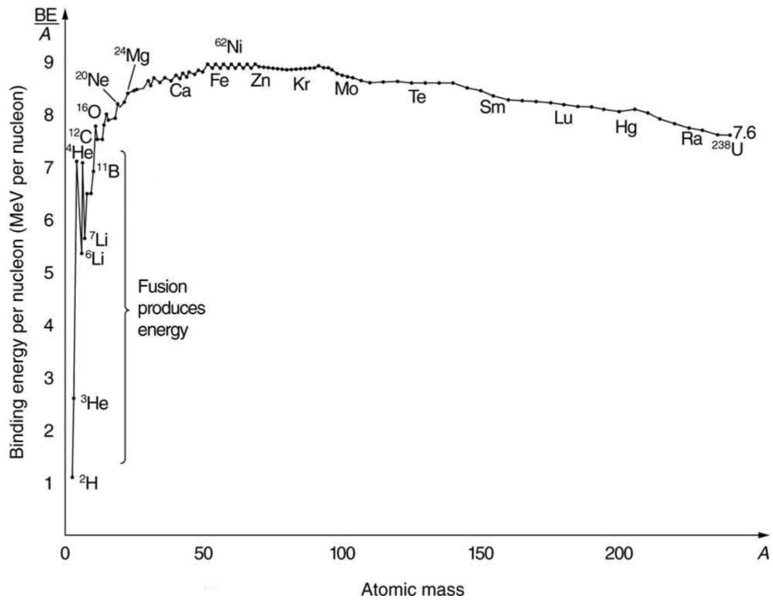
Fusion

- Define nuclear fusion.
- Discuss processes to achieve practical fusion energy generation.

While basking in the warmth of the summer sun, a student reads of the latest breakthrough in achieving sustained thermonuclear power and vaguely recalls hearing about the cold fusion controversy. The three are connected. The Sun’s energy is produced by nuclear fusion (see [link]). Thermonuclear power is the name given to the use of controlled nuclear fusion as an energy source. While research in the area of thermonuclear power is progressing, high temperatures and containment difficulties remain. The cold fusion controversy centered around unsubstantiated claims of practical fusion power at room temperatures.

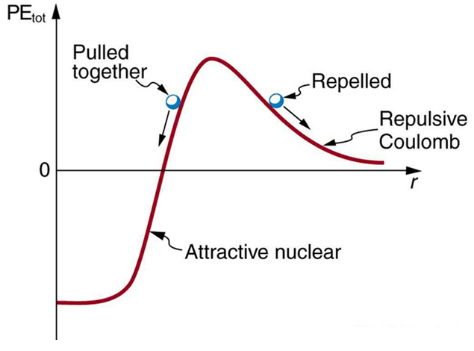
The Sun’s energy is produced by nuclear fusion. (credit: Spiralz)

Nuclear fusion is a reaction in which two nuclei are combined, or *fused*, to form a larger nucleus. We know that all nuclei have less mass than the sum of the masses of the protons and neutrons that form them. The missing mass times c^2 equals the binding energy of the nucleus—the greater the binding energy, the greater the missing mass. We also know that BE/A, the binding energy per nucleon, is greater for medium-mass nuclei and has a maximum at Fe (iron). This means that if two low-mass nuclei can be fused together to form a larger nucleus, energy can be released. The larger nucleus has a greater binding energy and less mass per nucleon than the two that combined. Thus mass is destroyed in the fusion reaction, and energy is released (see [link]). On average, fusion of low-mass nuclei releases energy, but the details depend on the actual nuclides involved.



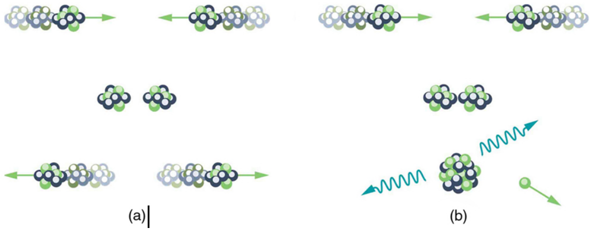
Fusion of light nuclei to form medium-mass nuclei destroys mass, because BE/A is greater for the product nuclei. The larger BE/A is, the less mass per nucleon, and so mass is converted to energy and released in these fusion reactions.

The major obstruction to fusion is the Coulomb repulsion between nuclei. Since the attractive nuclear force that can fuse nuclei together is short ranged, the repulsion of like positive charges must be overcome to get nuclei close enough to induce fusion. [link] shows an approximate graph of the potential energy between two nuclei as a function of the distance between their centers. The graph is analogous to a hill with a well in its center. A ball rolled from the right must have enough kinetic energy to get over the hump before it falls into the deeper well with a net gain in energy. So it is with fusion. If the nuclei are given enough kinetic energy to overcome the electric potential energy due to repulsion, then they can combine, release energy, and fall into a deep well. One way to accomplish this is to heat fusion fuel to high temperatures so that the kinetic energy of thermal motion is sufficient to get the nuclei together.



Potential energy between two light nuclei graphed as a function of distance between them. If the nuclei have enough kinetic energy to get over the Coulomb repulsion hump, they combine, release energy, and drop into a deep attractive well. Tunneling through the barrier is important in practice. The greater the kinetic energy and the higher the particles get up the barrier (or the lower the barrier), the more likely the tunneling.

You might think that, in the core of our Sun, nuclei are coming into contact and fusing. However, in fact, temperatures on the order of 10^8K are needed to actually get the nuclei in contact, exceeding the core temperature of the Sun. Quantum mechanical tunneling is what makes fusion in the Sun possible, and tunneling is an important process in most other practical applications of fusion, too. Since the probability of tunneling is extremely sensitive to barrier height and width, increasing the temperature greatly increases the rate of fusion. The closer reactants get to one another, the more likely they are to fuse (see [link]). Thus most fusion in the Sun and other stars takes place at their centers, where temperatures are highest. Moreover, high temperature is needed for thermonuclear power to be a practical source of energy.



(a) Two nuclei heading toward each other slow down, then stop, and then fly away without touching or fusing. (b) At higher energies, the two nuclei approach close enough for fusion via tunneling. The probability of tunneling increases as they approach,

but they do not have to touch for the
reaction to occur.

The Sun produces energy by fusing protons or hydrogen nuclei ^1H (by far the Sun's most abundant nuclide) into helium nuclei ^4He. The principal sequence of fusion reactions forms what is called the **proton-proton cycle**:

Equation:

$$^1H + {}^1H \rightarrow {}^2H + e^+ + v_e \qquad (0.42\ \text{MeV})$$

Equation:

$$^1H + {}^2H \rightarrow {}^3He + \gamma \qquad (5.49\ \text{MeV})$$

Equation:

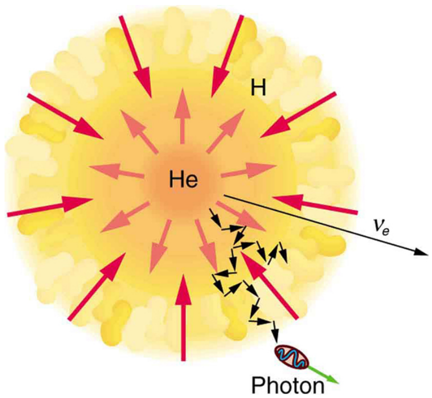
$$^3He + {}^3He \rightarrow {}^4He + {}^1H + {}^1H \qquad (12.86\ \text{MeV})$$

where e^+ stands for a positron and v_e is an electron neutrino. (The energy in parentheses is *released* by the reaction.) Note that the first two reactions must occur twice for the third to be possible, so that the cycle consumes six protons (^1H) but gives back two. Furthermore, the two positrons produced will find two electrons and annihilate to form four more γ rays, for a total of six. The overall effect of the cycle is thus

Equation:

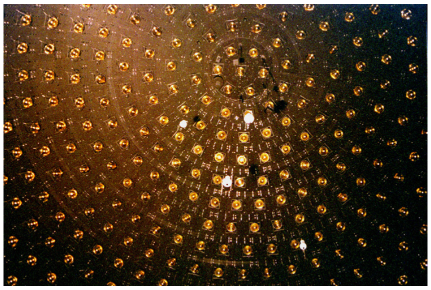
$$2e^- + 4{}^1H \rightarrow {}^4He + 2v_e + 6\gamma \qquad (26.7\ \text{MeV})$$

where the 26.7 MeV includes the annihilation energy of the positrons and electrons and is distributed among all the reaction products. The solar interior is dense, and the reactions occur deep in the Sun where temperatures are highest. It takes about 32,000 years for the energy to diffuse to the surface and radiate away. However, the neutrinos escape the Sun in less than two seconds, carrying their energy with them, because they interact so weakly that the Sun is transparent to them. Negative feedback in the Sun acts as a thermostat to regulate the overall energy output. For instance, if the interior of the Sun becomes hotter than normal, the reaction rate increases, producing energy that expands the interior. This cools it and lowers the reaction rate. Conversely, if the interior becomes too cool, it contracts, increasing the temperature and reaction rate. Stars like the Sun are stable for billions of years, until a significant fraction of their hydrogen has been depleted.

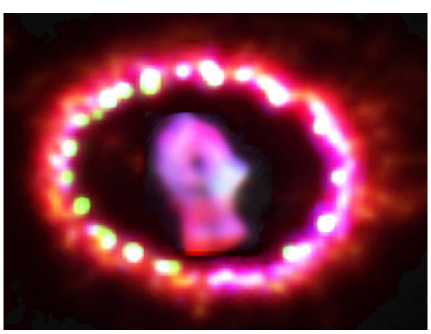


Nuclear fusion in the Sun converts hydrogen nuclei into helium; fusion occurs primarily at the boundary of the helium core, where temperature is highest and sufficient hydrogen remains. Energy released diffuses slowly to the surface, with the exception of neutrinos, which escape immediately. Energy production remains stable because of negative feedback effects.

Theories of the proton-proton cycle (and other energy-producing cycles in stars) were pioneered by the German-born, American physicist Hans Bethe (1906–2005), starting in 1938. He was awarded the 1967 Nobel Prize in physics for this work, and he has made many other contributions to physics and society. Neutrinos produced in these cycles escape so readily that they provide us an excellent means to test these theories and study stellar interiors. Detectors have been constructed and operated for more than four decades now to measure solar neutrinos (see [link]). Although solar neutrinos are detected and neutrinos were observed from Supernova 1987A ([link]), too few solar neutrinos were observed to be consistent with predictions of solar energy production. After many years, this solar neutrino problem was resolved with a blend of theory and experiment that showed that the neutrino does indeed have mass. It was also found that there are three types of neutrinos, each associated with a different type of nuclear decay.



This array of photomultiplier tubes is part of the large solar neutrino detector at the Fermi National Accelerator Laboratory in Illinois. In these experiments, the neutrinos interact with heavy water and produce flashes of light, which are detected by the photomultiplier tubes. In spite of its size and the huge flux of neutrinos that strike it, very few are detected each day since they interact so weakly. This, of course, is the same reason they escape the Sun so readily. (credit: Fred Ullrich)



Supernovas are the source of elements heavier than

iron. Energy released powers nucleosynthesis. Spectroscopic analysis of the ring of material ejected by Supernova 1987A observable in the southern hemisphere, shows evidence of heavy elements. The study of this supernova also provided indications that neutrinos might have mass. (credit: NASA, ESA, and P. Challis)

The proton-proton cycle is not a practical source of energy on Earth, in spite of the great abundance of hydrogen (^1H). The reaction $^1\text{H} + {}^1\text{H} \rightarrow {}^2\text{H} + e^+ + v_\text{e}$ has a very low probability of occurring. (This is why our Sun will last for about ten billion years.) However, a number of other fusion reactions are easier to induce. Among them are:

Equation:

$$^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{H} + {}^1\text{H} \qquad (4.03\ \text{MeV})$$

Equation:

$$^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + n \qquad (3.27\ \text{MeV})$$

Equation:

$$^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + n \qquad (17.59\ \text{MeV})$$

Equation:

$$^2\text{H} + {}^2\text{H} \rightarrow {}^4\text{He} + \gamma \qquad (23.85\ \text{MeV}).$$

Deuterium (^2H) is about 0.015% of natural hydrogen, so there is an immense amount of it in sea water alone. In addition to an abundance of deuterium fuel, these fusion reactions produce large energies per reaction (in parentheses), but they do not produce much radioactive waste. Tritium (^3H) is radioactive, but it is consumed as a fuel (the reaction $^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + n$), and the neutrons and γs can be shielded. The neutrons produced can also be used to create more energy and fuel in reactions like

Equation:

$$n + {}^{1}\text{H} \rightarrow {}^{2}\text{H} + \gamma \qquad (20.68\ \text{MeV})$$

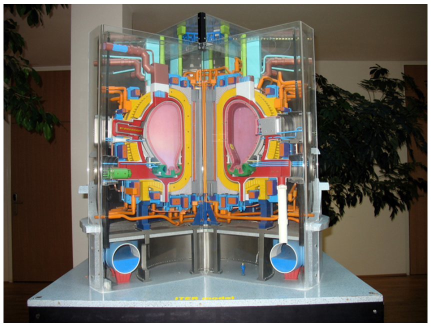
and
Equation:

$$n + {}^{1}\text{H} \rightarrow {}^{2}\text{H} + \gamma \qquad (2.22\ \text{MeV}).$$

Note that these last two reactions, and ${}^{2}\text{H} + {}^{2}\text{H} \rightarrow {}^{4}\text{He} + \gamma$, put most of their energy output into the γ ray, and such energy is difficult to utilize.

The three keys to practical fusion energy generation are to achieve the temperatures necessary to make the reactions likely, to raise the density of the fuel, and to confine it long enough to produce large amounts of energy. These three factors—temperature, density, and time—complement one another, and so a deficiency in one can be compensated for by the others. **Ignition** is defined to occur when the reactions produce enough energy to be self-sustaining after external energy input is cut off. This goal, which must be reached before commercial plants can be a reality, has not been achieved. Another milestone, called **break-even**, occurs when the fusion power produced equals the heating power input. Break-even has nearly been reached and gives hope that ignition and commercial plants may become a reality in a few decades.

Two techniques have shown considerable promise. The first of these is called **magnetic confinement** and uses the property that charged particles have difficulty crossing magnetic field lines. The tokamak, shown in [link], has shown particular promise. The tokamak's toroidal coil confines charged particles into a circular path with a helical twist due to the circulating ions themselves. In 1995, the Tokamak Fusion Test Reactor at Princeton in the US achieved world-record plasma temperatures as high as 500 million degrees Celsius. This facility operated between 1982 and 1997. A joint international effort is underway in France to build a tokamak-type reactor that will be the stepping stone to commercial power. ITER, as it is called, will be a full-scale device that aims to demonstrate the feasibility of fusion energy. It will generate 500 MW of power for extended periods of time and will achieve break-even conditions. It will study plasmas in conditions similar to those expected in a fusion power plant. Completion is scheduled for 2018.



(a) Artist’s rendition of ITER, a tokamak-type fusion reactor being built in southern France. It is hoped that this gigantic machine will reach the break-even point. Completion is scheduled for 2018. (credit: Stephan Mosel, Flickr)

The second promising technique aims multiple lasers at tiny fuel pellets filled with a mixture of deuterium and tritium. Huge power input heats the fuel, evaporating the confining pellet and crushing the fuel to high density with the expanding hot plasma produced. This technique is called **inertial confinement**, because the fuel’s inertia prevents it from escaping before significant fusion can take place. Higher densities have been reached than with tokamaks, but with smaller confinement times. In 2009, the Lawrence Livermore Laboratory (CA) completed a laser fusion device with 192 ultraviolet laser beams that are focused upon a D-T pellet (see [link]).

National Ignition Facility (CA). This image shows a laser bay where 192 laser beams will focus onto a small D-T target, producing fusion. (credit: Lawrence Livermore National Laboratory, Lawrence Livermore National Security, LLC, and the Department of Energy)

Example:
Calculating Energy and Power from Fusion
(a) Calculate the energy released by the fusion of a 1.00-kg mixture of deuterium and tritium, which produces helium. There are equal numbers of deuterium and tritium nuclei in the mixture.
(b) If this takes place continuously over a period of a year, what is the average power output?
Strategy
According to ${}^{2}\text{H} + {}^{3}\text{H} \rightarrow {}^{4}\text{He} + n$, the energy per reaction is 17.59 MeV. To find the total energy released, we must find the number of deuterium and tritium atoms in a kilogram. Deuterium has an atomic mass of about 2 and tritium has an atomic mass of about 3, for a total of about 5 g per mole of reactants or about 200 mol in 1.00 kg. To get a more precise figure, we will use the atomic masses from Appendix A. The power output is best expressed in watts, and so the energy output needs to be calculated in joules and then divided by the number of seconds in a year.
Solution for (a)
The atomic mass of deuterium (${}^{2}\text{H}$) is 2.014102 u, while that of tritium (${}^{3}\text{H}$) is 3.016049 u, for a total of 5.032151 u per reaction. So a mole of reactants has a mass of 5.03 g, and in 1.00 kg there are $(1000\text{ g})/(5.03\text{ g/mol})\text{=}198.8\text{ mol of reactants}$. The number of reactions that take place is therefore
Equation:

$$(198.8\text{ mol})\left(6.02\times 10^{23}\text{ mol}^{-1}\right) = 1.20\times 10^{26}\text{ reactions.}$$

The total energy output is the number of reactions times the energy per reaction:
Equation:

$$\begin{aligned} E = \left(1.20\times 10^{26}\text{ reactions}\right)(17.59\text{ MeV/reaction})\left(1.602\times 10^{-13}\text{ J/MeV}\right) \\ = 3.37\times 10^{14}\text{ J.} \end{aligned}$$

Solution for (b)
Power is energy per unit time. One year has 3.16×10^{7} s, so
Equation:

$$\begin{aligned} P &= \frac{E}{t} = \frac{3.37\times 10^{14}\text{ J}}{3.16\times 10^{7}\text{ s}} \\ &= 1.07\times 10^{7}\text{ W} = 10.7\text{ MW.} \end{aligned}$$

Discussion
By now we expect nuclear processes to yield large amounts of energy, and we are not disappointed here. The energy output of $3.37\times 10^{14}\text{ J}$ from fusing 1.00 kg of deuterium

and tritium is equivalent to 2.6 million gallons of gasoline and about eight times the energy output of the bomb that destroyed Hiroshima. Yet the average backyard swimming pool has about 6 kg of deuterium in it, so that fuel is plentiful if it can be utilized in a controlled manner. The average power output over a year is more than 10 MW, impressive but a bit small for a commercial power plant. About 32 times this power output would allow generation of 100 MW of electricity, assuming an efficiency of one-third in converting the fusion energy to electrical energy.

Section Summary

- Nuclear fusion is a reaction in which two nuclei are combined to form a larger nucleus. It releases energy when light nuclei are fused to form medium-mass nuclei.
- Fusion is the source of energy in stars, with the proton-proton cycle,
  Equation:

  $$^{1}\text{H} + {}^{1}\text{H} \rightarrow {}^{2}\text{H} + e^{+} + v_{\text{e}} \qquad (0.42\ \text{MeV})$$

  Equation:

  $$^{1}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{He} + \gamma \qquad (5.49\ \text{MeV})$$

  Equation:

  $$^{3}\text{He} + {}^{3}\text{He} \rightarrow {}^{4}\text{He} + {}^{1}\text{H} + {}^{1}\text{H} \qquad (12.86\ \text{MeV})$$

  being the principal sequence of energy-producing reactions in our Sun.
- The overall effect of the proton-proton cycle is
  Equation:

  $$2e^{-} + 4{}^{1}\text{H} \rightarrow {}^{4}\text{He} + 2v_{\text{e}} + 6\gamma \qquad (26.7\ \text{MeV}),$$

  where the 26.7 MeV includes the energy of the positrons emitted and annihilated.
- Attempts to utilize controlled fusion as an energy source on Earth are related to deuterium and tritium, and the reactions play important roles.
- Ignition is the condition under which controlled fusion is self-sustaining; it has not yet been achieved. Break-even, in which the fusion energy output is as great as the external energy input, has nearly been achieved.
- Magnetic confinement and inertial confinement are the two methods being developed for heating fuel to sufficiently high temperatures, at sufficient density, and for sufficiently long times to achieve ignition. The first method uses magnetic fields

and the second method uses the momentum of impinging laser beams for confinement.

Conceptual Questions

Exercise:

Problem: Why does the fusion of light nuclei into heavier nuclei release energy?

Exercise:

Problem:

Energy input is required to fuse medium-mass nuclei, such as iron or cobalt, into more massive nuclei. Explain why.

Exercise:

Problem:

In considering potential fusion reactions, what is the advantage of the reaction $^{2}\text{H} + {}^{3}\text{H} \rightarrow {}^{4}\text{He} + n$ over the reaction $^{2}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{He} + n$?

Exercise:

Problem:

Give reasons justifying the contention made in the text that energy from the fusion reaction $^{2}\text{H} + {}^{2}\text{H} \rightarrow {}^{4}\text{He} + \gamma$ is relatively difficult to capture and utilize.

Problems & Exercises

Exercise:

Problem:

Verify that the total number of nucleons, total charge, and electron family number are conserved for each of the fusion reactions in the proton-proton cycle in

Equation:

$$^{1}\text{H} + {}^{1}\text{H} \rightarrow {}^{2}\text{H} + e^{+} + v_{\text{e}},$$

Equation:

$$^{1}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{He} + \gamma,$$

and
Equation:

$$^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + {}^1\text{H} + {}^1\text{H}.$$

(List the value of each of the conserved quantities before and after each of the reactions.)

Solution:

(a) A=1+1=2, Z=1+1=1+1, efn $= 0 = -1 + 1$

(b) A=1+2=3, Z=1+1=2, efn=0=0

(c) A=3+3=4+1+1, Z=2+2=2+1+1, efn=0=0

Exercise:

Problem:

Calculate the energy output in each of the fusion reactions in the proton-proton cycle, and verify the values given in the above summary.

Exercise:

Problem:

Show that the total energy released in the proton-proton cycle is 26.7 MeV, considering the overall effect in $^1\text{H} + {}^1\text{H} \rightarrow {}^2\text{H} + e^+ + v_\text{e}$, $^1\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + \gamma$, and $^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + {}^1\text{H} + {}^1\text{H}$ and being certain to include the annihilation energy.

Solution:

$$\begin{aligned} E &= (m_\text{i} - m_\text{f})c^2 \\ &= \left[4m\left({}^1\text{H}\right) - m\left({}^4\text{He}\right)\right]c^2 \\ &= [4(1.007825) - 4.002603](931.5\text{ MeV}) \\ &= 26.73\text{ MeV} \end{aligned}$$

Exercise:

Problem:

Verify by listing the number of nucleons, total charge, and electron family number before and after the cycle that these quantities are conserved in the overall proton-proton cycle in $2e^- + 4{}^1\text{H} \rightarrow {}^4\text{He} + 2v_\text{e} + 6\gamma$.

Exercise:

Problem:

The energy produced by the fusion of a 1.00-kg mixture of deuterium and tritium was found in Example "Calculating Energy and Power from Fusion". Approximately how many kilograms would be required to supply the annual energy use in the United States?

Solution:

3.12×10^5 kg (about 200 tons)

Exercise:

Problem:

Tritium is naturally rare, but can be produced by the reaction $n + {}^2\text{H} \to {}^3\text{H} + \gamma$. How much energy in MeV is released in this neutron capture?

Exercise:

Problem: Two fusion reactions mentioned in the text are

$n + {}^3\text{He} \to {}^4\text{He} + \gamma$

and

$n + {}^1\text{H} \to {}^2\text{H} + \gamma$.

Both reactions release energy, but the second also creates more fuel. Confirm that the energies produced in the reactions are 20.58 and 2.22 MeV, respectively. Comment on which product nuclide is most tightly bound, ${}^4\text{He}$ or ${}^2\text{H}$.

Solution:

$$\begin{aligned} E &= (m_\text{i} - m_\text{f})c^2 \\ E_1 &= (1.008665 + 3.016030 - 4.002603)(931.5\text{ MeV}) \\ &= 20.58\text{ MeV} \\ E_2 &= (1.008665 + 1.007825 - 2.014102)(931.5\text{ MeV}) \\ &= 2.224\text{ MeV} \end{aligned}$$

${}^4\text{He}$ is more tightly bound, since this reaction gives off more energy per nucleon.

Exercise:

Problem:

(a) Calculate the number of grams of deuterium in an 80,000-L swimming pool, given deuterium is 0.0150% of natural hydrogen.

(b) Find the energy released in joules if this deuterium is fused via the reaction $^{2}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{He} + n$.

(c) Could the neutrons be used to create more energy?

(d) Discuss the amount of this type of energy in a swimming pool as compared to that in, say, a gallon of gasoline, also taking into consideration that water is far more abundant.

Exercise:

Problem:

How many kilograms of water are needed to obtain the 198.8 mol of deuterium, assuming that deuterium is 0.01500% (by number) of natural hydrogen?

Solution:

$1.19 \times 10^{4}\text{ kg}$

Exercise:

Problem: The power output of the Sun is $4 \times 10^{26}\text{ W}$.

(a) If 90% of this is supplied by the proton-proton cycle, how many protons are consumed per second?

(b) How many neutrinos per second should there be per square meter at the Earth from this process? This huge number is indicative of how rarely a neutrino interacts, since large detectors observe very few per day.

Exercise:

Problem:

Another set of reactions that result in the fusing of hydrogen into helium in the Sun and especially in hotter stars is called the carbon cycle. It is

Equation:

$$
\begin{aligned}
{}^{12}\mathrm{C}+{}^{1}\mathrm{H} &\rightarrow {}^{13}\mathrm{N}+\gamma, \\
{}^{13}\mathrm{N} &\rightarrow {}^{13}\mathrm{C}+e^{+}+v_e, \\
{}^{13}\mathrm{C}+{}^{1}\mathrm{H} &\rightarrow {}^{14}\mathrm{N}+\gamma, \\
{}^{14}\mathrm{N}+{}^{1}\mathrm{H} &\rightarrow {}^{15}\mathrm{O}+\gamma, \\
{}^{15}\mathrm{O} &\rightarrow {}^{15}\mathrm{N}+e^{+}+v_e, \\
{}^{15}\mathrm{N}+{}^{1}\mathrm{H} &\rightarrow {}^{12}\mathrm{C}+{}^{4}\mathrm{He}.
\end{aligned}
$$

Write down the overall effect of the carbon cycle (as was done for the proton-proton cycle in $2e^- + 4^1\mathrm{H} \rightarrow {}^4\mathrm{He} + 2v_e + 6\gamma$). Note the number of protons ($^1\mathrm{H}$) required and assume that the positrons (e^+) annihilate electrons to form more γ rays.

Solution:

$2e^- + 4^1\mathrm{H} \rightarrow {}^4\mathrm{He} + 7\gamma + 2v_e$

Exercise:

Problem:

(a) Find the total energy released in MeV in each carbon cycle (elaborated in the above problem) including the annihilation energy.

(b) How does this compare with the proton-proton cycle output?

Exercise:

Problem:

Verify that the total number of nucleons, total charge, and electron family number are conserved for each of the fusion reactions in the carbon cycle given in the above problem. (List the value of each of the conserved quantities before and after each of the reactions.)

Solution:

(a) A=12+1=13, Z=6+1=7, efn $= 0 = 0$

(b) A=13=13, Z=7=6+1, efn $= 0 = -1 + 1$

(c) A=13 + 1=14, Z=6+1=7, efn $= 0 = 0$

(d) A=14 + 1=15, Z=7+1=8, efn $= 0 = 0$

(e) A=15=15, Z=8=7+1, efn $= 0 = -1 + 1$

(f) $A = 15 + 1 = 12 + 4$, $Z = 7 + 1 = 6 + 2$, $\text{efn} = 0 = 0$

Exercise:

Problem: Integrated Concepts

The laser system tested for inertial confinement can produce a 100-kJ pulse only 1.00 ns in duration. (a) What is the power output of the laser system during the brief pulse?

(b) How many photons are in the pulse, given their wavelength is 1.06 µm?

(c) What is the total momentum of all these photons?

(d) How does the total photon momentum compare with that of a single 1.00 MeV deuterium nucleus?

Exercise:

Problem: Integrated Concepts

Find the amount of energy given to the ^{4}He nucleus and to the γ ray in the reaction $n + ^3\text{He} \rightarrow ^4\text{He} + \gamma$, using the conservation of momentum principle and taking the reactants to be initially at rest. This should confirm the contention that most of the energy goes to the γ ray.

Solution:

$E_{\gamma} = 20.6\ \text{MeV}$

$E_{^4\text{He}} = 5.68 \times 10^{-2}\ \text{MeV}$

Exercise:

Problem: Integrated Concepts

(a) What temperature gas would have atoms moving fast enough to bring two ^{3}He nuclei into contact? Note that, because both are moving, the average kinetic energy only needs to be half the electric potential energy of these doubly charged nuclei when just in contact with one another.

(b) Does this high temperature imply practical difficulties for doing this in controlled fusion?

Exercise:

Problem: Integrated Concepts

(a) Estimate the years that the deuterium fuel in the oceans could supply the energy needs of the world. Assume world energy consumption to be ten times that of the United States which is 8×10^{19} J/y and that the deuterium in the oceans could be converted to energy with an efficiency of 32%. You must estimate or look up the amount of water in the oceans and take the deuterium content to be 0.015% of natural hydrogen to find the mass of deuterium available. Note that approximate energy yield of deuterium is 3.37×10^{14} J/kg.

(b) Comment on how much time this is by any human measure. (It is not an unreasonable result, only an impressive one.)

Solution:

(a) 3×10^{9} y

(b) This is approximately half the lifetime of the Earth.

Glossary

break-even
: when fusion power produced equals the heating power input

ignition
: when a fusion reaction produces enough energy to be self-sustaining after external energy input is cut off

inertial confinement
: a technique that aims multiple lasers at tiny fuel pellets evaporating and crushing them to high density

magnetic confinement
: a technique in which charged particles are trapped in a small region because of difficulty in crossing magnetic field lines

nuclear fusion
: a reaction in which two nuclei are combined, or fused, to form a larger nucleus

proton-proton cycle
: the combined reactions $^{1}H+^{1}H \rightarrow ^{2}H+e^{+}+v_{e}$, $^{1}H+^{2}H \rightarrow ^{3}He+\gamma$, and $^{3}He+^{3}He \rightarrow ^{4}He+^{1}H+^{1}H$

Fission

- Define nuclear fission.
- Discuss how fission fuel reacts and describe what it produces.
- Describe controlled and uncontrolled chain reactions.

Nuclear fission is a reaction in which a nucleus is split (or *fissured*). Controlled fission is a reality, whereas controlled fusion is a hope for the future. Hundreds of nuclear fission power plants around the world attest to the fact that controlled fission is practical and, at least in the short term, economical, as seen in [link]. Whereas nuclear power was of little interest for decades following TMI and Chernobyl (and now Fukushima Daiichi), growing concerns over global warming has brought nuclear power back on the table as a viable energy alternative. By the end of 2009, there were 442 reactors operating in 30 countries, providing 15% of the world's electricity. France provides over 75% of its electricity with nuclear power, while the US has 104 operating reactors providing 20% of its electricity. Australia and New Zealand have none. China is building nuclear power plants at the rate of one start every month.

The people living near this nuclear power plant have no measurable exposure to radiation that is traceable to the plant. About 16% of the world's electrical power is generated by controlled nuclear fission in such plants. The cooling towers are the most prominent features but are not unique to nuclear

power. The reactor is in the small domed building to the left of the towers. (credit: Kalmthouts)

Fission is the opposite of fusion and releases energy only when heavy nuclei are split. As noted in "Fusion", energy is released if the products of a nuclear reaction have a greater binding energy per nucleon (BE/A) than the parent nuclei. [link] shows that BE/A is greater for medium-mass nuclei than heavy nuclei, implying that when a heavy nucleus is split, the products have less mass per nucleon, so that mass is destroyed and energy is released in the reaction. The amount of energy per fission reaction can be large, even by nuclear standards. The graph in [link] shows BE/A to be about 7.6 MeV/nucleon for the heaviest nuclei (A about 240), while BE/A is about 8.6 MeV/nucleon for nuclei having A about 120. Thus, if a heavy nucleus splits in half, then about 1 MeV per nucleon, or approximately 240 MeV per fission, is released. This is about 10 times the energy per fusion reaction, and about 100 times the energy of the average α, β, or γ decay.

Example:
Calculating Energy Released by Fission
Calculate the energy released in the following spontaneous fission reaction:
Equation:

$$^{238}\mathrm{U} \rightarrow {}^{95}\mathrm{Sr} + {}^{140}\mathrm{Xe} + 3n$$

given the atomic masses to be $m(^{238}\mathrm{U}) = 238.050784\ \mathrm{u}$, $m(^{95}\mathrm{Sr}) = 94.919388\ \mathrm{u}$, $m(^{140}\mathrm{Xe}) = 139.921610\ \mathrm{u}$, and $m(n) = 1.008665\ \mathrm{u}$.

Strategy
As always, the energy released is equal to the mass destroyed times c^2 , so we must find the difference in mass between the parent $^{238}\mathrm{U}$ and the fission products.

Solution
The products have a total mass of
Equation:

$$\begin{aligned} m_{\text{products}} &= 94.919388\text{ u} + 139.921610\text{ u} + 3(1.008665\text{ u}) \\ &= 237.866993\text{ u}. \end{aligned}$$

The mass lost is the mass of ^{238}U minus m_{products}, or

Equation:

$$\Delta m = 238.050784\text{ u} - 237.8669933\text{ u} = 0.183791\text{ u},$$

so the energy released is

Equation:

$$\begin{aligned} E &= (\Delta m)c^2 \\ &= (0.183791\text{ u})\frac{931.5\text{ MeV}/c^2}{\text{u}}c^2 = 171.2\text{ MeV}. \end{aligned}$$

Discussion

A number of important things arise in this example. The 171-MeV energy released is large, but a little less than the earlier estimated 240 MeV. This is because this fission reaction produces neutrons and does not split the nucleus into two equal parts. Fission of a given nuclide, such as ^{238}U , does not always produce the same products. Fission is a statistical process in which an entire range of products are produced with various probabilities. Most fission produces neutrons, although the number varies with each fission. This is an extremely important aspect of fission, because *neutrons can induce more fission*, enabling self-sustaining chain reactions.

Spontaneous fission can occur, but this is usually not the most common decay mode for a given nuclide. For example, ^{238}U can spontaneously fission, but it decays mostly by α emission. Neutron-induced fission is crucial as seen in [link]. Being chargeless, even low-energy neutrons can strike a nucleus and be absorbed once they feel the attractive nuclear force. Large nuclei are described by a **liquid drop model** with surface tension and oscillation modes, because the large number of nucleons act like atoms in a drop. The neutron is attracted and thus, deposits energy, causing the nucleus to deform as a liquid drop. If stretched enough, the nucleus narrows in the middle. The number of nucleons in contact and the strength of the nuclear force binding the nucleus together are reduced. Coulomb repulsion between the two ends then succeeds in fissioning the nucleus, which pops like a water drop into two large pieces and a few neutrons. **Neutron-induced fission** can be written as

Equation:

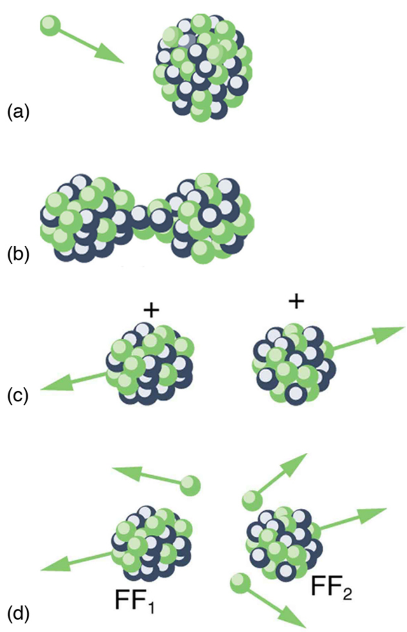
$$n + \ {}^{A}\mathrm{X} \rightarrow \mathrm{FF}_1 + \mathrm{FF}_2 + \mathrm{xn},$$

where FF_1 and FF_2 are the two daughter nuclei, called **fission fragments**, and x is the number of neutrons produced. Most often, the masses of the fission fragments are not the same. Most of the released energy goes into the kinetic energy of the fission fragments, with the remainder going into the neutrons and excited states of the fragments. Since neutrons can induce fission, a self-sustaining chain reaction is possible, provided more than one neutron is produced on average — that is, if $x > 1$ in $n + {}^{A}\mathrm{X} \rightarrow \mathrm{FF}_1 + \mathrm{FF}_2 + \mathrm{xn}$. This can also be seen in [link].

An example of a typical neutron-induced fission reaction is
Equation:

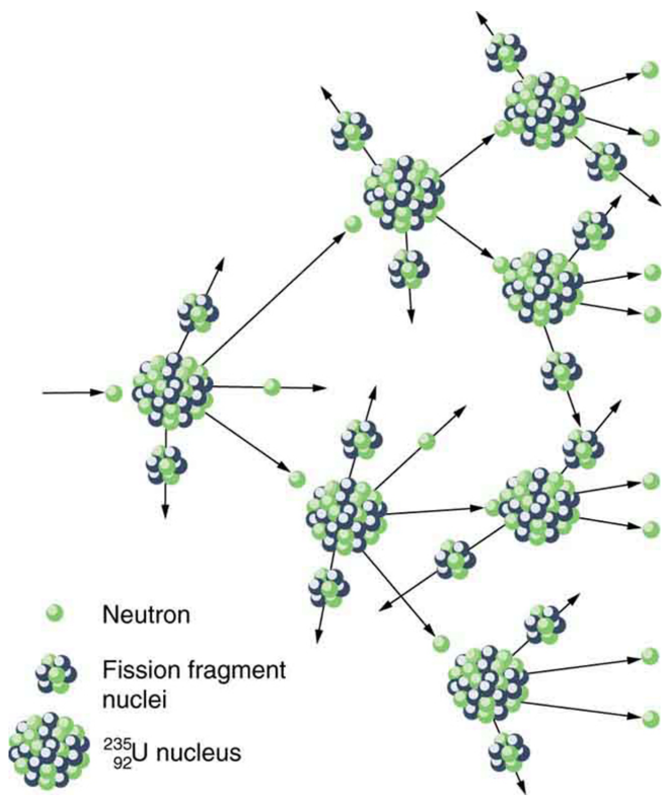
$$n + {}^{235}_{92}\mathrm{U} \rightarrow {}^{142}_{56}\mathrm{Ba} + {}^{91}_{36}\mathrm{Kr} + 3n.$$

Note that in this equation, the total charge remains the same (is conserved): $92 + 0 = 56 + 36$. Also, as far as whole numbers are concerned, the mass is constant: $1 + 235 = 142 + 91 + 3$. This is not true when we consider the masses out to 6 or 7 significant places, as in the previous example.



Neutron-induced

fission is shown. First, energy is put into this large nucleus when it absorbs a neutron. Acting like a struck liquid drop, the nucleus deforms and begins to narrow in the middle. Since fewer nucleons are in contact, the repulsive Coulomb force is able to break the nucleus into two parts with some neutrons also flying away.



A chain reaction can produce self-sustained fission if each fission

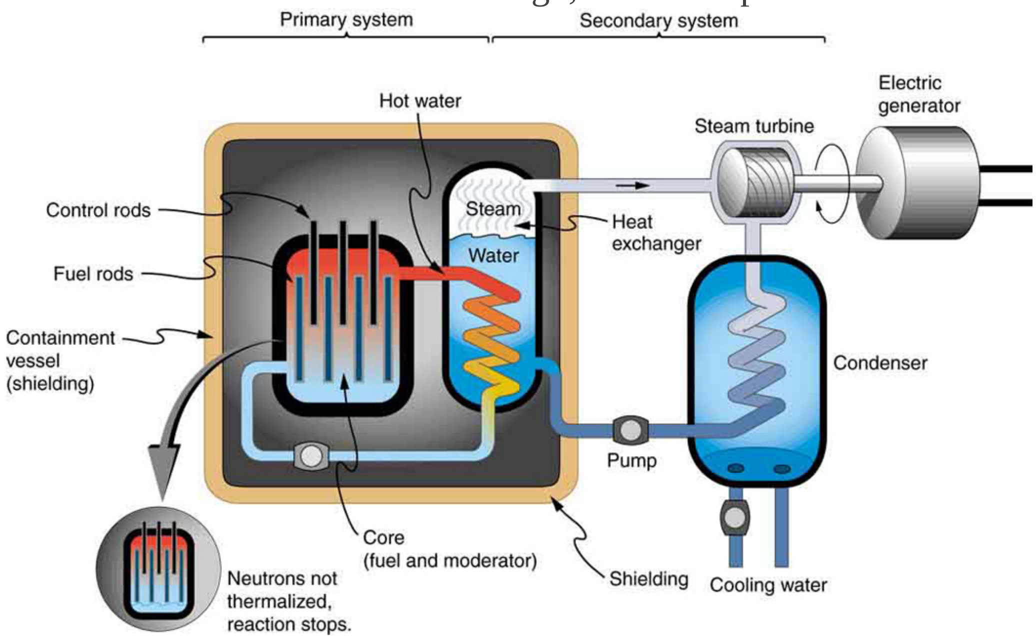
produces enough neutrons to induce at least one more fission. This depends on several factors, including how many neutrons are produced in an average fission and how easy it is to make a particular type of nuclide fission.

Not every neutron produced by fission induces fission. Some neutrons escape the fissionable material, while others interact with a nucleus without making it fission. We can enhance the number of fissions produced by neutrons by having a large amount of fissionable material. The minimum amount necessary for self-sustained fission of a given nuclide is called its **critical mass**. Some nuclides, such as $^{239}\mathrm{Pu}$, produce more neutrons per fission than others, such as $^{235}\mathrm{U}$. Additionally, some nuclides are easier to make fission than others. In particular, $^{235}\mathrm{U}$ and $^{239}\mathrm{Pu}$ are easier to fission than the much more abundant $^{238}\mathrm{U}$. Both factors affect critical mass, which is smallest for $^{239}\mathrm{Pu}$.

The reason $^{235}\mathrm{U}$ and $^{239}\mathrm{Pu}$ are easier to fission than $^{238}\mathrm{U}$ is that the nuclear force is more attractive for an even number of neutrons in a nucleus than for an odd number. Consider that $^{235}_{92}\mathrm{U}_{143}$ has 143 neutrons, and $^{239}_{94}\mathrm{P}_{145}$ has 145 neutrons, whereas $^{238}_{92}\mathrm{U}_{146}$ has 146. When a neutron encounters a nucleus with an odd number of neutrons, the nuclear force is more attractive, because the additional neutron will make the number even. About 2-MeV more energy is deposited in the resulting nucleus than would be the case if the number of neutrons was already even. This extra energy produces greater deformation, making fission more likely. Thus, $^{235}\mathrm{U}$ and $^{239}\mathrm{Pu}$ are superior fission fuels. The isotope $^{235}\mathrm{U}$ is only 0.72 % of natural uranium, while $^{238}\mathrm{U}$ is 99.27%, and $^{239}\mathrm{Pu}$ does not exist in nature. Australia has the largest deposits of uranium in the world, standing at 28% of the total. This is followed by Kazakhstan and Canada. The US has only 3% of global reserves.

Most fission reactors utilize $^{235}\mathrm{U}$, which is separated from $^{238}\mathrm{U}$ at some expense. This is called enrichment. The most common separation method is gaseous diffusion of uranium hexafluoride ($\mathrm{UF_6}$) through membranes. Since $^{235}\mathrm{U}$ has less mass than $^{238}\mathrm{U}$, its $\mathrm{UF_6}$ molecules have higher average velocity at the same temperature and diffuse faster. Another interesting characteristic of $^{235}\mathrm{U}$ is that it preferentially absorbs very slow moving neutrons (with energies a

fraction of an eV), whereas fission reactions produce fast neutrons with energies in the order of an MeV. To make a self-sustained fission reactor with ^{235}U , it is thus necessary to slow down ("thermalize") the neutrons. Water is very effective, since neutrons collide with protons in water molecules and lose energy. [link] shows a schematic of a reactor design, called the pressurized water reactor.



A pressurized water reactor is cleverly designed to control the fission of large amounts of ^{235}U , while using the heat produced in the fission reaction to create steam for generating electrical energy. Control rods adjust neutron flux so that criticality is obtained, but not exceeded. In case the reactor overheats and boils the water away, the chain reaction terminates, because water is needed to thermalize the neutrons. This inherent safety feature can be overwhelmed in extreme circumstances.

Control rods containing nuclides that very strongly absorb neutrons are used to adjust neutron flux. To produce large power, reactors contain hundreds to thousands of critical masses, and the chain reaction easily becomes self-sustaining, a condition called **criticality**. Neutron flux should be carefully regulated to avoid an exponential increase in fissions, a condition called **supercriticality**. Control rods help prevent overheating, perhaps even a meltdown or explosive disassembly. The water that is used to thermalize

neutrons, necessary to get them to induce fission in ^{235}U , and achieve criticality, provides a negative feedback for temperature increases. In case the reactor overheats and boils the water to steam or is breached, the absence of water kills the chain reaction. Considerable heat, however, can still be generated by the reactor's radioactive fission products. Other safety features, thus, need to be incorporated in the event of a *loss of coolant* accident, including auxiliary cooling water and pumps.

Example:
Calculating Energy from a Kilogram of Fissionable Fuel

Calculate the amount of energy produced by the fission of 1.00 kg of ^{235}U , given the average fission reaction of ^{235}U produces 200 MeV.

Strategy

The total energy produced is the number of ^{235}U atoms times the given energy per ^{235}U fission. We should therefore find the number of ^{235}U atoms in 1.00 kg.

Solution

The number of ^{235}U atoms in 1.00 kg is Avogadro's number times the number of moles. One mole of ^{235}U has a mass of 235.04 g; thus, there are $(1000\text{ g})/(235.04\text{ g/mol}) = 4.25\text{ mol}$. The number of ^{235}U atoms is therefore,

Equation:

$$(4.25\text{ mol})\left(6.02\times10^{23}\ {}^{235}\text{U/mol}\right) = 2.56\times10^{24}\ {}^{235}\text{U}.$$

So the total energy released is

Equation:

$$\begin{aligned} E &= \left(2.56\times10^{24}\ {}^{235}\text{U}\right)\left(\frac{200\text{ MeV}}{{}^{235}\text{U}}\right)\left(\frac{1.60\times10^{-13}\text{ J}}{\text{MeV}}\right) \\ &= 8.21\times10^{13}\text{ J}. \end{aligned}$$

Discussion

This is another impressively large amount of energy, equivalent to about 14,000 barrels of crude oil or 600,000 gallons of gasoline. But, it is only one-fourth the energy produced by the fusion of a kilogram mixture of deuterium and tritium as seen in the previous section. Even though each fission reaction yields about ten times the energy of a fusion reaction, the energy per kilogram of fission fuel is less, because there are far fewer moles per kilogram of the heavy nuclides.

Fission fuel is also much more scarce than fusion fuel, and less than 1% of uranium (the $^{235}\mathrm{U}$) is readily usable.

One nuclide already mentioned is $^{239}\mathrm{Pu}$, which has a 24,120-y half-life and does not exist in nature. Plutonium-239 is manufactured from $^{238}\mathrm{U}$ in reactors, and it provides an opportunity to utilize the other 99% of natural uranium as an energy source. The following reaction sequence, called **breeding**, produces $^{239}\mathrm{Pu}$. Breeding begins with neutron capture by $^{238}\mathrm{U}$:

Equation:

$$^{238}\mathrm{U}+n\rightarrow {}^{239}\mathrm{U}+\gamma.$$

Uranium-239 then β^{-} decays:

Equation:

$$^{239}\mathrm{U}\rightarrow {}^{239}\mathrm{Np}+\beta^{-}+v_{e}(\mathrm{t}_{1/2}=23\ \mathrm{min}).$$

Neptunium-239 also β^{-} decays:

Equation:

$$^{239}\mathrm{Np}\rightarrow {}^{239}\mathrm{Pu}+\beta^{-}+v_{e}(\mathrm{t}_{1/2}=2.4\ \mathrm{d}).$$

Plutonium-239 builds up in reactor fuel at a rate that depends on the probability of neutron capture by $^{238}\mathrm{U}$ (all reactor fuel contains more $^{238}\mathrm{U}$ than $^{235}\mathrm{U}$). Reactors designed specifically to make plutonium are called **breeder reactors**. They seem to be inherently more hazardous than conventional reactors, but it remains unknown whether their hazards can be made economically acceptable. The four reactors at Chernobyl, including the one that was destroyed, were built to breed plutonium and produce electricity. These reactors had a design that was significantly different from the pressurized water reactor illustrated above.

Plutonium-239 has advantages over $^{235}\mathrm{U}$ as a reactor fuel — it produces more neutrons per fission on average, and it is easier for a thermal neutron to cause it to fission. It is also chemically different from uranium, so it is inherently easier to separate from uranium ore. This means $^{239}\mathrm{Pu}$ has a particularly small critical mass, an advantage for nuclear weapons.

Section Summary

- Nuclear fission is a reaction in which a nucleus is split.
- Fission releases energy when heavy nuclei are split into medium-mass nuclei.
- Self-sustained fission is possible, because neutron-induced fission also produces neutrons that can induce other fissions, $n + {}^{A}X \rightarrow \mathrm{FF}_1 + \mathrm{FF}_2 + \mathrm{xn}$, where FF_1 and FF_2 are the two daughter nuclei, or fission fragments, and x is the number of neutrons produced.
- A minimum mass, called the critical mass, should be present to achieve criticality.
- More than a critical mass can produce supercriticality.
- The production of new or different isotopes (especially ${}^{239}\mathrm{Pu}$) by nuclear transformation is called breeding, and reactors designed for this purpose are called breeder reactors.

Conceptual Questions

Exercise:

Problem:

Explain why the fission of heavy nuclei releases energy. Similarly, why is it that energy input is required to fission light nuclei?

Exercise:

Problem:

Explain, in terms of conservation of momentum and energy, why collisions of neutrons with protons will thermalize neutrons better than collisions with oxygen.

Exercise:

Problem:

The ruins of the Chernobyl reactor are enclosed in a huge concrete structure built around it after the accident. Some rain penetrates the building in winter, and radioactivity from the building increases. What does this imply is happening inside?

Exercise:

Problem:

Since the uranium or plutonium nucleus fissions into several fission fragments whose mass distribution covers a wide range of pieces, would you expect more residual radioactivity from fission than fusion? Explain.

Exercise:

Problem:

The core of a nuclear reactor generates a large amount of thermal energy from the decay of fission products, even when the power-producing fission chain reaction is turned off. Would this residual heat be greatest after the reactor has run for a long time or short time? What if the reactor has been shut down for months?

Exercise:

Problem:

How can a nuclear reactor contain many critical masses and not go supercritical? What methods are used to control the fission in the reactor?

Exercise:

Problem:

Why can heavy nuclei with odd numbers of neutrons be induced to fission with thermal neutrons, whereas those with even numbers of neutrons require more energy input to induce fission?

Exercise:

Problem:

Why is a conventional fission nuclear reactor not able to explode as a bomb?

Problem Exercises

Exercise:

Problem:

(a) Calculate the energy released in the neutron-induced fission (similar to the spontaneous fission in [link])
Equation:

$$n + {}^{238}\text{U} \rightarrow {}^{96}\text{Sr} + {}^{140}\text{Xe} + 3n,$$

given $m({}^{96}\text{Sr}) = 95.921750\text{ u}$ and $m({}^{140}\text{Xe}) = 139.92164$. (b) This result is about 6 MeV greater than the result for spontaneous fission. Why? (c) Confirm that the total number of nucleons and total charge are conserved in this reaction.

Solution:

(a) 177.1 MeV

(b) Because the gain of an external neutron yields about 6 MeV, which is the average BE/A for heavy nuclei.

(c)
$A = 1 + 238 = 96 + 140 + 1 + 1 + 1,\ Z = 92 = 38 + 53,\ \text{efn} = 0 = 0$

Exercise:

Problem:

(a) Calculate the energy released in the neutron-induced fission reaction
Equation:

$$n + {}^{235}\text{U} \rightarrow {}^{92}\text{Kr} + {}^{142}\text{Ba} + 2n,$$

given $m({}^{92}\text{Kr}) = 91.926269\text{ u}$ and $m({}^{142}\text{Ba}) = 141.916361\text{ u}$.

(b) Confirm that the total number of nucleons and total charge are conserved in this reaction.

Exercise:

Problem:

(a) Calculate the energy released in the neutron-induced fission reaction
Equation:

$$n + {}^{239}\text{Pu} \rightarrow {}^{96}\text{Sr} + {}^{140}\text{Ba} + 4n,$$

given $m({}^{96}\text{Sr}) = 95.921750 \text{ u}$ and $m({}^{140}\text{Ba}) = 139.910581 \text{ u}.$

(b) Confirm that the total number of nucleons and total charge are conserved in this reaction.

Solution:

(a) 180.6 MeV

(b)
$A = 1 + 239 = 96 + 140 + 1 + 1 + 1 + 1,\ Z = 94 = 38 + 56,\ \text{efn} = 0 = 0$

Exercise:

Problem:

Confirm that each of the reactions listed for plutonium breeding just following [link] conserves the total number of nucleons, the total charge, and electron family number.

Exercise:

Problem:

Breeding plutonium produces energy even before any plutonium is fissioned. (The primary purpose of the four nuclear reactors at Chernobyl was breeding plutonium for weapons. Electrical power was a by-product used by the civilian population.) Calculate the energy produced in each of the reactions listed for plutonium breeding just following [link]. The pertinent masses are $m({}^{239}\text{U}) = 239.054289 \text{ u}$, $m({}^{239}\text{Np}) = 239.052932 \text{ u}$, and $m({}^{239}\text{Pu}) = 239.052157 \text{ u}$.

Solution:

${}^{238}\text{U} + n \ \rightarrow {}^{239}\text{U} + \gamma\ 4.81 \text{ MeV}$

$^{239}\text{U} \rightarrow {}^{239}\text{Np} + \beta^{-} + v_e$ 0.753 MeV

$^{239}\text{Np} \rightarrow {}^{239}\text{Pu} + \beta^{-} + v_e$ 0.211 MeV

Exercise:

Problem:

The naturally occurring radioactive isotope ^{232}Th does not make good fission fuel, because it has an even number of neutrons; however, it can be bred into a suitable fuel (much as ^{238}U is bred into ^{239}P).

(a) What are Z and N for ^{232}Th?

(b) Write the reaction equation for neutron captured by ^{232}Th and identify the nuclide ^{A}X produced in $n + {}^{232}\text{Th} \rightarrow {}^{A}X + \gamma$.

(c) The product nucleus β^{-} decays, as does its daughter. Write the decay equations for each, and identify the final nucleus.

(d) Confirm that the final nucleus has an odd number of neutrons, making it a better fission fuel.

(e) Look up the half-life of the final nucleus to see if it lives long enough to be a useful fuel.

Exercise:

Problem:

The electrical power output of a large nuclear reactor facility is 900 MW. It has a 35.0% efficiency in converting nuclear power to electrical.

(a) What is the thermal nuclear power output in megawatts?

(b) How many ^{235}U nuclei fission each second, assuming the average fission produces 200 MeV?

(c) What mass of ^{235}U is fissioned in one year of full-power operation?

Solution:

(a) 2.57×10^{3} MW

(b) 8.03×10^{19} fission/s

(c) 991 kg

Exercise:

Problem:

A large power reactor that has been in operation for some months is turned off, but residual activity in the core still produces 150 MW of power. If the average energy per decay of the fission products is 1.00 MeV, what is the core activity in curies?

Glossary

breeder reactors
: reactors that are designed specifically to make plutonium

breeding
: reaction process that produces ^{239}Pu

criticality
: condition in which a chain reaction easily becomes self-sustaining

critical mass
: minimum amount necessary for self-sustained fission of a given nuclide

fission fragments
: a daughter nuclei

liquid drop model
: a model of nucleus (only to understand some of its features) in which nucleons in a nucleus act like atoms in a drop

nuclear fission
: reaction in which a nucleus splits

neutron-induced fission
: fission that is initiated after the absorption of neutron

supercriticality

an exponential increase in fissions

Nuclear Weapons

- Discuss different types of fission and thermonuclear bombs.
- Explain the ill effects of nuclear explosion.

The world was in turmoil when fission was discovered in 1938. The discovery of fission, made by two German physicists, Otto Hahn and Fritz Strassman, was quickly verified by two Jewish refugees from Nazi Germany, Lise Meitner and her nephew Otto Frisch. Fermi, among others, soon found that not only did neutrons induce fission; more neutrons were produced during fission. The possibility of a self-sustained chain reaction was immediately recognized by leading scientists the world over. The enormous energy known to be in nuclei, but considered inaccessible, now seemed to be available on a large scale.

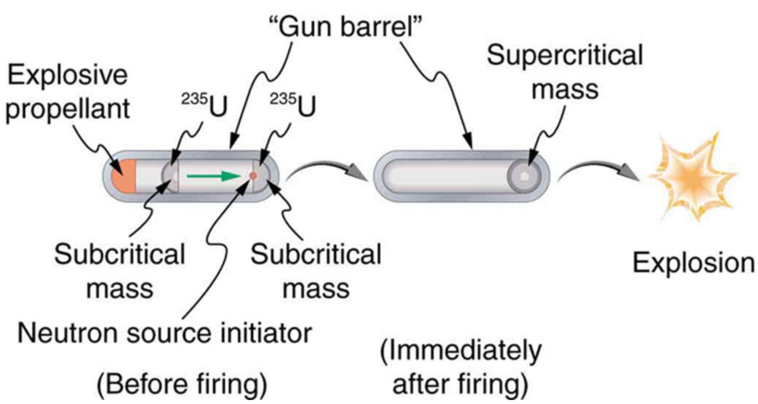
Within months after the announcement of the discovery of fission, Adolf Hitler banned the export of uranium from newly occupied Czechoslovakia. It seemed that the military value of uranium had been recognized in Nazi Germany, and that a serious effort to build a nuclear bomb had begun.

Alarmed scientists, many of them who fled Nazi Germany, decided to take action. None was more famous or revered than Einstein. It was felt that his help was needed to get the American government to make a serious effort at nuclear weapons as a matter of survival. Leo Szilard, an escaped Hungarian physicist, took a draft of a letter to Einstein, who, although pacifistic, signed the final version. The letter was for President Franklin Roosevelt, warning of the German potential to build extremely powerful bombs of a new type. It was sent in August of 1939, just before the German invasion of Poland that marked the start of World War II.

It was not until December 6, 1941, the day before the Japanese attack on Pearl Harbor, that the United States made a massive commitment to building a nuclear bomb. The top secret Manhattan Project was a crash program aimed at beating the Germans. It was carried out in remote locations, such as Los Alamos, New Mexico, whenever possible, and eventually came to cost billions of dollars and employ the efforts of more than 100,000 people. J. Robert Oppenheimer (1904–1967), whose talent and ambitions made him ideal, was chosen to head the project. The first

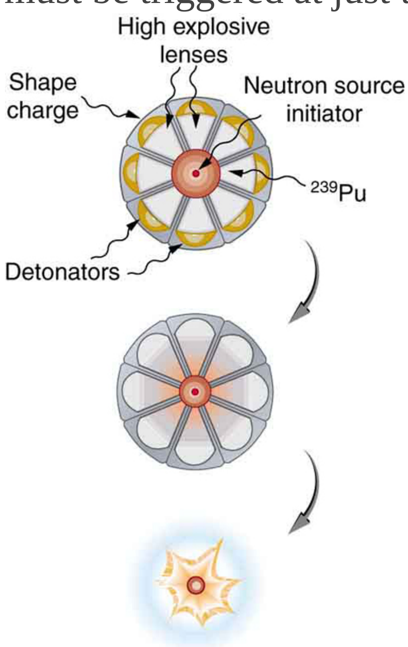
major step was made by Enrico Fermi and his group in December 1942, when they achieved the first self-sustained nuclear reactor. This first "atomic pile", built in a squash court at the University of Chicago, used carbon blocks to thermalize neutrons. It not only proved that the chain reaction was possible, it began the era of nuclear reactors. Glenn Seaborg, an American chemist and physicist, received the Nobel Prize in physics in 1951 for discovery of several transuranic elements, including plutonium. Carbon-moderated reactors are relatively inexpensive and simple in design and are still used for breeding plutonium, such as at Chernobyl, where two such reactors remain in operation.

Plutonium was recognized as easier to fission with neutrons and, hence, a superior fission material very early in the Manhattan Project. Plutonium availability was uncertain, and so a uranium bomb was developed simultaneously. [link] shows a gun-type bomb, which takes two subcritical uranium masses and blows them together. To get an appreciable yield, the critical mass must be held together by the explosive charges inside the cannon barrel for a few microseconds. Since the buildup of the uranium chain reaction is relatively slow, the device to hold the critical mass together can be relatively simple. Owing to the fact that the rate of spontaneous fission is low, a neutron source is triggered at the same time the critical mass is assembled.



A gun-type fission bomb for ^{235}U utilizes two subcritical masses forced together by explosive charges inside a cannon barrel. The energy yield depends on the amount of uranium and the time it can be held together before it disassembles itself.

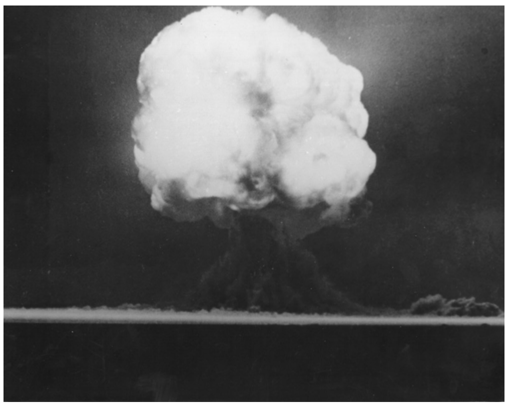
Plutonium’s special properties necessitated a more sophisticated critical mass assembly, shown schematically in [link]. A spherical mass of plutonium is surrounded by shape charges (high explosives that release most of their blast in one direction) that implode the plutonium, crushing it into a smaller volume to form a critical mass. The implosion technique is faster and more effective, because it compresses three-dimensionally rather than one-dimensionally as in the gun-type bomb. Again, a neutron source must be triggered at just the correct time to initiate the chain reaction.



An implosion created by high explosives compresses a sphere of ^{239}Pu into a critical mass. The superior fissionability of plutonium has made it the

universal bomb
material.

Owing to its complexity, the plutonium bomb needed to be tested before there could be any attempt to use it. On July 16, 1945, the test named Trinity was conducted in the isolated Alamogordo Desert about 200 miles south of Los Alamos (see [link]). A new age had begun. The yield of this device was about 10 kilotons (kT), the equivalent of 5000 of the largest conventional bombs.



Trinity test (1945), the first nuclear bomb (credit: United States Department of Energy)

Although Germany surrendered on May 7, 1945, Japan had been steadfastly refusing to surrender for many months, forcing large casualties. Invasion plans by the Allies estimated a million casualties of their own and untold losses of Japanese lives. The bomb was viewed as a way to end the war. The first was a uranium bomb dropped on Hiroshima on August 6. Its yield of about 15 kT destroyed the city and killed an estimated 80,000 people, with 100,000 more being seriously injured (see [link]). The second was a plutonium bomb dropped on Nagasaki only three days later, on August 9. Its 20 kT yield killed at least 50,000 people, something less than Hiroshima because of the hilly terrain and the fact that it was a few kilometers off target. The Japanese were told that one bomb a week would be dropped

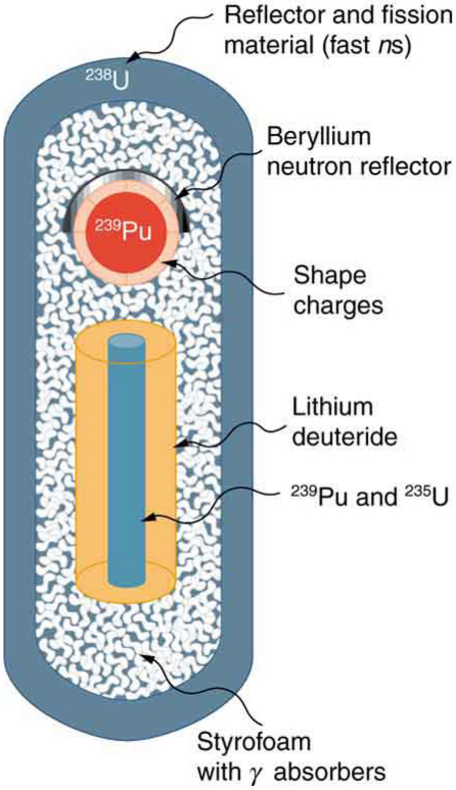
until they surrendered unconditionally, which they did on August 14. In actuality, the United States had only enough plutonium for one more and as yet unassembled bomb.

Destruction in Hiroshima (credit: United States Federal Government)

Knowing that fusion produces several times more energy per kilogram of fuel than fission, some scientists pushed the idea of a fusion bomb starting very early on. Calling this bomb the Super, they realized that it could have another advantage over fission—high-energy neutrons would aid fusion, while they are ineffective in $^{239}\mathrm{Pu}$ fission. Thus the fusion bomb could be virtually unlimited in energy release. The first such bomb was detonated by the United States on October 31, 1952, at Eniwetok Atoll with a yield of 10 megatons (MT), about 670 times that of the fission bomb that destroyed Hiroshima. The Soviets followed with a fusion device of their own in August 1953, and a weapons race, beyond the aim of this text to discuss, continued until the end of the Cold War.

[link] shows a simple diagram of how a thermonuclear bomb is constructed. A fission bomb is exploded next to fusion fuel in the solid form of lithium deuteride. Before the shock wave blows it apart, γ rays heat and compress the fuel, and neutrons create tritium through the reaction $n+^{6}\mathrm{Li}\rightarrow^{3}\mathrm{H}+^{4}\mathrm{He}$. Additional fusion and fission fuels are enclosed in a dense shell of $^{238}\mathrm{U}$. The shell reflects some of the neutrons back into the fuel to enhance its fusion, but at high internal temperatures fast neutrons are

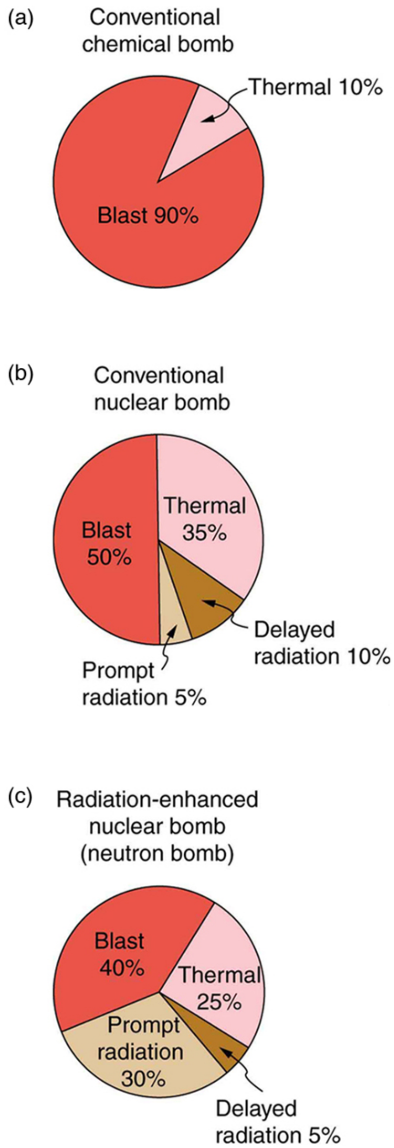
created that also cause the plentiful and inexpensive ^{238}U to fission, part of what allows thermonuclear bombs to be so large.



This schematic of a fusion bomb (H-bomb) gives some idea of how the ^{239}Pu fission trigger is used to ignite fusion fuel. Neutrons and γ rays transmit energy to the fusion fuel, create tritium from deuterium, and heat and compress the fusion fuel. The outer shell of ^{238}U serves to reflect some neutrons back into the fuel, causing more fusion,

and it boosts the
energy output by
fissioning itself when
neutron energies
become high enough.

The energy yield and the types of energy produced by nuclear bombs can be varied. Energy yields in current arsenals range from about 0.1 kT to 20 MT, although the Soviets once detonated a 67 MT device. Nuclear bombs differ from conventional explosives in more than size. [link] shows the approximate fraction of energy output in various forms for conventional explosives and for two types of nuclear bombs. Nuclear bombs put a much larger fraction of their output into thermal energy than do conventional bombs, which tend to concentrate the energy in blast. Another difference is the immediate and residual radiation energy from nuclear weapons. This can be adjusted to put more energy into radiation (the so-called neutron bomb) so that the bomb can be used to irradiate advancing troops without killing friendly troops with blast and heat.



Approximate fractions of energy output by conventional and two types of nuclear weapons. In addition to yielding more energy than conventional weapons, nuclear

bombs put a much larger fraction into thermal energy. This can be adjusted to enhance the radiation output to be more effective against troops. An enhanced radiation bomb is also called a neutron bomb.

At its peak in 1986, the combined arsenals of the United States and the Soviet Union totaled about 60,000 nuclear warheads. In addition, the British, French, and Chinese each have several hundred bombs of various sizes, and a few other countries have a small number. Nuclear weapons are generally divided into two categories. Strategic nuclear weapons are those intended for military targets, such as bases and missile complexes, and moderate to large cities. There were about 20,000 strategic weapons in 1988. Tactical weapons are intended for use in smaller battles. Since the collapse of the Soviet Union and the end of the Cold War in 1989, most of the 32,000 tactical weapons (including Cruise missiles, artillery shells, land mines, torpedoes, depth charges, and backpacks) have been demobilized, and parts of the strategic weapon systems are being dismantled with warheads and missiles being disassembled. According to the Treaty of Moscow of 2002, Russia and the United States have been required to reduce their strategic nuclear arsenal down to about 2000 warheads each.

A few small countries have built or are capable of building nuclear bombs, as are some terrorist groups. Two things are needed—a minimum level of technical expertise and sufficient fissionable material. The first is easy. Fissionable material is controlled but is also available. There are international agreements and organizations that attempt to control nuclear proliferation, but it is increasingly difficult given the availability of

fissionable material and the small amount needed for a crude bomb. The production of fissionable fuel itself is technologically difficult. However, the presence of large amounts of such material worldwide, though in the hands of a few, makes control and accountability crucial.

Section Summary

- There are two types of nuclear weapons—fission bombs use fission alone, whereas thermonuclear bombs use fission to ignite fusion.
- Both types of weapons produce huge numbers of nuclear reactions in a very short time.
- Energy yields are measured in kilotons or megatons of equivalent conventional explosives and range from 0.1 kT to more than 20 MT.
- Nuclear bombs are characterized by far more thermal output and nuclear radiation output than conventional explosives.

Conceptual Questions

Exercise:

Problem:

What are some of the reasons that plutonium rather than uranium is used in all fission bombs and as the trigger in all fusion bombs?

Exercise:

Problem:

Use the laws of conservation of momentum and energy to explain how a shape charge can direct most of the energy released in an explosion in a specific direction. (Note that this is similar to the situation in guns and cannons—most of the energy goes into the bullet.)

Exercise:

Problem:

How does the lithium deuteride in the thermonuclear bomb shown in [link] supply tritium $\left(^{3}\text{H}\right)$ as well as deuterium $\left(^{2}\text{H}\right)$?

Exercise:

Problem:

Fallout from nuclear weapons tests in the atmosphere is mainly ^{90}Sr and ^{137}Cs, which have 28.6- and 32.2-y half-lives, respectively. Atmospheric tests were terminated in most countries in 1963, although China only did so in 1980. It has been found that environmental activities of these two isotopes are decreasing faster than their half-lives. Why might this be?

Problems & Exercises

Exercise:

Problem: Find the mass converted into energy by a 12.0-kT bomb.

Solution:

0.56 g

Exercise:

Problem: What mass is converted into energy by a 1.00-MT bomb?

Exercise:

Problem:

Fusion bombs use neutrons from their fission trigger to create tritium fuel in the reaction $n + ^6\text{Li} \rightarrow ^3\text{H} + ^4\text{He}$. What is the energy released by this reaction in MeV?

Solution:

4.781 MeV

Exercise:

Problem:

It is estimated that the total explosive yield of all the nuclear bombs in existence currently is about 4,000 MT.

(a) Convert this amount of energy to kilowatt-hours, noting that $1\ \mathrm{kW} \cdot \mathrm{h} = 3.60 \times 10^{6}\ \mathrm{J}$.

(b) What would the monetary value of this energy be if it could be converted to electricity costing 10 cents per kW·h?

Exercise:

Problem:

A radiation-enhanced nuclear weapon (or neutron bomb) can have a smaller total yield and still produce more prompt radiation than a conventional nuclear bomb. This allows the use of neutron bombs to kill nearby advancing enemy forces with radiation without blowing up your own forces with the blast. For a 0.500-kT radiation-enhanced weapon and a 1.00-kT conventional nuclear bomb: (a) Compare the blast yields. (b) Compare the prompt radiation yields.

Solution:

(a) Blast yields $2.1 \times 10^{12}\ \mathrm{J}$ to $8.4 \times 10^{11}\ \mathrm{J}$, or 2.5 to 1, conventional to radiation enhanced.

(b) Prompt radiation yields $6.3 \times 10^{11}\ \mathrm{J}$ to $2.1 \times 10^{11}\ \mathrm{J}$, or 3 to 1, radiation enhanced to conventional.

Exercise:

Problem:

(a) How many $^{239}\mathrm{Pu}$ nuclei must fission to produce a 20.0-kT yield, assuming 200 MeV per fission? (b) What is the mass of this much $^{239}\mathrm{Pu}$?

Exercise:

Problem:

Assume one-fourth of the yield of a typical 320-kT strategic bomb comes from fission reactions averaging 200 MeV and the remainder from fusion reactions averaging 20 MeV.

(a) Calculate the number of fissions and the approximate mass of uranium and plutonium fissioned, taking the average atomic mass to be 238.

(b) Find the number of fusions and calculate the approximate mass of fusion fuel, assuming an average total atomic mass of the two nuclei in each reaction to be 5.

(c) Considering the masses found, does it seem reasonable that some missiles could carry 10 warheads? Discuss, noting that the nuclear fuel is only a part of the mass of a warhead.

Solution:

(a) 1.1×10^{25} fissions , 4.4 kg

(b) 3.2×10^{26} fusions , 2.7 kg

(c) The nuclear fuel totals only 6 kg, so it is quite reasonable that some missiles carry 10 overheads. The mass of the fuel would only be 60 kg and therefore the mass of the 10 warheads, weighing about 10 times the nuclear fuel, would be only 1500 lbs. If the fuel for the missiles weighs 5 times the total weight of the warheads, the missile would weigh about 9000 lbs or 4.5 tons. This is not an unreasonable weight for a missile.

Exercise:

Problem:

This problem gives some idea of the magnitude of the energy yield of a small tactical bomb. Assume that half the energy of a 1.00-kT nuclear depth charge set off under an aircraft carrier goes into lifting it out of the water—that is, into gravitational potential energy. How high is the carrier lifted if its mass is 90,000 tons?

Exercise:

Problem:

It is estimated that weapons tests in the atmosphere have deposited approximately 9 MCi of ^{90}Sr on the surface of the earth. Find the mass of this amount of ^{90}Sr.

Solution:

7×10^4 g

Exercise:

Problem:

A 1.00-MT bomb exploded a few kilometers above the ground deposits 25.0% of its energy into radiant heat.

(a) Find the calories per cm^2 at a distance of 10.0 km by assuming a uniform distribution over a spherical surface of that radius.

(b) If this heat falls on a person's body, what temperature increase does it cause in the affected tissue, assuming it is absorbed in a layer 1.00-cm deep?

Exercise:

Problem: **Integrated Concepts**

One scheme to put nuclear weapons to nonmilitary use is to explode them underground in a geologically stable region and extract the

geothermal energy for electricity production. There was a total yield of about 4,000 MT in the combined arsenals in 2006. If 1.00 MT per day could be converted to electricity with an efficiency of 10.0%:

(a) What would the average electrical power output be?

(b) How many years would the arsenal last at this rate?

Solution:

(a) 4.86×10^{9} W

(b) 11.0 y

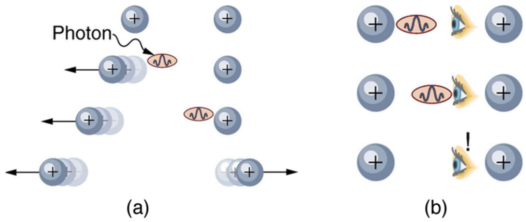
The Four Basic Forces

- State the four basic forces.
- Explain the Feynman diagram for the exchange of a virtual photon between two positive charges.
- Define QED.
- Describe the Feynman diagram for the exchange of a between a proton and a neutron.

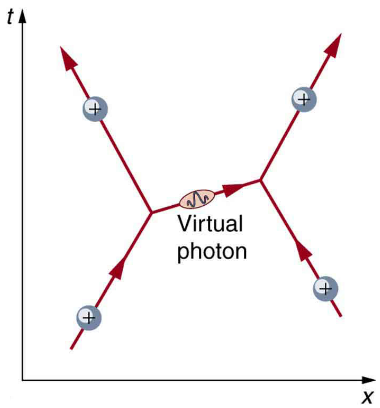
As first discussed in Problem-Solving Strategies and mentioned at various points in the text since then, there are only four distinct basic forces in all of nature. This is a remarkably small number considering the myriad phenomena they explain. Particle physics is intimately tied to these four forces. Certain fundamental particles, called carrier particles, carry these forces, and all particles can be classified according to which of the four forces they feel. The table given below summarizes important characteristics of the four basic forces.

Force	**Approximate relative strength**	**Range**	**+/− [footnote] + attractive; - repulsive; $+/-$ both.**	**Carrier particle**
Gravity	10^{-38}	∞	+ only	Graviton (conjectured)
Electromagnetic	10^{-2}	∞	$+/-$	Photon (observed)
Weak force	10^{-13}	$< 10^{-18}$ m	$+/-$	W^{+}, W^{-}, Z^{0} (observed[footnote]) Predicted by theory and first observed in 1983.
Strong force	1	$< 10^{-15}$ m	$+/-$	Gluons (conjectured[footnote]) Eight proposed—indirect evidence of existence. Underlie meson exchange.

Properties of the Four Basic Forces



The first image shows the exchange of a virtual photon transmitting the electromagnetic force between charges, just as virtual pion exchange carries the strong nuclear force between nucleons. The second image shows that the photon cannot be directly observed in its passage, because this would disrupt it and alter the force. In this case it does not get to the other charge.



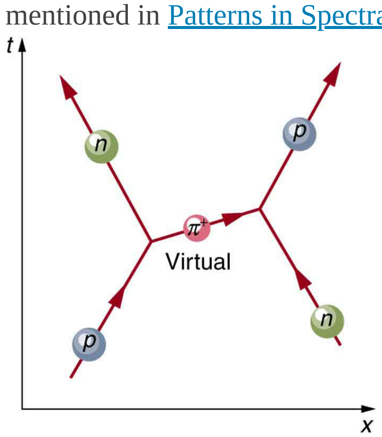
The Feynman diagram for the exchange of a virtual photon between two positive charges illustrates how the electromagnetic force is transmitted on a quantum mechanical scale. Time is graphed vertically while the distance is graphed horizontally. The two positive charges are seen to be repelled by the photon exchange.

Although these four forces are distinct and differ greatly from one another under all but the most extreme circumstances, we can see similarities among them. (In GUTs: the Unification of Forces, we will discuss how the four forces may be different manifestations of a single unified force.) Perhaps the most important characteristic among the forces is that they are all transmitted by the exchange of a carrier particle, exactly like what Yukawa had in mind for the strong nuclear force. Each carrier particle is a virtual particle—it cannot be directly observed while transmitting the force. [link] shows the exchange of a virtual photon between two positive charges. The photon cannot be directly observed in its passage, because this would disrupt it and alter the force.

[link] shows a way of graphing the exchange of a virtual photon between two positive charges. This graph of time versus position is called a **Feynman diagram**, after the brilliant American physicist Richard Feynman (1918–1988) who developed it.

[link] is a Feynman diagram for the exchange of a virtual pion between a proton and a neutron representing the same interaction as in [link]. Feynman diagrams are not only a useful tool for visualizing interactions at the quantum mechanical level, they are also used to calculate details of interactions, such as their strengths and probability of occurring. Feynman was one of the theorists who developed the field of **quantum electrodynamics** (QED), which is the quantum mechanics of electromagnetism. QED has been spectacularly successful in describing electromagnetic interactions on the submicroscopic scale. Feynman was an inspiring teacher, had a colorful personality, and made a profound impact on generations of physicists. He shared the 1965 Nobel Prize with Julian Schwinger and S. I. Tomonaga for work in QED with its deep implications for particle physics.

Why is it that particles called gluons are listed as the carrier particles for the strong nuclear force when, in The Yukawa Particle and the Heisenberg Uncertainty Principle Revisited, we saw that pions apparently carry that force? The answer is that pions are exchanged but they have a substructure and, as we explore it, we find that the strong force is actually related to the indirectly observed but more fundamental **gluons**. In fact, all the carrier particles are thought to be fundamental in the sense that they have no substructure. Another similarity among carrier particles is that they are all bosons (first mentioned in Patterns in Spectra Reveal More Quantization), having integral intrinsic spins.



The image shows a Feynman diagram for the exchange of a π^+ between a proton and a neutron, carrying the strong nuclear force between them. This diagram represents the

situation shown more pictorially in [link].

There is a relationship between the mass of the carrier particle and the range of the force. The photon is massless and has energy. So, the existence of (virtual) photons is possible only by virtue of the Heisenberg uncertainty principle and can travel an unlimited distance. Thus, the range of the electromagnetic force is infinite. This is also true for gravity. It is infinite in range because its carrier particle, the graviton, has zero rest mass. (Gravity is the most difficult of the four forces to understand on a quantum scale because it affects the space and time in which the others act. But gravity is so weak that its effects are extremely difficult to observe quantum mechanically. We shall explore it further in General Relativity and Quantum Gravity). The W^+, W^-, and Z^0 particles that carry the weak nuclear force have mass, accounting for the very short range of this force. In fact, the W^+, W^-, and Z^0 are about 1000 times more massive than pions, consistent with the fact that the range of the weak nuclear force is about 1/1000 that of the strong nuclear force. Gluons are actually massless, but since they act inside massive carrier particles like pions, the strong nuclear force is also short ranged.

The relative strengths of the forces given in the [link] are those for the most common situations. When particles are brought very close together, the relative strengths change, and they may become identical at extremely close range. As we shall see in GUTs: the Unification of Forces, carrier particles may be altered by the energy required to bring particles very close together—in such a manner that they become identical.

Summary

- The four basic forces and their carrier particles are summarized in the [link].
- Feynman diagrams are graphs of time versus position and are highly useful pictorial representations of particle processes.
- The theory of electromagnetism on the particle scale is called quantum electrodynamics (QED).

Problems & Exercises

Exercise:

Problem:

(a) Find the ratio of the strengths of the weak and electromagnetic forces under ordinary circumstances.

(b) What does that ratio become under circumstances in which the forces are unified?

Solution:

(a) 10^{-11} to 1, weak to EM

(b) 1 to 1

Exercise:

Problem:

The ratio of the strong to the weak force and the ratio of the strong force to the electromagnetic force become 1 under circumstances where they are unified. What are the ratios of the strong force to those two forces under normal circumstances?

Glossary

Feynman diagram
: a graph of time versus position that describes the exchange of virtual particles between subatomic particles

gluons
: exchange particles, analogous to the exchange of photons that gives rise to the electromagnetic force between two charged particles

quantum electrodynamics
: the theory of electromagnetism on the particle scale

Particles, Patterns, and Conservation Laws

- Define matter and antimatter.
- Outline the differences between hadrons and leptons.
- State the differences between mesons and baryons.

In the early 1930s only a small number of subatomic particles were known to exist—the proton, neutron, electron, photon and, indirectly, the neutrino. Nature seemed relatively simple in some ways, but mysterious in others. Why, for example, should the particle that carries positive charge be almost 2000 times as massive as the one carrying negative charge? Why does a neutral particle like the neutron have a magnetic moment? Does this imply an internal structure with a distribution of moving charges? Why is it that the electron seems to have no size other than its wavelength, while the proton and neutron are about 1 fermi in size? So, while the number of known particles was small and they explained a great deal of atomic and nuclear phenomena, there were many unexplained phenomena and hints of further substructures.

Things soon became more complicated, both in theory and in the prediction and discovery of new particles. In 1928, the British physicist P.A.M. Dirac (see [link]) developed a highly successful relativistic quantum theory that laid the foundations of quantum electrodynamics (QED). His theory, for example, explained electron spin and magnetic moment in a natural way. But Dirac's theory also predicted negative energy states for free electrons. By 1931, Dirac, along with Oppenheimer, realized this was a prediction of positively charged electrons (or positrons). In 1932, American physicist Carl Anderson discovered the positron in cosmic ray studies. The positron, or e^+ , is the same particle as emitted in β^+ decay and was the first antimatter that was discovered. In 1935, Yukawa predicted pions as the carriers of the strong nuclear force, and they were eventually discovered. Muons were discovered in cosmic ray experiments in 1937, and they seemed to be heavy, unstable versions of electrons and positrons. After World War II, accelerators energetic enough to create these particles were built. Not only were predicted and known particles created, but many unexpected particles were observed. Initially called elementary particles, their numbers proliferated to dozens and then hundreds, and the term "particle zoo" became the physicist's lament at the lack of simplicity. But patterns were observed in the particle zoo that led to simplifying ideas such as quarks, as we shall soon see.

P.A.M. Dirac's theory of relativistic quantum mechanics not only explained a great deal of what was known, it also predicted antimatter. (credit: Cambridge University, Cavendish Laboratory)

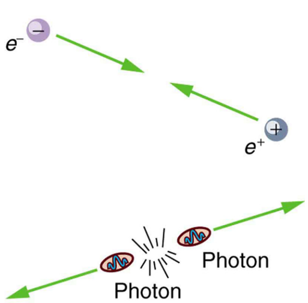
Matter and Antimatter

The positron was only the first example of antimatter. Every particle in nature has an antimatter counterpart, although some particles, like the photon, are their own antiparticles. Antimatter has charge opposite to that of matter (for example, the positron is positive while the electron is negative) but is nearly identical otherwise, having the same mass, intrinsic spin, half-life, and so on. When a particle and its antimatter counterpart interact, they annihilate one another, usually totally converting their masses to pure energy in the form of photons as seen in [link]. Neutral particles, such as neutrons, have neutral antimatter counterparts, which also annihilate when they interact. Certain neutral particles are their own antiparticle and live correspondingly short lives. For example, the neutral pion π^0 is its own antiparticle and has a half-life about 10^{-8} shorter than π^+ and π^-, which are each other's antiparticles. Without exception, nature is symmetric—all particles have antimatter counterparts. For example, antiprotons and antineutrons were first created in accelerator experiments in 1956 and the antiproton is negative. Antihydrogen atoms, consisting of an antiproton and antielectron, were observed in 1995 at CERN, too. It is possible to contain large-scale antimatter particles such as antiprotons by using electromagnetic traps that confine the particles within a magnetic field so that they don't annihilate with other particles. However, particles of the same charge repel each other, so the more particles that are contained in a trap, the more energy is needed to power the magnetic field that contains them. It is not currently possible to store a significant quantity of antiprotons. At any rate, we now see that negative charge is associated with both low-mass (electrons) and high-mass particles (antiprotons) and the apparent asymmetry is not there. But this knowledge does raise another question—why is there such a predominance of matter and so little antimatter? Possible explanations emerge later in this and the next chapter.

Hadrons and Leptons

Particles can also be revealingly grouped according to what forces they feel between them. All particles (even those that are massless) are affected by gravity, since gravity affects the space and time in which particles exist. All charged particles are affected by the electromagnetic force, as are neutral particles that have an internal distribution of charge (such as the neutron with its magnetic moment). Special names are given to particles that feel the strong and weak nuclear forces. **Hadrons** are particles that feel the strong nuclear force, whereas **leptons** are particles that do not. The proton, neutron, and the pions are examples of hadrons. The electron, positron, muons, and neutrinos are examples of leptons, the name meaning low mass. Leptons feel the weak nuclear force. In fact, all particles feel the weak nuclear force. This means that hadrons are distinguished by being able to feel both the strong and weak nuclear forces.

[link] lists the characteristics of some of the most important subatomic particles, including the directly observed carrier particles for the electromagnetic and weak nuclear forces, all leptons, and some hadrons. Several hints related to an underlying substructure emerge from an examination of these particle characteristics. Note that the carrier particles are called **gauge bosons**. First mentioned in Patterns in Spectra Reveal More Quantization, a **boson** is a particle with zero or an integer value of intrinsic spin (such as $s = 0, 1, 2, ...$), whereas a **fermion** is a particle with a half-integer value of intrinsic spin ($s = 1/2, 3/2, ...$). Fermions obey the Pauli exclusion principle whereas bosons do not. All the known and conjectured carrier particles are bosons.



When a particle

encounters its antiparticle, they annihilate, often producing pure energy in the form of photons. In this case, an electron and a positron convert all their mass into two identical energy rays, which move away in opposite directions to keep total momentum zero as it was before. Similar annihilations occur for other combinations of a particle with its antiparticle, sometimes producing more particles while obeying all conservation laws.

Category	**Particle name**	**Symbol**	**Antiparticle**	**Rest mass** (MeV/c^2)	B	L_e	L_μ	L_τ
Gauge	Photon	γ	Self	0	0	0	0	0
Bosons	W	W^+	W^-	80.39×10^3	0	0	0	0
	Z	Z^0	Self	91.19×10^3	0	0	0	0
Leptons	Electron	e^-	e^+	0.511	0	± 1	0	0

	Neutrino *(e)*	ν_e	$\bar{\nu}_e$	$0(7.0\mathrm{eV})$ [footnote] Neutrino masses may be zero. Experimental upper limits are given in parentheses.	0	±1	0	0
	Muon	μ^-	μ^+	105.7	0	0	±1	0
	Neutrino (μ)	ν_μ	$\bar{\nu}_\mu$	$0(< 0.27)$	0	0	±1	0
	Tau	τ^-	τ^+	1777	0	0	0	±1
	Neutrino (τ)	ν_τ	$\bar{\nu}_\tau$	$0(< 31)$	0	0	0	±1
Hadrons (selected)								
Mesons	Pion	π^+	π^-	139.6	0	0	0	0
		π^0	Self	135.0	0	0	0	0
	Kaon	K^+	K^-	493.7	0	0	0	0
		K^0	$\bar{K}^0$	497.6	0	0	0	0
	Eta	η^0	Self	547.9	0	0	0	0
(many other mesons known)								

Baryons	Proton	p	$\bar{p}$	938.3	±1	0	0	0
	Neutron	n	$\bar{n}$	939.6	±1	0	0	0
	Lambda	Λ^0	$\bar{\Lambda}^0$	1115.7	±1	0	0	0
	Sigma	Σ^+	$\bar{\Sigma}^-$	1189.4	±1	0	0	0
		Σ^0	$\bar{\Sigma}^0$	1192.6	±1	0	0	0
		Σ^-	$\bar{\Sigma}^+$	1197.4	±1	0	0	0
	Xi	Ξ^0	$\bar{\Xi}^0$	1314.9	±1	0	0	0
		Ξ^-	Ξ^+	1321.7	±1	0	0	0
	Omega	Ω^-	Ω^+	1672.5	±1	0	0	0
(many other baryons known)								

Selected Particle Characteristics[footnote]
The lower of the $\mp$ or $\pm$ symbols are the values for antiparticles.

All known leptons are listed in the table given above. There are only six leptons (and their antiparticles), and they seem to be fundamental in that they have no apparent underlying structure. Leptons have no discernible size other than their wavelength, so that we know they are pointlike down to about 10^{-18} m. The leptons fall into three families, implying three conservation laws for three quantum numbers. One of these was known from β decay, where the existence of the electron's neutrino implied that a new quantum number, called the **electron family number** L_e is conserved. Thus, in β decay, an antielectron's neutrino $\bar{v}_e$ must be created with $L_e = -1$ when an electron with L_e=+1 is created, so that the total remains 0 as it was before decay.

Once the muon was discovered in cosmic rays, its decay mode was found to be
Equation:

$$\mu^- \rightarrow e^- + \bar{v}_e + v_\mu,$$

which implied another “family” and associated conservation principle. The particle v_μ is a muon’s neutrino, and it is created to conserve **muon family number**L_μ. So muons are leptons with a family of their own, and **conservation of total** L_μ also seems to be obeyed in many experiments.

More recently, a third lepton family was discovered when τ particles were created and observed to decay in a manner similar to muons. One principal decay mode is
Equation:

$$\tau^- \rightarrow \mu^- + \bar{v}_\mu + v_\tau.$$

Conservation of total L_τ seems to be another law obeyed in many experiments. In fact, particle experiments have found that lepton family number is not universally conserved, due to neutrino “oscillations,” or transformations of neutrinos from one family type to another.

Mesons and Baryons

Now, note that the hadrons in the table given above are divided into two subgroups, called mesons (originally for medium mass) and baryons (the name originally meaning large mass). The division between mesons and baryons is actually based on their observed decay modes and is not strictly associated with their masses. **Mesons** are hadrons that can decay to leptons and leave no hadrons, which implies that mesons are not conserved in number. **Baryons** are hadrons that always decay to another baryon. A new physical quantity called **baryon number** B seems to always be conserved in nature and is listed for the various particles in the table given above. Mesons and leptons have $B = 0$ so that they can decay to other particles with $B = 0$. But baryons have $B{=}{+}1$ if they are matter, and $B = -1$ if they are antimatter. The **conservation of total baryon number** is a more general rule than first noted in nuclear physics, where it was observed that the total number of nucleons was always conserved in nuclear reactions and decays. That rule in nuclear physics is just one consequence of the conservation of the total baryon number.

Forces, Reactions, and Reaction Rates

The forces that act between particles regulate how they interact with other particles. For example, pions feel the strong force and do not penetrate as far in matter as do muons, which do not feel the strong force. (This was the way those who discovered the muon knew it could not be the particle that carries the strong force—its penetration or range was too great for it to be feeling the strong force.) Similarly, reactions that create other particles, like cosmic rays interacting with nuclei in the atmosphere, have greater probability if they are caused by the strong force than if they are caused by the weak force. Such knowledge has been useful to physicists while analyzing the particles produced by various accelerators.

The forces experienced by particles also govern how particles interact with themselves if they are unstable and decay. For example, the stronger the force, the faster they decay and the shorter is their lifetime. An example of a nuclear decay via the strong force is $^8\text{Be} \rightarrow \alpha + \alpha$ with a lifetime of about 10^{-16} s. The neutron is a good example of decay via the weak force. The process $n \rightarrow p + e^- + \bar{v}_e$ has a longer lifetime of 882 s. The weak force causes this decay, as it does all β decay. An important clue that the weak force is responsible for β decay is the creation of leptons, such as e^- and $\bar{v}_e$. None would be created if the strong force was responsible, just as no leptons are created in the decay of ^{8}Be. The systematics of particle lifetimes is a little simpler than nuclear lifetimes when hundreds of particles are examined (not just the ones in the table given above). Particles that decay via the weak force have lifetimes mostly in the range of 10^{-16} to 10^{-12} s, whereas those that decay via the strong force have lifetimes mostly in the range of 10^{-16} to 10^{-23} s. Turning this around, if we measure the lifetime of a particle, we can tell if it decays via the weak or strong force.

Yet another quantum number emerges from decay lifetimes and patterns. Note that the particles Λ, Σ, Ξ, and Ω decay with lifetimes on the order of 10^{-10} s (the exception is Σ^0, whose short lifetime is explained by its particular quark substructure.), implying that their decay is caused by the weak force alone, although they are hadrons and feel the strong force. The decay modes of these particles also show patterns—in particular, certain decays that should be possible within all the known conservation laws do not occur. Whenever something is possible in physics, it will happen. If something does not happen, it is forbidden by a rule. All this seemed strange to those studying these particles when they were first discovered, so they named a new quantum number **strangeness**, given the symbol S in the table given above. The values of strangeness assigned to various particles are based on the decay systematics. It is found that **strangeness is conserved by the strong force**, which governs the production of most of these particles in accelerator experiments. However, **strangeness is *not conserved* by the weak force**. This conclusion is reached from the fact that particles that have long lifetimes decay via the weak force and do not conserve strangeness. All of this also has implications for the carrier particles, since they transmit forces and are thus involved in these decays.

Example:
Calculating Quantum Numbers in Two Decays

(a) The most common decay mode of the Ξ^- particle is $\Xi^- \rightarrow \Lambda^0 + \pi^-$. Using the quantum numbers in the table given above, show that strangeness changes by 1, baryon number and charge are conserved, and lepton family numbers are unaffected.

(b) Is the decay $K^+ \rightarrow \mu^+ + \nu_\mu$ allowed, given the quantum numbers in the table given above?

Strategy

In part (a), the conservation laws can be examined by adding the quantum numbers of the decay products and comparing them with the parent particle. In part (b), the same procedure can reveal if a conservation law is broken or not.

Solution for (a)

Before the decay, the Ξ^- has strangeness $S = -2$. After the decay, the total strangeness is –1 for the Λ^0, plus 0 for the π^-. Thus, total strangeness has gone from –2 to –1 or a change of +1. Baryon number for the Ξ^- is $B = +1$ before the decay, and after the decay the Λ^0 has $B = +1$ and the π^- has $B = 0$ so that the total baryon number remains +1. Charge is –1 before the decay, and the total charge after is also $0 - 1 = -1$. Lepton numbers for all the particles are zero, and so lepton numbers are conserved.

Discussion for (a)

The Ξ^- decay is caused by the weak interaction, since strangeness changes, and it is consistent with the relatively long 1.64×10^{-10}-s lifetime of the Ξ^-.

Solution for (b)

The decay $K^+ \rightarrow \mu^+ + \nu_\mu$ is allowed if charge, baryon number, mass-energy, and lepton numbers are conserved. Strangeness can change due to the weak interaction. Charge is conserved as $s \rightarrow d$. Baryon number is conserved, since all particles have $B = 0$. Mass-energy is conserved in the sense that the K^+ has a greater mass than the products, so that the decay can be spontaneous. Lepton family numbers are conserved at 0 for the electron and tau family for all particles. The muon family number is $L_\mu = 0$ before and $L_\mu = -1 + 1 = 0$ after. Strangeness changes from +1 before to 0 + 0 after, for an allowed change of 1. The decay is allowed by all these measures.

Discussion for (b)

This decay is not only allowed by our reckoning, it is, in fact, the primary decay mode of the K^+ meson and is caused by the weak force, consistent with the long 1.24×10^{-8}-s lifetime.

There are hundreds of particles, all hadrons, not listed in [link], most of which have shorter lifetimes. The systematics of those particle lifetimes, their production probabilities, and decay products are completely consistent with the conservation laws noted for lepton families, baryon number, and strangeness, but they also imply other quantum numbers and conservation laws. There are a finite, and in fact relatively small, number of these conserved quantities, however, implying a finite set of substructures. Additionally, some of these short-lived particles resemble the excited states of other particles, implying an internal structure. All of this jigsaw puzzle can be tied together and explained relatively simply by the existence of fundamental substructures. Leptons seem to be

fundamental structures. Hadrons seem to have a substructure called quarks. [Quarks: Is That All There Is?](#) explores the basics of the underlying quark building blocks.

Murray Gell-Mann (b. 1929) proposed quarks as a substructure of hadrons in 1963 and was already known for his work on the concept of strangeness. Although quarks have never been directly observed, several predictions of the quark model were quickly confirmed, and their properties explain all known hadron characteristics. Gell-Mann was awarded the Nobel Prize in 1969. (credit: Luboš Motl)

Summary

- All particles of matter have an antimatter counterpart that has the opposite charge and certain other quantum numbers as seen in [link]. These matter-antimatter pairs are otherwise very similar but will annihilate when brought together. Known particles can be divided into three major groups—leptons, hadrons, and carrier particles (gauge bosons).

- Leptons do not feel the strong nuclear force and are further divided into three groups—electron family designated by electron family number L_e; muon family designated by muon family number L_μ; and tau family designated by tau family number L_τ. The family numbers are not universally conserved due to neutrino oscillations.
- Hadrons are particles that feel the strong nuclear force and are divided into baryons, with the baryon family number B being conserved, and mesons.

Conceptual Questions

Exercise:

Problem:

Large quantities of antimatter isolated from normal matter should behave exactly like normal matter. An antiatom, for example, composed of positrons, antiprotons, and antineutrons should have the same atomic spectrum as its matter counterpart. Would you be able to tell it is antimatter by its emission of antiphotons? Explain briefly.

Exercise:

Problem: Massless particles are not only neutral, they are chargeless (unlike the neutron). Why is this so?

Exercise:

Problem:

Massless particles must travel at the speed of light, while others cannot reach this speed. Why are all massless particles stable? If evidence is found that neutrinos spontaneously decay into other particles, would this imply they have mass?

Exercise:

Problem:

When a star erupts in a supernova explosion, huge numbers of electron neutrinos are formed in nuclear reactions. Such neutrinos from the 1987A supernova in the relatively nearby Magellanic Cloud were observed within hours of the initial brightening, indicating they traveled to earth at approximately the speed of light. Explain how this data can be used to set an upper limit on the mass of the neutrino, noting that if the mass is small the neutrinos could travel very close to the speed of light and have a reasonable energy (on the order of MeV).

Exercise:

Problem:

Theorists have had spectacular success in predicting previously unknown particles. Considering past theoretical triumphs, why should we bother to perform experiments?

Exercise:

Problem: What lifetime do you expect for an antineutron isolated from normal matter?

Exercise:

Problem: Why does the η^0 meson have such a short lifetime compared to most other mesons?

Exercise:

Problem: (a) Is a hadron always a baryon?

(b) Is a baryon always a hadron?

(c) Can an unstable baryon decay into a meson, leaving no other baryon?

Exercise:

Problem:

Explain how conservation of baryon number is responsible for conservation of total atomic mass (total number of nucleons) in nuclear decay and reactions.

Problems & Exercises

Exercise:

Problem:

The π^0 is its own antiparticle and decays in the following manner: $\pi^0 \rightarrow \gamma + \gamma$. What is the energy of each γ ray if the π^0 is at rest when it decays?

Solution:

67.5 MeV

Exercise:

Problem:

The primary decay mode for the negative pion is $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$. What is the energy release in MeV in this decay?

Exercise:

Problem:

The mass of a theoretical particle that may be associated with the unification of the electroweak and strong forces is $10^{14}\ \text{GeV}/c^2$.

(a) How many proton masses is this?

(b) How many electron masses is this? (This indicates how extremely relativistic the accelerator would have to be in order to make the particle, and how large the relativistic quantity γ would have to be.)

Solution:

(a) 1×10^{14}

(b) 2×10^{17}

Exercise:

Problem: The decay mode of the negative muon is $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$.

(a) Find the energy released in MeV.

(b) Verify that charge and lepton family numbers are conserved.

Exercise:

Problem: The decay mode of the positive tau is $\tau^+ \rightarrow \mu^+ + \nu_\mu + \bar{\nu}_\tau$.

(a) What energy is released?

(b) Verify that charge and lepton family numbers are conserved.

(c) The τ^+ is the antiparticle of the τ^-.Verify that all the decay products of the τ^+ are the antiparticles of those in the decay of the τ^- given in the text.

Solution:

(a) 1671 MeV

(b) $Q = 1$, $Q\prime = 1 + 0 + 0 = 1$. $L_\tau = -1$; $L\prime\tau = -1$; $L\mu = 0$; $L\prime\mu = -1 + 1 = 0$

(c) $\tau^- \rightarrow \mu^- + v_\mu + v_\tau$
$\Rightarrow \mu^- \text{ antiparticle of } \mu^+; v_\mu \text{ of } v_\mu; v_\tau \text{ of } v_\tau$

Exercise:

Problem: The principal decay mode of the sigma zero is $\Sigma^0 \rightarrow \Lambda^0 + \gamma$.

(a) What energy is released?

(b) Considering the quark structure of the two baryons, does it appear that the Σ^0 is an excited state of the Λ^0 ?

(c) Verify that strangeness, charge, and baryon number are conserved in the decay.

(d) Considering the preceding and the short lifetime, can the weak force be responsible? State why or why not.

Exercise:

Problem: (a) What is the uncertainty in the energy released in the decay of a π^0 due to its short lifetime?

(b) What fraction of the decay energy is this, noting that the decay mode is $\pi^0 \rightarrow \gamma + \gamma$ (so that all the π^0 mass is destroyed)?

Solution:

(a) 3.9 eV

(b) 2.9×10^{-8}

Exercise:

Problem: (a) What is the uncertainty in the energy released in the decay of a τ^- due to its short lifetime?

(b) Is the uncertainty in this energy greater than or less than the uncertainty in the mass of the tau neutrino? Discuss the source of the uncertainty.

Glossary

boson
: particle with zero or an integer value of intrinsic spin

baryons
: hadrons that always decay to another baryon

baryon number

a conserved physical quantity that is zero for mesons and leptons and ±1 for baryons and antibaryons, respectively

conservation of total baryon number
: a general rule based on the observation that the total number of nucleons was always conserved in nuclear reactions and decays

conservation of total electron family number
: a general rule stating that the total electron family number stays the same through an interaction

conservation of total muon family number
: a general rule stating that the total muon family number stays the same through an interaction

electron family number
: the number ±1 that is assigned to all members of the electron family, or the number 0 that is assigned to all particles not in the electron family

fermion
: particle with a half-integer value of intrinsic spin

gauge boson
: particle that carries one of the four forces

hadrons
: particles that feel the strong nuclear force

leptons
: particles that do not feel the strong nuclear force

meson
: hadrons that can decay to leptons and leave no hadrons

muon family number
: the number ±1 that is assigned to all members of the muon family, or the number 0 that is assigned to all particles not in the muon family

strangeness
: a physical quantity assigned to various particles based on decay systematics

tau family number
: the number ±1 that is assigned to all members of the tau family, or the number 0 that is assigned to all particles not in the tau family

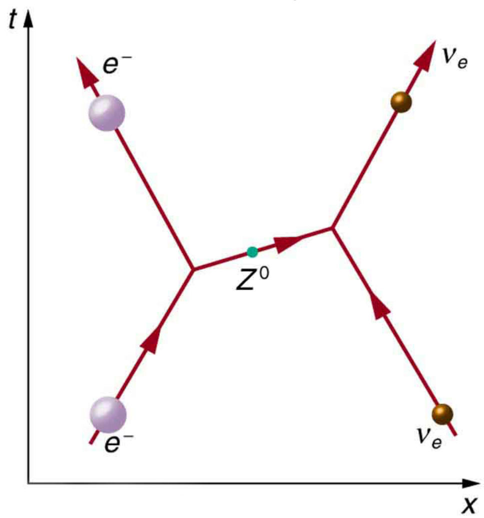
GUTs: The Unification of Forces

- State the grand unified theory.
- Explain the electroweak theory.
- Define gluons.
- Describe the principle of quantum chromodynamics.
- Define the standard model.

Present quests to show that the four basic forces are different manifestations of a single unified force follow a long tradition. In the 19th century, the distinct electric and magnetic forces were shown to be intimately connected and are now collectively called the electromagnetic force. More recently, the weak nuclear force has been shown to be connected to the electromagnetic force in a manner suggesting that a theory may be constructed in which all four forces are unified. Certainly, there are similarities in how forces are transmitted by the exchange of carrier particles, and the carrier particles themselves (the gauge bosons in [link]) are also similar in important ways. The analogy to the unification of electric and magnetic forces is quite good—the four forces are distinct under normal circumstances, but there are hints of connections even on the atomic scale, and there may be conditions under which the forces are intimately related and even indistinguishable. The search for a correct theory linking the forces, called the **Grand Unified Theory (GUT)**, is explored in this section in the realm of particle physics. Frontiers of Physics expands the story in making a connection with cosmology, on the opposite end of the distance scale.

[link] is a Feynman diagram showing how the weak nuclear force is transmitted by the carrier particle Z^0, similar to the diagrams in [link] and [link] for the electromagnetic and strong nuclear forces. In the 1960s, a gauge theory, called **electroweak theory**, was developed by Steven Weinberg, Sheldon Glashow, and Abdus Salam and proposed that the electromagnetic and weak forces are identical at sufficiently high energies. One of its predictions, in addition to describing both electromagnetic and weak force phenomena, was the existence of the W^+,W^-, and Z^0 carrier particles. Not only were three particles having spin 1 predicted, the mass of the W^+ and W^- was predicted to be 81 GeV/c^2, and that of the Z^0 was

predicted to be 90 GeV/c^2. (Their masses had to be about 1000 times that of the pion, or about 100 GeV/c^2, since the range of the weak force is about 1000 times less than the strong force carried by virtual pions.) In 1983, these carrier particles were observed at CERN with the predicted characteristics, including masses having the predicted values as seen in [link]. This was another triumph of particle theory and experimental effort, resulting in the 1984 Nobel Prize to the experiment's group leaders Carlo Rubbia and Simon van der Meer. Theorists Weinberg, Glashow, and Salam had already been honored with the 1979 Nobel Prize for other aspects of electroweak theory.

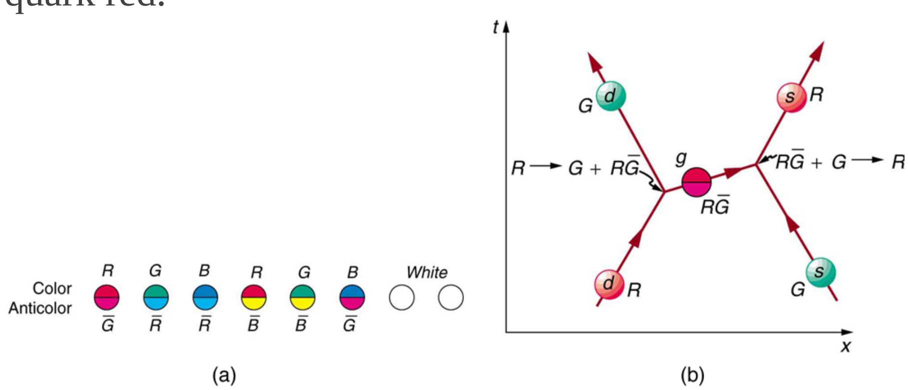


The exchange of a virtual Z^0 carries the weak nuclear force between an electron and a neutrino in this Feynman diagram. The Z^0 is one of the carrier particles for the weak nuclear force that has now been created in the laboratory with characteristics predicted by electroweak theory.

Although the weak nuclear force is very short ranged ($< 10^{-18}$ m, as indicated in [link]), its effects on atomic levels can be measured given the extreme precision of modern techniques. Since electrons spend some time in the nucleus, their energies are affected, and spectra can even indicate new aspects of the weak force, such as the possibility of other carrier particles. So systems many orders of magnitude larger than the range of the weak force supply evidence of electroweak unification in addition to evidence found at the particle scale.

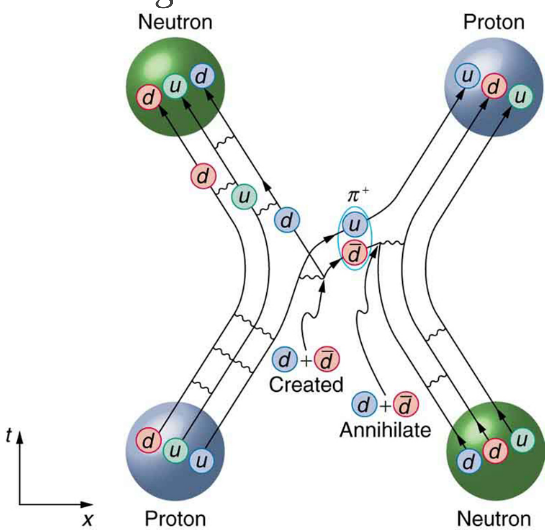
Gluons (g) are the proposed carrier particles for the strong nuclear force, although they are not directly observed. Like quarks, gluons may be confined to systems having a total color of white. Less is known about gluons than the fact that they are the carriers of the weak and certainly of the electromagnetic force. QCD theory calls for eight gluons, all massless and all spin 1. Six of the gluons carry a color and an anticolor, while two do not carry color, as illustrated in [link](a). There is indirect evidence of the existence of gluons in nucleons. When high-energy electrons are scattered from nucleons and evidence of quarks is seen, the momenta of the quarks are smaller than they would be if there were no gluons. That means that the gluons carrying force between quarks also carry some momentum, inferred by the already indirect quark momentum measurements. At any rate, the gluons carry color charge and can change the colors of quarks when exchanged, as seen in [link](b). In the figure, a red down quark interacts with a green strange quark by sending it a gluon. That gluon carries red

away from the down quark and leaves it green, because it is an RG (red-antigreen) gluon. (Taking antigreen away leaves you green.) Its antigreenness kills the green in the strange quark, and its redness turns the quark red.



In figure (a), the eight types of gluons that carry the strong nuclear force are divided into a group of six that carry color and a group of two that do not. Figure (b) shows that the exchange of gluons between quarks carries the strong force and may change the color of a quark.

The strong force is complicated, since observable particles that feel the strong force (hadrons) contain multiple quarks. [link] shows the quark and gluon details of pion exchange between a proton and a neutron as illustrated earlier in [link] and [link]. The quarks within the proton and neutron move along together exchanging gluons, until the proton and neutron get close together. As the u quark leaves the proton, a gluon creates a pair of virtual particles, a d quark and a $\bar{d}$ antiquark. The d quark stays behind and the proton turns into a neutron, while the u and $\bar{d}$ move together as a π^+ ([link] confirms the $u\bar{d}$ composition for the π^+.) The $\bar{d}$ annihilates a d quark in the neutron, the u joins the neutron, and the neutron becomes a proton. A pion is exchanged and a force is transmitted.



This Feynman diagram is the same interaction as shown in [link], but it shows

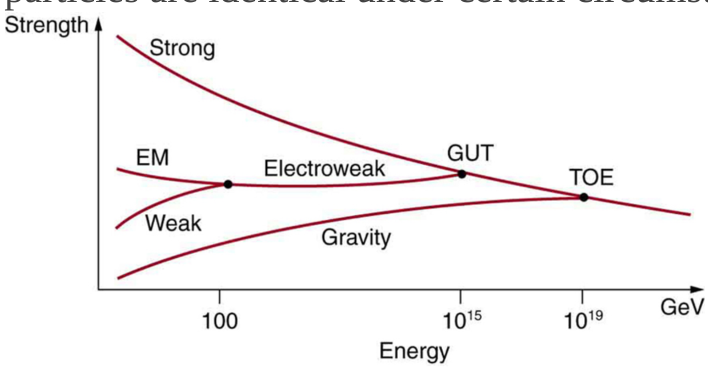
the quark and gluon details of the strong force interaction.

It is beyond the scope of this text to go into more detail on the types of quark and gluon interactions that underlie the observable particles, but the theory (**quantum chromodynamics** or QCD) is very self-consistent. So successful have QCD and the electroweak theory been that, taken together, they are called the **Standard Model**. Advances in knowledge are expected to modify, but not overthrow, the Standard Model of particle physics and forces.

Note:
Making Connections: Unification of Forces
Grand Unified Theory (GUT) is successful in describing the four forces as distinct under normal circumstances, but connected in fundamental ways. Experiments have verified that the weak and electromagnetic force become identical at very small distances and provide the GUT description of the carrier particles for the forces. GUT predicts that the other forces become identical under conditions so extreme that they cannot be tested in the laboratory, although there may be lingering evidence of them in the evolution of the universe. GUT is also successful in describing a system of carrier particles for all four forces, but there is much to be done, particularly in the realm of gravity.

How can forces be unified? They are definitely distinct under most circumstances, for example, being carried by different particles and having greatly different strengths. But experiments show that at extremely small distances, the strengths of the forces begin to become more similar. In fact, electroweak theory's prediction of the W^+, W^-, and Z^0 carrier particles was based on the strengths of the two forces being identical at extremely small distances as seen in [link]. As discussed in case of the creation of

virtual particles for extremely short times, the small distances or short ranges correspond to the large masses of the carrier particles and the correspondingly large energies needed to create them. Thus, the energy scale on the horizontal axis of [link] corresponds to smaller and smaller distances, with 100 GeV corresponding to approximately, 10^{-18}m for example. At that distance, the strengths of the EM and weak forces are the same. To test physics at that distance, energies of about 100 GeV must be put into the system, and that is sufficient to create and release the W^+, W^-, and Z^0 carrier particles. At those and higher energies, the masses of the carrier particles becomes less and less relevant, and the Z^0 in particular resembles the massless, chargeless, spin 1 photon. In fact, there is enough energy when things are pushed to even smaller distances to transform the, and Z^0 into massless carrier particles more similar to photons and gluons. These have not been observed experimentally, but there is a prediction of an associated particle called the **Higgs boson**. The mass of this particle is not predicted with nearly the certainty with which the mass of the W^+, W^-, and Z^0 particles were predicted, but it was hoped that the Higgs boson could be observed at the now-canceled Superconducting Super Collider (SSC). Ongoing experiments at the Large Hadron Collider at CERN have presented some evidence for a Higgs boson with a mass of 125 GeV, and there is a possibility of a direct discovery during 2012. The existence of this more massive particle would give validity to the theory that the carrier particles are identical under certain circumstances.



The relative strengths of the four basic forces vary with distance and, hence, energy is needed to probe small distances. At ordinary energies (a few eV or less), the forces differ greatly as

indicated in [link]. However, at energies available at accelerators, the weak and EM forces become identical, or unified. Unfortunately, the energies at which the strong and electroweak forces become the same are unreachable even in principle at any conceivable accelerator. The universe may provide a laboratory, and nature may show effects at ordinary energies that give us clues about the validity of this graph.

The small distances and high energies at which the electroweak force becomes identical with the strong nuclear force are not reachable with any conceivable human-built accelerator. At energies of about $10^{14}\ \mathrm{GeV}$ (16,000 J per particle), distances of about $10^{-30}\ \mathrm{m}$ can be probed. Such energies are needed to test theory directly, but these are about 10^{10} higher than the proposed giant SSC would have had, and the distances are about 10^{-12} smaller than any structure we have direct knowledge of. This would be the realm of various GUTs, of which there are many since there is no constraining evidence at these energies and distances. Past experience has shown that any time you probe so many orders of magnitude further (here, about 10^{12}), you find the unexpected. Even more extreme are the energies and distances at which gravity is thought to unify with the other forces in a TOE. Most speculative and least constrained by experiment are TOEs, one of which is called **Superstring theory**. Superstrings are entities that are $10^{-35}\ \mathrm{m}$ in scale and act like one-dimensional oscillating strings and are also proposed to underlie all particles, forces, and space itself.

At the energy of GUTs, the carrier particles of the weak force would become massless and identical to gluons. If that happens, then both lepton and baryon conservation would be violated. We do not see such violations, because we do not encounter such energies. However, there is a tiny probability that, at ordinary energies, the virtual particles that violate the

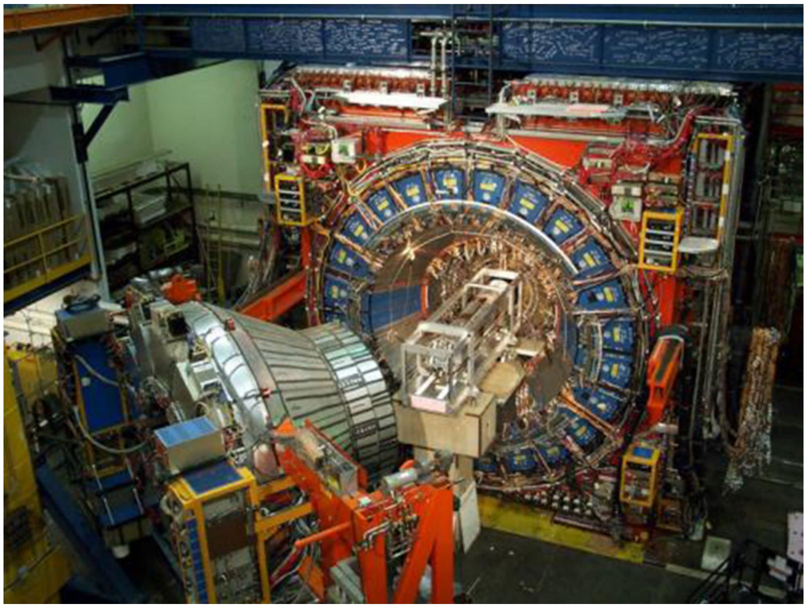
conservation of baryon number may exist for extremely small amounts of time (corresponding to very small ranges). All GUTs thus predict that the proton should be unstable, but would decay with an extremely long lifetime of about 10^{31} y. The predicted decay mode is

Equation:

$$p \rightarrow \pi^0 + e^+, \text{ (proposed proton decay)}$$

which violates both conservation of baryon number and electron family number. Although 10^{31} y is an extremely long time (about 10^{21} times the age of the universe), there are a lot of protons, and detectors have been constructed to look for the proposed decay mode as seen in [link]. It is somewhat comforting that proton decay has not been detected, and its experimental lifetime is now greater than 5×10^{32} y. This does not prove GUTs wrong, but it does place greater constraints on the theories, benefiting theorists in many ways.

From looking increasingly inward at smaller details for direct evidence of electroweak theory and GUTs, we turn around and look to the universe for evidence of the unification of forces. In the 1920s, the expansion of the universe was discovered. Thinking backward in time, the universe must once have been very small, dense, and extremely hot. At a tiny fraction of a second after the fabled Big Bang, forces would have been unified and may have left their fingerprint on the existing universe. This, one of the most exciting forefronts of physics, is the subject of Frontiers of Physics.



In the Tevatron accelerator at Fermilab, protons and antiprotons collide at high energies, and some of those collisions could result in the production of a Higgs boson in association with a *W* boson. When the *W* boson decays to a high-energy lepton and a neutrino, the detector triggers on the lepton, whether it is an electron or a muon. (credit: D. J. Miller)

Summary

- Attempts to show unification of the four forces are called Grand Unified Theories (GUTs) and have been partially successful, with connections proven between EM and weak forces in electroweak theory.
- The strong force is carried by eight proposed particles called gluons, which are intimately connected to a quantum number called color—their governing theory is thus called quantum chromodynamics

(QCD). Taken together, QCD and the electroweak theory are widely accepted as the Standard Model of particle physics.

- Unification of the strong force is expected at such high energies that it cannot be directly tested, but it may have observable consequences in the as-yet unobserved decay of the proton and topics to be discussed in the next chapter. Although unification of forces is generally anticipated, much remains to be done to prove its validity.

Conceptual Questions

Exercise:

Problem:

If a GUT is proven, and the four forces are unified, it will still be correct to say that the orbit of the moon is determined by the gravitational force. Explain why.

Exercise:

Problem:

If the Higgs boson is discovered and found to have mass, will it be considered the ultimate carrier of the weak force? Explain your response.

Exercise:

Problem:

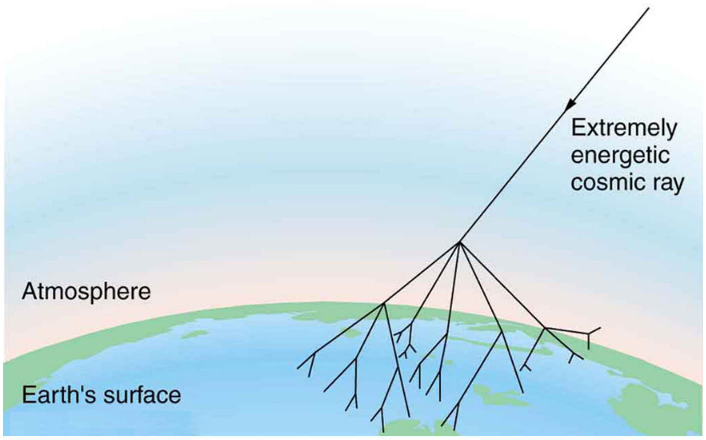
Gluons and the photon are massless. Does this imply that the W^+, W^-, and Z^0 are the ultimate carriers of the weak force?

Problems & Exercises

Exercise:

Problem: Integrated Concepts

The intensity of cosmic ray radiation decreases rapidly with increasing energy, but there are occasionally extremely energetic cosmic rays that create a shower of radiation from all the particles they create by striking a nucleus in the atmosphere as seen in the figure given below. Suppose a cosmic ray particle having an energy of 10^{10} GeV converts its energy into particles with masses averaging $200\ \text{MeV}/c^2$. (a) How many particles are created? (b) If the particles rain down on a 1.00-km^2 area, how many particles are there per square meter?



An extremely energetic cosmic ray creates a shower of particles on earth. The energy of these rare cosmic rays can approach a joule (about 10^{10} GeV) and, after multiple collisions, huge numbers of particles are created from this energy. Cosmic ray showers have been observed to extend over many square kilometers.

Solution:

(a) 5×10^{10}

(b) $5 \times 10^{4}\ \text{particles}/\text{m}^2$

Exercise:

Problem: Integrated Concepts

Assuming conservation of momentum, what is the energy of each γ ray produced in the decay of a neutral at rest pion, in the reaction $\pi^0 \rightarrow \gamma + \gamma$?

Exercise:

Problem: Integrated Concepts

What is the wavelength of a 50-GeV electron, which is produced at SLAC? This provides an idea of the limit to the detail it can probe.

Solution:

2.5×10^{-17} m

Exercise:

Problem: Integrated Concepts

(a) Calculate the relativistic quantity $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ for 1.00-TeV protons produced at Fermilab. (b) If such a proton created a π^+ having the same speed, how long would its life be in the laboratory? (c) How far could it travel in this time?

Exercise:

Problem: Integrated Concepts

The primary decay mode for the negative pion is $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$. (a) What is the energy release in MeV in this decay? (b) Using conservation of momentum, how much energy does each of the decay products receive, given the π^- is at rest when it decays? You may assume the muon antineutrino is massless and has momentum $p = E/c$, just like a photon.

Solution:

(a) 33.9 MeV

(b) Muon antineutrino 29.8 MeV, muon 4.1 MeV (kinetic energy)

Exercise:

Problem: Integrated Concepts

Plans for an accelerator that produces a secondary beam of *K*-mesons to scatter from nuclei, for the purpose of studying the strong force, call for them to have a kinetic energy of 500 MeV. (a) What would the relativistic quantity $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ be for these particles? (b) How long would their average lifetime be in the laboratory? (c) How far could they travel in this time?

Exercise:

Problem: Integrated Concepts

Suppose you are designing a proton decay experiment and you can detect 50 percent of the proton decays in a tank of water. (a) How many kilograms of water would you need to see one decay per month, assuming a lifetime of 10^{31} y? (b) How many cubic meters of water is this? (c) If the actual lifetime is 10^{33} y, how long would you have to wait on an average to see a single proton decay?

Solution:

(a) 7.2×10^5 kg

(b) 7.2×10^2 m^3

(c) 100 months

Exercise:

Problem: Integrated Concepts

In supernovas, neutrinos are produced in huge amounts. They were detected from the 1987A supernova in the Magellanic Cloud, which is about 120,000 light years away from the Earth (relatively close to our Milky Way galaxy). If neutrinos have a mass, they cannot travel at the speed of light, but if their mass is small, they can get close. (a) Suppose a neutrino with a $7\text{-eV}/c^2$ mass has a kinetic energy of 700 keV. Find the relativistic quantity $\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$ for it. (b) If the neutrino leaves the 1987A supernova at the same time as a photon and both travel to Earth, how much sooner does the photon arrive? This is not a large time difference, given that it is impossible to know which neutrino left with which photon and the poor efficiency of the neutrino detectors. Thus, the fact that neutrinos were observed within hours of the brightening of the supernova only places an upper limit on the neutrino's mass. (Hint: You may need to use a series expansion to find *v* for the neutrino, since its γ is so large.)

Exercise:

Problem: Construct Your Own Problem

Consider an ultrahigh-energy cosmic ray entering the Earth's atmosphere (some have energies approaching a joule). Construct a problem in which you calculate the energy of the particle based on the number of particles in an observed cosmic ray shower. Among the things to consider are the average mass of the shower particles, the average number per square meter, and the extent (number of square meters covered) of the shower. Express the energy in eV and joules.

Exercise:

Problem: Construct Your Own Problem

Consider a detector needed to observe the proposed, but extremely rare, decay of an electron. Construct a problem in which you calculate the amount of matter needed in the detector to be able to observe the decay, assuming that it has a signature that is clearly identifiable. Among the things to consider are the estimated half life (long for rare events), and the number of decays per unit time that you wish to observe, as well as the number of electrons in the detector substance.

Glossary

electroweak theory
: theory showing connections between EM and weak forces

grand unified theory
: theory that shows unification of the strong and electroweak forces

gluons
: eight proposed particles which carry the strong force

Higgs boson
: a massive particle that, if observed, would give validity to the theory that carrier particles are identical under certain circumstances

quantum chromodynamics
: the governing theory of connecting quantum number color to gluons

standard model
: combination of quantum chromodynamics and electroweak theory

superstring theory
: a theory of everything based on vibrating strings some 10^{-35} m in length

Atomic Masses

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
0	neutron	1	n	1.008 665	β^-	10.37 min
1	Hydrogen	1	^1H	1.007 825	99.985%	
	Deuterium	2	^2H or D	2.014 102	0.015%	
	Tritium	3	^3H or T	3.016 050	β^-	12.33 y
2	Helium	3	^3He	3.016 030	$1.38 \times 10^{-4}\%$	
		4	^4He	4.002 603	$\approx 100\%$	
3	Lithium	6	^6Li	6.015 121	7.5%	
		7	^7Li	7.016 003	92.5%	
4	Beryllium	7	^7Be	7.016 928	EC	53.29 d
		9	^9Be	9.012 182	100%	
5	Boron	10	^{10}B	10.012 937	19.9%	
		11	^{11}B	11.009 305	80.1%	
6	Carbon	11	^{11}C	11.011 432	EC, β^+	
		12	^{12}C	12.000 000	98.90%	
		13	^{13}C	13.003 355	1.10%	

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		14	^{14}C	14.003 241	β^-	5730 y
7	Nitrogen	13	^{13}N	13.005 738	β^+	9.96 min
		14	^{14}N	14.003 074	99.63%	
		15	^{15}N	15.000 108	0.37%	
8	Oxygen	15	^{15}O	15.003 065	EC, β^+	122 s
		16	^{16}O	15.994 915	99.76%	
		18	^{18}O	17.999 160	0.200%	
9	Fluorine	18	^{18}F	18.000 937	EC, β^+	1.83 h
		19	^{19}F	18.998 403	100%	
10	Neon	20	^{20}Ne	19.992 435	90.51%	
		22	^{22}Ne	21.991 383	9.22%	
11	Sodium	22	^{22}Na	21.994 434	β^+	2.602 y
		23	^{23}Na	22.989 767	100%	
		24	^{24}Na	23.990 961	β^-	14.96 h
12	Magnesium	24	^{24}Mg	23.985 042	78.99%	
13	Aluminum	27	^{27}Al	26.981 539	100%	
14	Silicon	28	^{28}Si	27.976 927	92.23%	2.62h

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		31	^{31}Si	30.975 362	β^-	
15	Phosphorus	31	^{31}P	30.973 762	100%	
		32	^{32}P	31.973 907	β^-	14.28 d
16	Sulfur	32	^{32}S	31.972 070	95.02%	
		35	^{35}S	34.969 031	β^-	87.4 d
17	Chlorine	35	^{35}Cl	34.968 852	75.77%	
		37	^{37}Cl	36.965 903	24.23%	
18	Argon	40	^{40}Ar	39.962 384	99.60%	
19	Potassium	39	^{39}K	38.963 707	93.26%	
		40	^{40}K	39.963 999	0.0117%, EC, β^-	1.28×10^9y
20	Calcium	40	^{40}Ca	39.962 591	96.94%	
21	Scandium	45	^{45}Sc	44.955 910	100%	
22	Titanium	48	^{48}Ti	47.947 947	73.8%	
23	Vanadium	51	^{51}V	50.943 962	99.75%	
24	Chromium	52	^{52}Cr	51.940 509	83.79%	
25	Manganese	55	^{55}Mn	54.938 047	100%	
26	Iron	56	^{56}Fe	55.934 939	91.72%	

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
27	Cobalt	59	^{59}Co	58.933 198	100%	
		60	^{60}Co	59.933 819	β^-	5.271 y
28	Nickel	58	^{58}Ni	57.935 346	68.27%	
		60	^{60}Ni	59.930 788	26.10%	
29	Copper	63	^{63}Cu	62.939 598	69.17%	
		65	^{65}Cu	64.927 793	30.83%	
30	Zinc	64	^{64}Zn	63.929 145	48.6%	
		66	^{66}Zn	65.926 034	27.9%	
31	Gallium	69	^{69}Ga	68.925 580	60.1%	
32	Germanium	72	^{72}Ge	71.922 079	27.4%	
		74	^{74}Ge	73.921 177	36.5%	
33	Arsenic	75	^{75}As	74.921 594	100%	
34	Selenium	80	^{80}Se	79.916 520	49.7%	
35	Bromine	79	^{79}Br	78.918 336	50.69%	
36	Krypton	84	^{84}Kr	83.911 507	57.0%	
37	Rubidium	85	^{85}Rb	84.911 794	72.17%	
38	Strontium	86	^{86}Sr	85.909 267	9.86%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		88	^{88}Sr	87.905 619	82.58%	
		90	^{90}Sr	89.907 738	β^-	28.8 y
39	Yttrium	89	^{89}Y	88.905 849	100%	
		90	^{90}Y	89.907 152	β^-	64.1 h
40	Zirconium	90	^{90}Zr	89.904 703	51.45%	
41	Niobium	93	^{93}Nb	92.906 377	100%	
42	Molybdenum	98	^{98}Mo	97.905 406	24.13%	
43	Technetium	98	^{98}Tc	97.907 215	β^-	4.2×10^6y
44	Ruthenium	102	^{102}Ru	101.904 348	31.6%	
45	Rhodium	103	^{103}Rh	102.905 500	100%	
46	Palladium	106	^{106}Pd	105.903 478	27.33%	
47	Silver	107	^{107}Ag	106.905 092	51.84%	
		109	^{109}Ag	108.904 757	48.16%	
48	Cadmium	114	^{114}Cd	113.903 357	28.73%	
49	Indium	115	^{115}In	114.903 880	95.7%, β^-	4.4×10^{14}y
50	Tin	120	^{120}Sn	119.902 200	32.59%	
51	Antimony	121	^{121}Sb	120.903 821	57.3%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
52	Tellurium	130	^{130}Te	129.906 229	33.8%, β^-	2.5×10^{21}y
53	Iodine	127	^{127}I	126.904 473	100%	
		131	^{131}I	130.906 114	β^-	8.040 d
54	Xenon	132	^{132}Xe	131.904 144	26.9%	
		136	^{136}Xe	135.907 214	8.9%	
55	Cesium	133	^{133}Cs	132.905 429	100%	
		134	^{134}Cs	133.906 696	EC, β^-	2.06 y
56	Barium	137	^{137}Ba	136.905 812	11.23%	
		138	^{138}Ba	137.905 232	71.70%	
57	Lanthanum	139	^{139}La	138.906 346	99.91%	
58	Cerium	140	^{140}Ce	139.905 433	88.48%	
59	Praseodymium	141	^{141}Pr	140.907 647	100%	
60	Neodymium	142	^{142}Nd	141.907 719	27.13%	
61	Promethium	145	^{145}Pm	144.912 743	EC, α	17.7 y
62	Samarium	152	^{152}Sm	151.919 729	26.7%	
63	Europium	153	^{153}Eu	152.921 225	52.2%	
64	Gadolinium	158	^{158}Gd	157.924 099	24.84%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
65	Terbium	159	^{159}Tb	158.925 342	100%	
66	Dysprosium	164	^{164}Dy	163.929 171	28.2%	
67	Holmium	165	^{165}Ho	164.930 319	100%	
68	Erbium	166	^{166}Er	165.930 290	33.6%	
69	Thulium	169	^{169}Tm	168.934 212	100%	
70	Ytterbium	174	^{174}Yb	173.938 859	31.8%	
71	Lutecium	175	^{175}Lu	174.940 770	97.41%	
72	Hafnium	180	^{180}Hf	179.946 545	35.10%	
73	Tantalum	181	^{181}Ta	180.947 992	99.98%	
74	Tungsten	184	^{184}W	183.950 928	30.67%	
75	Rhenium	187	^{187}Re	186.955 744	62.6%, β^-	$4.6 \times 10^{10}\text{y}$
76	Osmium	191	^{191}Os	190.960 920	β^-	15.4 d
		192	^{192}Os	191.961 467	41.0%	
77	Iridium	191	^{191}Ir	190.960 584	37.3%	
		193	^{193}Ir	192.962 917	62.7%	
78	Platinum	195	^{195}Pt	194.964 766	33.8%	
79	Gold	197	^{197}Au	196.966 543	100%	

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		198	$^{198}\mathrm{Au}$	197.968 217	β^-	2.696 d
80	Mercury	199	$^{199}\mathrm{Hg}$	198.968 253	16.87%	
		202	$^{202}\mathrm{Hg}$	201.970 617	29.86%	
81	Thallium	205	$^{205}\mathrm{Tl}$	204.974 401	70.48%	
82	Lead	206	$^{206}\mathrm{Pb}$	205.974 440	24.1%	
		207	$^{207}\mathrm{Pb}$	206.975 872	22.1%	
		208	$^{208}\mathrm{Pb}$	207.976 627	52.4%	
		210	$^{210}\mathrm{Pb}$	209.984 163	α, β^-	22.3 y
		211	$^{211}\mathrm{Pb}$	210.988 735	β^-	36.1 min
		212	$^{212}\mathrm{Pb}$	211.991 871	β^-	10.64 h
83	Bismuth	209	$^{209}\mathrm{Bi}$	208.980 374	100%	
		211	$^{211}\mathrm{Bi}$	210.987 255	α, β^-	2.14 min
84	Polonium	210	$^{210}\mathrm{Po}$	209.982 848	α	138.38 d
85	Astatine	218	$^{218}\mathrm{At}$	218.008 684	α, β^-	1.6 s
86	Radon	222	$^{222}\mathrm{Rn}$	222.017 570	α	3.82 d
87	Francium	223	$^{223}\mathrm{Fr}$	223.019 733	α, β^-	21.8 min
88	Radium	226	$^{226}\mathrm{Ra}$	226.025 402	α	1.60×10^3y

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
89	Actinium	227	^{227}Ac	227.027 750	α, β^-	21.8 y
90	Thorium	228	^{228}Th	228.028 715	α	1.91 y
		232	^{232}Th	232.038 054	100%, α	1.41×10^{10}y
91	Protactinium	231	^{231}Pa	231.035 880	α	3.28×10^4y
92	Uranium	233	^{233}U	233.039 628	α	1.59×10^3y
		235	^{235}U	235.043 924	0.720%, α	7.04×10^8y
		236	^{236}U	236.045 562	α	2.34×10^7y
		238	^{238}U	238.050 784	99.2745%, α	4.47×10^9y
		239	^{239}U	239.054 289	β^-	23.5 min
93	Neptunium	239	^{239}Np	239.052 933	β^-	2.355 d
94	Plutonium	239	^{239}Pu	239.052 157	α	2.41×10^4y
95	Americium	243	^{243}Am	243.061 375	α, fission	7.37×10^3y
96	Curium	245	^{245}Cm	245.065 483	α	8.50×10^3y
97	Berkelium	247	^{247}Bk	247.070 300	α	1.38×10^3y
98	Californium	249	^{249}Cf	249.074 844	α	351 y
99	Einsteinium	254	^{254}Es	254.088 019	α, β^-	276 d
100	Fermium	253	^{253}Fm	253.085 173	EC, α	3.00 d

Atomic Number, *Z*	Name	Atomic Mass Number, *A*	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
101	Mendelevium	255	^{255}Md	255.091 081	EC, α	27 min
102	Nobelium	255	^{255}No	255.093 260	EC, α	3.1 min
103	Lawrencium	257	^{257}Lr	257.099 480	EC, α	0.646 s
104	Rutherfordium	261	^{261}Rf	261.108 690	α	1.08 min
105	Dubnium	262	^{262}Db	262.113 760	α, fission	34 s
106	Seaborgium	263	^{263}Sg	263.11 86	α, fission	0.8 s
107	Bohrium	262	^{262}Bh	262.123 1	α	0.102 s
108	Hassium	264	^{264}Hs	264.128 5	α	0.08 ms
109	Meitnerium	266	^{266}Mt	266.137 8	α	3.4 ms

Atomic Masses

Selected Radioactive Isotopes

Decay modes are α, β^-, β^+, electron capture (EC) and isomeric transition (IT). EC results in the same daughter nucleus as would β^+ decay. IT is a transition from a metastable excited state. Energies for $\beta^\pm$ decays are the maxima; average energies are roughly one-half the maxima.

Isotope	$t_{1/2}$	**DecayMode(s)**	**Energy(MeV)**	**Percent**		γ**-Ray Energy(MeV)**
^{3}H	12.33 y	β^-	0.0186	100%		
^{14}C	5730 y	β^-	0.156	100%		
^{13}N	9.96 min	β^+	1.20	100%		
^{22}Na	2.602 y	β^+	0.55	90%	γ	1.27
^{32}P	14.28 d	β^-	1.71	100%		
^{35}S	87.4 d	β^-	0.167	100%		
^{36}Cl	3.00×10^5y	β^-	0.710	100%		
^{40}K	1.28×10^9y	β^-	1.31	89%		
^{43}K	22.3 h	β^-	0.827	87%	γs	0.373
						0.618
^{45}Ca	165 d	β^-	0.257	100%		
^{51}Cr	27.70 d	EC			γ	0.320
^{52}Mn	5.59d	β^+	3.69	28%	γs	1.33
						1.43
^{52}Fe	8.27 h	β^+	1.80	43%		0.169
						0.378
^{59}Fe	44.6 d	β^-s	0.273	45%	γs	1.10
			0.466	55%		1.29
^{60}Co	5.271 y	β^-	0.318	100%	γs	1.17
						1.33
^{65}Zn	244.1 d	EC			γ	1.12

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ-Ray Energy(MeV)
^{67}Ga	78.3 h	EC			γs	0.0933
						0.185
						0.300
						others
^{75}Se	118.5 d	EC			γs	0.121
						0.136
						0.265
						0.280
						others
^{86}Rb	18.8 d	β^-s	0.69	9%	γ	1.08
			1.77	91%		
^{85}Sr	64.8 d	EC			γ	0.514
^{90}Sr	28.8 y	β^-	0.546	100%		
^{90}Y	64.1 h	β^-	2.28	100%		
^{99m}Tc	6.02 h	IT			γ	0.142
^{113m}In	99.5 min	IT			γ	0.392
^{123}I	13.0 h	EC			γ	0.159
^{131}I	8.040 d	β^-s	0.248	7%	γs	0.364
			0.607	93%		others
			others			
^{129}Cs	32.3 h	EC			γs	0.0400
						0.372
						0.411
						others
^{137}Cs	30.17 y	β^-s	0.511	95%	γ	0.662
			1.17	5%		

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ-Ray Energy(MeV)
^{140}Ba	12.79 d	β^-	1.035	$\approx 100\%$	γs	0.030
						0.044
						0.537
						others
^{198}Au	2.696 d	β^-	1.161	$\approx 100\%$	γ	0.412
^{197}Hg	64.1 h	EC			γ	0.0733
^{210}Po	138.38 d	α	5.41	100%		
^{226}Ra	$1.60 \times 10^3 y$	αs	4.68	5%	γ	0.186
			4.87	95%		
^{235}U	$7.038 \times 10^8 y$	α	4.68	$\approx 100\%$	γs	numerous
^{238}U	$4.468 \times 10^9 y$	αs	4.22	23%	γ	0.050
			4.27	77%		
^{237}Np	$2.14 \times 10^6 y$	αs	numerous		γs	numerous
			4.96 (max.)			
^{239}Pu	$2.41 \times 10^4 y$	αs	5.19	11%	γs	7.5×10^{-5}
			5.23	15%		0.013
			5.24	73%		0.052
						others
^{243}Am	$7.37 \times 10^3 y$	αs	Max. 5.44		γs	0.075
			5.37	88%		others
			5.32	11%		
			others			

Selected Radioactive Isotopes

Useful Information

A collection of tables of useful information, including physical and numerical constants, metric prefixes, and unit conversion factors.

This appendix is broken into several tables.

- [link], Important Constants
- [link], Submicroscopic Masses
- [link], Solar System Data
- [link], Metric Prefixes for Powers of Ten and Their Symbols
- [link], The Greek Alphabet
- [link], SI units
- [link], Selected British Units
- [link], Other Units
- [link], Useful Formulae

Symbol	Meaning	Best Value	Approximate Value
c	Speed of light in vacuum	$2.99792458 \times 10^8\,\mathrm{m/s}$	$3.00 \times 10^8\,\mathrm{m/s}$
G	Gravitational constant	$6.67430(15) \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$	$6.67 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$
N_A	Avogadro’s number	$6.02214076 \times 10^{23}$	6.02×10^{23}
k	Boltzmann’s constant	$1.380649 \times 10^{-23}\,\mathrm{J/K}$	$1.38 \times 10^{-23}\,\mathrm{J/K}$
R	Gas constant	$8.314462618\,\mathrm{J/mol \cdot K}$	$8.31\,\mathrm{J/mol \cdot K}$
σ	Stefan-Boltzmann constant	$5.670374419 \times 10^{-8}\,\mathrm{W/m^2 \cdot K}$	$5.67 \times 10^{-8}\,\mathrm{W/m^2 \cdot K}$
k	Coulomb force constant	$8.9875517923(13) \times 10^9\,\mathrm{N \cdot m^2/C^2}$	$8.99 \times 10^9\,\mathrm{N \cdot m^2/C^2}$
e	Elementary charge	$1.602176634 \times 10^{-19}\,\mathrm{C}$	$1.60 \times 10^{-19}\,\mathrm{C}$
ε_0	Permittivity of free space	$8.8541878128(13) \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$	$8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$

Symbol	Meaning	Best Value	Approximate Value
μ_0	Permeability of free space	$4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$	$1.26 \times 10^{-6}\,\mathrm{T \cdot m/A}$
h	Planck's constant	$6.62607015 \times 10^{-34}\,\mathrm{J \cdot s}$	$6.63 \times 10^{-34}\,\mathrm{J \cdot s}$

Important Constants[footnote]
Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (2018 values). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

Symbol	Meaning	Best Value	Approximate Value
m_e	Electron mass	$9.1093837015(28) \times 10^{-31}\,\mathrm{kg}$	$9.11 \times 10^{-31}\,\mathrm{kg}$
m_p	Proton mass	$1.67262192369(51) \times 10^{-27}\,\mathrm{kg}$	$1.67 \times 10^{-27}\,\mathrm{kg}$
m_n	Neutron mass	$1.67492749804(95) \times 10^{-27}\,\mathrm{kg}$	$1.67 \times 10^{-27}\,\mathrm{kg}$
u	Atomic mass unit	$1.66053906660(50) \times 10^{-27}\,\mathrm{kg}$	$1.66 \times 10^{-27}\,\mathrm{kg}$

Submicroscopic Masses[footnote]
Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (2018 values). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

Sun	mass	$1.99 \times 10^{30}\mathrm{kg}$
	average radius	$6.96 \times 10^{8}\mathrm{m}$
	Earth-sun distance (average)	$1.496 \times 10^{11}\mathrm{m}$

Earth	mass	$5.9736 \times 10^{24}\mathrm{kg}$
	average radius	$6.376 \times 10^{6}\mathrm{m}$
	orbital period	$3.16 \times 10^{7}\mathrm{s}$
Moon	mass	$7.35 \times 10^{22}\mathrm{kg}$
	average radius	$1.74 \times 10^{6}\mathrm{m}$
	orbital period (average)	$2.36 \times 10^{6}\mathrm{s}$
	Earth-moon distance (average)	$3.84 \times 10^{8}\mathrm{m}$

Solar System Data

Prefix	**Symbol**	**Value**	**Prefix**	**Symbol**	**Value**
tera	T	10^{12}	deci	d	10^{-1}
giga	G	10^{9}	centi	c	10^{-2}
mega	M	10^{6}	milli	m	10^{-3}
kilo	k	10^{3}	micro	μ	10^{-6}
hecto	h	10^{2}	nano	n	10^{-9}
deka	da	10^{1}	pico	p	10^{-12}
—	—	$10^{0}(= 1)$	femto	f	10^{-15}

Metric Prefixes for Powers of Ten and Their Symbols

Alpha	A	α	Eta	H	η	Nu	N	ν	Tau	T	τ
Beta	B	β	Theta	Θ	θ	Xi	Ξ	ξ	Upsilon	Υ	υ
Gamma	Γ	γ	Iota	I	ι	Omicron	O	o	Phi	Φ	ϕ
Delta	Δ	δ	Kappa	K	κ	Pi	Π	π	Chi	X	χ
Epsilon	E	ε	Lambda	Λ	λ	Rho	P	ρ	Psi	Ψ	ψ
Zeta	Z	ζ	Mu	M	μ	Sigma	Σ	σ	Omega	Ω	ω

The Greek Alphabet

	Entity	Abbreviation	Name
Fundamental units	Length	m	meter
	Mass	kg	kilogram
	Time	s	second
	Current	A	ampere
Supplementary unit	Angle	rad	radian
Derived units	Force	$\mathrm{N} = \mathrm{kg} \cdot \mathrm{m}/\mathrm{s}^2$	newton
	Energy	$\mathrm{J} = \mathrm{kg} \cdot \mathrm{m}^2/\mathrm{s}^2$	joule
	Power	$\mathrm{W} = \mathrm{J}/\mathrm{s}$	watt
	Pressure	$\mathrm{Pa} = \mathrm{N}/\mathrm{m}^2$	pascal
	Frequency	$\mathrm{Hz} = 1/\mathrm{s}$	hertz
	Electronic potential	$\mathrm{V} = \mathrm{J}/\mathrm{C}$	volt

	Entity	Abbreviation	Name
	Capacitance	$\mathrm{F} = \mathrm{C}/\mathrm{V}$	farad
	Charge	$\mathrm{C} = \mathrm{s} \cdot \mathrm{A}$	coulomb
	Resistance	$\Omega = \mathrm{V}/\mathrm{A}$	ohm
	Magnetic field	$\mathrm{T} = \mathrm{N}/(\mathrm{A} \cdot \mathrm{m})$	tesla
	Nuclear decay rate	$\mathrm{Bq} = 1/\mathrm{s}$	becquerel

SI Units

Length	$1 \text{ inch (in)} = 2.54 \text{ cm (exactly)}$
	$1 \text{ foot (ft)} = 12 \text{ in} = 0.3048 \text{ m}$
	$1 \text{ yard (yd)} = 3 \text{ ft} = 0.9144 \text{ m}$
	$1 \text{ mile (mi)} = 1760 \text{ yd} = 1.609 \text{ km}$
Force	$1 \text{ pound (lb)} = 4.448 \text{ N}$
Energy	$1 \text{ British thermal unit (Btu)} = 1.055 \times 10^3 \text{ J}$
Power	$1 \text{ horsepower (hp)} = 746 \text{ W}$
Pressure	$1 \text{ lb/in}^2 = 6.895 \times 10^3 \text{ Pa}$

Selected British Units

Length	$1 \text{ light year (ly)} = 9.46 \times 10^{15} \text{ m}$
	$1 \text{ astronomical unit (au)} = 1.50 \times 10^{11} \text{ m}$

	1 nautical mile $= 1.852\,\text{km}$
	1 angstrom(Å) $= 10^{-10}\,\text{m}$
Area	1 acre $(\text{ac}) = 4.05 \times 10^3\,\text{m}^2$
	1 square foot $(\text{ft}^2) = 9.29 \times 10^{-2}\,\text{m}^2$
	1 barn $(b) = 10^{-28}\,\text{m}^2$
Volume	1 liter $(L) = 10^{-3}\,\text{m}^3$
	1 U.S. gallon $(\text{gal}) = 3.785 \times 10^{-3}\,\text{m}^3$
Mass	1 solar mass $= 1.99 \times 10^{30}\,\text{kg}$
	1 metric ton $= 10^3\,\text{kg}$
	1 atomic mass unit $(u) = 1.6605 \times 10^{-27}\,\text{kg}$
Time	1 year $(y) = 3.16 \times 10^7\,\text{s}$
	1 day $(d) = 86{,}400\,\text{s}$
Speed	1 mile per hour $(\text{mph}) = 1.609\,\text{km/h}$
	1 nautical mile per hour $(\text{naut}) = 1.852\,\text{km/h}$
Angle	1 degree $(^\circ) = 1.745 \times 10^{-2}\,\text{rad}$
	1 minute of arc $(') = 1/60$ degree
	1 second of arc $('') = 1/60$ minute of arc
	1 grad $= 1.571 \times 10^{-2}\,\text{rad}$
Energy	1 kiloton TNT $(\text{kT}) = 4.2 \times 10^{12}\,\text{J}$
	1 kilowatt hour $(\text{kW} \cdot h) = 3.60 \times 10^6\,\text{J}$
	1 food calorie $(\text{kcal}) = 4186\,\text{J}$
	1 calorie $(\text{cal}) = 4.186\,\text{J}$
	1 electron volt $(\text{eV}) = 1.60 \times 10^{-19}\,\text{J}$
Pressure	1 atmosphere $(\text{atm}) = 1.013 \times 10^5\,\text{Pa}$
	1 millimeter of mercury $(\text{mm Hg}) = 133.3\,\text{Pa}$
	1 torricelli $(\text{torr}) = 1\,\text{mm Hg} = 133.3\,\text{Pa}$
Nuclear decay rate	1 curie $(\text{Ci}) = 3.70 \times 10^{10}\,\text{Bq}$

Other Units

Circumference of a circle with radius r or diameter d	$C = 2\pi r = \pi d$
Area of a circle with radius r or diameter d	$A = \pi r^2 = \pi d^2/4$
Area of a sphere with radius r	$A = 4\pi r^2$
Volume of a sphere with radius r	$V = (4/3)\ \pi r^3$

Useful Formulae

Glossary of Key Symbols and Notation

In this glossary, key symbols and notation are briefly defined.

Symbol	**Definition**
any symbol	average (indicated by a bar over a symbol—e.g., v is average velocity)
$^\circ\text{C}$	Celsius degree
$^\circ\text{F}$	Fahrenheit degree
$//$	parallel
$\perp$	perpendicular
$\propto$	proportional to
$\pm$	plus or minus

Symbol	Definition
0	zero as a subscript denotes an initial value
α	alpha rays
α	angular acceleration
α	temperature coefficient(s) of resistivity
β	beta rays
β	sound level
β	volume coefficient of expansion
β^-	electron emitted in nuclear beta decay
β^+	positron decay
γ	gamma rays

Symbol	Definition
γ	surface tension
$\gamma = 1/\sqrt{1 - v^2/c^2}$	a constant used in relativity
Δ	change in whatever quantity follows
δ	uncertainty in whatever quantity follows
ΔE	change in energy between the initial and final orbits of an electron in an atom
ΔE	uncertainty in energy
Δm	difference in mass between initial and final products
ΔN	number of decays that occur
Δp	change in momentum

Symbol	Definition
Δp	uncertainty in momentum
$\Delta \mathrm{PE_g}$	change in gravitational potential energy
$\Delta\theta$	rotation angle
Δs	distance traveled along a circular path
Δt	uncertainty in time
Δt_0	proper time as measured by an observer at rest relative to the process
ΔV	potential difference
Δx	uncertainty in position
ε_0	permittivity of free space
η	viscosity

Symbol	Definition
θ	angle between the force vector and the displacement vector
θ	angle between two lines
θ	contact angle
θ	direction of the resultant
θ_b	Brewster's angle
θ_c	critical angle
κ	dielectric constant
λ	decay constant of a nuclide
λ	wavelength
λ_n	wavelength in a medium

Symbol	Definition
μ_0	permeability of free space
μ_k	coefficient of kinetic friction
μ_s	coefficient of static friction
v_e	electron neutrino
π^+	positive pion
π^-	negative pion
π^0	neutral pion
ρ	density
ρ_c	critical density, the density needed to just halt universal expansion
ρ_fl	fluid density

Symbol	Definition
ρ_{obj}	average density of an object
ρ/ρ_{w}	specific gravity
τ	characteristic time constant for a resistance and inductance (RL) or resistance and capacitance (RC) circuit
τ	characteristic time for a resistor and capacitor (RC) circuit
τ	torque
Υ	upsilon meson
Φ	magnetic flux
ϕ	phase angle
Ω	ohm (unit)
ω	angular velocity

Symbol	Definition
A	ampere (current unit)
A	area
A	cross-sectional area
A	total number of nucleons
a	acceleration
a_{B}	Bohr radius
a_{c}	centripetal acceleration
a_{t}	tangential acceleration
AC	alternating current
AM	amplitude modulation

Symbol	Definition
atm	atmosphere
B	baryon number
B	blue quark color
$\bar{B}$	antiblue (yellow) antiquark color
b	quark flavor bottom or beauty
B	bulk modulus
B	magnetic field strength
$\mathrm{B}_{\mathrm{int}}$	electron’s intrinsic magnetic field
$\mathrm{B}_{\mathrm{orb}}$	orbital magnetic field
BE	binding energy of a nucleus—it is the energy required to completely disassemble it into separate protons and neutrons

Symbol	Definition
BE/A	binding energy per nucleon
Bq	becquerel—one decay per second
C	capacitance (amount of charge stored per volt)
C	coulomb (a fundamental SI unit of charge)
C_{p}	total capacitance in parallel
C_{s}	total capacitance in series
CG	center of gravity
CM	center of mass
c	quark flavor charm
c	specific heat

Symbol	Definition
c	speed of light
Cal	kilocalorie
cal	calorie
COP_{hp}	heat pump’s coefficient of performance
COP_{ref}	coefficient of performance for refrigerators and air conditioners
$\cos\theta$	cosine
$\cot\theta$	cotangent
$\csc\theta$	cosecant
D	diffusion constant
d	displacement

Symbol	Definition
d	quark flavor down
dB	decibel
d_{i}	distance of an image from the center of a lens
d_{o}	distance of an object from the center of a lens
DC	direct current
E	electric field strength
ε	emf (voltage) or Hall electromotive force
emf	electromotive force
E	energy of a single photon
E	nuclear reaction energy

Symbol	Definition
E	relativistic total energy
E	total energy
E_0	ground state energy for hydrogen
E_0	rest energy
EC	electron capture
E_{cap}	energy stored in a capacitor
Eff	efficiency—the useful work output divided by the energy input
Eff_{C}	Carnot efficiency
E_{in}	energy consumed (food digested in humans)
E_{ind}	energy stored in an inductor

Symbol	Definition
E_{out}	energy output
e	emissivity of an object
e^+	antielectron or positron
eV	electron volt
F	farad (unit of capacitance, a coulomb per volt)
F	focal point of a lens
	force
F	magnitude of a force
F	restoring force
F_{B}	buoyant force

Symbol	Definition
F_{c}	centripetal force
F_{i}	force input
$_{\mathrm{net}}$	net force
F_{o}	force output
FM	frequency modulation
f	focal length
f	frequency
f_0	resonant frequency of a resistance, inductance, and capacitance (RLC) series circuit
f_0	threshold frequency for a particular material (photoelectric effect)

Symbol	Definition
f_1	fundamental
f_2	first overtone
f_3	second overtone
f_B	beat frequency
f_k	magnitude of kinetic friction
f_s	magnitude of static friction
G	gravitational constant
G	green quark color
G	antigreen (magenta) antiquark color

Symbol	Definition
g	acceleration due to gravity
g	gluons (carrier particles for strong nuclear force)
h	change in vertical position
h	height above some reference point
h	maximum height of a projectile
h	Planck's constant
hf	photon energy
h_{i}	height of the image
h_{o}	height of the object
I	electric current

Symbol	Definition
I	intensity
I	intensity of a transmitted wave
I	moment of inertia (also called rotational inertia)
I_0	intensity of a polarized wave before passing through a filter
I_{ave}	average intensity for a continuous sinusoidal electromagnetic wave
I_{rms}	average current
J	joule
J/Ψ	Joules/psi meson
K	kelvin
k	Boltzmann constant

Symbol	Definition
k	force constant of a spring
K_α	x rays created when an electron falls into an $n=1$ shell vacancy from the $n=3$ shell
K_β	x rays created when an electron falls into an $n=2$ shell vacancy from the $n=3$ shell
kcal	kilocalorie
KE	translational kinetic energy
$\mathrm{KE}+\mathrm{PE}$	mechanical energy
KE_e	kinetic energy of an ejected electron
$\mathrm{KE}_{\mathrm{rel}}$	relativistic kinetic energy
$\mathrm{KE}_{\mathrm{rot}}$	rotational kinetic energy
KE	thermal energy

Symbol	Definition
kg	kilogram (a fundamental SI unit of mass)
L	angular momentum
L	liter
L	magnitude of angular momentum
L	self-inductance
ℓ	angular momentum quantum number
L_α	x rays created when an electron falls into an $n=2$ shell from the $n=3$ shell
L_e	electron total family number
L_μ	muon family total number
L_τ	tau family total number

Symbol	Definition
L_{f}	heat of fusion
L_{f} and L_{v}	latent heat coefficients
$\mathrm{L}_{\mathrm{orb}}$	orbital angular momentum
L_{s}	heat of sublimation
L_{v}	heat of vaporization
L_z	*z* - component of the angular momentum
M	angular magnification
M	mutual inductance
m	indicates metastable state
m	magnification

Symbol	Definition
m	mass
m	mass of an object as measured by a person at rest relative to the object
m	meter (a fundamental SI unit of length)
m	order of interference
m	overall magnification (product of the individual magnifications)
$m\left({}^{A}\mathrm{X}\right)$	atomic mass of a nuclide
MA	mechanical advantage
m_{e}	magnification of the eyepiece
m_{e}	mass of the electron
m_{ℓ}	angular momentum projection quantum number

Symbol	Definition
m_n	mass of a neutron
m_{o}	magnification of the objective lens
mol	mole
m_p	mass of a proton
m_{s}	spin projection quantum number
N	magnitude of the normal force
N	newton
	normal force
N	number of neutrons
n	index of refraction

Symbol	Definition
n	number of free charges per unit volume
N_{A}	Avogadro's number
N_{r}	Reynolds number
$\mathrm{N \cdot m}$	newton-meter (work-energy unit)
$\mathrm{N \cdot m}$	newtons times meters (SI unit of torque)
OE	other energy
P	power
P	power of a lens
P	pressure
	momentum

Symbol	Definition
p	momentum magnitude
p	relativistic momentum
$_{\text{tot}}$	total momentum
$_{\text{tot}}$	total momentum some time later
P_{abs}	absolute pressure
P_{atm}	atmospheric pressure
P_{atm}	standard atmospheric pressure
PE	potential energy
PE_{el}	elastic potential energy
PE_{elec}	electric potential energy

Symbol	Definition
$\mathrm{PE_s}$	potential energy of a spring
P_{g}	gauge pressure
P_{in}	power consumption or input
P_{out}	useful power output going into useful work or a desired, form of energy
Q	latent heat
Q	net heat transferred into a system
Q	flow rate—volume per unit time flowing past a point
$+Q$	positive charge
$-Q$	negative charge

Symbol	Definition
q	electron charge
q_p	charge of a proton
q	test charge
QF	quality factor
R	activity, the rate of decay
R	radius of curvature of a spherical mirror
R	red quark color
R	antired (cyan) quark color
R	resistance
R	resultant or total displacement

Symbol	Definition
R	Rydberg constant
R	universal gas constant
r	distance from pivot point to the point where a force is applied
r	internal resistance
$r_{\perp}$	perpendicular lever arm
r	radius of a nucleus
r	radius of curvature
r	resistivity
r or rad	radiation dose unit
rem	roentgen equivalent man

Symbol	Definition
rad	radian
RBE	relative biological effectiveness
RC	resistor and capacitor circuit
rms	root mean square
r_n	radius of the *n*th H-atom orbit
R_{p}	total resistance of a parallel connection
R_{s}	total resistance of a series connection
R_{s}	Schwarzschild radius
S	entropy
	intrinsic spin (intrinsic angular momentum)

Symbol	Definition
S	magnitude of the intrinsic (internal) spin angular momentum
S	shear modulus
S	strangeness quantum number
s	quark flavor strange
s	second (fundamental SI unit of time)
s	spin quantum number
	total displacement
$\sec\theta$	secant
$\sin\theta$	sine
s_z	*z*-component of spin angular momentum

Symbol	Definition
T	period—time to complete one oscillation
T	temperature
T_{c}	critical temperature—temperature below which a material becomes a superconductor
T	tension
T	tesla (magnetic field strength *B*)
t	quark flavor top or truth
t	time
$t_{1/2}$	half-life—the time in which half of the original nuclei decay
$\tan\theta$	tangent
U	internal energy

Symbol	Definition
u	quark flavor up
u	unified atomic mass unit
	velocity of an object relative to an observer
	velocity relative to another observer
V	electric potential
V	terminal voltage
V	volt (unit)
V	volume
	relative velocity between two observers
v	speed of light in a material

Symbol	Definition
	velocity
	average fluid velocity
$V_{\mathrm{B}} - V_{\mathrm{A}}$	change in potential
d	drift velocity
V_{p}	transformer input voltage
V_{rms}	rms voltage
V_{s}	transformer output voltage
tot	total velocity
v_{w}	propagation speed of sound or other wave
w	wave velocity

Symbol	Definition
W	work
W	net work done by a system
W	watt
w	weight
w_{fl}	weight of the fluid displaced by an object
W_{c}	total work done by all conservative forces
W_{nc}	total work done by all nonconservative forces
W_{out}	useful work output
X	amplitude
X	symbol for an element

Symbol	Definition
${}^{Z}_{A}X_{N}$	notation for a particular nuclide
x	deformation or displacement from equilibrium
x	displacement of a spring from its undeformed position
x	horizontal axis
X_{C}	capacitive reactance
X_{L}	inductive reactance
x_{rms}	root mean square diffusion distance
y	vertical axis
Y	elastic modulus or Young's modulus
Z	atomic number (number of protons in a nucleus)

Symbol	Definition
Z	impedance